JEECUP Group A Mathematics Sample Paper – 15

Duration: 60 Minutes **Maximum Marks: 200**

Instructions

- This paper contains **50** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**. No marks will be deducted for incorrect answers. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. If HCF of two positive integers a and b is 12 and their product is 2160, then their LCM is:

(A) 120

(B) 180

(C) 240

(D) 360

Q2. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is:

(A) 10

(B) -10

(C) -7

(D) -2

Q3. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has:

(A) a unique solution

(B) exactly two solutions

(C) infinitely many solutions

(D) no solution

Q4. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then:

(A) $k < 4$

(B) $k > 4$

(C) $k \leq 4$

(D) $k \geq 4$

Q5. The 11th term of the AP: $-3, -\frac{1}{2}, 2, \ldots$ is:

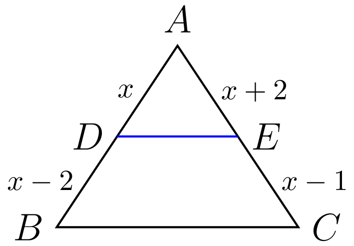
(A) 28

(B) 22

(C) -38

(D) $-46\frac{1}{2}$

Q6. In $\triangle ABC$, $DE \parallel BC$ such that $AD = x$, $DB = x - 2$, $AE = x + 2$, and $EC = x - 1$. The value of x is:



(A) 4

(B) 2

(C) 1

(D) 3

Q7. The distance of the point $P(-6, 8)$ from the origin is:

(A) 8

(B) $2\sqrt{7}$

(C) 10

(D) 6

Q8. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $\tan\theta$ is:

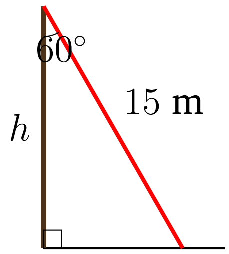
(A) $\sqrt{2} - 1$

(B) $\sqrt{2} + 1$

(C) $\frac{1}{\sqrt{2}}$

(D) $\sqrt{3}$

Q9. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is:



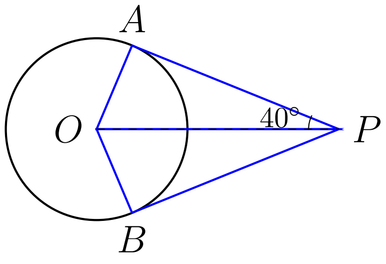
(A) $15\sqrt{3}$ m

(B) $\frac{15\sqrt{3}}{2}$ m

(C) $\frac{15}{2}$ m

(D) 15 m

Q10. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then $\angle POA$ is equal to:



(A) 50°

(B) 60°

(C) 70°

(D) 80°

Q11. To divide a line segment AB in the ratio $5:7$, first a ray AX is drawn so that $\angle BAX$ is an acute angle and then points $A_1, A_2, A_3, \ldots$ are located at equal distances on the ray AX. The minimum number of these points is:

(A) 5

(B) 7

(C) 12

(D) 35

Q12. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then:

(A) $R_1 + R_2 = R$

(B) $R_1^2 + R_2^2 = R^2$

(C) $R_1 + R_2 < R$

(D) $R_1^2 + R_2^2 < R^2$

Q13. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. The height of the cylinder is:

(A) 2.74 cm

(B) 3.24 cm

(C) 1.37 cm

(D) 4.50 cm

Q14. For the following distribution, the modal class is:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	3	12	20	15	5

(A) $10 - 20$

(B) $20 - 30$

(C) $30 - 40$

(D) $40 - 50$

Q15. Which of the following cannot be the probability of an event?

(A) $\frac{2}{3}$

(B) -1.5

(C) 15%

(D) 0.7

Q16. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(A) one decimal place

(B) two decimal places

(C) three decimal places

(D) four decimal places

Q17. If one of the zeroes of a cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is:

(A) $b - a + 1$

(B) $b - a - 1$

(C) $a - b + 1$

(D) $a - b - 1$

Q18. Aruna has only Rs. 1 and Rs. 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs. 75, then the number of Rs. 1 and Rs. 2 coins are, respectively:

(A) 35 and 15

(B) 35 and 20

(C) 15 and 35

(D) 25 and 25

Q19. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are:

(A) $-2, \frac{3}{2}$

(B) $2, -\frac{3}{2}$

(C) $-2, -\frac{3}{2}$

(D) $2, \frac{3}{2}$

Q20. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

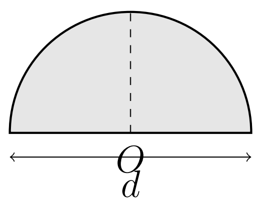
(A) 5

(B) 20

(C) 25

(D) 30

Q21. If the perimeter of a semi-circular protractor is 36 cm, then its diameter is:



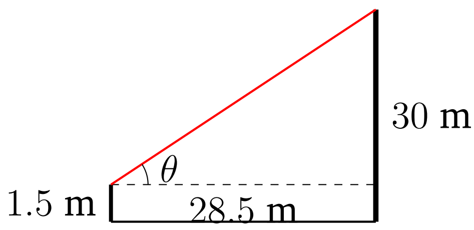
(A) 10 cm

(B) 12 cm

(C) 14 cm

(D) 16 cm

Q22. An observer 1.5 m tall is 28.5 m away from a tower 30 m high. The angle of elevation of the top of the tower from the eye of the observer is:



(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q23. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is:

(A) 2 units

(B) π units

(C) 4 units

(D) 7 units

Q24. If a card is selected at random from a well-shuffled deck of 52 cards, the probability of its being a red face card is:

(A) $\frac{3}{26}$

(B) $\frac{3}{13}$

(C) $\frac{2}{13}$

(D) $\frac{1}{2}$

Q25. If three coins are tossed simultaneously, then the probability of getting exactly two heads is:

(A) $\frac{1}{8}$

(B) $\frac{3}{8}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

Q26. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is:

(A) 13

(B) 65

(C) 875

(D) 1750

Q27. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is:

(A) 2

(B) 4

(C) -2

(D) -4

Q28. The value of k for which the system of linear equations $kx + 3y = k - 3$ and $12x + ky = k$ has infinitely many solutions is:

(A) 6

(B) -6

(C) 0

(D) 12

Q29. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is:

(A) 2

(B) -2

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Q30. If the sum of first n terms of an AP is $3n^2 + 5n$, then its m-th term is:

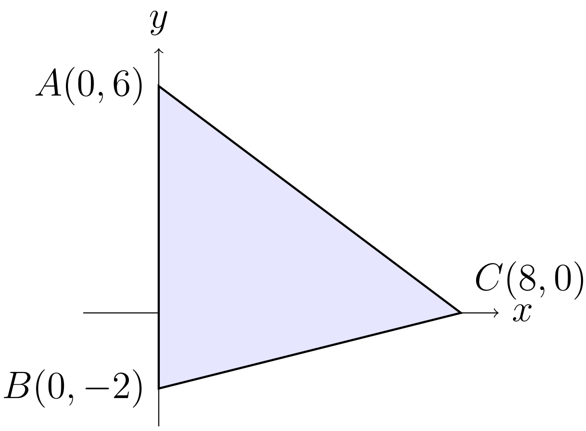
(A) $6m + 2$

(B) $6m - 2$

(C) $5m + 3$

(D) $3m^2 + 5m$

Q31. The area of the triangle formed by the points $A(0, 6)$, $B(0, -2)$, and $C(8, 0)$ is:



(A) 16 sq. units

(B) 32 sq. units

(C) 4 sq. units

(D) 8 sq. units

Q32. If $\sin A = \frac{1}{2}$, then the value of $3\cos A - 4\cos^3 A$ is:

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) -1

Q33. From a point on the ground, which is 15 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is found to be $60°$:

(A) $15\sqrt{3}$ m

(B) $\frac{15}{\sqrt{3}}$ m

(C) 15 m

(D) 30 m

Q34. If the area of a circle is 154 cm^2, then its perimeter is:

(A) 11 cm

(B) 22 cm

(C) 44 cm

(D) 55 cm

Q35. A solid piece of iron in the form of a cuboid of dimensions 49 cm$\times$33 cm$\times$ 24 cm, is melted to form a solid sphere. The radius of the sphere is:

(A) 21 cm

(B) 28 cm

(C) 35 cm

(D) 14 cm

Q36. Relationship among the measures of central tendency is:

(A) 3 Median = Mode + 2 Mean

(B) 3 Mean = Median + 2 Mode

(C) Mode = 3 Mean − 2 Median

(D) Median = Mode + 2 Mean

Q37. A die is thrown once. The probability of getting a prime number is:

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{6}$

Q38. The product of a non-zero rational and an irrational number is:

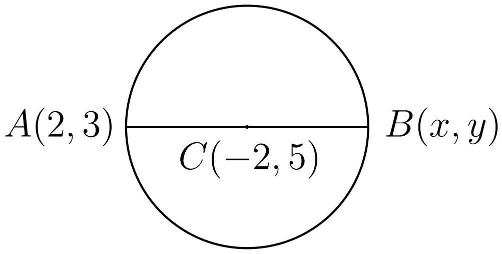
(A) always irrational

(B) always rational

(C) rational or irrational

(D) one

Q39. If the coordinates of one end of a diameter of a circle are $(2, 3)$ and the coordinates of its centre are $(-2, 5)$, then the coordinates of the other end of the diameter are:



(A) $(-6, 7)$

(B) $(6, -7)$

(C) $(0, 8)$

(D) $(0, 4)$

Q40. If $\tan\theta = \frac{4}{3}$, then $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} =$

(A) 7

(B) 1

(C) -7

(D) $\frac{1}{7}$

Q41. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to:

(A) $\frac{3\sqrt{3}}{2}$ cm

(B) $3\sqrt{3}$ cm

(C) 6 cm

(D) 3 cm

Q42. If the radius of a circle is diminished by 10%, then its area is diminished by:

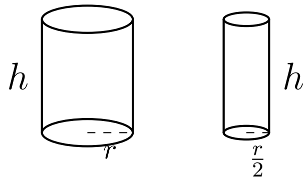
(A) 10%

(B) 19%

(C) 20%

(D) 36%

Q43. A repository contains volumes of cylinders. If the radius of the base of a right circular cylinder is halved, keeping the height same, the ratio of the volume of the reduced cylinder to that of the original cylinder is:



(A) $2:1$

(B) $1:2$

(C) $1:4$

(D) $4:1$

Q44. While computing mean of grouped data, we assume that the frequencies are:

(A) evenly distributed over all the classes

(B) centered at the classmarks of the classes

(C) centered at the upper limits of the classes

(D) centered at the lower limits of the classes

Q45. Two dice are thrown together. The probability that the sum of the two numbers will be a multiple of 4 is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{9}$

Q46. For which natural number n, the number 6^n ends with the digit zero?

