JEECUP Group A Mathematics Sample Paper – 3

Duration: 60 Minutes **Maximum Marks: 200**

Instructions

- This paper contains **50** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**. No marks will be deducted for incorrect answers. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then the HCF(a, b) is:

(A) xy

(B) xy^2

(C) x^3y^3

(D) x^2y^2

Q2. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is:

(A) 10

(B) -10

(C) -7

(D) -2

Q3. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has:

(A) a unique solution

(B) exactly two solutions

(C) infinitely many solutions

(D) no solution

Q4. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then:

(A) $k < 4$

(B) $k > 4$

(C) $k \leq 4$

(D) $k \geq 4$

Q5. The 11^{th} term of the AP: $-3, -\frac{1}{2}, 2, \ldots$ is:

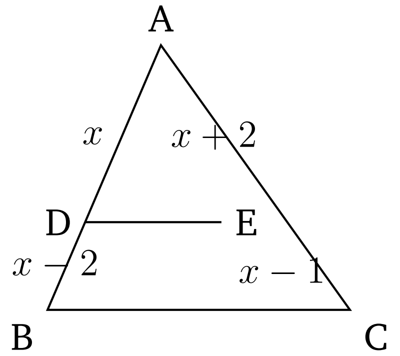
(A) 28

(B) 22

(C) -38

(D) $-46\frac{1}{2}$

Q6. In $\triangle ABC$, $DE \parallel BC$ such that $AD = x$, $DB = x - 2$, $AE = x + 2$, and $EC = x - 1$. Then the value of x is:



(A) 4

(B) 3

(C) 2

(D) 5

Q7. The distance of the point $P(-6, 8)$ from the origin is:

(A) 8

(B) $2\sqrt{7}$

(C) 10

(D) 6

Q8. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $\tan\theta$ is:

(A) $\sqrt{2} - 1$

(B) $\sqrt{2} + 1$

(C) $\frac{1}{\sqrt{2}}$

(D) $\sqrt{3}$

Q9. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is:

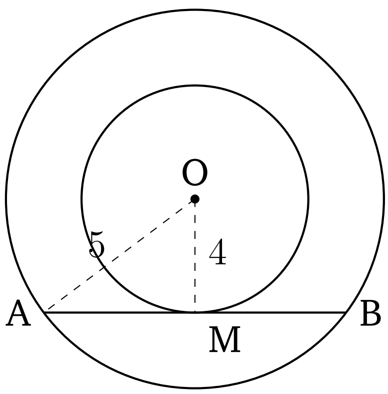
(A) $15\sqrt{3}$ m

(B) $\frac{15\sqrt{3}}{2}$ m

(C) $\frac{15}{2}$ m

(D) 15 m

Q10. If radii of two concentric circles are 4 cm and 5 cm, then the length of the chord of one circle which is tangent to the other circle is:



(A) 3 cm

(B) 6 cm

(C) 9 cm

(D) 1 cm

Q11. To divide a line segment AB in the ratio $5 : 7$, first a ray AX is drawn so that $\angle BAX$ is an acute angle and then points $A_1, A_2, A_3, \ldots$ are located at equal distances on the ray AX. The minimum number of these points is:

(A) 5

(B) 7

(C) 12

(D) 10

Q12. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is:

(A) 2 units

(B) π units

(C) 4 units

(D) 7 units

Q13. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. The height of the cylinder is:

(A) 2.74 cm

(B) 2.14 cm

(C) 1.37 cm

(D) 2.45 cm

Q14. For the following distribution, the modal class is:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	3	12	20	15	5

(A) $10 - 20$

(B) $20 - 30$

(C) $30 - 40$

(D) $40 - 50$

Q15. Which of the following cannot be the probability of an event?

(A) $\frac{2}{3}$

(B) -1.5

(C) 15%

(D) 0.7

Q16. The exponent of 2 in the prime factorization of 144 is:

(A) 4

(B) 5

(C) 6

(D) 3

Q17. If one of the zeroes of a cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is:

(A) $b - a + 1$

(B) $b - a - 1$

(C) $a - b + 1$

(D) $a - b - 1$

Q18. Aruna has only Rs. 1 and Rs. 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs. 75, then the number of Rs. 1 and Rs. 2 coins are, respectively:

(A) 35 and 15

(B) 35 and 20

(C) 15 and 35

(D) 25 and 25

Q19. The non-zero value of k for which the quadratic equation $kx^2+6x+k=0$ has equal roots is:

(A) 3

(B) ± 3

(C) -3

(D) ± 6

Q20. The sum of first 16 terms of the AP: $10, 6, 2, \dots$ is:

(A) -320

(B) 320

(C) -352

(D) -400

Q21. The areas of two similar triangles are 81 cm^2 and 49 cm^2 respectively. If the altitude of the bigger triangle is 4.5 cm, then the corresponding altitude of the smaller triangle is:

(A) 3.5 cm

(B) 2.7 cm

(C) 3.0 cm

(D) 2.5 cm

Q22. The area of a triangle with vertices $A(3,0)$, $B(7,0)$ and $C(8,4)$ is:

(A) 14

(B) 28

(C) 8

(D) 6

Q23. If $\sin A = \frac{1}{2}$, then the value of $\frac{3\cos A - 4\cos^3 A}{\sin A}$ is:

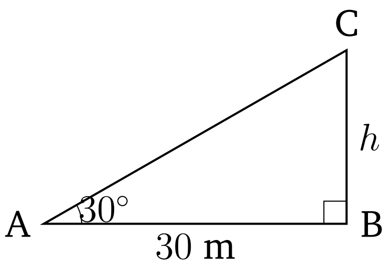
(A) 1

(B) 0

(C) -1

(D) $\frac{1}{2}$

Q24. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is $30°$. The height of the tower is:



(A) 10 m

(B) $10\sqrt{3}$ m

(C) $30\sqrt{3}$ m

(D) 20 m

Q25. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral $PQOR$ is:

(A) 60 cm^2

(B) 65 cm^2

(C) 30 cm^2

(D) 32.5 cm^2

Q26. If the area of a circle is 154 cm^2, then its perimeter is:

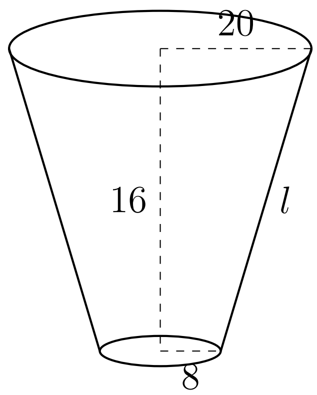
(A) 11 cm

(B) 22 cm

(C) 44 cm

(D) 55 cm

Q27. The radii of the top and bottom circles of a bucket are 20 cm and 8 cm respectively. If the depth of the bucket is 16 cm, its slant height is:



(A) 20 cm

(B) 12 cm

(C) 16 cm

(D) 18 cm

Q28. If the mean of the observations $x, x+3, x+5, x+7$, and $x+10$ is 9, then the mean of the last three observations is:

(A) $10\frac{1}{3}$

(B) $10\frac{2}{3}$

(C) $11\frac{1}{3}$

(D) $11\frac{2}{3}$

Q29. A card is drawn from a well shuffled deck of 52 playing cards. The probability of getting a black face card is:

(A) $\frac{3}{13}$

(B) $\frac{3}{26}$

(C) $\frac{1}{26}$

(D) $\frac{3}{52}$

Q30. Three bells toll at intervals of $9, 12, 15$ minutes respectively. If they start tolling together, after what time will they next toll together?

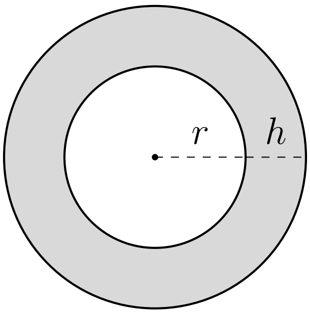
(A) 3 hours

(B) 6 hours

(C) 2 hours

(D) 1 hour

Q31. The area of the circular path of uniform width h surrounding a circular region of radius r is:



(A) $\pi h(2r + h)$

(B) $\pi h(2r - h)$

(C) $\pi h(r + h)$

(D) $\pi h(r - h)$

Q32. If a pair of linear equations is consistent, then the lines will be:

(A) parallel

(B) always coincident

(C) intersecting or coincident

(D) always intersecting

Q33. If the sum of the roots of the equation $kx^2 + 2x + 3k = 0$ is equal to their product, then k is equal to:

(A) $\frac{1}{3}$

(B) $-\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $-\frac{2}{3}$

Q34. The 10^{th} term from the end of the AP: $4, 9, 14, \ldots, 254$ is:

(A) 209

(B) 204

(C) 214

(D) 184

Q35. If the mid-point of the line segment joining the points $A(3, 4)$ and $B(k, 6)$ is $P(x, y)$ and $x + y - 10 = 0$, then the value of k is:

(A) 2

(B) 3

(C) 4

(D) 7

Q36. If $\tan\theta = \frac{4}{3}$, then $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} =$

