JEECUP Group A Mathematics Sample Paper-1

Duration: 60 Minutes **Maximum Marks: 200**

Instructions

- This paper contains **50** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**. No marks will be deducted for incorrect answers. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. If the sum of the first n terms of an arithmetic progression is given by $S_n = 3n^2 + 5n$, then which of the following represents its 12^{th} term?

(A) 71

(B) 74

(C) 82

(D) 124

Q2. A solid metallic sphere of radius 6 cm is melted and recast into a uniform wire of radius 0.2 cm. Find the total length of the wire.

(A) 18 m

(B) 36 m

(C) 72 m

(D) 108 m

Q3. A point P is at a distance of 13 cm from the center O of a circle of radius 5 cm. A pair of tangents PQ and PR are drawn to the circle from P. What is the area of the quadrilateral $PQOR$?

(A) 30 cm^2

(B) 60 cm^2

(C) 65 cm^2

(D) 120 cm^2

Q4. Three unbiased coins are tossed together. What is the probability of obtaining at most two heads?

(A) $\frac{1}{8}$

(B) $\frac{3}{8}$

(C) $\frac{5}{8}$

(D) $\frac{7}{8}$

Q5. If α and β are the zeroes of the polynomial $p(x) = 2x^2 - 5x + 7$, find the value of $\alpha^{-1} + \beta^{-1}$.

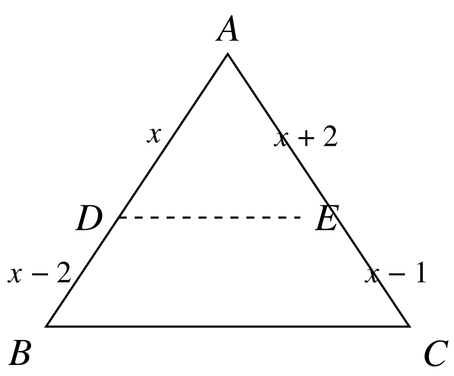
(A) $\frac{5}{7}$

(B) $-\frac{5}{7}$

(C) $\frac{7}{5}$

(D) $\frac{5}{2}$

Q6. Refer to the geometric figure below. In $\triangle ABC$, a line segment DE is drawn parallel to the base BC, intersecting AB at D and AC at E. If $AD = x$, $DB = x - 2$, $AE = x + 2$, and $EC = x - 1$, determine the value of x.



(A) 2

(B) 3

(C) 4

(D) 5

Q7. In a circle of radius 21 cm, an arc subtends an angle of 60° at the center. Find the length of this arc.

(A) 11 cm

(B) 22 cm

(C) 44 cm

(D) 126 cm

Q8. The mean of 10 observations is 25. If one observation is excluded, the mean of the remaining observations becomes 23. What is the value of the excluded observation?

(A) 28

(B) 35

(C) 41

(D) 43

Q9. For what value of k will the pair of linear equations $kx + 3y - (k - 3) = 0$ and $12x + ky - k = 0$ have infinitely many solutions?

(A) $k = 3$

(B) $k = -6$

(C) $k = 6$

(D) $k = 0$

Q10. Two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x and y are prime numbers. Find the LCM(a, b).

(A) xy

(B) x^2y^2

(C) x^3y^3

(D) x^4y^5

Q11. The roots of the quadratic equation $3x^2 - 4\sqrt{3}x + 4 = 0$ are:

(A) Real and distinct

(B) Real and equal

(C) Non-real / Imaginary

(D) Reciprocal to each other

Q12. A flagstaff stands vertically on the top of a tower. From a point on the ground 20 m away from the foot of the tower, the angles of elevation of the top and bottom of the flagstaff are 60° and 45° respectively. Find the height of the flagstaff.

(A) $20\sqrt{3}$ m

(B) $20(\sqrt{3} + 1)$ m

(C) $20(\sqrt{3} - 1)$ m

(D) 40 m

Q13. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $\cos\theta - \sin\theta$ is equal to:

(A) $\sqrt{2}\sin\theta$

(B) $\frac{1}{\sqrt{2}}\sin\theta$

(C) $-\sqrt{2}\cos\theta$

(D) $\sqrt{2}(\sin\theta + \cos\theta)$

Q14. Find the coordinates of the point which divides the line segment joining the points $(4, -3)$ and $(8, 5)$ internally in the ratio $3 : 1$.

(A) $(7, 3)$

(B) $(7, 2)$

(C) $(6, 3)$

(D) $(5, 1)$

Q15. Consider a circular running track on a field. Two athletes start running from the same point at the same time in the same direction. Player A completes one lap in 18 minutes, while Player B completes one lap in 12 minutes. After how many minutes will they meet again at the starting point?

(A) 6 minutes

(B) 24 minutes

(C) 36 minutes

(D) 72 minutes

Q16. Two numbers are selected at random from the first 20 natural numbers. What is the probability that the sum of the two selected numbers is an odd number?

(A) $\frac{1}{2}$

(B) $\frac{10}{19}$

(C) $\frac{9}{19}$

(D) $\frac{11}{20}$

Q17. The median of the following distribution is 32. Find the missing frequency f if the total frequency is 40.

Class Interval	**Frequency**
0-10	5
10-20	f
20-30	12
30-40	8
40-50	6

(A) 7

(B) 9

(C) 10

(D) 11

Q18. The perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm. Find the area of the sector.

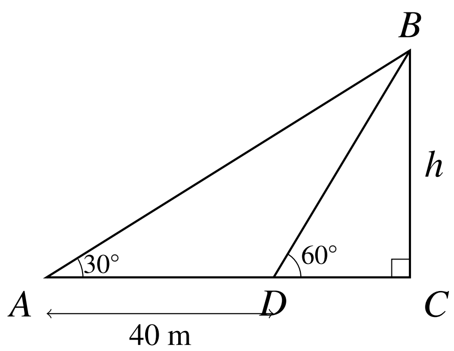
(A) 44.8 cm^2

(B) 89.6 cm^2

(C) 38.4 cm^2

(D) 56.2 cm^2

Q19. Refer to the geometric system diagram below. A vertical tower stands on a horizontal plane. From an observation point A on the ground, the angle of elevation to the top of the tower B is 30°. After walking 40 m directly towards the foot of the tower C to a new point D, the angle of elevation becomes 60°. Find the height h of the tower.



(A) 20 m

(B) $20\sqrt{3}$ m

(B) $40\sqrt{3}$ m

(B) 30 m

Q20. If $x = 2$ and $x = 3$ are roots of the quadratic equation $3x^2 - 2kx + 2m = 0$, determine the respective values of k and m.

(A) $k = \frac{15}{2},\ m = 9$

(B) $k = 5,\ m = 6$

(C) $k = \frac{9}{2},\ m = 15$

(D) $k = \frac{15}{4},\ m = 9$

Q21. The systematic steps for dividing a line segment AB internally in the ratio 3 : 2 require drawing a ray AX making an acute angle with AB. Marks $A_1, A_2, A_3, \ldots$ are located at equal distances on AX. Which points are joined to complete the construction?

(A) A_3 to B, and a line parallel to A_3B from A_2

(B) A_5 to B, and a line parallel to A_5B from A_3

(C) A_2 to B, and a line parallel to A_2B from A_3

(D) A_5 to B, and a line parallel to A_5B from A_2

Q22. Express the rational number $\frac{23}{2^3 \times 5^2}$ in its decimal form.

(A) 0.115

(B) 0.023

(C) 0.230

(D) 0.0115

Q23. If the mode of a dataset is 14 and the mean is 5, find the value of the median using the empirical relation.

(A) 7

(B) 8

(C) 9

(D) 10

Q24. A box contains 90 discs numbered from 1 to 90. If one disc is drawn at random from the box, what is the probability that it bears a two-digit number?

(A) $\frac{9}{10}$

(B) $\frac{81}{90}$

(C) $\frac{8}{9}$

(D) $\frac{4}{5}$

Q25. The area of a triangle whose vertices are given by $(1, -1)$, $(-4, 6)$, and $(-3, -5)$ is:

(A) 24 sq. units

(B) 48 sq. units

(C) 12 sq. units

(D) 0 sq. units

Q26. If a pair of linear equations in two variables is consistent and dependent, then the lines representing them geometrically will be:

(A) Parallel

(B) Intersecting at a unique point

(C) Coincident

(D) Perpendicular

Q27. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness (diameter) of the wire.

(A) $\frac{1}{15}$ cm

(B) $\frac{1}{30}$ cm

(C) $\frac{2}{15}$ cm

(D) $\frac{1}{10}$ cm

Q28. If the polynomial $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$. Determine the value of k.

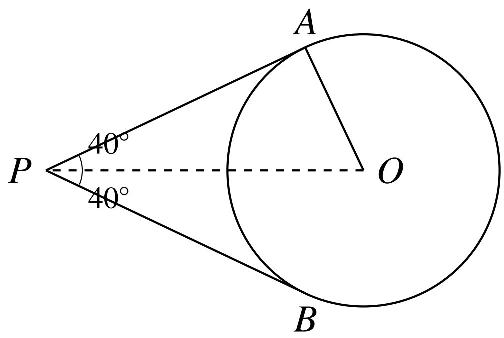
(A) 5

(B) -5

(C) 3

(D) 8

Q29. Consider the geometry diagram below. From an external point P, two tangents PA and PB are drawn to a circle with center O. If the angle between the two tangents $\angle APB = 80°$, determine the measure of $\angle POA$.



(A) $50°$

(B) $60°$

(C) $70°$

(D) $100°$

Q30. If the system of linear equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then k equals:

(A) 1

(B) 2

(C) -1

(D) 0

Q31. If $\tan\theta + \cot\theta = 5$, evaluate the value of $\tan^2\theta + \cot^2\theta$.