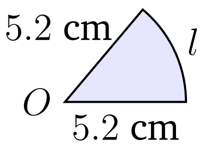
(A) $n = 5$

(B) $n = 10$

(C) For any even number n

(D) No such value of n exists

Q47. If the perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm, then the area of the sector is:



(A) 15.6 cm^2

(B) 15 cm^2

(C) 16 cm^2

(D) 16.6 cm^2

Q48. A conical vessel, with internal radius 5 cm and height 24 cm, is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. The height to which the water rises in the cylindrical vessel is:

(A) 2 cm

(B) 4 cm

(C) 1 cm

(D) 6 cm

Q49. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, for finding the mean of grouped data, d_i's are deviations from a of:

(A) lower limits of the classes

(B) upper limits of the classes

(C) classmarks of the classes

(D) frequencies of the class intervals

Q50. If the areas of two similar triangles are in the ratio $25 : 64$, then their corresponding sides are in the ratio:

(A) $25 : 64$

(B) $64 : 25$

(C) $5 : 8$

(D) $8 : 5$

Detailed Solutions

Q1.

Solution

Concept: The fundamental relationship between two positive integers a and b, their Highest Common Factor (HCF), and their Least Common Multiple (LCM) is given by the product formula:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Given the product of the two numbers and their HCF, we can rearrange this formula to isolate and solve for the unknown LCM.

Solution: Step 1: Identify the given values from the problem statement. We are given the Highest Common Factor of the two numbers, HCF $= 12$. We are also given the total product of the two positive integers, $a \times b = 2160$.

Step 2: Write down the governing mathematical formula that relates these three quantities together:

$$\text{HCF} \times \text{LCM} = \text{Product of the two numbers}$$

Step 3: Substitute the known values into the equation to set up our algebraic expression:

$$12 \times \text{LCM} = 2160$$

Step 4: Isolate the LCM by dividing both sides of the equation by 12:

$$\text{LCM} = \frac{2160}{12}$$

Step 5: Perform the division carefully to calculate the final numerical value:

$$\text{LCM} = 180$$

Thus, the Least Common Multiple of the two numbers is 180, which corresponds to option B.

Final Answer: 180

Answer: (B) Go Back to Question 1

Q2.

Solution

Concept: If a real number α is a zero of a polynomial $P(x)$, then substituting $x = \alpha$ into the polynomial must yield a total value of zero, meaning $P(\alpha) = 0$. For a quadratic polynomial, this condition allows us to set up a linear equation to solve for any unknown constant coefficient.

Solution: Step 1: Define the given quadratic polynomial as a function of x:

$$P(x) = x^2 + 3x + k$$

Step 2: Use the given information that one of the zeroes of this polynomial is 2. According to the factor theorem and definition of zeroes, we can substitute $x = 2$ directly into the polynomial function:

$$P(2) = 0$$

Step 3: Replace x with 2 in the algebraic expression for $P(x)$:

$$(2)^2 + 3(2) + k = 0$$

Step 4: Simplify the numeric squares and products sequentially:

$$4 + 6 + k = 0$$

$$10 + k = 0$$

Step 5: Isolate the unknown variable k by subtracting 10 from both sides of the equation:

$$k = -10$$

Therefore, the value of the constant k must be -10, matching option B.

Final Answer: -10

Answer: (B) Go Back to Question 2

Q3.

Solution

Concept: For a pair of linear equations in two variables written in the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, the nature of the system can be determined by comparing the ratios of their coefficients:

$$\text{If } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The two lines are completely parallel to each other and never intersect, meaning the system has no solution.

Solution: Step 1: Write down both given linear equations and identify their respective coefficients:

$$\text{Equation 1: } x + 2y + 5 = 0 \implies a_1 = 1, b_1 = 2, c_1 = 5$$

$$\text{Equation 2: } -3x - 6y + 1 = 0 \implies a_2 = -3, b_2 = -6, c_2 = 1$$

Step 2: Compute the ratio of the coefficients of the variable x:

$$\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}$$

Step 3: Compute the ratio of the coefficients of the variable y:

$$\frac{b_1}{b_2} = \frac{2}{-6} = -\frac{1}{3}$$

Step 4: Compute the ratio of the constant terms:

$$\frac{c_1}{c_2} = \frac{5}{1} = 5$$

Step 5: Analyze and compare all three computed ratios:

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \implies \left(-\frac{1}{3} = -\frac{1}{3} \neq 5\right)$$

This geometric condition confirms that the lines are parallel, and therefore the system has no solution.

Final Answer: no solution

Answer: (D) Go Back to Question 3

Q4.

Solution

Concept: The nature of the roots of a standard quadratic equation $ax^2 + bx + c = 0$ is governed entirely by its discriminant, denoted by D. The mathematical formula for the discriminant is:

$$D = b^2 - 4ac$$

For a quadratic equation to possess real and distinct (unequal) roots, the discriminant must be strictly greater than zero ($D > 0$).

Solution: Step 1: Identify the coefficients of the given quadratic equation $x^2+4x+k = 0$ by comparing it with the standard form $ax^2 + bx + c = 0$:

$$a = 1, \quad b = 4, \quad c = k$$

Step 2: Write down the formal algebraic condition required for the roots to be real and distinct:

$$D > 0 \implies b^2 - 4ac > 0$$

Step 3: Substitute the numerical values of the coefficients into this inequality:

$$(4)^2 - 4(1)(k) > 0$$

Step 4: Simplify the terms and basic arithmetic operations:

$$16 - 4k > 0$$

Step 5: Rearrange the inequality to isolate the variable k. Add $4k$ to both sides:

$$16 > 4k$$

Divide both sides of the inequality by the positive integer 4:

$$4 > k \implies k < 4$$

Hence, the parameter k must be strictly less than 4.

Final Answer: $\boxed{k < 4}$

Answer: (A) Go Back to Question 4

Q5.

Solution

Concept: The general n-th term of an Arithmetic Progression (AP) can be determined using the standard sequence formula:

$$a_n = a + (n-1)d$$

where a represents the first term of the progression, d represents the common difference between consecutive terms, and n represents the specific position of the term in the sequence.

Solution: Step 1: Identify the first term (a) directly from the given sequence $-3, -\frac{1}{2}, 2, \ldots$:

$$a = -3$$

Step 2: Compute the common difference (d) by subtracting the first term from the second term:

$$d = \left(-\frac{1}{2}\right) - (-3) = -\frac{1}{2} + 3 = \frac{-1+6}{2} = \frac{5}{2}$$

Step 3: Note the required term position specified in the question, which is the 11th term, so $n = 11$.

Step 4: Substitute the values of a, d, and n into the general formula for the n-th term:

$$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$$

Step 5: Simplify the bracketed term and perform the fraction multiplication:

$$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$$

$$a_{11} = -3 + 5 \times 5 = -3 + 25 = 22$$

Thus, the 11th term of the progression is 22.

Final Answer: 22

Answer: (B) Go Back to Question 5

Q6.

Solution

Concept: According to Thales's Theorem, also widely known as the Basic Proportionality Theorem (BPT), if a straight line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in the exact same ratio:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Solution: Step 1: Use the given information that line segment DE is parallel to base BC ($DE \parallel BC$). Apply the Basic Proportionality Theorem directly to set up a ratio equation:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 2: Substitute the algebraic expressions given for each segment into the theorem's formula:

$$\frac{x}{x-2} = \frac{x+2}{x-1}$$

Step 3: Cross-multiply the fractions to clear the denominators and set up a polynomial equation:

$$x(x-1) = (x+2)(x-2)$$

Step 4: Expand the expressions on both sides. Use the algebraic identity $(a+b)(a-b) = a^2 - b^2$ on the right side:

$$x^2 - x = x^2 - 4$$

Step 5: Cancel the x^2 term from both sides of the equation to simplify it to a linear form:

$$-x = -4 \implies x = 4$$

Thus, the numerical value of x is 4.

Final Answer: 4

Answer: (A) Go Back to Question 6

Q7.