(A) 7

(B) 1

(C) -7

(D) $\frac{1}{7}$

Q37. A shadow of a vertical tower on a level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. The height of the tower is:

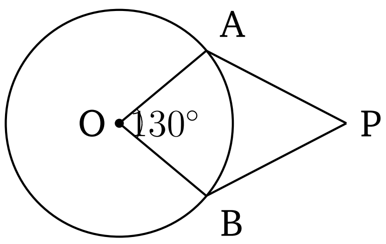
(A) $5(\sqrt{3}+1)$ m

(B) $5(\sqrt{3}-1)$ m

(C) $10(\sqrt{3}+1)$ m

(D) $10(\sqrt{3}-1)$ m

Q38. If the angle between two radii of a circle is $130°$, the angle between the tangents at the ends of the radii is:



(A) $90°$

(B) $50°$

(C) $70°$

(D) $40°$

Q39. If the area of a sector of a circle of radius 6 cm is 13.2 cm^2, then the length of the corresponding arc of the sector is:

(A) 2.2 cm

(B) 4.4 cm

(C) 6.6 cm

(D) 8.8 cm

Q40. A solid piece of iron in the form of a cuboid of dimensions 49 cm $\times$ 33 cm $\times$ 24 cm, is melted to form a solid sphere of radius r. The radius r is:

(A) 21 cm

(B) 28 cm

(C) 35 cm

(D) 14 cm

Q41. If the median of the data: $24, 25, 26, x+2, x+3, 30, 31, 34$ (arranged in ascending order) is 27.5, then the value of x is:

(A) 25

(B) 26

(C) 27

(D) 28

Q42. Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is a prime number?

(A) $\frac{5}{12}$

(B) $\frac{7}{12}$

(C) $\frac{13}{36}$

(D) $\frac{11}{36}$

Q43. The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is:

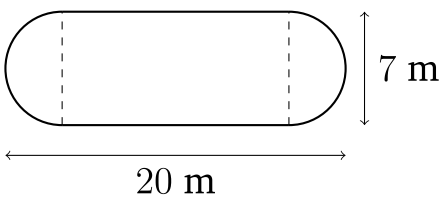
(A) 13

(B) 65

(C) 875

(D) 1750

Q44. The shape of a garden is rectangular in the middle and semi-circular at the ends as shown in the diagram. If the total length of the garden is 20 m and its width is 7 m, find the total area of the garden.



(A) 115.5 m^2

(B) 129.5 m^2

(C) 168.0 m^2

(D) 91.0 m^2

Q45. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to their product, then k is equal to:

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) $-\frac{1}{3}$

(D) $\frac{1}{3}$

Q46. A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being $240°$. If another circle of area equal to the area of this sector is formed, its radius must be:

(A) $\frac{100\sqrt{6}}{3}$ cm

(B) $\frac{100\sqrt{2}}{3}$ cm

(C) 80 cm

(D) 60 cm

Q47. If the volume of a right circular cone is 9856 cm^3 and the diameter of its base is 28 cm, the slant height of the cone is:

(A) 48 cm

(B) 50 cm

(C) 52 cm

(D) 25 cm

Q48. Relationship among mean, median and mode is:

(A) Mode $= 3$ Median $- 2$ Mean

(B) Mode $=2$ Median -3 Mean

(C) Median $=3$ Mode -2 Mean

(D) Mean $=3$ Median -2 Mode

Q49. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it bears a perfect square number is:

(A) $\frac{1}{10}$

(B) $\frac{9}{89}$

(C) $\frac{1}{9}$

(D) $\frac{1}{11}$

Q50. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(A) one decimal place

(B) two decimal places

(C) three decimal places

(D) four decimal places

Detailed Solutions

Q1.

Solution

Concept: The Highest Common Factor (HCF) of two or more algebraic expressions is obtained by taking the product of the lowest powers of all common prime factors present in the expressions.

Solution: Step 1: Write down the prime factorization of the given numbers a and b.
The first positive integer is expressed as:

$$a = x^3 \cdot y^2$$

The second positive integer is expressed as:

$$b = x^1 \cdot y^3$$

Here, it is given that both x and y are prime numbers.

Step 2: Identify the common prime bases in both expressions. The common prime bases are x and y.

Step 3: Determine the lowest power of each common prime factor.
For the prime factor x, the powers are 3 in a and 1 in b. The minimum value is $\min(3, 1) = 1$. Therefore, the lowest power of x is x^1.
For the prime factor y, the powers are 2 in a and 3 in b. The minimum value is $\min(2, 3) = 2$. Therefore, the lowest power of y is y^2.

Step 4: Multiply these lowest powers together to find the Highest Common Factor.

$$\text{HCF}(a, b) = x^1 \cdot y^2 = xy^2$$

Hence, the highest common factor of a and b is xy^2.

Final Answer: xy^2

Answer: (B) Go Back to Question 1

Q2.

Solution

Concept: If α is a zero of a polynomial $p(x)$, then $p(\alpha) = 0$. By substituting the given zero into the quadratic polynomial, we can form a linear equation in terms of the unknown variable k and solve for it.

Solution: Step 1: Let the given quadratic polynomial be denoted as $p(x) = x^2 + 3x + k$.

Step 2: It is given that one of the zeroes of this polynomial is 2. According to the definition of zeroes of a polynomial, substituting $x = 2$ into $p(x)$ must yield a value of zero.
Therefore, we set:

$$p(2) = 0$$

Step 3: Substitute $x = 2$ into the expression for $p(x)$:

$$(2)^2 + 3(2) + k = 0$$

Step 4: Simplify the numerical terms in the equation.
Evaluating the square of 2 gives 4, and multiplying 3 by 2 gives 6.

$$4 + 6 + k = 0$$

$$10 + k = 0$$

Step 5: Isolate the variable k by subtracting 10 from both sides of the equation.

$$k = -10$$

Thus, the value of the unknown constant k is -10.

Final Answer: $\boxed{-10}$

Answer: (B) Go Back to Question 2

Q3.

Solution

Concept: For a pair of linear equations in two variables $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we analyze the ratios of the coefficients. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel and have no solution.

Solution: Step 1: Identify and list the coefficients from the given pair of linear equations.
The first equation is $x + 2y + 5 = 0$. Here, we have:

$$a_1 = 1, \quad b_1 = 2, \quad c_1 = 5$$

The second equation is $-3x - 6y + 1 = 0$. Here, we have:

$$a_2 = -3, \quad b_2 = -6, \quad c_2 = 1$$

Step 2: Compute the ratio of the coefficients of x, which is $\frac{a_1}{a_2}$:

$$\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}$$

Step 3: Compute the ratio of the coefficients of y, which is $\frac{b_1}{b_2}$:

$$\frac{b_1}{b_2} = \frac{2}{-6} = -\frac{1}{3}$$

Step 4: Compute the ratio of the constant terms, which is $\frac{c_1}{c_2}$:

$$\frac{c_1}{c_2} = \frac{5}{1} = 5$$

Step 5: Compare the obtained ratios. We observe that:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \left(-\frac{1}{3} = -\frac{1}{3} \neq 5\right)$$

This mathematical condition confirms that the two lines represented by the equations are strictly parallel to each other. Consequently, they will never intersect, meaning there is no solution.

Final Answer: no solution

Answer: (D) Go Back to Question 3

Q4.

Solution

Concept: For a standard quadratic equation $ax^2 + bx + c = 0$, the nature of its roots is determined by the discriminant, denoted as $D = b^2 - 4ac$. For the roots to be real and distinct, the condition $D > 0$ must be strictly satisfied.

Solution: Step 1: Compare the given quadratic equation $x^2 + 4x + k = 0$ with the standard form $ax^2 + bx + c = 0$ to identify the coefficients.
Here, the coefficients are:

$$a = 1, \quad b = 4, \quad c = k$$

Step 2: Write down the mathematical formula for the discriminant D:

$$D = b^2 - 4ac$$

Step 3: Substitute the identified values of a, b, and c into the discriminant formula.

$$D = (4)^2 - 4(1)(k)$$

$$D = 16 - 4k$$

Step 4: Apply the given condition that the roots of the quadratic equation must be real and distinct. This requires that the discriminant must be strictly greater than zero.

$$D > 0 \implies 16 - 4k > 0$$

Step 5: Solve the linear inequality for the variable k.
Subtract 16 from both sides of the inequality:

$$-4k > -16$$

Divide both sides of the inequality by -4. Remember that dividing or multiplying an inequality by a negative number reverses the direction of the inequality sign.

$$k < \frac{-16}{-4} \implies k < 4$$

Thus, for the roots to be real and distinct, k must be strictly less than 4.

Final Answer: $\boxed{k < 4}$

Answer: (A) Go Back to Question 4

Q5.

Solution

Concept: The n^{th} term of an Arithmetic Progression (AP) is given by $a_n = a + (n-1)d$, where a is the first term, d is the common difference, and n is the term index.