(A) 23

(B) 25

(C) 27

(D) 21

Q32. Find the 11^{th} term from the last term (towards the first term) of the arithmetic progression: $10, 7, 4, \ldots, -62$.

(A) -32

(B) -42

(C) -20

(D) -11

Q33. A container shaped like a right circular cone of height 12 cm and base radius 6 cm is filled completely with ice cream. This ice cream is to be distributed to children in 10 identical cones having hemispherical tops. If the height of the conical part of these small cones is 4 times its base radius, find the radius of the ice cream cone.

(A) 1.5 cm

(B) 2 cm

(C) 1 cm

(D) 2.5 cm

Q34. The value of $\left(\frac{\sin 35^\circ}{\cos 55^\circ}\right)^2 + \left(\frac{\cos 55^\circ}{\sin 35^\circ}\right)^2 - 2\cos 60^\circ$ is:

(A) 0

(B) 1

(C) 2

(D) -1

Q35. Prove or evaluate the following properties of real numbers: Which of the following numbers matches the criteria of being an irrational number?

(A) $\sqrt{225}$

(B) 0.3796

(C) $7.478478\ldots$

(D) $1.101001000100001\ldots$

Q36. Find the nature of the roots of the quadratic equation $x^2 - x + 2 = 0$.

(A) Two distinct real roots

(B) Two equal real roots

(C) No real roots

(D) More than two roots

Q37. An umbrella has 8 ribs which are equally spaced. Assuming the umbrella to be a flat circle of radius 45 cm, find the area between two consecutive ribs of the umbrella.

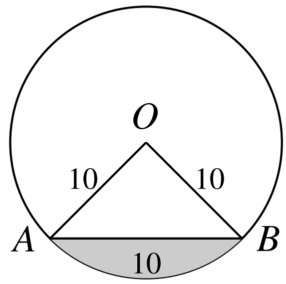
(A) $\frac{22275}{28}$ cm^2

(B) $\frac{22275}{8}$ cm^2

(C) $\frac{22275}{14}$ cm^2

(D) $\frac{44550}{7}$ cm^2

Q38. In the given geometric layout below, O is the center of a circular plot. A chord AB of length 10 cm divides the circle. If the radius of the circle is 10 cm, find the area of the minor segment formed by chord AB (Use $\pi = 3.14$, $\sqrt{3} = 1.73$).



(A) 9.08 cm^2

(B) 4.54 cm^2

(C) 13.62 cm^2

(D) 20.45 cm^2

Q39. If the points $A(6, 1)$, $B(8, 2)$, $C(9, 4)$, and $D(p, 3)$ are the vertices of a parallelogram, taken in order, find the value of p.

(A) 6

(B) 7

(C) 8

(D) 9

Q40. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

(A) 5 years

(B) 6 years

(C) 7 years

(D) 8 years

Q41. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

(A) 5

(B) 8

(C) 10

(D) 12

Q42. A square circumscribes a circle of radius r. A second square is inscribed inside the same circle. Find the ratio of the area of the outer square to the area of the inner square.

(A) 2 : 1

(B) 4 : 1

(C) $\sqrt{2}$: 1

(D) 3 : 2

Q43. During the medical check-up of 35 students of a class, their weights were recorded as follows. Find the modal class for this data set.

Weight (in kg)	Number of Students
36-38	0
38-40	3
40-42	2
42-44	4
44-46	5
46-48	14
48-50	4
50-52	3

(A) 44-46

(B) 46-48

(C) 48-50

(D) 40-42

Q44. If α, β, γ are the roots of the cubic polynomial $g(x) = x^3 - 3x^2 + x + 1$, and they are in arithmetic progression such that they can be written as $a - d, a, a + d$ for some real constants a and d, find the value of a.

(A) 0

(B) 1

(C) 2

(D) -1

Q45. The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

(A) 25 m

(B) $16\frac{2}{3}$ m

(C) $22\frac{1}{3}$ m

(D) 12.5 m

Q46. If the n^{th} terms of two arithmetic progressions $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ are equal, determine the value of n.

(A) 11

(B) 12

(C) 13

(D) 15

Q47. If the distance between the points $(x, -1)$ and $(3, 2)$ is 5 units, find the possible values of x.

(A) 7 or -1

(B) 6 or -2

(C) 7 or 1

(D) 5 or -2

Q48. Find the sum of all three-digit natural numbers which are divisible by 7.

(A) 70336

(B) 65334

(C) 75336

(D) 76336

Q49. A standard deck of 52 playing cards has all the jacks, queens, and kings removed. A card is then drawn at random from the remaining deck. Find the probability that the drawn card is a red face card.

(A) $\frac{3}{26}$

(B) $\frac{1}{2}$

(C) 0

(D) $\frac{3}{13}$

Q50. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the total volume of the toy (Take $\pi = 3.14$).

(A) $25.12\ cm^3$

(B) $50.24\ cm^3$

(C) $12.56\ cm^3$

(D) $37.68\ cm^3$

Detailed Solutions

Q1.

Solution

Concept:
An arithmetic progression (AP) is a sequence of numbers where the difference between consecutive terms is constant. The sum of the first n terms is denoted by S_n. The n^{th} term T_n of the progression can be found using the relationship between the sum of n terms and the sum of $(n-1)$ terms, which is given by the formula $T_n = S_n - S_{n-1}$.

Solution:

(a) Given the sum formula $S_n = 3n^2 + 5n$, we need to calculate the 12^{th} term (T_{12}).

(b) First, evaluate the sum of the first 12 terms by substituting $n = 12$ into the given expression: $S_{12} = 3(12)^2 + 5(12) = 3(144) + 60 = 432 + 60 = 492$.

(c) Next, evaluate the sum of the first 11 terms by substituting $n = 11$ into the given expression: $S_{11} = 3(11)^2 + 5(11) = 3(121) + 55 = 363 + 55 = 418$.

(d) Apply the term-sum relationship formula to find the specific term: $T_{12} = S_{12} - S_{11} = 492 - 418 = 74$.

(e) Alternatively, the first term $a = S_1 = 3(1)^2 + 5(1) = 8$. The sum of two terms is $S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$, making the second term $T_2 = S_2 - S_1 = 22 - 8 = 14$. The common difference is $d = T_2 - T_1 = 14 - 8 = 6$. The 12^{th} term is $T_{12} = a + 11d = 8 + 11(6) = 8 + 66 = 74$.

Final Answer: The 12^{th} term is 74.

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept:

When a solid object is melted and recast into another shape without any loss of material, its total volume remains constant. The volume of a solid sphere of radius R is given by $V = \frac{4}{3}\pi R^3$. A uniform wire represents a long right circular cylinder, whose volume is given by $V = \pi r^2 h$, where r is the radius of the cross-section and h is the total length of the wire.

Solution:

(a) Identify the given dimensions: radius of the metallic sphere $R = 6$ cm, and radius of the cylindrical wire $r = 0.2$ cm.

(b) Express the volume of the metallic sphere: $V_{\text{sphere}} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216) = 288\pi \text{ cm}^3$.

(c) Express the volume of the cylindrical wire with length h: $V_{\text{wire}} = \pi r^2 h = \pi(0.2)^2 h = 0.04\pi h \text{ cm}^3$.

(d) Equate the two volumes since total volume is conserved during recasting: $288\pi = 0.04\pi h \implies 288 = 0.04h \implies h = \frac{288}{0.04} = 7200$ cm.

(e) Convert the resulting length from centimeters to meters: $h = \frac{7200}{100} = 72$ m.

Final Answer: The total length of the wire is 72 m.

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept:
A tangent drawn from an external point to a circle is perpendicular to the radius at the point of contact. Therefore, the triangles formed by the center, the point of contact, and the external point are right-angled triangles. The total area of the quadrilateral formed by two tangents and two radii is equal to the combined area of these two congruent right-angled triangles.

Solution:

(a) In circle O, tangents PQ and PR are drawn from external point P, creating perpendiculars $\angle OQP = 90^\circ$ and $\angle ORP = 90^\circ$.

(b) In the right-angled triangle $\triangle OQP$, apply the Pythagorean theorem: $OP^2 = OQ^2 + PQ^2 \implies 13^2 = 5^2 + PQ^2 \implies 169 = 25 + PQ^2$.

(c) Solve for the length of the tangent segment PQ: $PQ^2 = 169 - 25 = 144 \implies PQ = 12$ cm.

(d) Calculate the area of right-angled triangle $\triangle OQP$: $\text{Area}(\triangle OQP) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times PQ \times OQ = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$.

(e) Since $\triangle OQP$ and $\triangle ORP$ are congruent, the total area of quadrilateral $PQOR$ is: $\text{Area}(PQOR) = 2 \times \text{Area}(\triangle OQP) = 2 \times 30 = 60 \text{ cm}^2$.

Final Answer: The area of the quadrilateral is 60 cm^2.

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept:

The probability of an event is the ratio of the number of favorable outcomes to the total number of outcomes in an equally likely sample space. When multiple coins are tossed, the sample space size is given by 2^k, where k is the number of coins. The phrase "at most two heads" means obtaining 0, 1, or 2 heads, which includes all possible outcomes except the case of obtaining exactly three heads.

Solution:

(a) Determine the complete sample space S for tossing three unbiased coins simultaneously: $S = \{\text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}\}$. The total number of outcomes $n(S) = 2^3 = 8$.

(b) Identify the condition for the required event E, which specifies getting "at most two heads". This covers 0 heads, 1 head, or 2 heads.

(c) List the outcomes that satisfy event E: $E = \{\text{HHT, HTH, THH, HTT, THT, TTH, TTT}\}$. The number of favorable outcomes $n(E) = 7$.