Solution

Concept: The straight-line distance of any given point $P(x, y)$ in a Cartesian coordinate system from the origin $O(0, 0)$ is derived from the standard distance formula and is mathematically expressed as:

$$\text{Distance } OP = \sqrt{x^2 + y^2}$$

This is a direct application of the Pythagorean theorem on the coordinates of the point.

Solution: Step 1: Identify the coordinates of the given point $P(x, y)$ from the problem statement:

$$x = -6, \quad y = 8$$

Step 2: Write down the origin distance formula to establish the structure of the calculation:

$$\text{Distance} = \sqrt{x^2 + y^2}$$

Step 3: Substitute the specific values of x and y into the radical expression:

$$\text{Distance} = \sqrt{(-6)^2 + (8)^2}$$

Step 4: Compute the squares of both numbers independently. Remember that the square of a negative number is positive:

$$(-6)^2 = 36, \quad (8)^2 = 64$$

$$\text{Distance} = \sqrt{36 + 64}$$

Step 5: Sum the values under the radical and extract the principal square root:

$$\text{Distance} = \sqrt{100} = 10 \text{ units}$$

Hence, the total distance of the point from the origin is 10.

Final Answer: $\boxed{10}$

Answer: (C) Go Back to Question 7

Q8.

Solution

Concept: The trigonometric functions $\sin\theta$ and $\cos\theta$ are related to $\tan\theta$ by the fundamental identity definition $\tan\theta = \frac{\sin\theta}{\cos\theta}$. By rearranging the given linear equation, we can express all terms containing $\cos\theta$ on one side and solve directly for the ratio.

Solution: Step 1: Start with the given trigonometric equality:

$$\sin\theta + \cos\theta = \sqrt{2}\cos\theta$$

Step 2: Rearrange the equation to gather all terms involving the cosine function onto the right-hand side:

$$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

Step 3: Factor out the common term $\cos\theta$ from the expression on the right side:

$$\sin\theta = (\sqrt{2} - 1)\cos\theta$$

Step 4: Divide both sides of the equation by $\cos\theta$ to construct the tangent definition ratio on the left side:

$$\frac{\sin\theta}{\cos\theta} = \sqrt{2} - 1$$

Step 5: Substitute the fundamental identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ into the equation:

$$\tan\theta = \sqrt{2} - 1$$

Therefore, the value of $\tan\theta$ is equal to $\sqrt{2} - 1$.

Final Answer: $\boxed{\sqrt{2} - 1}$

Answer: (A) Go Back to Question 8

Q9.

Solution

Concept: In a right-angled triangle, the basic trigonometric ratios can link internal angles to side lengths. The cosine of an angle ($\cos\theta$) is defined as the ratio of the length of the adjacent side to the length of the hypotenuse:

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

Using this relationship helps find unknown vertical heights when given a slanted length.

Solution: Step 1: Visualize the physical scenario as a right-angled triangle where the vertical wall represents the perpendicular height h, the ladder forms the hypotenuse of length 15 m, and the angle between the ladder and the vertical wall is 60°.

Step 2: Identify the trigonometric relation that connects the given angle, its adjacent side (h), and the hypotenuse (15 m):

$$\cos(60^\circ) = \frac{\text{Height of the wall } (h)}{\text{Length of the ladder}}$$

Step 3: Substitute the known length of the ladder into the fraction:

$$\cos(60^\circ) = \frac{h}{15}$$

Step 4: Recall the exact standard trigonometric value for $\cos(60^\circ)$, which is $\frac{1}{2}$:

$$\frac{1}{2} = \frac{h}{15}$$

Step 5: Solve for the height h by cross-multiplying:

$$h = \frac{15}{2}\text{ m}$$

The total vertical height of the wall is therefore $\frac{15}{2}$ m.

Final Answer: $\boxed{\frac{15}{2}\text{ m}}$

Answer: (C) Go Back to Question 9

Q10.

Solution

Concept: Tangents drawn from an external point to a circle subtend equal angles at the center, and the line segment connecting the center to that external point acts as an angle bisector for both the angle between the tangents and the angle at the center. Additionally, a radius is perpendicular to the tangent at its point of contact (90°).

Solution: Step 1: Note that the angle between the two tangents PA and PB is given as $\angle APB = 80^\circ$. The line segment OP joining the center to the external point bisects this angle:

$$\angle APO = \frac{\angle APB}{2} = \frac{80^\circ}{2} = 40^\circ$$

Step 2: Use the standard circle theorem which states that a tangent is perpendicular to the radius at the point of tangency. This gives:

$$\angle OAP = 90^\circ$$

Step 3: Focus on the right-angled triangle $\triangle OAP$. The sum of all interior angles inside any triangle is always 180°:

$$\angle POA + \angle OAP + \angle APO = 180^\circ$$

Step 4: Substitute the known values into the triangle angle sum equation:

$$\angle POA + 90^\circ + 40^\circ = 180^\circ$$

Step 5: Add the values and isolate $\angle POA$:

$$\angle POA + 130^\circ = 180^\circ$$

$$\angle POA = 180^\circ - 130^\circ = 50^\circ$$

Thus, the measure of $\angle POA$ is 50°.

Final Answer: $\boxed{50^\circ}$

Answer: (A) Go Back to Question 10

Q11.

Solution

Concept: To geometrically divide a given line segment AB internally in a ratio $m : n$ using the standard ruler-and-compass construction method, an acute ray AX is drawn first. A total number of equidistant marks are plotted along this ray. The minimum number of points required on this ray is equal to the sum of the components of the given ratio $(m + n)$.

Solution: Step 1: Identify the given internal division ratio components from the problem statement:

$$m = 5, \quad n = 7$$

Step 2: Understand the geometric construction rule for dividing a segment. The method requires marking points $A_1, A_2, \ldots, A_k$ on ray AX such that the total number of points k is equal to the sum of the ratio parts.

Step 3: Set up the addition to find the minimum number of equidistant points needed:

$$\text{Minimum points} = m + n$$

Step 4: Substitute the values 5 and 7 into the construction formula:

$$\text{Minimum points} = 5 + 7$$

Step 5: Calculate the sum:

$$\text{Minimum points} = 12$$

Hence, a minimum of 12 points must be located on the ray AX.

Final Answer: 12

Answer: (C) Go Back to Question 11

Q12.

Solution

Concept: The area A of any circle with a radius r is calculated using the formula $A = \pi r^2$. If the sum of the areas of two small distinct circles equals the area of a single larger circle, we can set up an equation using this area formula and simplify it by eliminating common factors.

Solution: Step 1: Express the areas of the two individual small circles with radii R_1 and R_2:

$$\text{Area of first circle} = \pi R_1^2$$

$$\text{Area of second circle} = \pi R_2^2$$

Step 2: Express the area of the large target circle with radius R:

$$\text{Area of large circle} = \pi R^2$$

Step 3: Set up the equation according to the condition given in the question statement:

$$\text{Area of first circle} + \text{Area of second circle} = \text{Area of large circle}$$

$$\pi R_1^2 + \pi R_2^2 = \pi R^2$$

Step 4: Factor out the common constant π from the left side of the equation:

$$\pi(R_1^2 + R_2^2) = \pi R^2$$

Step 5: Divide both sides by π to obtain the final simplified relationship between the radii:

$$R_1^2 + R_2^2 = R^2$$

This matches the relation given in option B.

Final Answer: $\boxed{R_1^2 + R_2^2 = R^2}$

Answer: (B) Go Back to Question 12

Q13.

Solution

Concept: When a solid geometric object is melted and recast into another solid shape, the total volume of material remains constant throughout the process. Therefore, the volume of the original sphere must equal the volume of the newly formed cylinder:

$$V_{\text{sphere}} = V_{\text{cylinder}} \implies \frac{4}{3}\pi r^3 = \pi R^2 h$$

Solution: Step 1: Write down the dimensions given for both shapes:

$$\text{Radius of the sphere } (r) = 4.2 \text{ cm}$$

$$\text{Radius of the cylinder } (R) = 6 \text{ cm}$$

Let h be the unknown height of the cylinder.

Step 2: Equate the two volume formulas since the total volume is conserved:

$$\frac{4}{3}\pi r^3 = \pi R^2 h$$

Step 3: Divide both sides of the equation by π to eliminate the constant:

$$\frac{4}{3}r^3 = R^2 h$$

Step 4: Substitute the numerical values into the simplified expression:

$$\frac{4}{3} \times (4.2)^3 = (6)^2 \times h$$

$$\frac{4}{3} \times 4.2 \times 4.2 \times 4.2 = 36 \times h$$

$$4 \times 1.4 \times 4.2 \times 4.2 = 36 \times h$$

$$5.6 \times 17.64 = 36 \times h \implies 98.784 = 36 \times h$$

Step 5: Solve for h by dividing both sides by 36:

$$h = \frac{98.784}{36} = 2.744 \text{ cm}$$

Rounding to two decimal places gives a height of 2.74 cm.

Final Answer: $\boxed{2.74 \text{ cm}}$

Answer: (A) Go Back to Question 13

Q14.

Solution

Concept: In a grouped frequency distribution table, the modal class is defined as the specific class interval that possesses the highest frequency value among all the classes listed. Identifying this class is the first step in calculating the mode of grouped data.

Solution: Step 1: Examine the given frequency distribution table carefully and list out each class interval alongside its corresponding frequency value:

$$\text{Class } 0 - 10 \implies \text{Frequency} = 3$$

$$\text{Class } 10 - 20 \implies \text{Frequency} = 12$$

$$\text{Class } 20 - 30 \implies \text{Frequency} = 20$$

$$\text{Class } 30 - 40 \implies \text{Frequency} = 15$$

$$\text{Class } 40 - 50 \implies \text{Frequency} = 5$$

Step 2: Compare all the individual frequency numbers to find the maximum value:

$$\text{Frequencies: } 3, 12, 20, 15, 5$$

The largest number in this set of frequencies is 20.