Solution: Step 1: Identify the parameters from the sequence $-3, -\frac{1}{2}, 2, \ldots$ for $n = 11$.

$$a = -3, \quad d = \left(-\frac{1}{2}\right) - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}, \quad n = 11$$

Step 2: Substitute a, d, and n into the general n^{th} term formula.

$$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$$

Step 3: Simplify the expression to compute the final value.

$$a_{11} = -3 + 10\left(\frac{5}{2}\right) = -3 + 5(5) = -3 + 25 = 22$$

Thus, the 11^{th} term of the given AP is 22.

Final Answer: $\boxed{22}$

Answer: (B) Go Back to Question 5

Q6.

Solution

Concept: By the Basic Proportionality Theorem (Thales' Theorem), if a line $DE \parallel BC$ intersects the sides AB and AC of $\triangle ABC$, it divides them in the same ratio: $\frac{AD}{DB} = \frac{AE}{EC}$.

Solution: Step 1: State the BPT ratio equation and substitute the given side lengths.

$$\frac{AD}{DB} = \frac{AE}{EC} \implies \frac{x}{x-2} = \frac{x+2}{x-1}$$

Step 2: Cross-multiply the fractions to form an algebraic equation.

$$x(x-1) = (x+2)(x-2)$$

Step 3: Expand both sides using distributive properties and the difference of squares identity.

$$x^2 - x = x^2 - 4$$

Step 4: Cancel x^2 from both sides to find the value of x.

$$-x = -4 \implies x = 4$$

Since lengths are positive ($x - 2 = 2 > 0$ and $x - 1 = 3 > 0$), $x = 4$ is valid.

Final Answer: $\boxed{4}$

Answer: (A) Go Back to Question 6

Q7.

Solution

Concept: The distance of any point $P(x, y)$ from the origin $O(0, 0)$ in a Cartesian coordinate system is derived from the standard distance formula and is given by the expression $d = \sqrt{x^2 + y^2}$.

Solution: Step 1: Identify the coordinates of the given point P and the origin O.
The given point is $P(-6, 8)$, which implies:

$$x = -6, \quad y = 8$$

The origin coordinates are $O(0, 0)$.

Step 2: Write down the specialized distance formula from the origin.

$$\text{Distance } (OP) = \sqrt{x^2 + y^2}$$

Step 3: Substitute the values of x and y into the formula.

$$OP = \sqrt{(-6)^2 + (8)^2}$$

Step 4: Calculate the squares of the numbers inside the radical sign.
The square of -6 is 36, and the square of 8 is 64.

$$OP = \sqrt{36 + 64}$$

Step 5: Add the values together and compute the square root of the final sum.

$$OP = \sqrt{100} = 10 \text{ units}$$

Therefore, the straight-line distance of the point $P(-6, 8)$ from the origin is exactly 10 units.

Final Answer: $\boxed{10}$

Answer: (C) Go Back to Question 7

Q8.

Solution

Concept: The trigonometric functions can be manipulated using algebraic rules. Since $\tan\theta = \frac{\sin\theta}{\cos\theta}$, we can find the value of $\tan\theta$ by dividing the entire given equation by $\cos\theta$ and isolating the term containing $\tan\theta$.

Solution: Step 1: Write down the given trigonometric equation.

$$\sin\theta + \cos\theta = \sqrt{2}\cos\theta$$

Step 2: To convert the terms into $\tan\theta$, divide every term on both sides of the equation by $\cos\theta$ (assuming $\cos\theta \neq 0$).

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \frac{\sqrt{2}\cos\theta}{\cos\theta}$$

Step 3: Simplify each fraction using standard trigonometric definitions.
We know that $\frac{\sin\theta}{\cos\theta} = \tan\theta$ and $\frac{\cos\theta}{\cos\theta} = 1$.

$$\tan\theta + 1 = \sqrt{2}$$

Step 4: Isolate the term $\tan\theta$ by shifting 1 to the right side of the equation.

$$\tan\theta = \sqrt{2} - 1$$

Thus, the value of $\tan\theta$ is equal to $\sqrt{2} - 1$.

Final Answer: $\boxed{\sqrt{2} - 1}$

Answer: (A) Go Back to Question 8

Q9.

Solution

Concept: In a right-angled triangle, the cosine ratio links an angle to its adjacent side and hypotenuse: $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$. Here, the ladder forms the hypotenuse, and the wall forms the adjacent side to the angle with the vertical.

Solution: Step 1: Identify the given dimensions from the ladder problem.

$$\text{Hypotenuse (Ladder Length, } AC) = 15 \text{ m}, \quad \text{Angle with vertical } (\theta) = 60^\circ$$

Step 2: Relate the vertical wall height (AB) to the hypotenuse using the cosine ratio.

$$\cos 60^\circ = \frac{AB}{AC} \implies \frac{1}{2} = \frac{AB}{15}$$

Step 3: Solve for the height of the wall AB.

$$AB = \frac{15}{2} \text{ m}$$

Hence, the height of the wall is $\frac{15}{2}$ m.

Final Answer: $\boxed{\frac{15}{2} \text{ m}}$

Answer: (C) Go Back to Question 9

Q10.

Solution

Concept: A chord of an outer concentric circle that is tangent to the inner circle is bisected at the point of contact. The radius of the inner circle (r), radius of the outer circle (R), and half the chord length (MB) form a right-angled triangle satisfying $R^2 = r^2 + MB^2$.

Solution: Step 1: Identify the given radii parameters of the concentric circles.

$$\text{Inner radius } (r) = 4 \text{ cm}, \quad \text{Outer radius } (R) = 5 \text{ cm}$$

Step 2: Apply Pythagoras' Theorem to the right triangle $\triangle OMB$ to find half-chord length MB.

$$R^2 = r^2 + MB^2 \implies 5^2 = 4^2 + MB^2$$

$$25 = 16 + MB^2 \implies MB^2 = 9 \implies MB = 3 \text{ cm}$$

Step 3: Double the bisected segment length to determine the full length of the chord AB.

$$AB = 2 \cdot MB = 2 \cdot 3 = 6 \text{ cm}$$

Thus, the length of the chord is 6 cm.

Final Answer: 6 cm

Answer: (B) Go Back to Question 10

Q11.

Solution

Concept: To divide a line segment geometrically in a given ratio $m : n$ using an acute ray method, the total minimum number of equidistant points that need to be marked on the ray is equal to the sum of the terms of the ratio, which is $m + n$.

Solution: Step 1: Identify the given ratio in which the line segment AB needs to be divided. The specified ratio is:

$$m : n = 5 : 7$$

Step 2: Understand the geometric construction procedure. A ray AX is drawn making an acute angle with the original line segment AB.
To carry out the division according to the ratio $5 : 7$, we need to locate points at equal intervals along this ray so that the points can represent fractional parts of the whole.

Step 3: The standard mathematical rule for this type of construction requires marking a total number of points equal to the sum of the ratio components.

$$\text{Minimum number of points} = m + n$$

Step 4: Substitute the values $m = 5$ and $n = 7$ into the sum formula.

$$\text{Minimum number of points} = 5 + 7 = 12$$

Therefore, the minimum number of equidistant points to be marked on the ray AX is 12 (denoted from A_1 up to A_{12}).

Final Answer: $\boxed{12}$

Answer: (C) Go Back to Question 11

Q12.

Solution

Concept: The perimeter (circumference) of a circle is $2\pi r$ and its area is πr^2. Equating these two properties allows us to solve directly for the radius r.

Solution: Step 1: Set the perimeter formula equal to the area formula.

$$2\pi r = \pi r^2$$

Step 2: Divide both sides by πr (since radius $r \neq 0$ for a physical circle).

$$2 = r \implies r = 2 \text{ units}$$

Step 3: Confirm the solution. A radius of 2 units gives a perimeter of 4π and an area of 4π, matching the criteria perfectly.
Thus, the radius of the circle is 2 units.

Final Answer: $\boxed{2 \text{ units}}$

Answer: (A) Go Back to Question 12

Q13.

Solution

Concept: When a solid object is melted and completely recast into a different shape, its total volume remains constant. Therefore, the volume of the original solid sphere must be equated to the volume of the newly formed cylinder to determine the unknown height.

Solution: Step 1: Write down the formula for the volume of a sphere of radius r_1.

$$V_{\text{sphere}} = \frac{4}{3}\pi r_1^3$$

Given the radius of the metallic sphere is $r_1 = 4.2$ cm.

Step 2: Write down the formula for the volume of a cylinder of radius r_2 and height h.

$$V_{\text{cylinder}} = \pi r_2^2 h$$

Given the radius of the cylinder is $r_2 = 6$ cm.

Step 3: Equate the two volumes since the material is preserved during recasting.

$$\pi r_2^2 h = \frac{4}{3}\pi r_1^3 \implies r_2^2 h = \frac{4}{3} r_1^3$$

Step 4: Substitute the given numerical values into the equation.

$$(6)^2 \cdot h = \frac{4}{3} \cdot (4.2)^3$$

$$36 \cdot h = \frac{4}{3} \cdot 74.088 = 98.784$$

Step 5: Solve for the height h by dividing by 36.

$$h = \frac{98.784}{36} = 2.744 \text{ cm}$$

Rounding to two decimal places gives 2.74 cm.