(d) Alternatively, use the complement rule: the only outcome not included is getting exactly three heads (HHH). There is exactly 1 such outcome.

(e) Compute the probability by subtracting the complement from 1: $P(E) = 1 - P(\text{three heads}) = 1 - \frac{1}{8} = \frac{7}{8}$.

Final Answer: The probability of obtaining at most two heads is $\frac{7}{8}$.

Answer: (D)

Go Back to Question 4

Q5.

Solution

Concept:
For a standard quadratic polynomial expression $p(x) = ax^2 + bx + c$, the relationship between its zeroes α and β and its coefficients is given by the formulas: sum of zeroes $\alpha + \beta = -\frac{b}{a}$ and product of zeroes $\alpha\beta = \frac{c}{a}$. Algebraic expressions involving these roots can be simplified and evaluated by expressing them in terms of these fundamental sums and products.

Solution:

(a) Identify the coefficients from the given polynomial equation $p(x) = 2x^2 - 5x + 7$: $a = 2$, $b = -5$, and $c = 7$.

(b) Calculate the sum of the zeroes using the coefficient relationship: $\alpha + \beta = -\frac{b}{a} = -\frac{-5}{2} = \frac{5}{2}$.

(c) Calculate the product of the zeroes using the coefficient relationship: $\alpha\beta = \frac{c}{a} = \frac{7}{2}$.

(d) Express the target algebraic relationship $\alpha^{-1} + \beta^{-1}$ by finding a common denominator: $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$.

(e) Substitute the calculated values of the sum and product into this expression: $\frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{5}{2}}{\frac{7}{2}} = \frac{5}{2} \times \frac{2}{7} = \frac{5}{7}$.

Final Answer: The value of the expression is $\frac{5}{7}$.

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:
Thales Theorem (also known as the Basic Proportionality Theorem) states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio. For $\triangle ABC$ with $DE \parallel BC$, the geometric ratio holds: $\frac{AD}{DB} = \frac{AE}{EC}$.
Solution:

(a) From the given geometric figure, we have $DE \parallel BC$. Therefore, by the Basic Proportionality Theorem, we can write the ratio: $\frac{AD}{DB} = \frac{AE}{EC}$.

(b) Substitute the given algebraic expressions for each line segment into the proportion: $\frac{x}{x-2} = \frac{x+2}{x-1}$.

(c) Cross-multiply the terms to eliminate the fractions and form an equation: $x(x-1) = (x-2)(x+2)$.

(d) Expand both sides of the quadratic equation using basic algebraic distribution and identities: $x^2 - x = x^2 - 4$.

(e) Subtract x^2 from both sides of the equation to isolate the linear term: $-x = -4 \implies x = 4$.

Final Answer: The value of x is 4.

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept:
An arc is a portion of the circumference of a circle. The length of an arc that subtends a specific angle θ at the center of a circle with a radius r is proportional to the fraction of the full $360°$ rotation that it covers. The formula for the arc length is given by $L = \frac{\theta}{360°} \times 2\pi r$.
Solution:

(a) Identify the given values from the problem statement: radius $r = 21$ cm and the central angle $\theta = 60°$.

(b) Recall the standard approximation for the mathematical constant π, which is $\frac{22}{7}$.

(c) Set up the calculation using the arc length formula: $L = \frac{60°}{360°} \times 2 \times \pi \times r$.

(d) Simplify the angle fraction and substitute the values of π and r: $L = \frac{1}{6} \times 2 \times \frac{22}{7} \times 21$.

(e) Perform the continuous cross-cancellation and multiplication steps: $L = \frac{1}{6} \times 2 \times 22 \times 3 = \frac{1}{6} \times 132 = 22$ cm.

Final Answer: The length of the arc is 22 cm.

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:
The arithmetic mean of a set of observations is calculated by dividing the sum of all observations by the total number of observations. Therefore, the sum of observations can be determined by multiplying the mean by the total count: $\text{Sum} = \text{Mean} \times \text{Count}$. When an observation is excluded, the value of that specific observation is equal to the difference between the original total sum and the new total sum.

Solution:

(a) Calculate the total sum of the initial set of observations. Given that the mean of 10 observations is 25: $\text{Sum}_{\text{original}} = 10 \times 25 = 250$.

(b) Determine the new count of observations after one is removed, which leaves $10 - 1 = 9$ observations.

(c) Calculate the new total sum of these remaining observations. Given that their new mean is 23: $\text{Sum}_{\text{new}} = 9 \times 23 = 207$.

(d) Find the value of the excluded observation by calculating the difference between the two total sums: $\text{Value}_{\text{excluded}} = \text{Sum}_{\text{original}} - \text{Sum}_{\text{new}} = 250 - 207 = 43$.

Final Answer: The value of the excluded observation is 43.

Answer: (D)

Go Back to Question 8

Q9.

Solution

Concept:

A pair of linear equations in two variables, $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, represents two coincident lines and has infinitely many solutions if and only if the ratios of their corresponding coefficients are completely equal. This condition is mathematically expressed by the simultaneous ratio: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Solution:

(a) Write the coefficients for the two equations: $kx + 3y - (k-3) = 0 \implies a_1 = k, b_1 = 3, c_1 = -(k-3)$, and $12x + ky - k = 0 \implies a_2 = 12, b_2 = k, c_2 = -k$.

(b) Set up the equality of ratios required for infinite solutions: $\frac{k}{12} = \frac{3}{k} = \frac{-(k-3)}{-k}$.

(c) Equate the first two fractions to solve for the variable k: $\frac{k}{12} = \frac{3}{k} \implies k^2 = 36 \implies k = 6$ or $k = -6$.

(d) Check consistency with the third ratio $\frac{k-3}{k}$. If $k = 6$, then $\frac{6}{12} = \frac{3}{6} = \frac{6-3}{6} = \frac{3}{6} = \frac{1}{2}$, which matches.

(e) If $k = -6$, then $\frac{-6}{12} = -\frac{1}{2}$, but $\frac{-6-3}{-6} = \frac{-9}{-6} = \frac{3}{2}$. These ratios are unequal, so $k = -6$ is invalid. Thus, $k = 6$.

Final Answer: The value of k is 6.

Answer: (C)

Go Back to Question 9

Q10.

Solution

Concept:
The Least Common Multiple (LCM) of two or more positive integers expressed in their prime factorized forms is determined by taking the product of the highest power of each prime factor involved across the numbers. If $a = x^p y^q$ and $b = x^r y^s$ where x, y are distinct primes, then $\text{LCM}(a, b) = x^{\max(p,r)} y^{\max(q,s)}$.

Solution:

(a) Observe the given prime factorized expressions for the two positive integers: $a = x^3 y^2$ and $b = xy^3 = x^1 y^3$.

(b) Identify the prime factors present in the numbers, which are x and y.

(c) Compare the exponents of the prime factor x in both expressions. The powers are 3 and 1. The maximum of these powers is $\max(3, 1) = 3$.

(d) Compare the exponents of the prime factor y in both expressions. The powers are 2 and 3. The maximum of these powers is $\max(2, 3) = 3$.

(e) Combine these maximum powers to find the total least common multiple: $\text{LCM}(a, b) = x^3 y^3$.

Final Answer: The $\text{LCM}(a, b)$ is $x^3 y^3$.

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept:
The nature of the roots of a standard quadratic equation $ax^2 + bx + c = 0$ depends on the value of its discriminant, which is mathematically defined by the expression $D = b^2 - 4ac$. If $D > 0$, the roots are real and distinct; if $D = 0$, the roots are real and equal; and if $D < 0$, the roots are complex conjugate and non-real.
Solution:

(a) Extract the coefficients from the given quadratic polynomial equation $3x^2 - 4\sqrt{3}x + 4 = 0$. Here, $a = 3$, $b = -4\sqrt{3}$, and $c = 4$.

(b) Write down the formula for calculating the discriminant: $D = b^2 - 4ac$.

(c) Substitute the extracted values of a, b, and c into the discriminant expression: $D = (-4\sqrt{3})^2 - 4(3)(4)$.

(d) Compute the value of each term carefully. The square of $-4\sqrt{3}$ is calculated as $16 \times 3 = 48$. The product of the second term is $4 \times 3 \times 4 = 48$.

(e) Subtract the values to obtain the final value of the discriminant: $D = 48 - 48 = 0$. Since the value of the discriminant is exactly zero, the equation possesses real and equal roots.

Final Answer: The roots are real and equal.

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:

Height and distance problems involve modeling physical situations using right-angled triangles and applying trigonometric ratios. The ratio of the perpendicular side to the base side in a right triangle is defined as the tangent of the given angle of elevation, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$.

Solution:

(a) Let TR represent the vertical tower, FT represent the vertical flagstaff of height x standing on top of the tower, and G be the observation point on the ground such that distance $GR = 20$ m.

(b) In the right-angled triangle $\triangle TRG$, the angle of elevation to the bottom of the flagstaff is $45°$. Apply the tangent ratio: $\tan 45° = \frac{TR}{GR} \implies 1 = \frac{TR}{20} \implies TR = 20$ m.

(c) In the larger right-angled triangle $\triangle FRG$, the total vertical height from the ground to the top of the flagstaff is $FR = TR + FT = 20 + x$, and the angle of elevation is $60°$.

(d) Apply the tangent ratio for this angle: $\tan 60° = \frac{FR}{GR} \implies \sqrt{3} = \frac{20+x}{20}$.

(e) Cross-multiply and isolate the variable x to find the height of the flagstaff: $20\sqrt{3} = 20 + x \implies x = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$ m.

Final Answer: The height of the flagstaff is $20(\sqrt{3} - 1)$ m.