Step 3: Identify the class interval linked to this maximum frequency of 20. Looking back at the table, the class interval corresponding to 20 is $20 - 30$.

Step 4: Conclude that since $20 - 30$ has the highest frequency, it fits the definition of the modal class.

Final Answer: 20-30

Answer: (B) Go Back to Question 14

Q15.

Solution

Concept: By definition, the probability $P(E)$ of any random event E must always be a real number bounded within the inclusive range from 0 to 1. Mathematically, this probability boundary condition is written as:

$$0 \leq P(E) \leq 1$$

Therefore, a probability can never be negative, nor can it ever exceed 1.

Solution: Step 1: Evaluate each given option individually to check if it satisfies the probability range constraint $0 \leq P(E) \leq 1$.

Step 2: Analyze option A: The fraction is $\frac{2}{3}$, which evaluates to approximately 0.6667. Since $0 \leq 0.6667 \leq 1$, this is a valid probability.

Step 3: Analyze option B: The value is -1.5. Since this number is strictly less than 0, it is a negative value, which violates the fundamental condition that probability cannot be negative.

Step 4: Analyze option C: The value is 15%, which can be rewritten as a decimal fraction $\frac{15}{100} = 0.15$. Since $0 \leq 0.15 \leq 1$, this is a valid probability.

Step 5: Analyze option D: The value is 0.7. Since $0 \leq 0.7 \leq 1$, this is also a valid probability value. Thus, -1.5 cannot be a probability.

Final Answer: -1.5

Answer: (B) Go Back to Question 15

Q16.

Solution

Concept: The decimal expansion of a rational number $\frac{p}{q}$ (in its lowest terms) terminates if the prime factorization of the denominator q contains only powers of 2 and/or 5. If $q = 2^m \times 5^n$, the decimal expansion terminates after a number of places equal to the maximum of m and n, mathematically denoted as $\max(m, n)$.

Solution: Step 1: Write down the given fraction and isolate its denominator for prime factorization:

$$\text{Fraction} = \frac{14587}{1250} \implies \text{Denominator } (q) = 1250$$

Step 2: Find the prime factors of the denominator 1250 through successive division:

$$1250 = 2 \times 625 = 2 \times 5 \times 125 = 2 \times 5 \times 5 \times 25 = 2^1 \times 5^4$$

Step 3: Express the denominator in the standard index form $2^m \times 5^n$:

$$m = 1, \quad n = 4$$

Step 4: Determine the maximum exponent between the prime factors 2 and 5:

$$\text{Number of terminating places} = \max(m, n) = \max(1, 4) = 4$$

Step 5: Conclude that the decimal expansion will terminate after exactly four decimal places.

Final Answer: four decimal places

Answer: (D) Go Back to Question 16

Q17.

Solution

Concept: For a cubic polynomial $x^3 + ax^2 + bx + c$, let its three roots be α, β, and γ. According to Vieta's formulas, the product of all three roots is related to the constant term by:

$$\alpha\beta\gamma = -c$$

If one zero is known, substituting it into the polynomial also helps find an expression for the constant term.

Solution: Step 1: Let the three zeroes of the cubic polynomial $f(x) = x^3 + ax^2 + bx + c$ be $\alpha = -1$, β, and γ. We need to find the product of the other two zeroes, which is $\beta\gamma$.

Step 2: Since -1 is a zero of the polynomial, substituting $x = -1$ into $f(x)$ must equal zero:

$$f(-1) = (-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$-1 + a - b + c = 0 \implies c = 1 - a + b$$

Step 3: Use Vieta's relationship for the product of all three roots of a cubic polynomial:

$$\alpha\beta\gamma = -\frac{\text{Constant term}}{\text{Coefficient of } x^3} = \frac{-c}{1} = -c$$

Step 4: Substitute the known value of $\alpha = -1$ into this product equation:

$$(-1)\beta\gamma = -c \implies \beta\gamma = c$$

Step 5: Substitute the expression for c obtained in Step 2 into this equation:

$$\beta\gamma = 1 - a + b = b - a + 1$$

Thus, the product of the other two roots is $b - a + 1$.

Final Answer: $\boxed{b - a + 1}$

Answer: (A) Go Back to Question 17

Q18.

Solution

Concept: Problems involving different denominations of money can be modeled as a system of linear equations in two variables. Let the number of coins of each denomination be represented by variables. We can then set up two independent linear equations: one for the total number of coins and one for the total monetary value.

Solution: Step 1: Define the variables for the problem. Let the number of Rs. 1 coins be x, and let the number of Rs. 2 coins be y.

Step 2: Set up the first linear equation based on the total number of coins provided:

$$x + y = 50 \quad \text{— (Equation 1)}$$

Step 3: Set up the second linear equation based on the total monetary value of the coins:

$$1x + 2y = 75 \implies x + 2y = 75 \quad \text{— (Equation 2)}$$

Step 4: Solve the system of equations by subtracting Equation 1 from Equation 2 to eliminate the variable x:

$$(x + 2y) - (x + y) = 75 - 50$$

$$y = 25$$

Step 5: Substitute the value of $y = 25$ back into Equation 1 to find x:

$$x + 25 = 50 \implies x = 25$$

So, Aruna has 25 coins of Rs. 1 and 25 coins of Rs. 2.

Final Answer: 25 and 25

Answer: (D) Go Back to Question 18

Q19.

Solution

Concept: The roots of a quadratic equation $ax^2 + bx + c = 0$ can be found using the method of splitting the middle term (factorization). We look for two numbers whose product equals $a \times c$ and whose sum equals the middle coefficient b.

Solution: Step 1: Write down the given quadratic equation and note its coefficients:

$$2x^2 - x - 6 = 0 \implies a = 2, \quad b = -1, \quad c = -6$$

Step 2: Find the target product for splitting the middle term:

$$\text{Product} = a \times c = 2 \times (-6) = -12$$

We need two numbers that multiply to -12 and add up to -1. These numbers are -4 and 3.

Step 3: Rewrite the middle term of the quadratic equation using these two numbers:

$$2x^2 - 4x + 3x - 6 = 0$$

Step 4: Group the terms into pairs and factor out the common terms from each group:

$$2x(x - 2) + 3(x - 2) = 0$$

Factor out the common binomial expression $(x - 2)$:

$$(2x + 3)(x - 2) = 0$$

Step 5: Set each linear factor to zero to find the roots:

$$2x + 3 = 0 \implies x = -\frac{3}{2}$$

$$x - 2 = 0 \implies x = 2$$

The roots are 2 and $-\frac{3}{2}$, which matches option B.

Final Answer: $\boxed{2, -\frac{3}{2}}$

Answer: (B) Go Back to Question 19

Q20.

Solution

Concept: In an Arithmetic Progression (AP), the general formula for any term is $a_n = a + (n-1)d$. When calculating the difference between two specific terms, the first term a cancels out, meaning the difference depends only on the common difference d and the distance between their positions.

Solution: Step 1: Write down the general expression for the two required terms, a_{18} and a_{13}, using the standard AP term formula:

$$a_{18} = a + (18-1)d = a + 17d$$

$$a_{13} = a + (13-1)d = a + 12d$$

Step 2: Set up the subtraction expression for the difference between these two terms:

$$a_{18} - a_{13} = (a + 17d) - (a + 12d)$$

Step 3: Expand the subtraction to cancel out the first term a:

$$a_{18} - a_{13} = a + 17d - a - 12d$$

$$a_{18} - a_{13} = 17d - 12d = 5d$$

Step 4: Identify the value of the common difference d given in the question statement:

$$d = 5$$

Step 5: Substitute $d = 5$ into our simplified difference expression:

$$a_{18} - a_{13} = 5(5) = 25$$

The difference between the two terms is 25.

Final Answer: 25

Answer: (C) Go Back to Question 20

Q21.

Solution

Concept: The total perimeter of a semi-circular protractor includes both the curved semi-circular arc length and the straight baseline diameter. If r represents the radius of the semi-circle, the formula for the total perimeter is:

$$\text{Perimeter} = \pi r + 2r = r(\pi + 2)$$

Solution: Step 1: Write down the total perimeter formula for a semi-circular protractor and equate it to the given value:

$$\text{Perimeter} = 36 \text{ cm} \implies r(\pi + 2) = 36$$

Step 2: Substitute the standard fractional approximation for π, which is $\frac{22}{7}$, into the equation:

$$r\left(\frac{22}{7} + 2\right) = 36$$

Step 3: Simplify the expression inside the brackets by finding a common denominator:

$$r\left(\frac{22 + 14}{7}\right) = 36$$

$$r\left(\frac{36}{7}\right) = 36$$

Step 4: Solve for the radius r by isolating it on one side of the equation:

$$r = 36 \times \frac{7}{36} \implies r = 7 \text{ cm}$$

Step 5: Calculate the diameter (d) of the protractor, which is twice the radius length:

$$d = 2r = 2 \times 7 = 14 \text{ cm}$$

The total diameter of the protractor is 14 cm.

Final Answer: 14 cm

Answer: (C) **Go Back to Question 21**

Q22.