Final Answer: $\boxed{2.74 \text{ cm}}$

Answer: (A) Go Back to Question 13

Q14.

Solution

Concept: In a grouped frequency distribution, the modal class is defined as the specific class interval that possesses the highest frequency among all the given classes.

Solution: Step 1: Analyze the given frequency distribution table carefully by mapping each class interval to its corresponding frequency value.

Marks	Number of students
$0 - 10$	3
$10 - 20$	12
$20 - 30$	20
$30 - 40$	15
$40 - 50$	5

Step 2: Scan the frequency column to identify the maximum frequency value.
The frequency values are $3, 12, 20, 15$, and 5. The largest numerical value among these is 20.

Step 3: Locate the specific class interval corresponding to this maximum frequency of 20.
Looking at the table, the class interval associated with the frequency 20 is $20 - 30$.

Step 4: Conclude based on the definition that this interval represents the modal class.
Therefore, the modal class for this marks distribution is $20 - 30$.

Final Answer: $\boxed{20 - 30}$

Answer: (B) Go Back to Question 14

Q15.

Solution

Concept: The probability of any event E, denoted as $P(E)$, is a measure constrained by axioms. It must satisfy the strict inequality condition $0 \leq P(E) \leq 1$. A probability can never be negative and can never exceed 1.

Solution: Step 1: Recall the fundamental bounding rule of probability. The probability of an event must always lie within the inclusive range of 0 to 1. Let us evaluate each option individually against this rule.

Step 2: Analyze Option (A): The value is $\frac{2}{3}$. Converting to a decimal gives approximately 0.667. Since $0 \leq 0.667 \leq 1$, this is a valid probability.

Step 3: Analyze Option (B): The value is -1.5. This value is strictly less than 0, representing a negative probability, which is mathematically impossible under standard probability axioms.

Step 4: Analyze Option (C): The value is 15%. Converting a percentage to a fraction or decimal gives $\frac{15}{100} = 0.15$. Since $0 \leq 0.15 \leq 1$, this is a valid probability.

Step 5: Analyze Option (D): The value is 0.7. Since $0 \leq 0.7 \leq 1$, this is also a valid probability.
Consequently, -1.5 is the only value that cannot represent the probability of an event.

Final Answer: $\boxed{-1.5}$

Answer: (B) Go Back to Question 15

Q16.

Solution

Concept: The prime factorization of a composite number involves breaking it down into a product of its prime factors. The exponent of a particular prime factor represents how many times that prime divides the number.

Solution: Step 1: Perform the systematic prime factorization of the given integer 144 by dividing it by the smallest prime numbers successively.
Since 144 is even, divide it by 2:

$$144 \div 2 = 72$$

Since 72 is even, divide it by 2 again:

$$72 \div 2 = 36$$

Since 36 is even, divide it by 2 again:

$$36 \div 2 = 18 \implies 18 \div 2 = 9$$

Step 2: Now, 9 is an odd number, so divide by the next prime number, which is 3:

$$9 \div 3 = 3 \implies 3 \div 3 = 1$$

Step 3: Collect all the prime factors obtained in the steps above to express the number in exponential notation.

$$144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^4 \cdot 3^2$$

Step 4: Identify the exponent of the prime base 2 from the exponential expression. The power attached to the base 2 is 4.
Thus, the exponent of 2 in the prime factorization of 144 is 4.

Final Answer: $\boxed{4}$

Answer: (A) Go Back to Question 16

Q17.

Solution

Concept:
For a cubic polynomial $p(x) = x^3 + ax^2 + bx + c$ with roots α, β, γ, the product of the roots satisfies $\alpha\beta\gamma = -c$. Additionally, since -1 is a root, substituting $x = -1$ gives $p(-1) = 0$, which allows us to find a relation for c.

Solution: Step 1: Let the three zeroes of the cubic polynomial $p(x) = x^3 + ax^2 + bx + c$ be denoted as α, β, and γ. Let one of the zeroes be given as $\alpha = -1$.

Step 2: Use the relationship between the coefficients and the product of all three zeroes of a cubic polynomial:

$$\alpha \cdot \beta \cdot \gamma = -c$$

Step 3: Substitute the value of $\alpha = -1$ into the product equation:

$$(-1) \cdot \beta \cdot \gamma = -c \implies \beta \cdot \gamma = c$$

This shows that the product of the other two zeroes is equal to the constant term c.

Step 4: Find an expression for c by utilizing the fact that -1 is a zero of the polynomial, meaning $p(-1) = 0$.

$$(-1)^3 + a(-1)^2 + b(-1) + c = 0 \implies -1 + a - b + c = 0$$

Step 5: Isolate c from this linear equation to express it in terms of a and b.

$$c = b - a + 1$$

Since $\beta\gamma = c$, the product of the other two zeroes is $b - a + 1$.

Final Answer: $\boxed{b - a + 1}$

Answer: (A) Go Back to Question 17

Q18.

Solution

Concept: This word problem can be modeled as a system of two linear equations in two variables. By defining variables for the counts of each coin type, we can create equations for the total count and the total monetary value, and solve them simultaneously.

Solution: Step 1: Define the variables for the unknown quantities.
Let the number of Rs. 1 coins be x.
Let the number of Rs. 2 coins be y.

Step 2: Frame the first linear equation based on the total number of coins with Aruna.

$$x + y = 50 \quad \text{— (Equation 1)}$$

Step 3: Frame the second linear equation based on the total monetary value of the coins.

$$1(x) + 2(y) = 75 \implies x + 2y = 75 \quad \text{— (Equation 2)}$$

Step 4: Solve the system of linear equations using the elimination method. Subtract Equation 1 from Equation 2 to eliminate x:

$$(x + 2y) - (x + y) = 75 - 50 \implies y = 25$$

Step 5: Substitute the value of $y = 25$ back into Equation 1 to find x.

$$x + 25 = 50 \implies x = 25$$

Thus, Aruna has 25 coins of Rs. 1 and 25 coins of Rs. 2.

Final Answer: 25 and 25

Answer: (D) Go Back to Question 18

Q19.

Solution

Concept: A quadratic equation $ax^2 + bx + c = 0$ possesses real and equal roots if and only if its discriminant is exactly equal to zero ($D = b^2 - 4ac = 0$).

Solution: Step 1: Identify the coefficients of the given quadratic equation $kx^2 + 6x + k = 0$. By comparison with the standard form, we have:

$$a = k, \quad b = 6, \quad c = k$$

Step 2: Set up the formula for the discriminant D and set it to zero to satisfy the equal roots condition.

$$D = b^2 - 4ac = 0 \implies (6)^2 - 4(k)(k) = 0$$

Step 3: Simplify the resulting algebraic equation.

$$36 - 4k^2 = 0$$

Step 4: Solve for the variable k^2 by isolating the term.

$$4k^2 = 36 \implies k^2 = 9$$

Step 5: Take the square root on both sides to find the values of k.

$$k = \pm\sqrt{9} = \pm 3$$

The problem specifies finding the non-zero value of k, which yields ± 3.

Final Answer: $\boxed{\pm 3}$

Answer: (B) Go Back to Question 19

Q20.

Solution

Concept: The sum of the first n terms of an Arithmetic Progression can be determined using the standard summation formula $S_n = \frac{n}{2}[2a + (n-1)d]$, where a is the first term and d is the common difference.

Solution: Step 1: Extract the relevant parameters from the given AP: $10, 6, 2, \ldots$
The first term $a = 10$.
The common difference $d = 6 - 10 = -4$.
The number of terms to sum is $n = 16$.

Step 2: Write out the arithmetic series summation formula.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Step 3: Substitute the parameters $n = 16$, $a = 10$, and $d = -4$ into the formula.

$$S_{16} = \frac{16}{2}[2(10) + (16-1)(-4)]$$

Step 4: Evaluate the operations inside the square brackets step-by-step.

$$S_{16} = 8[20 + (15)(-4)] = 8[20 - 60]$$

Step 5: Perform the final multiplication to get the total sum.

$$S_{16} = 8[-40] = -320$$

Thus, the sum of the first 16 terms of the given progression is -320.

Final Answer: $\boxed{-320}$

Answer: (A) Go Back to Question 20

Q21.

Solution

Concept: For two similar triangles, the ratio of their areas is equal to the square of the ratio of their corresponding altitudes. Mathematically, $\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{h_1}{h_2}\right)^2$.

Solution: Step 1: State the given components. Let the area of the bigger triangle be $\text{Area}_1 = 81$ cm^2 and its altitude be $h_1 = 4.5$ cm. Let the area of the smaller triangle be $\text{Area}_2 = 49$ cm^2 and its corresponding altitude be h_2.

Step 2: Set up the theorem relating the areas and altitudes of similar triangles.