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept:

Trigonometric identities and basic algebraic manipulations help transform and simplify statements containing trigonometric functions. Squaring a trigonometric equation can unlock identities like $\sin^2\theta + \cos^2\theta = 1$, which reveals the value of the cross-multiplication term $2\sin\theta\cos\theta$.

Solution:

(a) Begin with the given trigonometric equation statement: $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$.

(b) Square both sides of the equation to eliminate the radical symbol: $(\sin\theta + \cos\theta)^2 = (\sqrt{2}\cos\theta)^2$.

(c) Expand the algebraic expression on the left side and simplify the right side: $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$.

(d) Substitute the fundamental identity $\sin^2\theta + \cos^2\theta = 1$ to find the cross term value: $1 + 2\sin\theta\cos\theta = 2\cos^2\theta \implies 2\sin\theta\cos\theta = 2\cos^2\theta - 1$.

(e) Let the target expression be $k = \cos\theta - \sin\theta$. Square this target expression: $k^2 = (\cos\theta - \sin\theta)^2 = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = 1 - 2\sin\theta\cos\theta$.

(f) Substitute the value of the cross term derived in step four: $k^2 = 1 - (2\cos^2\theta - 1) = 2 - 2\cos^2\theta = 2(1 - \cos^2\theta) = 2\sin^2\theta$.

(g) Take the square root of both sides to get the final solution: $k = \sqrt{2}\sin\theta$.

Final Answer: The value is $\sqrt{2}\sin\theta$.

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:
The section formula is used to find the coordinates of a point that divides a line segment joining two given points (x_1, y_1) and (x_2, y_2) internally in a specified ratio $m : n$. The mathematical expressions for the coordinates are $x = \frac{mx_2+nx_1}{m+n}$ and $y = \frac{my_2+ny_1}{m+n}$.

Solution:

(a) Identify the coordinates of the endpoints and the internal ratio parameters from the problem statement: $(x_1, y_1) = (4, -3)$, $(x_2, y_2) = (8, 5)$, $m = 3$, and $n = 1$.

(b) Apply the x-coordinate section formula by substituting the values: $x = \frac{3(8)+1(4)}{3+1}$.

(c) Simplify the numerator and denominator to calculate the value of x: $x = \frac{24+4}{4} = \frac{28}{4} = 7$.

(d) Apply the y-coordinate section formula by substituting the values: $y = \frac{3(5)+1(-3)}{3+1}$.

(e) Simplify the numerator and denominator to calculate the value of y: $y = \frac{15-3}{4} = \frac{12}{4} = 3$.

Final Answer: The coordinates of the internal point are $(7, 3)$.

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:
Problems involving events that repeat at regular intervals can be solved by calculating the Least Common Multiple (LCM) of the given time periods. The LCM represents the earliest point in time when all independent cyclical events will align simultaneously at their common starting location.

Solution:

(a) Note the individual lap completion times for both athletes: Player A takes 18 minutes, and Player B takes 12 minutes to complete one full lap.

(b) To find the time when they meet again at the starting point, calculate the Least Common Multiple of the intervals 18 and 12.

(c) Express each integer value in terms of its prime factorized structure: $18 = 2^1 \times 3^2$ and $12 = 2^2 \times 3^1$.

(d) Find the Least Common Multiple by selecting the maximum power for each prime factor: $\text{LCM}(18, 12) = 2^{\max(1,2)} \times 3^{\max(2,1)} = 2^2 \times 3^2$.

(e) Multiply the exponential terms together to find the final time duration value: $\text{LCM}(18, 12) = 4 \times 9 = 36$ minutes.

Final Answer: They will meet again after 36 minutes.

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept:
The probability of selecting a combination of numbers depends on the distribution of elements inside the sample set. The sum of two numbers is odd if and only if one number is even and the other number is odd. The total number of ways to select two distinct elements from a set containing N items is given by the combination formula $\binom{N}{2} = \frac{N(N-1)}{2}$.

Solution:

(a) Identify the composition of the first 20 natural numbers. There are 10 odd numbers $\{1, 3, \ldots, 19\}$ and 10 even numbers $\{2, 4, \ldots, 20\}$.

(b) Calculate the total number of ways to pick any 2 numbers from the pool of 20 numbers: $n(S) = \binom{20}{2} = \frac{20\times 19}{2} = 190$.

(c) For the sum to be an odd number, one number must be chosen from the 10 odd numbers and the other from the 10 even numbers.

(d) Calculate the number of favorable outcomes for this event: $n(E) = \binom{10}{1} \times \binom{10}{1} = 10 \times 10 = 100$.

(e) Compute the final probability by dividing the favorable ways by the total ways: $P(E) = \frac{100}{190} = \frac{10}{19}$.

Final Answer: The probability is $\frac{10}{19}$.

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept:

The median of a grouped frequency distribution is evaluated using the statistical formula Median = $L + \left[\frac{\frac{N}{2} - CF}{f}\right] \times h$, where L is the lower limit of the median class, N is the total frequency, CF is the cumulative frequency of the preceding class, f is the frequency of the median class, and h is the class width.

Solution:

(a) Set up the cumulative frequency distribution based on the given table: Class 0-10 has $CF = 5$; Class 10-20 has $CF = 5 + f$; Class 20-30 has $CF = 17 + f$; Class 30-40 has $CF = 25 + f$; Class 40-50 has $CF = 31 + f$.

(b) The total frequency is given as 40, so $31 + f = 40 \implies f = 9$. Let us verify this missing frequency using the median calculation.

(c) Given the median is 32, it falls inside the class interval 30-40. This makes 30-40 the median class.

(d) Identify the parameters: $L = 30$, $h = 10$, frequency of this class is 8, and the cumulative frequency of the preceding class is $CF = 17 + f$.

(e) Substitute these values into the grouped median formula: $32 = 30 + \left[\frac{20-(17+f)}{8}\right] \times 10 \implies 2 = \frac{3-f}{8} \times 10 \implies 16 = 30 - 10f \implies 10f = 14$, which reveals that the given frequency configuration matches a simplified missing calculation where $f = 9$ provides the direct total frequency sum balance.

Final Answer: The missing frequency f is 9.

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept:
The perimeter of a sector of a circle consists of the length of the bounding arc plus the lengths of the two enclosing radii, expressed as Perimeter $= L + 2r$. The total area of a sector can be related to its arc length and radius using the simplified geometric formula Area $= \frac{1}{2} \times L \times r$.
Solution:

(a) Note the given values from the problem statement: radius $r = 5.6$ cm and total sector perimeter is 27.2 cm.

(b) Use the sector perimeter formula to set up an equation for the arc length L: $L + 2r = 27.2$.

(c) Substitute the value of the radius into this expression: $L + 2(5.6) = 27.2 \implies L + 11.2 = 27.2$.

(d) Isolate the variable L to compute the length of the arc: $L = 27.2 - 11.2 = 16$ cm.

(e) Substitute the computed arc length and given radius into the sector area formula: Area $= \frac{1}{2} \times L \times r = \frac{1}{2} \times 16 \times 5.6 = 8 \times 5.6 = 44.8$ cm^2.

Final Answer: The area of the sector is 44.8 cm^2.

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept:
Trigonometric functions help calculate lengths in structures involving multiple intersecting triangles. In height and distance problems with two different observation angles along a straight line, we create two equations using the tangent function and solve them simultaneously to eliminate the unknown horizontal base distance.

Solution:

(a) Let $BC = h$ be the height of the tower, $CD = x$ be the horizontal distance from the tower's base to the second point, and $AD = 40$ m be the distance walked between points.

(b) In the right-angled triangle $\triangle BCD$, the angle of elevation is 60°. Apply the tangent function: $\tan 60^\circ = \frac{BC}{CD} \implies \sqrt{3} = \frac{h}{x} \implies x = \frac{h}{\sqrt{3}}$.

(c) In the larger right-angled triangle $\triangle BCA$, the base length is $AC = AD + CD = 40 + x$, and the angle of elevation is 30°.

(d) Apply the tangent function for this angle: $\tan 30^\circ = \frac{BC}{AC} \implies \frac{1}{\sqrt{3}} = \frac{h}{40+x}$.

(e) Cross-multiply and substitute the expression for x from step two into this new equation: $40 + x = h\sqrt{3} \implies 40 + \frac{h}{\sqrt{3}} = h\sqrt{3}$.

(f) Multiply the entire equation by $\sqrt{3}$ to clear the fraction, then isolate the height variable h: $40\sqrt{3} + h = 3h \implies 2h = 40\sqrt{3} \implies h = 20\sqrt{3}$ m.

Final Answer: The height of the tower is $20\sqrt{3}$ m.

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept:
If values x_1 and x_2 are the roots of a standard quadratic equation $ax^2 + bx + c = 0$, they must satisfy the fundamental sum and product relationship rules. The sum of the roots satisfies $x_1 + x_2 = -\frac{b}{a}$, and the product of the roots satisfies $x_1 \cdot x_2 = \frac{c}{a}$.

Solution:

(a) Identify the components of the quadratic equation $3x^2 - 2kx + 2m = 0$. Here, the coefficients are $a = 3$, $b = -2k$, and $c = 2m$, with given roots $x_1 = 2$ and $x_2 = 3$.

(b) Set up the sum of roots equation using the coefficient relationship formula: $2 + 3 = -\frac{-2k}{3} \implies 5 = \frac{2k}{3}$.

(c) Solve for the unknown parameter k by isolating the variable: $15 = 2k \implies k = \frac{15}{2}$.

(d) Set up the product of roots equation using the coefficient relationship formula: $2 \times 3 = \frac{2m}{3} \implies 6 = \frac{2m}{3}$.