Solution

Concept: In applications of trigonometry, problems involving heights and distances can be modeled using right-angled triangles. The tangent of an angle ($\tan\theta$) is defined as the ratio of the perpendicular side to the adjacent side:

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$$

We must account for the observer's height by subtracting it from the total tower height to find the net perpendicular side.

Solution: Step 1: Calculate the net height of the tower above the observer's eye level. Subtract the observer's height from the total height of the tower:

$$\text{Net height } (p) = 30 \text{ m} - 1.5 \text{ m} = 28.5 \text{ m}$$

Step 2: Identify the horizontal distance from the observer to the base of the tower, which forms the base of our right triangle:

$$\text{Horizontal distance } (b) = 28.5 \text{ m}$$

Step 3: Set up the tangent trigonometric ratio for the angle of elevation θ:

$$\tan\theta = \frac{\text{Net height of tower}}{\text{Horizontal distance}}$$

Step 4: Substitute the values calculated into the trigonometric fraction:

$$\tan\theta = \frac{28.5}{28.5} \implies \tan\theta = 1$$

Step 5: Find the angle θ whose tangent value is equal to 1:

$$\tan\theta = \tan(45^\circ) \implies \theta = 45^\circ$$

Thus, the angle of elevation of the top of the tower from the observer's eye is 45°.

Final Answer: $\boxed{45^\circ}$

Answer: (B) Go Back to Question 22

Q23.

Solution

Concept: The perimeter (circumference) C of a circle with radius r is given by $C = 2\pi r$, and its area A is given by $A = \pi r^2$. If these two quantities are numerically equal, we can set up an equation equating the two formulas and solve for the radius r.

Solution: Step 1: Write down the mathematical formulas for both the perimeter and the area of a circle in terms of its radius r:

$$\text{Perimeter} = 2\pi r$$

$$\text{Area} = \pi r^2$$

Step 2: Set the two expressions equal to each other as specified by the problem condition:

$$\text{Area} = \text{Perimeter} \implies \pi r^2 = 2\pi r$$

Step 3: Divide both sides of the equation by π to eliminate the constant:

$$r^2 = 2r$$

Step 4: Rearrange the equation into a standard quadratic form to avoid losing potential solutions:

$$r^2 - 2r = 0$$

Factor out the radius variable r:

$$r(r-2) = 0$$

Step 5: Solve for r. Since a circle's radius must be a positive length ($r \neq 0$), we choose the non-zero root:

$$r - 2 = 0 \implies r = 2 \text{ units}$$

The radius of the circle is 2 units.

Final Answer: $\boxed{2 \text{ units}}$

Answer: (A) Go Back to Question 23

Q24.

Solution

Concept: The classical probability of an event is calculated as the ratio of the number of favorable outcomes to the total number of possible outcomes in the sample space:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

A standard deck contains 52 cards split into 4 suits, where face cards are the Jacks, Queens, and Kings.

Solution: Step 1: Determine the total number of outcomes in the sample space, which is the total number of cards in a standard deck:

$$\text{Total cards} = 52$$

Step 2: Calculate the number of favorable outcomes, which is the number of red face cards. A deck has two red suits (Hearts and Diamonds), and each suit contains exactly 3 face cards (Jack, Queen, King).

$$\text{Number of red face cards} = 2 \times 3 = 6$$

Step 3: Set up the probability fraction using the definition ratio:

$$P(\text{Red Face Card}) = \frac{\text{Number of red face cards}}{\text{Total number of cards}} = \frac{6}{52}$$

Step 4: Simplify the fraction by dividing both the numerator and the denominator by their greatest common divisor, which is 2:

$$P(\text{Red Face Card}) = \frac{6 \div 2}{52 \div 2} = \frac{3}{26}$$

Thus, the probability of drawing a red face card is $\frac{3}{26}$.

Final Answer: $\boxed{\frac{3}{26}}$

Answer: (A) Go Back to Question 24

Q25.

Solution

Concept: When multiple coins are tossed simultaneously, we can list the entire sample space of outcomes. The probability of getting a specific result is then found by counting the outcomes that match the criteria and dividing by the total number of outcomes in the sample space.

Solution: Step 1: List the entire sample space S for tossing three distinct coins simultaneously. Each coin has 2 possible outcomes, so the total number of outcomes is $2^3 = 8$:

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$\text{Total number of possible outcomes} = 8$$

Step 2: Identify the favorable outcomes that contain exactly two heads (H). Let this event be E. Looking at the sample space, we find:

$$E = \{HHT, HTH, THH\}$$

Step 3: Count the number of elements in the favorable event set:

$$\text{Number of favorable outcomes} = 3$$

Step 4: Apply the standard probability formula:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} = \frac{3}{8}$$

Step 5: Conclude that the probability of obtaining exactly two heads is $\frac{3}{8}$.

Final Answer: $\boxed{\frac{3}{8}}$

Answer: (B) Go Back to Question 25

Q26.

Solution

Concept: According to Euclid's division lemma, if a number divides multiple values leaving specific remainders, that number must be a common factor of the differences obtained by subtracting the remainders from those respective values. The largest such number is the Highest Common Factor (HCF) of these differences.

Solution: Step 1: Subtract the respective remainders from the given numbers to find the numbers that are exactly divisible by the required factor:

$$\text{First number} = 70 - 5 = 65$$

$$\text{Second number} = 125 - 8 = 117$$

Step 2: The problem requires finding the largest number that divides both 65 and 117 exactly. This means we need to find HCF$(65, 117)$.

Step 3: Find the prime factorization of both numbers:

$$65 = 5 \times 13$$

$$117 = 3 \times 3 \times 13 = 3^2 \times 13$$

Step 4: Identify the common prime factors between the two factorizations and choose the lowest power of each common factor:

$$\text{Common prime factor} = 13$$

$$\text{HCF}(65, 117) = 13$$

Step 5: Conclude that the largest number which divides 70 and 125 leaving remainders 5 and 8 is 13.

Final Answer: $\boxed{13}$

Answer: (A) Go Back to Question 26

Q27.

Solution

Concept: For a standard cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$, let the roots be α, β, and γ. According to Vieta's relations, the sum of the zeroes is given by the ratio of coefficients:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

By substituting the given polynomial coefficients into this relationship, we can set up a linear equation to find the value of the unknown parameter k.

Solution: Step 1: Identify the coefficients of the given cubic polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ by comparing it with the standard form:

$$a = 2, \quad b = -3k, \quad c = 4, \quad d = -5$$

Step 2: Write down the formula for the sum of the zeroes of a cubic polynomial:

$$\text{Sum of zeroes} = -\frac{b}{a}$$

Step 3: Substitute the coefficient expressions into this formula:

$$\text{Sum of zeroes} = -\frac{-3k}{2} = \frac{3k}{2}$$

Step 4: Equate this expression to the actual sum of zeroes given in the question, which is 6:

$$\frac{3k}{2} = 6$$

Step 5: Solve for the unknown variable k by cross-multiplying and dividing:

$$3k = 6 \times 2 \implies 3k = 12$$

$$k = \frac{12}{3} \implies k = 4$$

Thus, the value of the constant k must be 4.

Final Answer: 4

Answer: (B) Go Back to Question 27

Q28.

Solution

Concept: A system of two linear equations $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$ has infinitely many solutions (coincident lines) if and only if the ratios of all their corresponding coefficients are equal:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Solution: Step 1: Express both equations in the standard linear format and list their coefficients:

$$\text{Equation 1: } kx+3y=k-3 \implies a_1=k,\ b_1=3,\ c_1=k-3$$

$$\text{Equation 2: } 12x+ky=k \implies a_2=12,\ b_2=k,\ c_2=k$$

Step 2: Set up the ratio equality condition for infinitely many solutions:

$$\frac{k}{12}=\frac{3}{k}=\frac{k-3}{k}$$

Step 3: Solve the first pair of fractions to find possible values for k:

$$\frac{k}{12}=\frac{3}{k} \implies k^2=36 \implies k=6 \text{ or } k=-6$$

Step 4: Verify which value of k satisfies the remaining ratio equality $\frac{3}{k}=\frac{k-3}{k}$.

If $k=6$:

$$\frac{3}{6}=\frac{6-3}{6} \implies \frac{1}{2}=\frac{3}{6}=\frac{1}{2} \quad \text{(True)}$$

If $k=-6$:

$$\frac{3}{-6}=\frac{-6-3}{-6} \implies -\frac{1}{2}=\frac{-9}{-6}=\frac{3}{2} \quad \text{(False)}$$

Step 5: Conclude that $k=6$ is the unique solution that satisfies all coefficient conditions.

Final Answer: $\boxed{6}$

Answer: (A) Go Back to Question 28

Q29.

Solution

Concept: If a specific number α is a root of a quadratic equation $ax^2 + bx + c = 0$, substituting $x = \alpha$ into the equation must satisfy the equality. This substitution transforms the quadratic equation into a simple linear equation in terms of any unknown parameter.

Solution: Step 1: Write down the given quadratic equation from the problem statement:

$$x^2 + kx - \frac{5}{4} = 0$$

Step 2: Use the fact that $x = \frac{1}{2}$ is a given root of this equation. Substitute $x = \frac{1}{2}$ directly into the expression:

$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

Step 3: Simplify the squared term and group the numerical fractions together:

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{k}{2} + \left(\frac{1}{4} - \frac{5}{4}\right) = 0$$

Step 4: Combine the fractions with the same denominator:

$$\frac{k}{2} + \left(\frac{-4}{4}\right) = 0 \implies \frac{k}{2} - 1 = 0$$

Step 5: Isolate the unknown variable k:

$$\frac{k}{2} = 1 \implies k = 2$$

Therefore, the value of the constant k must be 2.