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{h_1}{h_2}\right)^2$$

Step 3: Substitute the known values into the ratio equation.

$$\frac{81}{49} = \left(\frac{4.5}{h_2}\right)^2$$

Step 4: Take the positive square root on both sides to simplify the equation.

$$\frac{9}{7} = \frac{4.5}{h_2}$$

Step 5: Solve for the unknown altitude h_2 by cross-multiplication.

$$9 \cdot h_2 = 4.5 \cdot 7 \implies h_2 = \frac{31.5}{9} = 3.5 \text{ cm}$$

Hence, the corresponding altitude of the smaller triangle is 3.5 cm.

Final Answer: $\boxed{3.5 \text{ cm}}$

Answer: (A) Go Back to Question 21

Q22.

Solution

Concept: The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) can be calculated using the coordinate geometry formula: Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

Solution: Step 1: Identify the coordinates of the vertices from the given points $A(3, 0)$, $B(7, 0)$, and $C(8, 4)$.

$$x_1 = 3, y_1 = 0; \quad x_2 = 7, y_2 = 0; \quad x_3 = 8, y_3 = 4$$

Step 2: Write out the standard area formula for a triangle in a Cartesian plane.

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Step 3: Substitute the coordinates into the formula.

$$\text{Area} = \frac{1}{2}|3(0 - 4) + 7(4 - 0) + 8(0 - 0)|$$

Step 4: Simplify the terms inside the absolute value brackets sequentially.

$$\text{Area} = \frac{1}{2}|3(-4) + 7(4) + 8(0)| = \frac{1}{2}| - 12 + 28 + 0|$$

Step 5: Compute the final numerical value.

$$\text{Area} = \frac{1}{2}|16| = 8 \text{ sq. units}$$

Thus, the area of the triangle is 8.

Final Answer: $\boxed{8}$

Answer: (C) Go Back to Question 22

Q23.

Solution

Concept: This problem can be solved by using trigonometric identities or by finding $\cos A$ from $\sin A$ and substituting it into the expression. Alternatively, we can recognize the triple-angle identity $\cos 3A = 4\cos^3 A - 3\cos A$.

Solution: Step 1: Write down the expression to be evaluated.

$$E = \frac{3\cos A - 4\cos^3 A}{\sin A}$$

Step 2: Factor out a negative sign from the numerator to match a standard trigonometric identity.

$$E = \frac{-(4\cos^3 A - 3\cos A)}{\sin A} = \frac{-\cos 3A}{\sin A}$$

Step 3: It is given that $\sin A = \frac{1}{2}$. In the standard primary quadrant, this corresponds to an angle of $A = 30^\circ$.

Step 4: Substitute $A = 30^\circ$ into the simplified expression.

$$E = \frac{-\cos(3 \cdot 30^\circ)}{\sin 30^\circ} = \frac{-\cos 90^\circ}{\sin 30^\circ}$$

Step 5: Use standard exact values: $\cos 90^\circ = 0$ and $\sin 30^\circ = \frac{1}{2}$.

$$E = \frac{-0}{1/2} = 0$$

Thus, the total value of the given expression is 0.

Final Answer: $\boxed{0}$

Answer: (B) Go Back to Question 23

Q24.

Solution

Concept: In a right-angled triangle, the tangent of an angle is defined as the ratio of the length of the opposite side to the length of the adjacent side ($\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$).

Solution: Step 1: Let h be the height of the vertical tower, and let the distance from the foot of the tower to the observation point on the ground be 30 m. The angle of elevation to the top of the tower is given as $\theta = 30^\circ$.

Step 2: Set up the tangent ratio for the right-angled triangle formed by the tower and the ground line.

$$\tan\theta = \frac{\text{Height of tower}}{\text{Distance from foot}} \implies \tan 30^\circ = \frac{h}{30}$$

Step 3: Substitute the standard trigonometric value $\tan 30^\circ = \frac{1}{\sqrt{3}}$ into the equation.

$$\frac{1}{\sqrt{3}} = \frac{h}{30}$$

Step 4: Isolate the variable h representing the height.

$$h = \frac{30}{\sqrt{3}}$$

Step 5: Rationalize the denominator by multiplying the numerator and denominator by $\sqrt{3}$.

$$h = \frac{30 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}\text{ m}$$

Thus, the exact height of the tower is $10\sqrt{3}$ m.

Final Answer: $\boxed{10\sqrt{3}\text{ m}}$

Answer: (B) Go Back to Question 24

Q25.

Solution

Concept: The tangents drawn from an external point to a circle are perpendicular to the radii at the points of contact, forming two congruent right-angled triangles. The total area of the quadrilateral formed is twice the area of one right triangle.

Solution: Step 1: Understand the geometry of the system. Point P is an external point at a distance of $OP = 13$ cm from the center O. The radius of the circle is $OQ = OR = 5$ cm.
PQ and PR are tangents, so $\angle PQO = \angle PRO = 90°$.

Step 2: Apply Pythagoras' Theorem to the right triangle $\triangle PQO$ to find the length of the tangent segment PQ.

$$OP^2 = OQ^2 + PQ^2 \implies (13)^2 = (5)^2 + PQ^2$$

$$169 = 25 + PQ^2 \implies PQ^2 = 144 \implies PQ = 12 \text{ cm}$$

Step 3: Calculate the area of the right-angled triangle $\triangle PQO$.

$$\text{Area}(\triangle PQO) = \frac{1}{2} \cdot PQ \cdot OQ = \frac{1}{2} \cdot 12 \cdot 5 = 30 \text{ cm}^2$$

Step 4: The quadrilateral $PQOR$ is made up of two congruent right triangles, $\triangle PQO$ and $\triangle PRO$. Therefore, its total area is exactly twice the area of $\triangle PQO$.

$$\text{Area}(PQOR) = 2 \cdot \text{Area}(\triangle PQO) = 2 \cdot 30 = 60 \text{ cm}^2$$

Thus, the total area of the quadrilateral is 60 cm^2.

Final Answer: $\boxed{60 \text{ cm}^2}$

Answer: (A) Go Back to Question 25

Q26.

Solution

Concept: The area of a circle with radius r is given by $A = \pi r^2$, and its perimeter (circumference) is given by $C = 2\pi r$. We can find the radius from the given area and substitute it into the perimeter formula.

Solution: Step 1: State the given area of the circle.

$$A = 154 \text{ cm}^2$$

Step 2: Use the area formula to find the radius r. Let $\pi = \frac{22}{7}$.

$$\pi r^2 = 154 \implies \frac{22}{7} r^2 = 154$$

Step 3: Isolate r^2 and solve for r.

$$r^2 = 154 \cdot \frac{7}{22} = 7 \cdot 7 = 49$$

$$r = \sqrt{49} = 7 \text{ cm}$$

Step 4: Use the value of the radius to calculate the perimeter (circumference) C.

$$C = 2\pi r = 2 \cdot \frac{22}{7} \cdot 7$$

Step 5: Cancel the common factor of 7 to find the final value.

$$C = 2 \cdot 22 = 44 \text{ cm}$$

Thus, the perimeter of the circle is 44 cm.

Final Answer: $\boxed{44 \text{ cm}}$

Answer: (C) Go Back to Question 26

Q27.

Solution

Concept: A bucket forms a frustum of a cone. Its slant height l is calculated using the right-angled triangle relation: $l = \sqrt{h^2 + (R - r)^2}$, where h is the depth, R is the upper radius, and r is the lower radius.

Solution: Step 1: Extract the given structural dimensions from the diagram.

$$R = 20 \text{ cm}, \quad r = 8 \text{ cm}, \quad h = 16 \text{ cm}$$

Step 2: Calculate the difference between the upper and lower radii.

$$R - r = 20 - 8 = 12 \text{ cm}$$

Step 3: Substitute h and $(R - r)$ into the slant height formula and simplify.

$$l = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20 \text{ cm}$$

Hence, the slant height of the bucket is 20 cm.

Final Answer: $\boxed{20 \text{ cm}}$

Answer: (A) Go Back to Question 27

Q28.

Solution

Concept: The mean of a dataset is computed by dividing the sum of all individual observations by the total count of observations. We can determine the value of x from the first condition and use it to find the mean of the specified subsets.

Solution: Step 1: Count the given observations: x, $x+3$, $x+5$, $x+7$, and $x+10$. The total count is $n = 5$.

Step 2: Set up the equation for the mean of these 5 observations, which is given as 9.

$$\frac{x + (x+3) + (x+5) + (x+7) + (x+10)}{5} = 9$$

Step 3: Simplify the numerator and solve for x.

$$\frac{5x+25}{5} = 9 \implies x+5 = 9 \implies x = 4$$

Step 4: Identify the last three observations from the original sequence. These are:

$$x+5, \quad x+7, \quad \text{and} \quad x+10$$

Substitute $x = 4$ into these terms to find their numerical values:

$$4+5 = 9, \quad 4+7 = 11, \quad 4+10 = 14$$

Step 5: Calculate the arithmetic mean of these three numerical values.

$$\text{Mean} = \frac{9+11+14}{3} = \frac{34}{3} = 11\frac{1}{3}$$

Thus, the mean of the last three observations is $11\frac{1}{3}$.