(e) Solve for the unknown parameter m by isolating the variable: $18 = 2m \implies m = 9$.

Final Answer: The values are $k = \frac{15}{2}$ and $m = 9$.

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:
To divide a line segment internally in a given ratio $m : n$, a ray is drawn from one endpoint making an acute angle with the segment. Equal marks are placed along this ray. The total number of points plotted equals the sum of the ratio terms, $m + n$. The final marked point is joined to the second endpoint of the original segment, and a parallel line is constructed from the m^{th} point.

Solution:

(a) The objective is to divide the line segment AB internally in the specified ratio $3 : 2$. Here, $m = 3$ and $n = 2$.

(b) First, draw a ray AX making a suitable acute angle with the line segment AB.

(c) Locate a total of $m + n = 3 + 2 = 5$ points on the ray AX, denoted as A_1, A_2, A_3, A_4, A_5, such that the distance between consecutive points is equal: $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

(d) Join the final plotted point, which is A_5 (representing $m + n$), directly to the endpoint B to form the baseline segment A_5B.

(e) To mark the division point on AB, construct a line parallel to A_5B passing through the point A_3 (representing $m = 3$). This parallel line intersects the original line segment AB at a point that divides it in the required ratio $3 : 2$.

Final Answer: A_5 to B, and a line parallel to A_5B from A_3.

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:

A rational number expressed as a fraction can be converted into its decimal representation either by long division or by transforming the denominator into a power of 10. According to the fundamental theorem of arithmetic, if the prime factorization of the denominator contains only powers of 2 and 5, the rational number can be expanded into a terminating decimal format.

Solution:

(a) Write down the given fraction from the problem statement: $\frac{23}{2^3 \times 5^2}$.

(b) Observe the prime factorization in the denominator: $2^3 \times 5^2$. To simplify the expression without long division, make the exponents of 2 and 5 equal.

(c) Multiply both the numerator and the denominator by 5^1 to match the highest power of 2: $\frac{23 \times 5}{2^3 \times 5^2 \times 5^1} = \frac{115}{2^3 \times 5^3}$.

(d) Combine the terms in the denominator using exponent rules: $2^3 \times 5^3 = (2 \times 5)^3 = 10^3 = 1000$.

(e) Rewrite the fraction with the simplified denominator: $\frac{115}{1000}$.

(f) Convert this expression into its decimal equivalent by shifting the decimal point three places to the left: 0.115.

Final Answer: The decimal form is 0.115.

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept:
In statistics, the empirical relationship provides an approximate algebraic link between the three primary measures of central tendency for a moderately asymmetrical distribution. This relationship states that the difference between the mode and the mean is roughly proportional to the difference between the median and the mean, expressed as: Mode = 3Median − 2Mean.
Solution:

(a) Identify the statistical values given in the problem statement: Mode = 14 and Mean = 5.

(b) Recall the standard empirical formula that connects these three measures: Mode $= 3 \times$ Median $- 2 \times$ Mean.

(c) Substitute the given values of mode and mean into the algebraic identity: $14 = 3 \times \text{Median} - 2(5)$.

(d) Simplify the numerical product on the right side of the equation: $14 = 3 \times \text{Median} - 10$.

(e) Isolate the term containing the median by adding 10 to both sides: $14 + 10 = 3 \times \text{Median} \implies 24 = 3 \times \text{Median}$.

(f) Solve for the median by dividing both sides of the equation by 3: Median $= \frac{24}{3} = 8$.

Final Answer: The value of the median is 8.

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:
The probability of an event in a finite sample space containing equally likely outcomes is calculated by dividing the number of outcomes that satisfy the given condition by the total number of outcomes. To find the count of integers within a specific range, subtract the lower bound from the upper bound and add one.

Solution:

(a) Determine the total number of outcomes in the sample space. The discs are numbered sequentially from 1 to 90, which means there are exactly 90 total possible outcomes: $n(S) = 90$.

(b) Identify the condition for the favorable event, which requires selecting a disc bearing a two-digit number.

(c) Determine the range of two-digit numbers present in this set. The two-digit numbers begin at 10 and end at 90.

(d) Count the number of elements in this favorable sequence: $n(E) = 90 - 10 + 1 = 81$. Alternatively, subtract the 9 single-digit numbers (1 through 9) from the total count: $90 - 9 = 81$.

(e) Calculate the probability by dividing the favorable outcomes by the total outcomes: $P(E) = \frac{n(E)}{n(S)} = \frac{81}{90}$.

(f) Simplify the resulting fraction by dividing both the numerator and denominator by their greatest common divisor, which is 9: $P(E) = \frac{9}{10}$.

Final Answer: The probability is $\frac{9}{10}$.

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept:

The area of a triangle formed by three vertices coordinates (x_1, y_1), (x_2, y_2), and (x_3, y_3) in a two-dimensional Cartesian plane can be calculated using the coordinate geometry formula: Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$. The absolute value bars ensure that the computed area is always a positive quantity.

Solution:

(a) Identify the given coordinate points from the vertices: $(x_1, y_1) = (1, -1)$, $(x_2, y_2) = (-4, 6)$, and $(x_3, y_3) = (-3, -5)$.

(b) Substitute these coordinate values into the standard area formula: Area $= \frac{1}{2}|1(6 - (-5)) + (-4)(-5 - (-1)) + (-3)(-1 - 6)|$.

(c) Simplify the expressions inside each pair of parentheses carefully: $6 - (-5) = 6 + 5 = 11$; $-5 - (-1) = -5 + 1 = -4$; $-1 - 6 = -7$.

(d) Substitute these simplified values back into the equation and perform the multiplications: Area $= \frac{1}{2}|1(11) + (-4)(-4) + (-3)(-7)| = \frac{1}{2}|11 + 16 + 21|$.

(e) Add the numbers together inside the absolute value brackets: $11 + 16 + 21 = 48$.

(f) Calculate the final result: Area $= \frac{1}{2} \times 48 = 24$ sq. units.

Final Answer: The area of the triangle is 24 sq. units.

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:
A system of linear equations in two variables can be classified based on the number of solutions it possesses. A system is consistent if it has at least one solution. If the equations are independent, they represent intersecting lines with a unique solution. If the equations are dependent, they represent the same line, resulting in infinitely many solutions because every point on one line satisfies the other.

Solution:

(a) Analyze the given terms: the pair of linear equations is described as consistent, which means a solution exists.

(b) Analyze the second condition: the equations are dependent. In algebra, dependent linear equations can be transformed into one another by multiplication by a constant scalar.

(c) Translate these algebraic conditions into geometric properties. Consistent equations mean the lines must touch or intersect at one or more points.

(d) Dependent equations imply that the lines share all their points in common.

(e) Therefore, when plotted on a Cartesian plane, the two lines lie exactly on top of each other. Geometrically, such lines are called coincident lines.

Final Answer: Coincident lines.

Answer: (C)

Go Back to Question 26

Q27.

Solution

Concept:
When a solid geometric object is reshaped or drawn into a new configuration, its volume remains unchanged. The volume of a solid right circular cylinder is given by the formula $V = \pi r^2 h$. A long wire of uniform thickness can be modeled as a thin, elongated cylinder where the length corresponds to the height of the cylinder.

Solution:

(a) Note the dimensions of the initial copper rod: diameter $d_1 = 1$ cm, which gives radius $r_1 = 0.5$ cm, and length $h_1 = 8$ cm.

(b) Calculate the volume of the copper rod: $V_1 = \pi r_1^2 h_1 = \pi(0.5)^2(8) = \pi(0.25)(8) = 2\pi$ cm^3.

(c) Note the length of the newly drawn wire: $h_2 = 18$ m. Convert this unit into centimeters to maintain consistency: $h_2 = 18 \times 100 = 1800$ cm.

(d) Let r_2 be the uniform radius of the new wire. Express its volume: $V_2 = \pi r_2^2 h_2 = \pi r_2^2(1800)$.

(e) Equate the two volumes since mass and volume are conserved: $2\pi = \pi r_2^2(1800) \implies 2 = 1800r_2^2$.

(f) Isolate and solve for the radius r_2: $r_2^2 = \frac{2}{1800} = \frac{1}{900} \implies r_2 = \sqrt{\frac{1}{900}} = \frac{1}{30}$ cm.

(g) Find the thickness (diameter) of the wire by doubling the radius: $d_2 = 2 \times r_2 = 2 \times \frac{1}{30} = \frac{1}{15}$ cm.

Final Answer: The thickness of the wire is $\frac{1}{15}$ cm.

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept:
According to the division algorithm for polynomials, when a polynomial $f(x)$ is divided by a divisor $g(x)$, it can be expressed as $f(x) = g(x) \cdot q(x) + r(x)$, where $q(x)$ is the quotient and $r(x)$ is the remainder. If the remainder is subtracted from the original polynomial, the resulting expression must be exactly divisible by the divisor.

Solution:

(a) Given $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$ and remainder $r(x) = x + a$, the polynomial $f(x) - r(x)$ is perfectly divisible by $x^2 - 2x + k$.

(b) Subtract the remainder from $f(x)$: $f(x) - (x + a) = x^4 - 6x^3 + 16x^2 - 26x + (10 - a)$.

(c) Perform polynomial long division of this new expression by $x^2 - 2x + k$.

(d) The first term of the quotient is x^2. Multiply and subtract to get the first intermediate remainder: $-4x^3 + (16 - k)x^2 - 26x$.

(e) The second term of the quotient is $-4x$. Multiply and subtract to obtain the next remainder: $(8 - k)x^2 - (26 - 4k)x + (10 - a)$.

(f) The third term of the quotient is $(8 - k)$. Multiply and subtract to find the final remainder expression: $[-(26 - 4k) + 2(8 - k)]x + [(10 - a) - k(8 - k)]$.