Final Answer: $\boxed{2}$

Answer: (A) Go Back to Question 29

Q30.

Solution

Concept: In an Arithmetic Progression (AP), the m-th term (a_m) can be derived from the sum of the first n terms (S_n) using the standard sequence relation:

$$a_m = S_m - S_{m-1}$$

This relationship holds because subtracting the sum of $(m-1)$ terms from the sum of m terms leaves only the final m-th term.

Solution: Step 1: Write down the given formula for the sum of the first n terms of the sequence:

$$S_n = 3n^2 + 5n$$

Step 2: Replace n with m to write down the expression for the sum of m terms (S_m):

$$S_m = 3m^2 + 5m$$

Step 3: Replace n with $(m-1)$ to find the sum of the first $(m-1)$ terms (S_{m-1}):

$$S_{m-1} = 3(m-1)^2 + 5(m-1)$$

Expand the algebraic expression carefully using $(m-1)^2 = m^2 - 2m + 1$:

$$S_{m-1} = 3(m^2 - 2m + 1) + 5m - 5$$

$$S_{m-1} = 3m^2 - 6m + 3 + 5m - 5 = 3m^2 - m - 2$$

Step 4: Use the term formula $a_m = S_m - S_{m-1}$ to calculate the general m-th term:

$$a_m = (3m^2 + 5m) - (3m^2 - m - 2)$$

Step 5: Distribute the negative sign and simplify by combining like terms:

$$a_m = 3m^2 + 5m - 3m^2 + m + 2$$

$$a_m = 6m + 2$$

Thus, the m-th term of the progression is $6m + 2$.
Final Answer: 6m + 2

Answer: (A) Go Back to Question 30

Q31.

Solution

Concept: The coordinate area formula for a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is:

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Alternatively, if two vertices lie on one coordinate axis, the length along that axis can be used as the base, and the coordinate distance of the third vertex from that axis serves as the height.

Solution: Step 1: Identify the coordinates of the three given vertices of the triangle:

$$(x_1, y_1) = (0, 6), \quad (x_2, y_2) = (0, -2), \quad (x_3, y_3) = (8, 0)$$

Step 2: Notice that vertices $A(0, 6)$ and $B(0, -2)$ both lie entirely on the y-axis. We can calculate the length of the base AB by taking the absolute difference between their y-coordinates:

$$\text{Base } (AB) = |6 - (-2)| = |6 + 2| = 8 \text{ units}$$

Step 3: The third vertex $C(8, 0)$ lies on the x-axis. The perpendicular height from C to the line AB (the y-axis) is simply the absolute value of the x-coordinate of point C:

$$\text{Height} = |8| = 8 \text{ units}$$

Step 4: Use the classic geometric triangle area formula:

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

Step 5: Substitute the base and height values into this formula:

$$\text{Area} = \frac{1}{2} \times 8 \times 8 = 4 \times 8 = 32 \text{ sq. units}$$

Hence, the total area of the triangle is 32 square units.

Final Answer: $\boxed{32 \text{ sq. units}}$

Answer: (B) Go Back to Question 31

Q32.

Solution

Concept: The problem can be solved either by determining the value of angle A from the given sine value or by applying the standard triple-angle trigonometric identity for cosine:

$$\cos(3A) = 4\cos^3 A - 3\cos A$$

By recognizing that the given expression is exactly equal to $-\cos(3A)$, we can simplify the calculation.

Solution: Step 1: Note the given condition $\sin A = \frac{1}{2}$. We know from standard trigonometric values that for an acute angle, $\sin(30^\circ) = \frac{1}{2}$. Therefore, we can find angle A:

$$A = 30^\circ$$

Step 2: Examine the algebraic expression that we need to evaluate:

$$E = 3\cos A - 4\cos^3 A$$

Step 3: Factor out a negative sign from the expression to match it with a standard identity:

$$E = -(4\cos^3 A - 3\cos A)$$

Step 4: Substitute the triple-angle identity $\cos(3A) = 4\cos^3 A - 3\cos A$ into the expression:

$$E = -\cos(3A)$$

Step 5: Substitute the value of angle $A = 30^\circ$ into this simplified form:

$$E = -\cos(3 \times 30^\circ) = -\cos(90^\circ)$$

Recall that $\cos(90^\circ) = 0$. Therefore:

$$E = -0 = 0$$

The numerical value of the expression is 0.

Final Answer: $\boxed{0}$

Answer: (A) Go Back to Question 32

Q33.

Solution

Concept: In right-angled triangle trigonometry, the tangent of an interior angle is defined as the ratio of the length of the opposite side (perpendicular) to the length of the adjacent side (base):

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

This trigonometric ratio allows us to calculate unknown heights of objects when the distance to their base and the angle of elevation are known.

Solution: Step 1: Model the problem as a right-angled triangle where h is the unknown height of the vertical tower, the horizontal base distance from the foot of the tower is 15 m, and the angle of elevation is $\theta = 60°$.

Step 2: Write down the tangent ratio formula linking these parameters together:

$$\tan(60°) = \frac{\text{Height of the tower }(h)}{\text{Horizontal distance}}$$

Step 3: Substitute the known horizontal distance into the denominator of the fraction:

$$\tan(60°) = \frac{h}{15}$$

Step 4: Recall that the exact standard mathematical value for $\tan(60°)$ is $\sqrt{3}$:

$$\sqrt{3} = \frac{h}{15}$$

Step 5: Isolate the height variable h by multiplying both sides of the equation by 15:

$$h = 15\sqrt{3}\text{ m}$$

The height of the vertical tower is $15\sqrt{3}$ m.

Final Answer: $\boxed{15\sqrt{3}m}$

Answer: (A) Go Back to Question 33

Q34.

Solution

Concept: The area A of a circle with a radius r is calculated using the formula $A = \pi r^2$, and its perimeter (or circumference) C is calculated using the formula $C = 2\pi r$. Given the area, we can first find the value of the radius r and then use it to compute the perimeter.

Solution: Step 1: Write down the circle area formula and set it equal to the given numerical value:

$$\text{Area} = \pi r^2 = 154\ \text{cm}^2$$

Step 2: Substitute the standard fractional value of $\frac{22}{7}$ for π into the area equation:

$$\frac{22}{7} \times r^2 = 154$$

Step 3: Isolate the term r^2 by multiplying both sides by the reciprocal fraction $\frac{7}{22}$:

$$r^2 = 154 \times \frac{7}{22}$$

$$r^2 = 7 \times 7 \implies r^2 = 49$$

Step 4: Take the positive square root to find the radius of the circle:

$$r = \sqrt{49} = 7\ \text{cm}$$

Step 5: Use the radius value $r = 7$ cm to calculate the perimeter using the circumference formula:

$$\text{Perimeter} = 2\pi r = 2 \times \frac{22}{7} \times 7 = 2 \times 22 = 44\ \text{cm}$$

The total perimeter of the circle is 44 cm.

Final Answer: $\boxed{44\ \text{cm}}$

Answer: (C) Go Back to Question 34

Q35.

Solution

Concept: When a solid object is melted and recast into another shape without any loss of material, the total volume remains conserved. Therefore, the volume of the newly formed solid sphere must equal the volume of the original cuboid:

$$V_{\text{sphere}} = V_{\text{cuboid}} \implies \frac{4}{3}\pi R^3 = l \times b \times h$$

Solution: Step 1: Calculate the total volume of the original cuboid using its given dimensions:

$$\text{Volume of cuboid} = 49 \times 33 \times 24 \text{ cm}^3$$

Step 2: Write down the volume formula for a solid sphere with an unknown radius R:

$$\text{Volume of sphere} = \frac{4}{3}\pi R^3$$

Step 3: Equate the two volumes based on the principle of conservation of volume:

$$\frac{4}{3}\pi R^3 = 49 \times 33 \times 24$$

Step 4: Substitute the fractional value $\frac{22}{7}$ for π into the equation:

$$\frac{4}{3} \times \frac{22}{7} \times R^3 = 49 \times 33 \times 24$$

$$\frac{88}{21} \times R^3 = 49 \times 33 \times 24$$

Step 5: Isolate R^3 and simplify the resulting expression using prime factorization:

$$R^3 = \frac{49 \times 33 \times 24 \times 21}{88}$$

$$R^3 = \frac{49 \times 33 \times 24 \times 21}{11 \times 8} = 49 \times 3 \times 3 \times 21$$

$$R^3 = 7^2 \times 3^2 \times (3 \times 7) = 7^3 \times 3^3 = (7 \times 3)^3 = 21^3$$

Taking the cube root of both sides gives $R = 21$ cm.

Final Answer: 21 cm

Answer: (A) Go Back to Question 35

Q36.

Solution

Concept: In statistics, the three primary measures of central tendency—Mean, Median, and Mode—are related by an empirical formula for moderately skewed distributions. This fundamental relationship is expressed as:

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

This formula can be rearranged algebraically into different equivalent forms.

Solution: Step 1: Write down the standard empirical relationship that connects the three measures of central tendency:

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

Step 2: Analyze each given option to see which one can be algebraically rearranged to match this standard empirical formula.

Step 3: Examine option A: $3\,\text{Median} = \text{Mode} + 2\,\text{Mean}$. Let us rearrange this equation by subtracting 2 Mean from both sides:

$$\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$$

This perfectly matches the standard empirical formula.