Final Answer: $\boxed{11\frac{1}{3}}$

Answer: (C) Go Back to Question 28

Q29.

Solution

Concept: The probability of drawing a specific type of card is the ratio of the number of favorable cards to the total number of cards in a deck (52). Face cards are Kings, Queens, and Jacks. There are 3 face cards per suit.

Solution: Step 1: Identify the size of the entire sample space. A standard deck contains:

$$\text{Total number of cards} = 52$$

Step 2: Determine the number of suits that match the color condition. The black suits are Spades and Clubs (2 suits total).

Step 3: Count the face cards present in each suit. Each suit contains 3 face cards (King, Queen, Jack).

Step 4: Calculate the total number of favorable outcomes (black face cards).

$$\text{Favorable cards} = 2 \text{ suits} \times 3 \text{ face cards/suit} = 6 \text{ black face cards}$$

Step 5: Compute the probability by dividing the favorable outcomes by the sample space size and simplifying.

$$P(\text{Black Face Card}) = \frac{6}{52} = \frac{3}{26}$$

Hence, the probability of getting a black face card is $\frac{3}{26}$.

Final Answer: $\boxed{\frac{3}{26}}$

Answer: (B) **Go Back to Question 29**

Q30.

Solution

Concept: To find the next time multiple events occurring at regular intervals happen simultaneously, we compute the Least Common Multiple (LCM) of their individual interval times.

Solution: Step 1: Write down the intervals given for the three bells.

$$\text{Intervals} = 9 \text{ minutes}, \quad 12 \text{ minutes}, \quad 15 \text{ minutes}$$

Step 2: Find the prime factorization of each individual number.

$$9 = 3^2$$

$$12 = 2^2 \cdot 3$$

$$15 = 3 \cdot 5$$

Step 3: Calculate the LCM by multiplying the highest powers of all prime factors present.

$$\text{LCM}(9, 12, 15) = 2^2 \cdot 3^2 \cdot 5 = 4 \cdot 9 \cdot 5$$

$$\text{LCM} = 36 \cdot 5 = 180 \text{ minutes}$$

Step 4: Convert the interval value from minutes into hours.

$$\text{Time in hours} = \frac{180}{60} = 3 \text{ hours}$$

Step 5: Conclude that the bells will toll together again after exactly 3 hours.

Final Answer: 3 hours

Answer: (A) Go Back to Question 30

Q31.

Solution

Concept: The area of a circular path of uniform width is the difference between the areas of the outer circle and the inner circle: Area $= \pi R^2 - \pi r^2$.

Solution: Step 1: Identify the radii dimensions from the provided diagram.

$$\text{Inner radius} = r, \quad \text{Outer radius } (R) = r + h$$

Step 2: Set up the area subtraction formula and substitute the expression for R.

$$\text{Area} = \pi R^2 - \pi r^2 = \pi\left[(r+h)^2 - r^2\right]$$

Step 3: Expand the squared term and simplify the algebraic expression.

$$\text{Area} = \pi\left[r^2 + 2rh + h^2 - r^2\right] = \pi(2rh + h^2)$$

Factor out the common width variable h:

$$\text{Area} = \pi h(2r + h)$$

Final Answer: $\boxed{\pi h(2r+h)}$

Answer: (A) Go Back to Question 31

Q32.

Solution

Concept: A system of linear equations is defined as consistent if it possesses at least one valid solution. This occurs when the lines either intersect at a unique point or lie completely on top of each other.

Solution: Step 1: Understand the definitions of consistency in coordinate geometry. A pair of lines is consistent if there is at least one set of coordinates (x, y) that satisfies both equations simultaneously.

Step 2: Analyze the geometric scenarios for intersecting lines. Intersecting lines meet at exactly one point, providing a unique solution. Therefore, intersecting lines are consistent.

Step 3: Analyze the geometric scenarios for coincident lines. Coincident lines lie directly on top of each other, meaning they intersect at infinitely many points, yielding infinite solutions. Thus, coincident lines are also consistent.

Step 4: Analyze the geometric scenario for parallel lines. Parallel lines never cross, meaning they have no solution, which defines an inconsistent system.

Step 5: Combine the consistent cases. A consistent pair of linear equations can represent lines that are either intersecting or coincident.

Final Answer: intersecting or coincident

Answer: (C) Go Back to Question 32

Q33.

Solution

Concept: For a standard quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is given by $-\frac{b}{a}$ and the product of the roots is given by $\frac{c}{a}$. Equating these two expressions allows us to find unknown constants.

Solution: Step 1: Compare the given equation $kx^2 + 2x + 3k = 0$ with the standard form to find the coefficients.

$$a = k, \quad b = 2, \quad c = 3k$$

Step 2: Express the sum of the roots using the coefficient ratios.

$$\text{Sum} = -\frac{b}{a} = -\frac{2}{k}$$

Step 3: Express the product of the roots using the coefficient ratios.

$$\text{Product} = \frac{c}{a} = \frac{3k}{k} = 3 \quad (\text{given } k \neq 0)$$

Step 4: Set the sum equal to the product as specified by the problem statement.

$$-\frac{2}{k} = 3$$

Step 5: Solve the equation for the variable k.

$$-2 = 3k \implies k = -\frac{2}{3}$$

Final Answer: $\boxed{-\frac{2}{3}}$

Answer: (D) Go Back to Question 33

Q34.

Solution

Concept: The n^{th} term from the end of an Arithmetic Progression with last term l and common difference d can be directly calculated using the specialized formula $a'_n = l - (n-1)d$.

Solution: Step 1: Identify the parameter values of the given AP: $4, 9, 14, \ldots, 254$.
The last term l is:

$$l = 254$$

Step 2: Find the common difference d by subtracting the first term from the second term.

$$d = 9 - 4 = 5$$

Step 3: State the specific term index required from the end. We need the 10^{th} term, so:

$$n = 10$$

Step 4: Substitute the identified parameters into the reverse term formula.

$$a'_{10} = l - (10-1)d$$

$$a'_{10} = 254 - 9(5)$$

Step 5: Complete the arithmetic operations to find the final value.

$$a'_{10} = 254 - 45 = 209$$

Thus, the 10^{th} term from the end of the sequence is 209.

Final Answer: $\boxed{209}$

Answer: (A) Go Back to Question 34

Q35.

Solution

Concept: The midpoint coordinates (x, y) of a segment connecting $A(x_1, y_1)$ and $B(x_2, y_2)$ are found via $x = \frac{x_1+x_2}{2}$ and $y = \frac{y_1+y_2}{2}$. These coordinates must satisfy the given linear relationship equation.

Solution: Step 1: Set up the midpoint expressions for points $A(3, 4)$ and $B(k, 6)$.

$$x = \frac{3+k}{2}$$

$$y = \frac{4+6}{2} = \frac{10}{2} = 5$$

Step 2: Write down the linear equation that the midpoint coordinates $P(x, y)$ must satisfy.

$$x + y - 10 = 0$$

Step 3: Substitute the expressions for x and y obtained in Step 1 into this linear equation.

$$\left(\frac{3+k}{2}\right) + 5 - 10 = 0$$

Step 4: Simplify the numerical terms in the linear statement.

$$\frac{3+k}{2} - 5 = 0 \implies \frac{3+k}{2} = 5$$

Step 5: Solve for the unknown constant k by cross-multiplying.

$$3 + k = 10 \implies k = 10 - 3 = 7$$

Hence, the value of k is 7.

Final Answer: $\boxed{7}$

Answer: (D) Go Back to Question 35

Q36.

Solution

Concept: An algebraic expression containing sine and cosine functions can be simplified by dividing both the numerator and the denominator by $\cos\theta$. This converts the expression entirely into terms of $\tan\theta$.

Solution: Step 1: Write down the trigonometric expression to be evaluated.

$$E = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

Step 2: Divide every single term in both the numerator and denominator by $\cos\theta$.

$$E = \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}$$

Step 3: Apply the basic trigonometric definition $\frac{\sin\theta}{\cos\theta} = \tan\theta$.

$$E = \frac{\tan\theta + 1}{\tan\theta - 1}$$

Step 4: Substitute the given value $\tan\theta = \frac{4}{3}$ into the restructured equation.

$$E = \frac{\frac{4}{3} + 1}{\frac{4}{3} - 1}$$

Step 5: Simplify the fractions in the numerator and denominator to get the final integer value.

$$E = \frac{\frac{7}{3}}{\frac{1}{3}} = \frac{7}{3} \cdot \frac{3}{1} = 7$$

Final Answer: $\boxed{7}$

Answer: (A) Go Back to Question 36

Q37.

Solution

Concept: This problem can be modeled using two right-angled triangles sharing a common vertical side (the height of the tower, h). Tangent ratios link the height to the shifting shadow lengths on the ground base.