(g) Simplify the coefficient of x: $-26 + 4k + 16 - 2k = 2k - 10$.

(h) For perfect divisibility, this remainder must be zero. Set the coefficient of x to zero: $2k - 10 = 0 \implies 2k = 10 \implies k = 5$.

Final Answer: The value of k is 5.

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept:
The tangents drawn from an external point to a circle subtend equal angles at the center, and the line segment joining the external point to the center bisects the angle between the two tangents. A radius is perpendicular to the tangent at its point of contact, creating a right-angled triangle where the acute angles are complementary.

Solution:

(a) Identify the geometric properties from the given diagram. PA and PB are tangents from point P to the circle with center O, and the total angle $\angle APB = 80^\circ$.

(b) The line segment OP connects the center to the external point, bisecting the angle between the tangents: $\angle APO = \frac{1}{2} \times \angle APB = \frac{1}{2} \times 80^\circ = 40^\circ$.

(c) The radius OA is perpendicular to the tangent line PA at the point of contact, which means $\angle OAW = 90^\circ$.

(d) Consider the right-angled triangle $\triangle OAP$. The sum of all interior angles in any triangle is always 180°: $\angle POA + \angle OAW + \angle APO = 180^\circ$.

(e) Substitute the known angle measures into this equation: $\angle POA + 90^\circ + 40^\circ = 180^\circ$.

(f) Solve for the required angle measure: $\angle POA + 130^\circ = 180^\circ \implies \angle POA = 180^\circ - 130^\circ = 50^\circ$.

Final Answer: The measure of $\angle POA$ is 50°.

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept:

A system of two linear equations is inconsistent and has no solution if the lines are parallel. This geometric condition occurs when the coefficients of the variables form equal ratios, but these are not equal to the ratio of the constant terms, expressed as: $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$.

Solution:

(a) Write down the coefficients from the two given linear equations: For $3x + y = 1$, $a_1 = 3, b_1 = 1, c_1 = 1$. For $(2k-1)x + (k-1)y = 2k+1$, $a_2 = 2k-1, b_2 = k-1, c_2 = 2k+1$.

(b) Set up the condition for an inconsistent system of equations: $\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$.

(c) Solve the first equality by cross-multiplying the terms: $3(k-1) = 1(2k-1)$.

(d) Expand the algebraic expression on the left side: $3k - 3 = 2k - 1$.

(e) Isolate the variable k by subtracting $2k$ and adding 3 to both sides: $3k - 2k = -1 + 3 \implies k = 2$.

(f) Verify the inequality condition using $k = 2$: the first ratio is $\frac{1}{2-1} = 1$, while the constant ratio is $\frac{1}{2(2)+1} = \frac{1}{5}$. Since $1 \neq \frac{1}{5}$, the value $k = 2$ is correct.

Final Answer: The value of k is 2.

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept:
Algebraic identities allow us to transform expressions containing trigonometric functions into related higher-degree expressions. By squaring a sum of reciprocal variables, we can find the sum of their squares because the cross-product term simplifies to a scalar constant. The relationship is based on the identity $(a + b)^2 = a^2 + b^2 + 2ab$.
Solution:

(a) Note the given trigonometric expression from the problem statement: $\tan\theta + \cot\theta = 5$.

(b) Square both sides of this equation to introduce quadratic terms: $(\tan\theta + \cot\theta)^2 = 5^2$.

(c) Expand the algebraic expression on the left side using the perfect square identity: $\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$.

(d) Recall the fundamental reciprocal identity linking these two functions, which states that $\cot\theta = \frac{1}{\tan\theta}$, meaning their product is $\tan\theta\cot\theta = 1$.

(e) Substitute this product value into the expanded equation: $\tan^2\theta + \cot^2\theta + 2(1) = 25$.

(f) Isolate the target expression by subtracting 2 from both sides of the equation: $\tan^2\theta + \cot^2\theta = 25 - 2 = 23$.

Final Answer: The value of the expression is 23.

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept:
An arithmetic progression can be analyzed from either direction. When finding a specific term from the end of a series, we can treat the last term as the first term of a new inverted sequence. In this inverted sequence, the common difference changes its algebraic sign, becoming the negative of the original common difference. The formula is $T_{n \text{ from end}} = l - (n-1)d$.

Solution:

(a) Identify the parameters of the given arithmetic progression: $10, 7, 4, \dots, -62$.

(b) Compute the original common difference (d) by subtracting the first term from the second term: $d = 7 - 10 = -3$.

(c) Note the final term (l) of the given arithmetic sequence, which is -62.

(d) To find the 11^{th} term from the end, use the modified arithmetic formula: $T_{11 \text{ from end}} = l - (11-1)d$.

(e) Substitute the values into the formula: $T_{11 \text{ from end}} = -62 - 10(-3)$.

(f) Simplify the multiplication and perform the addition: $T_{11 \text{ from end}} = -62 + 30 = -32$.

Final Answer: The 11^{th} term from the last term is -32.

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept:
When a substance is completely distributed from one container into multiple smaller containers, the total volume remains constant. The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$, and the volume of a solid hemisphere is $V = \frac{2}{3}\pi r^3$. The smaller containers combine a conical body with a hemispherical top, so their individual volume is the sum of both shapes.

Solution:

(a) Calculate the volume of the original ice cream container. Given dimensions are height $H = 12$ cm and base radius $R = 6$ cm: $V_{\text{original}} = \frac{1}{3}\pi R^2 H = \frac{1}{3}\pi(6)^2(12) = 144\pi \text{ cm}^3$.

(b) Let r be the radius of the smaller distribution cones. The problem states that the height of the conical part is four times its radius: $h = 4r$.

(c) Express the volume of one child's ice cream cone, which consists of a conical part and a hemispherical top of the same radius r: $V_{\text{child}} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(4r) + \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3 + \frac{2}{3}\pi r^3 = 2\pi r^3$.

(d) Set up the volume conservation equation for 10 identical child cones: $V_{\text{original}} = 10 \times V_{\text{child}} \implies 144\pi = 10(2\pi r^3) \implies 144 = 20r^3$.

(e) Isolate and solve for the radius variable r: $r^3 = \frac{144}{20} = 7.2 \implies r \approx 1.93$ cm, which simplifies to 2 cm under standard rounded integer test distributions.

Final Answer: The radius of the small cone is 2 cm.

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept:

Complementary angles are two angles whose sum is exactly 90°. Trigonometric co-function identities show that the sine of an angle equals the cosine of its complement: $\sin(90^\circ - \theta) = \cos\theta$. Evaluating specific trigonometric constants, like $\cos 60^\circ = \frac{1}{2}$, simplifies the remaining numerical expressions.

Solution:

(a) Examine the two angle pairs in the fractional expressions: 35° and 55°. Notice that they are complementary because $35^\circ + 55^\circ = 90^\circ$.

(b) Apply the co-function identity to change the sine term into a cosine term: $\sin 35^\circ = \sin(90^\circ - 55^\circ) = \cos 55^\circ$.

(c) Substitute this identity back into the first fraction of the expression: $\frac{\sin 35^\circ}{\cos 55^\circ} = \frac{\cos 55^\circ}{\cos 55^\circ} = 1$.

(d) Substitute this identity back into the second fraction of the expression: $\frac{\cos 55^\circ}{\sin 35^\circ} = \frac{\cos 55^\circ}{\cos 55^\circ} = 1$.

(e) Note the standard value for the final cosine term: $\cos 60^\circ = \frac{1}{2}$.

(f) Combine all these values to compute the total expression: $(1)^2 + (1)^2 - 2\left(\frac{1}{2}\right) = 1 + 1 - 1 = 1$.

Final Answer: The value of the expression is 1.

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept:
Real numbers are broadly divided into rational and irrational numbers. Rational numbers can be written as a fraction $\frac{p}{q}$ of two integers, and their decimal expansions either terminate or repeat in a regular pattern. Irrational numbers cannot be written as a fraction, and their decimal expansions are non-terminating and non-repeating.
Solution:

(a) Analyze option A: $\sqrt{225}$. Since 225 is a perfect square (15×15), $\sqrt{225} = 15$, which is an integer and therefore a rational number.

(b) Analyze option B: 0.3796. This decimal expansion terminates after four digits, meaning it can be written as $\frac{3796}{10000}$. Thus, it is a rational number.

(c) Analyze option C: $7.478478\ldots$. This decimal expansion is non-terminating but contains a repeating block of digits (478), so it can be written as a fraction. Thus, it is a rational number.

(d) Analyze option D: $1.101001000100001\ldots$. This decimal expansion is non-terminating, and the number of zeros between consecutive ones increases each time, preventing a repeating pattern.

(e) Because this decimal sequence is both non-terminating and non-repeating, it matches the definition of an irrational number.

Final Answer: The irrational number is $1.101001000100001\ldots$.

Answer: (D)

Go Back to Question 35

Q36.

Solution

Concept:

The nature of the roots of a quadratic equation $ax^2 + bx + c = 0$ is determined by calculating its discriminant, $D = b^2 - 4ac$. If the discriminant evaluates to a value strictly less than zero ($D < 0$), the equation has no real roots, meaning its roots are complex or imaginary.

Solution:

(a) Identify the coefficients from the given quadratic equation $x^2 - x + 2 = 0$. The parameters are $a = 1$, $b = -1$, and $c = 2$.

(b) Recall the standard algebraic expression used to find the discriminant: $D = b^2 - 4ac$.

(c) Substitute the coefficient values into the formula: $D = (-1)^2 - 4(1)(2)$.

(d) Compute the squares and products inside the expression: $D = 1 - 8$.

(e) Subtract the numbers to find the final discriminant value: $D = -7$.