Step 4: Verify that the other options lead to incorrect relationships that do not match the standard formula.

Step 5: Conclude that option A is the correct mathematical statement of the relationship.

Final Answer: $\boxed{3\text{ Median} = \text{Mode} + 2\text{ Mean}}$

Answer: (A) Go Back to Question 36

Q37.

Solution

Concept: The probability of an event is defined as the number of favorable outcomes divided by the total number of possible outcomes in the sample space. For a standard fair die, the sample space contains six outcomes, and we must identify which of these numbers are prime numbers.

Solution: Step 1: Write down the complete sample space S for throwing a single standard die:

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$\text{Total number of possible outcomes} = 6$$

Step 2: Identify all the prime numbers within this sample space. A prime number is a natural number strictly greater than 1 that has no positive divisors other than 1 and itself. Note that 1 is not a prime number:

$$\text{Prime numbers on a die} = \{2, 3, 5\}$$

Step 3: Count the total number of favorable prime outcomes:

$$\text{Number of favorable outcomes} = 3$$

Step 4: Use the standard probability formula to calculate the probability:

$$P(\text{Prime}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} = \frac{3}{6}$$

Step 5: Simplify the fraction by dividing both the numerator and denominator by 3:

$$P(\text{Prime}) = \frac{1}{2}$$

The probability of rolling a prime number is $\frac{1}{2}$.

Final Answer: $\boxed{1/2}$

Answer: (C) Go Back to Question 37

Q38.

Solution

Concept: A rational number can be expressed as a fraction of two integers $\frac{p}{q}$ (where $q \neq 0$), while an irrational number cannot. A mathematical theorem states that the product of any non-zero rational number and an irrational number is always an irrational number.

Solution: Step 1: Let r be a non-zero rational number, which means $r = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $p, q \neq 0$. Let x be an irrational number. We want to analyze the nature of their product, $y = r \times x$.

Step 2: Use a proof by contradiction. Assume that the product y is a rational number. This means we can write $y = \frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.

Step 3: Set up the algebraic equation based on this assumption:

$$r \times x = y \implies \left(\frac{p}{q}\right) \times x = \frac{a}{b}$$

Step 4: Isolate the irrational variable x by multiplying both sides by the reciprocal fraction $\frac{q}{p}$ (which is valid since $p \neq 0$):

$$x = \frac{a \times q}{b \times p}$$

Since the product of integers results in another integer, the right-hand side of the equation is a ratio of two integers, which defines a rational number.

Step 5: This creates a contradiction, because the irrational number x cannot equal a rational number. Therefore, our initial assumption must be false, proving that the product is always irrational.

Final Answer: always irrational

Answer: (A) Go Back to Question 38

Q39.

Solution

Concept: The centre of a circle is the exact midpoint of any of its diameters. According to the standard coordinate geometry midpoint formula, the coordinates of the midpoint $M(x_m, y_m)$ between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ are given by:

$$x_m = \frac{x_1 + x_2}{2}, \quad y_m = \frac{y_1 + y_2}{2}$$

Solution: Step 1: Let the known end of the diameter be $A(x_1, y_1) = (2, 3)$. Let the unknown other end of the diameter be $B(x_2, y_2)$. The given centre of the circle is $C(x_m, y_m) = (-2, 5)$.

Step 2: Set up the midpoint equation for the x-coordinates:

$$x_m = \frac{x_1 + x_2}{2} \implies -2 = \frac{2 + x_2}{2}$$

Step 3: Solve for the unknown coordinate x_2 by multiplying both sides by 2:

$$-4 = 2 + x_2 \implies x_2 = -4 - 2 = -6$$

Step 4: Set up the midpoint equation for the y-coordinates:

$$y_m = \frac{y_1 + y_2}{2} \implies 5 = \frac{3 + y_2}{2}$$

Step 5: Solve for the unknown coordinate y_2 by multiplying both sides by 2:

$$10 = 3 + y_2 \implies y_2 = 10 - 3 = 7$$

The coordinates of the other end of the diameter are $(-6, 7)$.

Final Answer: (-6, 7)

Answer: (A) Go Back to Question 39

Q40.

Solution

Concept: An algebraic expression containing both $\sin\theta$ and $\cos\theta$ can be simplified by dividing every term in both the numerator and the denominator by $\cos\theta$. This conversion replaces the sine and cosine functions with the tangent function ($\tan\theta = \frac{\sin\theta}{\cos\theta}$), allowing for direct substitution.

Solution: Step 1: Write down the given expression that needs to be evaluated:

$$E = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

Step 2: Divide every single term in both the numerator and the denominator by $\cos\theta$:

$$E = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}$$

Step 3: Substitute the trigonometric identity definition $\tan\theta = \frac{\sin\theta}{\cos\theta}$ into the expression:

$$E = \frac{\tan\theta + 1}{\tan\theta - 1}$$

Step 4: Substitute the given numerical value $\tan\theta = \frac{4}{3}$ into this simplified formula:

$$E = \frac{\frac{4}{3} + 1}{\frac{4}{3} - 1}$$

Step 5: Simplify the fractions in both the numerator and the denominator:

$$E = \frac{\frac{4+3}{3}}{\frac{4-3}{3}} = \frac{\frac{7}{3}}{\frac{1}{3}} = \frac{7}{1} = 7$$

The final numerical value of the expression is 7.

Final Answer: $\boxed{7}$

Answer: (A) Go Back to Question 40

Q41.

Solution

Concept: The line connecting an external point to the center of a circle bisects the angle between the two tangents drawn from that point. Furthermore, the radius drawn to the point of tangency forms a right angle (90°) with the tangent line, creating a right-angled triangle where:

$$\tan(\theta) = \frac{\text{Radius}}{\text{Tangent Length}}$$

Solution: Step 1: Let P be the external point, O be the center of the circle, and A be the point of tangency. The total angle between the two tangents is given as 60°. The line segment OP bisects this angle:

$$\angle APO = \frac{60^\circ}{2} = 30^\circ$$

Step 2: Identify the right-angled triangle $\triangle OAP$, where $\angle OAP = 90^\circ$ because the radius is perpendicular to the tangent at the point of contact.

Step 3: Use the tangent trigonometric ratio for the angle $\angle APO = 30^\circ$ in $\triangle OAP$:

$$\tan(30^\circ) = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{\text{Radius } (OA)}{\text{Tangent length } (PA)}$$

Step 4: Substitute the given radius value $OA = 3$ cm into the trigonometric equation:

$$\tan(30^\circ) = \frac{3}{PA}$$

Step 5: Substitute the standard trigonometric value $\tan(30^\circ) = \frac{1}{\sqrt{3}}$ and solve for the tangent length PA:

$$\frac{1}{\sqrt{3}} = \frac{3}{PA} \implies PA = 3\sqrt{3} \text{ cm}$$

The length of each tangent line segment is $3\sqrt{3}$ cm.

Final Answer: $\boxed{3\sqrt{3}cm}$

Answer: (B) Go Back to Question 41

Q42.

Solution

Concept: The area A of a circle is proportional to the square of its radius r, according to the formula $A = \pi r^2$. When the radius changes by a specific percentage, we can calculate the new radius, determine the new area expression, and find the percentage decrease relative to the original area.

Solution: Step 1: Let the original radius of the circle be r. The original area of this circle is:

$$A_1 = \pi r^2$$

Step 2: Calculate the new radius r' after it is diminished by 10%:

$$r' = r - 10\% \text{ of } r = r - 0.10r = 0.90r$$

Step 3: Compute the new area A_2 using the diminished radius value r':

$$A_2 = \pi(r')^2 = \pi(0.90r)^2 = 0.81\pi r^2$$

Step 4: Find the absolute decrease in the area by subtracting the new area from the original area:

$$\text{Decrease in Area} = A_1 - A_2 = \pi r^2 - 0.81\pi r^2 = 0.19\pi r^2$$

Step 5: Calculate the percentage decrease relative to the original area A_1:

$$\text{Percentage Decrease} = \left(\frac{\text{Decrease in Area}}{\text{Original Area}}\right) \times 100\%$$

$$\text{Percentage Decrease} = \left(\frac{0.19\pi r^2}{\pi r^2}\right) \times 100\% = 0.19 \times 100\% = 19\%$$

Thus, the area of the circle is diminished by exactly 19%.

Final Answer: $\boxed{19\%}$

Answer: (B) Go Back to Question 42

Q43.

Solution

Concept: The volume V of a right circular cylinder is calculated using the formula $V = \pi r^2 h$, where r represents the radius of the circular base and h represents the vertical height. By changing the radius while keeping the height constant, we can find the ratio of the new volume to the original volume.

Solution: Step 1: Write down the volume formula for the original cylinder with a base radius r and height h:

$$V_{\text{original}} = \pi r^2 h$$

Step 2: Determine the new radius r' after it is halved, as specified in the problem statement:

$$r' = \frac{r}{2}$$

Step 3: Write down the volume expression for the modified cylinder, keeping the height h the same:

$$V_{\text{reduced}} = \pi (r')^2 h = \pi \left(\frac{r}{2}\right)^2 h$$

Step 4: Expand the squared term to simplify the volume formula for the reduced cylinder:

$$V_{\text{reduced}} = \pi \left(\frac{r^2}{4}\right) h = \frac{1}{4}\pi r^2 h$$

Step 5: Set up the required ratio of the volume of the reduced cylinder to the original cylinder:

$$\text{Ratio} = \frac{V_{\text{reduced}}}{V_{\text{original}}} = \frac{\frac{1}{4}\pi r^2 h}{\pi r^2 h} = \frac{1}{4}$$

Thus, the volume ratio of the reduced cylinder to the original cylinder is $1:4$.