Solution: Step 1: Let h be the height of the vertical tower. Let the initial shadow length at a 45° elevation angle be x. When the angle changes to 30°, the shadow increases by 10 m, making the new length $x + 10$.

Step 2: In the first right triangle corresponding to the 45° angle of elevation:

$$\tan 45^\circ = \frac{h}{x} \implies 1 = \frac{h}{x} \implies x = h$$

Step 3: In the second right triangle corresponding to the 30° angle of elevation:

$$\tan 30^\circ = \frac{h}{x + 10}$$

Step 4: Substitute $\tan 30^\circ = \frac{1}{\sqrt{3}}$ and replace x with h from Step 2.

$$\frac{1}{\sqrt{3}} = \frac{h}{h + 10} \implies h + 10 = h\sqrt{3}$$

Step 5: Isolate the variable h and rationalize the expression.

$$h\sqrt{3} - h = 10 \implies h(\sqrt{3} - 1) = 10$$

$$h = \frac{10}{\sqrt{3} - 1} = \frac{10(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{10(\sqrt{3} + 1)}{2} = 5(\sqrt{3} + 1) \text{ m}$$

Final Answer: $\boxed{5(\sqrt{3} + 1) \text{ m}}$

Answer: (A) Go Back to Question 37

Q38.

Solution

Concept: The radii drawn to the points of contact of two tangents from an external point are perpendicular to the tangents. Consequently, the angle between the two radii and the angle between the tangents are supplementary (their sum is $180°$).

Solution: Step 1: Identify the geometric shape formed by the center O, contact points A and B, and external point P. The shape $OAPB$ is a quadrilateral.

Step 2: Use the property that tangents are perpendicular to the radii at the points of contact.

$$\angle OAP = 90°, \quad \angle OBP = 90°$$

Step 3: State the sum of all interior angles of any quadrilateral, which is always $360°$.

$$\angle AOB + \angle OAP + \angle APB + \angle OBP = 360°$$

Step 4: Substitute the known angle values into the sum equation. The angle between the radii is given as $\angle AOB = 130°$.

$$130° + 90° + \angle APB + 90° = 360°$$

$$310° + \angle APB = 360°$$

Step 5: Solve for the angle between the tangents ($\angle APB$).

$$\angle APB = 360° - 310° = 50°$$

Final Answer: $\boxed{50°}$

Answer: (B) Go Back to Question 38

Q39.

Solution

Concept: The area of a sector of radius r can be related directly to its corresponding arc length l through the specialized formula $A = \frac{1}{2} \cdot l \cdot r$. This avoids the need to explicitly calculate the central angle first.

Solution: Step 1: Write down the given parameter metrics for the sector.

$$\text{Radius } (r) = 6 \text{ cm}$$

$$\text{Area } (A) = 13.2 \text{ cm}^2$$

Step 2: State the direct relationship equation connecting Area, Arc Length (l), and Radius.

$$A = \frac{1}{2} \cdot l \cdot r$$

Step 3: Substitute the known numerical values into the formula.

$$13.2 = \frac{1}{2} \cdot l \cdot 6$$

Step 4: Simplify the multiplication on the right side of the equation.

$$13.2 = 3l$$

Step 5: Isolate and solve for the arc length l by dividing by 3.

$$l = \frac{13.2}{3} = 4.4 \text{ cm}$$

Hence, the corresponding arc length of the sector is 4.4 cm.

Final Answer: $\boxed{4.4 \text{ cm}}$

Answer: (B) Go Back to Question 39

Q40.

Solution

Concept: When a solid object is melted and transformed into a new shape, the total volume remains conserved. We equate the volume of the original cuboid ($V = L \cdot W \cdot H$) to the volume of the new sphere ($V = \frac{4}{3}\pi r^3$) to find the unknown radius.

Solution: Step 1: Compute the volume of the solid iron cuboid using its dimensions.

$$V_{\text{cuboid}} = 49 \cdot 33 \cdot 24 \text{ cm}^3$$

Step 2: Write out the mathematical formula for the volume of a sphere.

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

Step 3: Equate the two volumes due to conservation of mass and material. Use $\pi = \frac{22}{7}$.

$$\frac{4}{3} \cdot \frac{22}{7} \cdot r^3 = 49 \cdot 33 \cdot 24$$

Step 4: Isolate r^3 by moving all scalar factor coefficients to the opposite side.

$$r^3 = \frac{49 \cdot 33 \cdot 24 \cdot 3 \cdot 7}{4 \cdot 22}$$

Simplify by dividing common factors ($33 \div 11 = 3$, $22 \div 11 = 2$, and $24 \div 8 = 3$):

$$r^3 = 49 \cdot 3 \cdot 3 \cdot 3 \cdot 7 = (7 \cdot 7 \cdot 7) \cdot (3 \cdot 3 \cdot 3)$$

$$r^3 = 7^3 \cdot 3^3 = (21)^3$$

Step 5: Take the cube root on both sides to find the radius r.

$$r = 21 \text{ cm}$$

Final Answer: $\boxed{21 \text{ cm}}$

Answer: (A) Go Back to Question 40

Q41.

Solution

Concept: For an even number of sorted observations N, the median is calculated as the average of the two middlemost terms, located at positions $\frac{N}{2}$ and $\frac{N}{2}+1$.

Solution: Step 1: Count the total number of entries in the pre-sorted dataset. The elements are $24, 25, 26, x+2, x+3, 30, 31, 34$.

$$\text{Total entries } (N) = 8$$

Step 2: Because 8 is an even number, locate the indices of the two central elements.

$$\text{First middle position} = \frac{8}{2} = 4^{\text{th}} \text{ element} \implies x+2$$

$$\text{Second middle position} = 4+1 = 5^{\text{th}} \text{ element} \implies x+3$$

Step 3: Express the median as the arithmetic mean of these two central elements.

$$\text{Median} = \frac{(x+2)+(x+3)}{2} = \frac{2x+5}{2}$$

Step 4: Set this expression equal to the given numerical median value of 27.5.

$$\frac{2x+5}{2} = 27.5$$

Step 5: Solve for the variable x.

$$2x+5 = 55 \implies 2x = 50 \implies x = 25$$

Final Answer: $\boxed{25}$

Answer: (A) Go Back to Question 41

Q42.

Solution

Concept: When rolling two standard fair dice, the total number of outcomes in the sample space is $6 \times 6 = 36$. We count the specific pair outcomes whose sum matches a prime number value $(2, 3, 5, 7, 11)$.

Solution: Step 1: Determine the total number of outcomes in the sample space.

$$\text{Total outcomes} = 36$$

Step 2: List the possible prime values for the sum of two dice, which can range from 2 to 12. The primes are $2, 3, 5, 7$, and 11. Step 3: Enumerate the favorable outcomes for each prime sum value:

$$\text{Sum} = 2\text{: } (1,1) \implies 1 \text{ outcome}$$

$$\text{Sum} = 3\text{: } (1,2), (2,1) \implies 2 \text{ outcomes}$$

$$\text{Sum} = 5\text{: } (1,4), (2,3), (3,2), (4,1) \implies 4 \text{ outcomes}$$

$$\text{Sum} = 7\text{: } (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \implies 6 \text{ outcomes}$$

$$\text{Sum} = 11\text{: } (5,6), (6,5) \implies 2 \text{ outcomes}$$

Step 4: Sum the number of favorable outcomes.

$$\text{Total favorable outcomes} = 1 + 2 + 4 + 6 + 2 = 15$$

Step 5: Divide by the sample space size and reduce the fraction to simplest form.

$$\text{Probability} = \frac{15}{36} = \frac{5}{12}$$

Final Answer: $\boxed{\frac{5}{12}}$

Answer: (A) Go Back to Question 42

Q43.

Solution

Concept: The largest number that divides given numbers leaving specific remainders is found by subtracting the respective remainders from the original numbers and computing the Highest Common Factor (HCF) of the resulting integers.

Solution: Step 1: Subtract the required remainder values from each corresponding starting number.

$$\text{First number: } 70 - 5 = 65$$

$$\text{Second number: } 125 - 8 = 117$$

Step 2: Find the prime factorization of the first adjusted integer, 65.

$$65 = 5 \cdot 13$$

Step 3: Find the prime factorization of the second adjusted integer, 117.

$$117 = 3 \cdot 39 = 3^2 \cdot 13$$

Step 4: Identify the highest common factor (HCF) by taking the lowest power of all common prime factors.
The only common prime factor present in both factorizations is 13.

$$\text{HCF}(65, 117) = 13$$

Step 5: Conclude that 13 is the largest integer that satisfies the given division criteria.

Final Answer: $\boxed{13}$

Answer: (A) Go Back to Question 43

Q44.

Solution

Concept: The total area of the garden is computed by adding the area of the central rectangular region to the areas of the two semi-circular ends. Combining two identical semi-circles forms one complete circle.