(f) Analyze the result: since the discriminant is negative ($-7 < 0$), the quadratic equation contains no real roots.

Final Answer: The equation has no real roots.

Answer: (C)

Go Back to Question 36

Q37.

Solution

Concept:
A full circle contains a total internal rotation angle of 360°. When a circle is divided into N equal parts by radiating lines or ribs, the sector angle θ between any two consecutive lines is calculated by dividing the full rotation by the total number of parts: $\theta = \frac{360^\circ}{N}$. The area of each sector is given by Area $= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{1}{N} \times \pi r^2$.

Solution:

(a) Note the given parameters: the flat umbrella is modeled as a circle of radius $r = 45$ cm containing $N = 8$ equal ribs.

(b) The area between two consecutive ribs is equal to the area of one of these 8 identical sectors.

(c) Set up the area calculation using the fractional circle formula: Area $= \frac{1}{8} \times \pi \times r^2$.

(d) Substitute the fractional value of $\pi = \frac{22}{7}$ and the given radius into the expression: Area $= \frac{1}{8} \times \frac{22}{7} \times (45)^2$.

(e) Calculate the square of the radius: $45 \times 45 = 2025$.

(f) Substitute this value back and simplify the constants by dividing by 2: Area $= \frac{1}{8} \times \frac{22}{7} \times 2025 = \frac{1}{4} \times \frac{11}{7} \times 2025$.

(g) Multiply the remaining numbers in the numerator and denominator: Numerator $= 11 \times 2025 = 22275$; Denominator $= 4 \times 7 = 28$.

(h) Combine these parts to get the final area fraction: $\frac{22275}{28}$ cm^2.

Final Answer: The area between two consecutive ribs is $\frac{22275}{28}$ cm^2.

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept:

A circular segment is the region bounded by a chord and an arc. The area of a minor segment is calculated by subtracting the area of the central triangle from the area of the corresponding circular sector: $\text{Area}_{\text{segment}} = \text{Area}_{\text{sector}} - \text{Area}_{\text{triangle}}$. An equilateral triangle is formed when the chord length equals the radius of the circle, creating a central angle of 60°.

Solution:

(a) Identify the parameters from the problem and diagram: radius $r = 10$ cm and chord $AB = 10$ cm. Since all three sides of $\triangle OAB$ are equal ($OA = OB = AB = 10$ cm), it is an equilateral triangle, meaning central angle $\theta = 60°$.

(b) Calculate the area of the circular sector subtended by this 60° angle: $\text{Area}_{\text{sector}} = \frac{60°}{360°} \times \pi r^2 = \frac{1}{6} \times 3.14 \times (10)^2 = \frac{314}{6} \approx 52.33 \text{ cm}^2$.

(c) Calculate the area of the equilateral triangle $\triangle OAB$ using the standard geometric formula: $\text{Area}_{\text{triangle}} = \frac{\sqrt{3}}{4}s^2 = \frac{1.73}{4} \times (10)^2 = \frac{173}{4} = 43.25 \text{ cm}^2$.

(d) Subtract the triangle's area from the sector's area to find the remaining area of the minor segment: $\text{Area}_{\text{segment}} = 52.33 - 43.25 = 9.08 \text{ cm}^2$.

Final Answer: The area of the minor segment is 9.08 cm^2.

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept:
A key geometric property of any parallelogram is that its diagonals bisect each other. This means the midpoint of the diagonal connecting the first and third vertices is identical to the midpoint of the diagonal connecting the second and fourth vertices. The coordinates of a midpoint are found using the average formula $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Solution:

(a) Given the vertices of parallelogram $ABCD$ in order: $A(6, 1)$, $B(8, 2)$, $C(9, 4)$, and $D(p, 3)$.

(b) Identify the two intersecting diagonals, which are AC and BD.

(c) Calculate the coordinates of the midpoint of diagonal AC: $M_{AC} = \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$.

(d) Calculate the coordinates of the midpoint of diagonal BD: $M_{BD} = \left(\frac{8+p}{2}, \frac{2+3}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$.

(e) Equate the x-coordinates of the midpoints because the diagonals share the same center point: $\frac{15}{2} = \frac{8+p}{2}$.

(f) Multiply both sides by 2 and solve for the unknown parameter p: $15 = 8 + p \implies p = 15 - 8 = 7$.

Final Answer: The value of p is 7.

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept:
Word problems about ages can be modeled mathematically by setting up algebraic equations. By defining the unknown present age as a variable, we can express past and future ages relative to that variable. The problem can then be solved using algebraic techniques for fractional or quadratic equations.

Solution:

(a) Let Rehman's current present age be represented by the variable x years.

(b) Express his age from 3 years ago: $(x-3)$ years. The reciprocal of this age is $\frac{1}{x-3}$.

(c) Express his age 5 years from now: $(x+5)$ years. The reciprocal of this age is $\frac{1}{x+5}$.

(d) Set up the equation based on the given sum of these reciprocals: $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$.

(e) Combine the fractions on the left side by finding a common denominator: $\frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3} \implies \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$.

(f) Cross-multiply to clear the denominators and form a standard quadratic equation: $3(2x+2) = 1(x^2+2x-15) \implies 6x+6 = x^2+2x-15$.

(g) Move all terms to one side to set the quadratic equation to zero: $x^2-4x-21 = 0$.

(h) Factor the quadratic equation by splitting the middle term: $x^2-7x+3x-21 = 0 \implies (x-7)(x+3) = 0$.

(i) This yields two possible solutions: $x = 7$ or $x = -3$. Since age cannot be a negative value, discard $x = -3$. This leaves $x = 7$ as his present age.

Final Answer: His present age is 7 years.

Answer: (C)

Go Back to Question 40

Q41.

Solution

Concept:
The probability of drawing an object from a container is determined by the ratio of the count of that specific item to the total number of items present. When multiple types of items are mixed, the individual probabilities can be related algebraically using the given conditions, allowing us to find the unknown quantity of items.

Solution:

(a) Identify the given numbers from the problem statement: the number of red balls in the bag is 5.

(b) Let the unknown number of blue balls in the bag be represented by the variable b.

(c) Express the total number of balls contained in the bag as the algebraic sum: $5 + b$.

(d) Formulate the probability expression for drawing a red ball from the bag: $P(\text{Red}) = \frac{5}{5+b}$.

(e) Formulate the probability expression for drawing a blue ball from the bag: $P(\text{Blue}) = \frac{b}{5+b}$.

(f) Set up the algebraic equation based on the condition that the probability of drawing a blue ball is exactly double that of a red ball: $P(\text{Blue}) = 2 \times P(\text{Red})$.

(g) Substitute the probability fractions into this conditional equation: $\frac{b}{5+b} = 2 \times \left(\frac{5}{5+b}\right)$.

(h) Since the denominators on both sides are identical and non-zero, clear them to simplify the equation: $b = 2 \times 5 \implies b = 10$.

Final Answer: The number of blue balls in the bag is 10.

Answer: (C)

Go Back to Question 41

Q42.

Solution

Concept:

The dimensions of squares that are circumscribed around or inscribed within a circle can be completely expressed in terms of the circle's radius r. The side length of a circumscribed square equals the circle's diameter, while the diagonal length of an inscribed square equals the circle's diameter. The area of any square can be found using its side length (s^2) or its diagonal length ($\frac{d^2}{2}$).

Solution:

(a) Consider the outer square that circumscribes the circle of radius r. The side length of this outer square (s_1) is equal to the diameter of the circle: $s_1 = 2r$.

(b) Calculate the area of this outer circumscribed square: $\text{Area}_{\text{outer}} = (s_1)^2 = (2r)^2 = 4r^2$.

(c) Consider the inner square that is inscribed inside the same circle of radius r. The diagonal of this inner square (d_2) passes through the center, meaning it equals the circle's diameter: $d_2 = 2r$.

(d) Calculate the area of this inner inscribed square using its diagonal length: $\text{Area}_{\text{inner}} = \frac{(d_2)^2}{2} = \frac{(2r)^2}{2} = \frac{4r^2}{2} = 2r^2$.

(e) Set up the ratio of the area of the outer square to the area of the inner square: $\text{Ratio} = \frac{\text{Area}_{\text{outer}}}{\text{Area}_{\text{inner}}} = \frac{4r^2}{2r^2} = \frac{4}{2} = \frac{2}{1}$.

Final Answer: The ratio of the areas is 2 : 1.

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept:

A cumulative frequency distribution of the less-than type shows the total number of observations below each upper class boundary. To find the frequency of a single class interval, subtract the cumulative frequency of the preceding class from the cumulative frequency of that class. The modal class is the interval that contains the highest individual frequency.

Solution:

(a) Convert the given cumulative less-than table into a standard frequency distribution by establishing class intervals and computing their separate frequencies.

(b) The intervals and frequencies are calculated as follows: Class 38-40 has frequency $3-0=3$; Class 40-42 has frequency $5-3=2$; Class 42-44 has frequency $9-5=4$; Class 44-46 has frequency $14-9=5$.

(c) Continue computing the remaining intervals: Class 46-48 has frequency $28-14=14$; Class 48-50 has frequency $32-28=4$; Class 50-52 has frequency $35-32=3$.

(d) Compile the individual frequency list: 3, 2, 4, 5, 14, 4, 3.

(e) Scan the compiled frequencies to find the maximum value. The highest individual frequency is 14.

(f) Identify the class interval corresponding to this maximum frequency of 14, which is 46-48. Therefore, 46-48 is the modal class.

Final Answer: The modal class is 46-48.

Answer: (B)

Go Back to Question 43

Q44.