Final Answer: 1:4

Answer: (C) **Go Back to Question 43**

Q44.

Solution

Concept: When calculating statistical measures like the mean from grouped data tables, individual values within a class interval are not known. To simplify the calculations, a standard mathematical assumption is made regarding how frequencies are distributed within each class interval.

Solution: Step 1: Recall the standard formula used to compute the mean of grouped data, known as the Direct Method:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

where f_i represents the frequency of a class interval and x_i represents the corresponding value representing that class.

Step 2: Understand the definition of x_i in this formula. The value x_i is defined as the classmark (or midpoint) of the specific class interval, calculated as:

$$x_i = \frac{\text{Lower Limit} + \text{Upper Limit}}{2}$$

Step 3: Analyze why the midpoint is used in this calculation. Since the exact data points within a class interval are unknown, the standard statistical assumption is that all the frequencies within a given class are concentrated precisely at its central value.

Step 4: Match this assumption with the definitions provided in the multiple-choice options. The central value of a class interval is its classmark.

Step 5: Conclude that we assume the frequencies are centered at the classmarks of the classes, which corresponds to option B.

Final Answer: centered at the classmarks of the classes

Answer: (B) Go Back to Question 44

Q45.

Solution

Concept: When rolling two standard fair dice simultaneously, the total number of outcomes in the sample space is $6 \times 6 = 36$. To find the probability of an event, we list and count all the specific outcomes where the sum of the two numbers matches the given condition (in this case, being a multiple of 4).

Solution: Step 1: Determine the total number of outcomes in the sample space when two dice are rolled:

$$\text{Total outcomes} = 6 \times 6 = 36$$

Step 2: Identify the possible values for the sum of two dice that are multiples of 4. The minimum possible sum is $1 + 1 = 2$ and the maximum possible sum is $6 + 6 = 12$. The multiples of 4 within this range are 4, 8, and 12.

Step 3: List all the favorable pairs (d_1, d_2) that produce these target sums:

$$\text{Pairs that sum to 4: } \{(1,3), (2,2), (3,1)\}$$

$$\text{Pairs that sum to 8: } \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$$

$$\text{Pairs that sum to 12: } \{(6,6)\}$$

Step 4: Count the total number of favorable outcomes by adding the number of pairs for each sum:

$$\text{Total favorable outcomes} = 3 + 5 + 1 = 9$$

Step 5: Calculate the final probability by dividing the number of favorable outcomes by the total outcomes:

$$\text{Probability} = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{9}{36} = \frac{1}{4}$$

Thus, the probability that the sum is a multiple of 4 is $\frac{1}{4}$.

Final Answer: $\boxed{1/4}$

Answer: (C) **Go Back to Question 45**

Q46.

Solution

Concept: According to the Fundamental Theorem of Arithmetic, every composite number can be uniquely factored into a product of prime numbers. For any positive integer power of a number to end with the digit zero, its prime factorization must contain both 2 and 5 as prime factors.

Solution: Step 1: Write down the given expression 6^n and find the prime factorization of its base number, 6:

$$6 = 2 \times 3$$

Step 2: Raise this prime factorization to the power of the natural number n using exponent rules:

$$6^n = (2 \times 3)^n = 2^n \times 3^n$$

Step 3: Analyze the prime factors present in the factorization of 6^n. The only prime factors are 2 and 3, regardless of the value of the exponent n.

Step 4: Recall the mathematical condition required for a number's expansion to end with a zero digit. The number must be divisible by 10, which means its prime factorization must contain at least one power of the prime factor 5.

Step 5: Since the prime factorization of 6^n contains only 2 and 3 and completely lacks the prime factor 5, there is no natural number n for which 6^n can end with the digit zero.

Final Answer: No such value of n exists

Answer: (D) Go Back to Question 46

Q47.

Solution

Concept: The total perimeter P of a sector of a circle consists of two straight radius lengths plus the curved arc length l:

$$P = 2r + l$$

Once the arc length l is determined from this perimeter relation, the total area A of the sector can be calculated using the alternative formula:

$$A = \frac{1}{2} \times l \times r$$

Solution: Step 1: Write down the given dimensions for the sector from the problem statement:

$$\text{Radius } (r) = 5.2 \text{ cm}$$

$$\text{Total Perimeter } (P) = 16.4 \text{ cm}$$

Step 2: Use the sector perimeter formula to set up an equation to find the unknown arc length l:

$$P = 2r + l \implies 16.4 = 2(5.2) + l$$

Step 3: Simplify the multiplication and isolate the variable l:

$$16.4 = 10.4 + l$$

$$l = 16.4 - 10.4 = 6.0 \text{ cm}$$

Step 4: Use the sector area formula that links the radius and arc length directly:

$$\text{Area} = \frac{1}{2} \times l \times r$$

Step 5: Substitute the values $l = 6$ cm and $r = 5.2$ cm into this area formula:

$$\text{Area} = \frac{1}{2} \times 6 \times 5.2 = 3 \times 5.2 = 15.6 \text{ cm}^2$$

The total area of the sector is 15.6 cm^2.

Final Answer: $\boxed{15.6 \text{ cm}^2}$

Answer: (A) Go Back to Question 47

Q48.

Solution

Concept: When water is poured from one vessel into another, the total volume of water remains constant. Therefore, the volume of the conical vessel filled with water must equal the volume of the water inside the cylindrical vessel:

$$V_{\text{cone}} = V_{\text{cylinder}} \implies \frac{1}{3}\pi r^2 h = \pi R^2 H$$

Solution: Step 1: Note down the given internal dimensions for both vessels from the problem statement:

$$\text{Conical vessel: Radius } r = 5 \text{ cm}, \text{ Height } h = 24 \text{ cm}$$

$$\text{Cylindrical vessel: Radius } R = 10 \text{ cm}$$

Let H be the unknown height to which the water rises in the cylinder.

Step 2: Equate the two volume formulas since the total volume of water is conserved:

$$\frac{1}{3}\pi r^2 h = \pi R^2 H$$

Step 3: Divide both sides of the equation by π to eliminate the common constant:

$$\frac{1}{3}r^2 h = R^2 H$$

Step 4: Substitute the known numerical values into this simplified volume equation:

$$\frac{1}{3} \times (5)^2 \times 24 = (10)^2 \times H$$

$$\frac{1}{3} \times 25 \times 24 = 100 \times H$$

Step 5: Simplify the arithmetic operations and solve for H:

$$25 \times 8 = 100 \times H \implies 200 = 100 \times H$$

$$H = \frac{200}{100} = 2 \text{ cm}$$

The water rises to a height of 2 cm in the cylindrical vessel.

Final Answer: $\boxed{2 \text{ cm}}$

Answer: (A) Go Back to Question 48

Q49.

Solution

Concept: In statistics, the Assumed Mean Method is used to calculate the mean of a grouped frequency distribution. The formula is given by $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, where a represents the chosen assumed mean value and d_i represents the deviation value calculated for each class.

Solution: Step 1: Recall the complete definition of the terms used in the Assumed Mean Method formula for grouped datasets:

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

Step 2: Identify what each variable in this formula represents. The variable f_i represents the frequency of the i-th class interval, and a is the assumed mean, which is chosen from the central data values.

Step 3: Examine the formula used to calculate the individual deviations d_i:

$$d_i = x_i - a$$

Step 4: Define what the variable x_i represents in grouped statistical data. In grouped frequency tables, x_i is defined as the classmark (or the midpoint) of each respective class interval.

Step 5: Conclude that the terms d_i represent the mathematical deviations of the class-marks of the classes from the assumed mean a, which corresponds to option C.

Final Answer: classmarks of the classes

Answer: (C) Go Back to Question 49

Q50.

Solution

Concept: According to the properties of similar triangles, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding side lengths:

$$\frac{\text{Area}(\triangle 1)}{\text{Area}(\triangle 2)} = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2$$

Taking the principal square root of both sides allows us to find the ratio of their corresponding sides.

Solution: Step 1: Write down the geometric theorem relating the areas and sides of two similar triangles:

$$\frac{\text{Area of first triangle}}{\text{Area of second triangle}} = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2$$

Step 2: Substitute the given numerical ratio of the areas $(25 : 64)$ into this formula:

$$\frac{25}{64} = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2$$

Step 3: Express the fractions on the left-hand side of the equation as perfect squares:

$$25 = 5^2, \quad 64 = 8^2 \implies \frac{25}{64} = \left(\frac{5}{8}\right)^2$$

Step 4: Set up the modified equation equating the squared terms:

$$\left(\frac{5}{8}\right)^2 = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2$$

Step 5: Take the positive square root of both sides to isolate the side ratio:

$$\frac{\text{Side}_1}{\text{Side}_2} = \frac{5}{8}$$

Thus, the corresponding sides of the two triangles are in the ratio $5 : 8$.

Final Answer: 5:8

Answer: (C) Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	D	4	A	5	B
6	A	7	C	8	A	9	C	10	A
11	C	12	B	13	A	14	B	15	B
16	D	17	A	18	D	19	B	20	C
21	C	22	B	23	A	24	A	25	B
26	A	27	B	28	A	29	A	30	A
31	B	32	A	33	A	34	C	35	A
36	A	37	C	38	A	39	A	40	A
41	B	42	B	43	C	44	B	45	C
46	D	47	A	48	A	49	C	50	C