Solution: Step 1: Analyze the dimensions from the provided diagram. The total length of the garden is 20 m and the total width is 7 m.
The width of the garden corresponds to the diameter of the semi-circular ends, so:

$$\text{Diameter } (d) = 7 \text{ m} \implies \text{Radius } (r) = \frac{7}{2} = 3.5 \text{ m}$$

Step 2: Determine the straight length of the middle rectangular portion by subtracting the radius of both circular ends from the total length.

$$\text{Length of rectangle } (L) = 20 - (3.5 + 3.5) = 20 - 7 = 13 \text{ m}$$

Step 3: Calculate the area of the central rectangular portion.

$$\text{Area}_{\text{rectangle}} = \text{Length} \times \text{Width} = 13 \cdot 7 = 91 \text{ m}^2$$

Step 4: Calculate the combined area of the two semi-circular ends, which equals one full circle of radius $r = 3.5$ m.

$$\text{Area}_{\text{circles}} = \pi r^2 = \frac{22}{7} \cdot (3.5)^2 = \frac{22}{7} \cdot 12.25 = 38.5 \text{ m}^2$$

Step 5: Add the two component areas together to find the total area of the garden.

$$\text{Total Area} = \text{Area}_{\text{rectangle}} + \text{Area}_{\text{circles}} = 91 + 38.5 = 129.5 \text{ m}^2$$

Final Answer: $\boxed{129.5 \text{ m}^2}$

Answer: (B) Go Back to Question 44

Q45.

Solution

Concept: For any quadratic polynomial $ax^2 + bx + c$, the sum of its zeroes is equal to $-\frac{b}{a}$ and the product of its zeroes is equal to $\frac{c}{a}$. We equate these expressions to solve for the unknown constant k.

Solution: Step 1: Identify the coefficients of the polynomial $kx^2 + 2x + 3k$ by comparing it to the standard form.

$$a = k, \quad b = 2, \quad c = 3k$$

Step 2: Write down the expression for the sum of the zeroes.

$$\text{Sum} = -\frac{b}{a} = -\frac{2}{k}$$

Step 3: Write down the expression for the product of the zeroes.

$$\text{Product} = \frac{c}{a} = \frac{3k}{k} = 3 \quad (\text{assuming } k \neq 0)$$

Step 4: Set the sum equal to the product according to the given problem criteria.

$$-\frac{2}{k} = 3$$

Step 5: Isolate and solve for the variable k.

$$-2 = 3k \implies k = -\frac{2}{3}$$

Final Answer: $\boxed{-\frac{2}{3}}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: The area of a sector with radius R and central angle θ is given by $A = \frac{\theta}{360^\circ}\pi R^2$. We set this area equal to the area formula of a new circle (πr^2) to find its radius r.

Solution: Step 1: Write down the given parameters of the cut sector.

$$\text{Radius of sector } (R) = 100 \text{ cm}$$

$$\text{Sector angle } (\theta) = 240^\circ$$

Step 2: Calculate the area of this sector using the fractional circle area formula.

$$A_{\text{sector}} = \frac{240^\circ}{360^\circ} \cdot \pi \cdot (100)^2 = \frac{2}{3} \cdot \pi \cdot 10000 = \frac{20000\pi}{3}$$

Step 3: Set up the equation equating the area of the new circle of radius r to the calculated sector area.

$$\pi r^2 = A_{\text{sector}} \implies \pi r^2 = \frac{20000\pi}{3}$$

Step 4: Cancel the common factor of π on both sides and isolate r^2.

$$r^2 = \frac{20000}{3}$$

Step 5: Take the square root of both sides and simplify the radical expression.

$$r = \sqrt{\frac{20000}{3}} = \sqrt{\frac{10000 \cdot 2}{3}} = 100\sqrt{\frac{2}{3}}$$

Multiply the numerator and denominator inside the radical by 3 to rationalize:

$$r = 100\frac{\sqrt{6}}{\sqrt{3} \cdot \sqrt{3}} = \frac{100\sqrt{6}}{3} \text{ cm}$$

Final Answer: $\boxed{\frac{100\sqrt{6}}{3} \text{ cm}}$

Answer: (A) Go Back to Question 46

Q47.

Solution

Concept: The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. We can use the given volume and base radius to find the vertical height h, and then apply Pythagoras' Theorem ($l = \sqrt{r^2 + h^2}$) to determine the slant height l.

Solution: Step 1: Determine the base radius r from the given diameter of 28 cm.

$$r = \frac{28}{2} = 14 \text{ cm}$$

Step 2: Substitute the known values into the volume formula to solve for the vertical height h. Let $\pi = \frac{22}{7}$.

$$9856 = \frac{1}{3} \cdot \frac{22}{7} \cdot (14)^2 \cdot h$$

$$9856 = \frac{1}{3} \cdot \frac{22}{7} \cdot 196 \cdot h = \frac{1}{3} \cdot 22 \cdot 28 \cdot h$$

$$9856 = \frac{616}{3} \cdot h$$

Step 3: Isolate and solve for the vertical height variable h.

$$h = \frac{9856 \cdot 3}{616} = 16 \cdot 3 = 48 \text{ cm}$$

Step 4: Set up Pythagoras' Theorem to calculate the slant height l.

$$l = \sqrt{r^2 + h^2} = \sqrt{(14)^2 + (48)^2}$$

Step 5: Simplify the terms inside the square root and evaluate.

$$l = \sqrt{196 + 2304} = \sqrt{2500} = 50 \text{ cm}$$

Thus, the slant height of the cone is 50 cm.

Final Answer: $\boxed{50 \text{ cm}}$

Answer: (B) Go Back to Question 47

Q48.

Solution

Concept: In statistics, the empirical relationship between the three main measures of central tendency for a moderately asymmetrical distribution is defined by the fixed linear equation: Mode $=3$ Median -2 Mean.

Solution: Step 1: Recall the standard empirical formula derived by Karl Pearson that connects Mean, Median, and Mode.

Step 2: The established mathematical formula states that the mode value can be approximated by subtracting twice the mean value from three times the median value.

$$\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}$$

Step 3: Compare this formula directly with the given multiple-choice options.
Option (A) matches this exact mathematical relation: Mode $=3$ Median -2 Mean.

Step 4: Verify that the other options represent incorrect algebraic rearrangements of this relation. Thus, option (A) is the correct statement.

Final Answer: $\boxed{\text{Mode} = 3\ \text{Median} - 2\ \text{Mean}}$

Answer: (A) Go Back to Question 48

Q49.

Solution

Concept: The probability of selecting a perfect square number is found by determining the count of perfect squares that fall within the specified numerical inclusive range (1 to 90) and dividing it by the total number of discs.

Solution: Step 1: Identify the total number of possible outcomes in the sample space.

$$\text{Total number of discs} = 90$$

Step 2: List all the perfect square integers that exist within the range from 1 to 90.

$$1^2 = 1, \quad 2^2 = 4, \quad 3^2 = 9, \quad 4^2 = 16, \quad 5^2 = 25$$

$$6^2 = 36, \quad 7^2 = 49, \quad 8^2 = 64, \quad 9^2 = 81$$

Note that the next perfect square is $10^2 = 100$, which lies outside the given range.

Step 3: Count the total number of valid perfect squares listed in Step 2.

$$\text{Number of favorable outcomes} = 9$$

Step 4: Set up the probability fraction by dividing the favorable count by the total sample space size.

$$\text{Probability} = \frac{9}{90}$$

Step 5: Reduce the fraction to its simplest form.

$$\text{Probability} = \frac{1}{10}$$

Final Answer: $\boxed{\frac{1}{10}}$

Answer: (A) Go Back to Question 49

Q50.

Solution

Concept: The decimal expansion of a rational fraction $\frac{p}{q}$ terminates if the prime factorization of the denominator q contains only prime factors of 2 and/or 5 (expressed as $2^n \cdot 5^m$). The number of terminating decimal places is equal to $\max(n, m)$.

Solution: Step 1: Write down the given rational number fraction.

$$\frac{14587}{1250}$$

Step 2: Perform the prime factorization of the denominator, 1250.

$$1250 = 10 \cdot 125 = (2 \cdot 5) \cdot 5^3 = 2^1 \cdot 5^4$$

Step 3: Identify the exponents of the prime bases 2 and 5 in the factorization.

$$\text{Exponent of } 2 \implies n = 1$$

$$\text{Exponent of } 5 \implies m = 4$$

Step 4: Determine the maximum value between the two identified exponents.

$$\max(n, m) = \max(1, 4) = 4$$

Step 5: Conclude based on the terminating decimal theorem that the decimal expansion will terminate after exactly 4 decimal places.

Final Answer: four decimal places

Answer: (D) Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	D	4	A	5	B
6	A	7	C	8	A	9	C	10	B
11	C	12	A	13	A	14	B	15	B
16	A	17	A	18	D	19	B	20	A
21	A	22	C	23	B	24	B	25	A
26	C	27	A	28	C	29	B	30	A
31	A	32	C	33	D	34	A	35	D
36	A	37	A	38	B	39	B	40	A
41	A	42	A	43	A	44	B	45	A
46	A	47	B	48	A	49	A	50	D