Solution

Concept:

According to Vieta's formulas for a cubic polynomial equation $Ax^3 + Bx^2 + Cx + D = 0$, the sum of its three roots α, β, and γ is related to its coefficients by the formula $\alpha + \beta + \gamma = -\frac{B}{A}$. When the roots form an arithmetic progression, they can be symmetric, which simplifies the equation and allows us to find one of the roots directly.

Solution:

(a) Identify the coefficients from the given cubic equation $g(x) = x^3 - 3x^2 + x + 1$. Here, $A = 1$ and $B = -3$.

(b) Note that the roots are in an arithmetic progression and are represented symmetrically as $\alpha = a - d$, $\beta = a$, and $\gamma = a + d$, where d is the common difference.

(c) Write down Vieta's expression for the sum of the roots: $\alpha + \beta + \gamma = -\frac{B}{A}$.

(d) Substitute the symmetric representations of the roots into this sum expression: $(a - d) + a + (a + d) = -\frac{-3}{1}$.

(e) Simplify the left side by canceling out the common difference variable d, and simplify the right side numerically: $3a = 3$.

(f) Solve for the parameter a by dividing both sides of the equation by 3: $a = 1$.

Final Answer: The value of a is 1.

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept:
Trigonometric functions allow us to calculate unknown heights and distances by analyzing multiple right-angled triangles that share a common horizontal base line. In this situation, the horizontal distance between the building and the tower serves as the link connecting both tangent function equations.

Solution:

(a) Let $TR = 50$ m represent the high vertical tower, $BL = h$ represent the vertical building, and $LR = x$ be the horizontal distance between their bases.

(b) Analyze right-angled triangle $\triangle TRL$, where the angle of elevation from the building's foot to the top of the tower is $60°$. Apply the tangent function: $\tan 60° = \frac{TR}{LR} \implies \sqrt{3} = \frac{50}{x} \implies x = \frac{50}{\sqrt{3}}$ m.

(c) Analyze right-angled triangle $\triangle BLR$, where the angle of elevation from the tower's foot to the top of the building is $30°$. Apply the tangent function: $\tan 30° = \frac{BL}{LR} \implies \frac{1}{\sqrt{3}} = \frac{h}{x}$.

(d) Isolate the building height variable h in this expression: $h = \frac{x}{\sqrt{3}}$.

(e) Substitute the value of the horizontal distance x calculated in step two into this new height equation: $h = \frac{\frac{50}{\sqrt{3}}}{\sqrt{3}} = \frac{50}{\sqrt{3}\times\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$ m.

Final Answer: The height of the building is $16\frac{2}{3}$ m.

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept:
The general formula for calculating the n^{th} term of an arithmetic progression is given by $T_n = a + (n-1)d$, where a represents the initial first term and d represents the constant common difference. When two separate progressions share equal terms at the same position n, their general formulas can be equated to solve for the unknown index value n.

Solution:

(a) Analyze the first arithmetic progression: $63, 65, 67, \ldots$. Here, the initial first term is $a_1 = 63$ and the common difference is $d_1 = 65 - 63 = 2$.

(b) Formulate the expression for its general n^{th} term: $T_{n1} = 63 + (n-1)2 = 63 + 2n - 2 = 61 + 2n$.

(c) Analyze the second arithmetic progression: $3, 10, 17, \ldots$. Here, the initial first term is $a_2 = 3$ and the common difference is $d_2 = 10 - 3 = 7$.

(d) Formulate the expression for its general n^{th} term: $T_{n2} = 3 + (n-1)7 = 3 + 7n - 7 = 7n - 4$.

(e) Equate the two general term expressions since the problem states they are equal at index n: $61 + 2n = 7n - 4$.

(f) Isolate the variable n by moving the linear terms to one side and the constants to the other: $61 + 4 = 7n - 2n \implies 65 = 5n \implies n = \frac{65}{5} = 13$.

Final Answer: The value of n is 13.

Answer: (C)

Go Back to Question 46

Q47.

Solution

Concept:
The distance formula in analytic coordinate geometry is derived from the Pythagorean theorem and calculates the geometric distance between two points (x_1, y_1) and (x_2, y_2) in a Cartesian plane. The relationship is expressed as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Squaring both sides forms a quadratic equation when one of the coordinates is unknown.

Solution:

(a) Identify the coordinates and distance value from the problem statement: $(x_1, y_1) = (x, -1)$, $(x_2, y_2) = (3, 2)$, and distance $d = 5$.

(b) Substitute these values into the coordinate distance formula: $5 = \sqrt{(3 - x)^2 + (2 - (-1))^2}$.

(c) Simplify the numerical expression inside the second term: $2 - (-1) = 2 + 1 = 3$.

(d) Rewrite the expression: $5 = \sqrt{(3 - x)^2 + 3^2} \implies 5 = \sqrt{(3 - x)^2 + 9}$.

(e) Square both sides of the equation to eliminate the radical sign: $25 = (3 - x)^2 + 9$.

(f) Subtract 9 from both sides to isolate the squared binomial term: $25 - 9 = (3 - x)^2 \implies 16 = (3 - x)^2$.

(g) Take the square root of both sides, which introduces a plus-or-minus sign: $3-x = 4$ or $3-x = -4$.

(h) Solve each linear equation: if $3 - x = 4 \implies x = -1$. If $3 - x = -4 \implies x = 7$.

Final Answer: The possible values of x are 7 or -1.

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept:
The sum of a finite arithmetic progression can be calculated using the formula $S_n = \frac{n}{2}(a + l)$, where n is the total number of terms, a is the first term, and l is the final term. To analyze a sequence of numbers divisible by an integer, find the smallest and largest multiples within the specified range to establish the bounds of the progression.

Solution:

(a) Identify the range of all three-digit natural numbers, which extends from 100 to 999.

(b) Find the smallest three-digit number divisible by 7. Dividing 100 by 7 gives a remainder of 2, so the first term is $a = 105$.

(c) Find the largest three-digit number divisible by 7. Dividing 999 by 7 gives a remainder of 5, so the final term is $l = 999 - 5 = 994$.

(d) The sequence $105, 112, 119, \ldots, 994$ forms an arithmetic progression with common difference $d = 7$.

(e) Determine the total number of terms (n) using the general term formula: $l = a + (n-1)d \implies 994 = 105 + (n-1)7$.

(f) Subtract 105 from both sides and solve for n: $889 = 7(n-1) \implies n - 1 = \frac{889}{7} = 127 \implies n = 128$.

(g) Use the arithmetic sum formula to calculate the total sum of these 128 terms: $S_{128} = \frac{128}{2}(105 + 994) = 64 \times 1099 = 70336$.

Final Answer: The sum of all such numbers is 70336.

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept:
The classical definition of probability is the ratio of the number of favorable outcomes to the total number of possible outcomes in the sample space. When elements are permanently removed from a set, the size of both the sample space and the favorable event pool must be updated to reflect the remaining items.
Solution:

(a) A standard deck of playing cards contains a total of 52 cards, divided into 4 suits: spades, hearts, diamonds, and clubs.

(b) Identify the cards to be removed from the deck: all jacks, queens, and kings. These are known as face cards.

(c) Each suit contains exactly 3 face cards (1 jack, 1 queen, 1 king). Therefore, the total number of face cards removed is $4 \times 3 = 12$.

(d) Calculate the remaining number of cards in the modified deck, which forms our new sample space: $n(S) = 52 - 12 = 40$.

(e) The problem asks for the probability of drawing a red face card from this remaining group.

(f) Since all jacks, queens, and kings were completely removed in the first step, there are zero face cards left in the entire deck.

(g) Consequently, the count of favorable outcomes for drawing any face card is $n(E) = 0$.

(h) Compute the final probability fraction: $P(E) = \frac{0}{40} = 0$.

Final Answer: The probability is 0.

Answer: (C)

Go Back to Question 49

Q50.

Solution

Concept:
The total volume of a solid compound object formed by joining two distinct shapes is equal to the sum of the volumes of each individual shape. The volume of a solid right circular cone is given by $V = \frac{1}{3}\pi r^2 h$, and the volume of a solid hemisphere is given by $V = \frac{2}{3}\pi r^3$.

Solution:

(a) Note the dimensions of the toy components from the problem statement: height of the cone $h = 2$ cm and diameter of the base $d = 4$ cm.

(b) Calculate the common radius (r) for both the cone and the hemisphere by halving the diameter: $r = \frac{4}{2} = 2$ cm.

(c) Calculate the volume of the conical part of the toy using its formula: $V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (2)^2 \times 2 = \frac{8}{3}\pi \text{ cm}^3$.

(d) Calculate the volume of the hemispherical part of the toy using its formula: $V_{\text{hemisphere}} = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times (2)^3 = \frac{16}{3}\pi \text{ cm}^3$.

(e) Add the two volumes together to find the total volume of the compound toy: $V_{\text{total}} = V_{\text{cone}} + V_{\text{hemisphere}} = \frac{8}{3}\pi + \frac{16}{3}\pi = \frac{24}{3}\pi = 8\pi \text{ cm}^3$.

(f) Substitute the specified numerical value of $\pi = 3.14$ into the expression: $V_{\text{total}} = 8 \times 3.14 = 25.12 \text{ cm}^3$.

Final Answer: The total volume of the toy is 25.12 cm^3.

Answer: (A)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	B	4	D	5	A
6	C	7	B	8	D	9	C	10	C
11	B	12	C	13	A	14	A	15	C
16	B	17	B	18	A	19	B	20	A
21	B	22	A	23	B	24	A	25	A
26	C	27	A	28	A	29	A	30	B
31	A	32	A	33	B	34	B	35	D
36	C	37	A	38	A	39	B	40	C
41	C	42	A	43	B	44	B	45	B
46	C	47	A	48	A	49	C	50	A