JEECUP Group A Mathematics Sample Paper-6

Duration: 60 Minutes **Maximum Marks: 200**

Instructions

- This paper contains **50** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+4 marks**. No marks will be deducted for incorrect answers. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. If the least number which when divided by 18, 24 and 30 leaves remainder 7 in each case is N, then the value of $\frac{N-7}{6}$ is:

(A) 60

(B) 120

(C) 180

(D) 240

Q2. The product of two numbers is 8640 and their HCF is 12. If their LCM is 720, then one possible pair of numbers is:

(A) 96, 90

(B) 120, 72

(C) 144, 60

(D) 108, 80

Q3. If $\sqrt{7+4\sqrt{3}} = a + \sqrt{3}$, where a is a positive integer, then the value of a is:

(A) 1

(B) 2

(C) 3

(D) 4

Q4. The decimal expansion of the rational number $\frac{29}{2^3 \times 5^2}$ will terminate after:

(A) 2 decimal places

(B) 3 decimal places

(C) 4 decimal places

(D) 5 decimal places

Q5. If the HCF of two numbers is 16 and their product is 18432, then the least possible value of their LCM is:

(A) 576

(B) 720

(C) 864

(D) 1152

Q6. If α and β are the zeroes of the polynomial $x^2 - 9x + 14$, then the value of $\alpha^2 + \beta^2$ is:

(A) 25

(B) 49

(C) 53

(D) 81

Q7. The polynomial $x^3 - 6x^2 + 11x - 6$ has roots α, β, γ. The value of $\alpha\beta + \beta\gamma + \gamma\alpha$ is:

(A) 5

(B) 6

(C) 11

(D) 18

Q8. If one zero of the polynomial $2x^2 - kx + 3$ is reciprocal of the other, then the value of k is:

(A) ± 5

(B) ± 6

(C) ± 7

(D) ± 8

Q9. If $2x + 3y = 17$ and $3x - 2y = 6$, then the value of $x^2 + y^2$ is:

(A) 25

(B) 29

(C) 34

(D) 41

Q10. The pair of equations $(k - 1)x + 3y = 2$ and $4x + (k + 2)y = 5$ has infinitely many solutions if the value of k is:

(A) 2

(B) 4

(C) -1

(D) No such value

Q11. A number consists of two digits whose sum is 11. If the digits are interchanged, the number decreases by 27. The original number is:

(A) 74

(B) 83

(C) 92

(D) 65

Q12. If $\frac{3}{x} + \frac{2}{y} = 7$ and $\frac{2}{x} - \frac{1}{y} = 1$, then the value of $\frac{1}{x} + \frac{1}{y}$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q13. The roots of the quadratic equation $x^2 - 8x + k = 0$ differ by 2. Then the value of k is:

(A) 12

(B) 15

(C) 16

(D) 18

Q14. If one root of the equation $x^2 - 5x + p = 0$ is square of the other root, then the value of p is:

(A) 2

(B) 4

(C) 6

(D) 8

Q15. The equation $x^2 - (k + 3)x + 2k = 0$ has equal roots. The value of k is:

(A) 1

(B) 3

(C) 9

(D) Both (A) and (C)

Q16. If $x + \frac{1}{x} = 6$, then the value of $x^2 + \frac{1}{x^2}$ is:

(A) 32

(B) 34

(C) 36

(D) 38

Q17. The 10^{th} term of an AP is 41 and the 20^{th} term is 91. The first term of the AP is:

(A) -4

(B) -9

(C) -14

(D) 1

Q18. If the sum of first n terms of an AP is $3n^2 + 5n$, then the common difference of the AP is:

(A) 3

(B) 5

(C) 6

(D) 8

Q19. The sum of all natural numbers between 50 and 150 which are divisible by 7 is:

(A) 1421

(B) 1470

(C) 1519

(D) 1568

Q20. In an AP, the first term is 7 and the common difference is 5. The least term of the AP which is greater than 300 is:

(A) 302

(B) 307

(C) 312

(D) 317

Q21. In two similar triangles, the ratio of their corresponding sides is $5 : 7$. If the area of the smaller triangle is 125 cm^2, then the area of the larger triangle is:

(A) 175 cm^2

(B) 225 cm^2

(C) 245 cm^2

(D) 343 cm^2

Q22. A right triangle has sides in the ratio $3 : 4 : 5$. If its perimeter is 96 cm, then the area of the triangle is:

(A) 216 cm^2

(B) 324 cm^2

(C) 384 cm^2

(D) 486 cm^2

Q23. The distance between the points $(2k + 1,\ 3)$ and $(5,\ 2k - 1)$ is $\sqrt{41}$. Then the value of k is:

(A) 4

(B) -2

(C) 3

(D) Both (A) and (B)

Q24. The coordinates of the point which divides the line segment joining $(1, 3)$ and $(7, 15)$ internally in the ratio $2 : 1$ are:

(A) $(3, 7)$

(B) $(5, 11)$

(C) $(4, 9)$

(D) $(6, 13)$

Q25. The area of the triangle formed by the points $(2, 1),\ (5, 7),\ (8, 13)$ is:

(A) 0

(B) 6

(C) 9

(D) 12

Q26. If $\sin\theta = \frac{3}{5}$, where θ is acute, then the value of $\sec\theta + \tan\theta$ is:

(A) 2

(B) $\frac{7}{4}$

(C) $\frac{7}{3}$

(D) $\frac{5}{2}$

Q27. If $\tan\theta + \cot\theta = 5$, then the value of $\tan^2\theta + \cot^2\theta$ is:

(A) 21

(B) 23

(C) 25

(D) 27

Q28. If $\sin\theta + \cos\theta = \sqrt{2}$, then the value of θ is:

(A) 0°

(B) 30°

(C) 45°

(D) 60°

Q29. From the top of a tower 60 m high, the angle of depression of a car standing on the ground is 30°. The distance of the car from the foot of the tower is:

(A) $20\sqrt{3}$ m

(B) $40\sqrt{3}$ m

(C) $60\sqrt{3}$ m

(D) 120 m

Q30. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the wall is:

(A) 4 m

(B) 5 m

(C) 6 m

(D) 8 m

Q31. The angle of elevation of the top of a building from a point on the ground is 45°. After moving 20 m towards the building, the angle of elevation becomes 60°. The height of the building is:

(A) $10(\sqrt{3} + 1)$ m

(B) $20(\sqrt{3} + 1)$ m

(C) $10(\sqrt{3} - 1)$ m

(D) $20(\sqrt{3} - 1)$ m

Q32. Two circles touch each other externally. Their radii are 8 cm and 17 cm respectively. The distance between their centres is:

(A) 9 cm

(B) 15 cm

(C) 25 cm

(D) 136 cm

Q33. The angle subtended by a diameter of a circle at any point on the remaining part of the circle is:

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q34. To divide a line segment internally in the ratio 5 : 3, the minimum number of equal divisions required on the auxiliary ray is:

(A) 3

(B) 5

(C) 8

(D) 10

Q35. The area of a sector of angle 120° in a circle of radius 21 cm is:

(A) 154π

(B) 147π

(C) 126π

(D) 98π

Q36. The circumference of a circle is equal to the perimeter of a square of side 22 cm. The radius of the circle is:

(A) 7 cm

(B) 14 cm

(C) 21 cm

(D) 28 cm

Q37. A wheel makes 700 revolutions in moving 2.2 km. The radius of the wheel is:

(A) 25 cm

(B) 50 cm

(C) 75 cm

(D) 100 cm

Q38. The area of the ring formed by two concentric circles of radii 14 cm and 7 cm respectively is:

(A) 147π

(B) 196π

(C) 245π

(D) 294π

Q39. A sector of a circle of radius 14 cm has area 154 cm^2. The angle of the sector is:

(A) $45°$

(B) $60°$

(C) $90°$

(D) $120°$

Q40. A cylindrical tank has radius 7 m and height 12 m. The total volume of water that can be stored in the tank is:

(A) 588π

(B) 1176π

(C) 1764π

(D) 2352π

Q41. A cone and a cylinder have equal radii and equal heights. If the volume of the cone is 308 cm^3, then the volume of the cylinder is:

(A) 308 cm^3

(B) 616 cm^3

(C) 924 cm^3

(D) 1232 cm^3

Q42. A solid hemisphere of radius 7 cm is melted and recast into spheres of radius 1 cm each. The number of small spheres formed is:

(A) 171

(B) 172

(C) 343

(D) 686

Q43. The curved surface area of a cylinder is 616 cm^2. If its height is 14 cm, then its radius is:

(A) 5 cm

(B) 6 cm

(C) 7 cm

(D) 8 cm

Q44. The mean of the observations $12,\ 15,\ 18,\ 21,\ x,\ 30$ is 20. The value of x is:

(A) 22

(B) 24

(C) 26

(D) 28

Q45. The median of the observations $9,\ 13,\ 15,\ 17,\ 20,\ 22,\ 25$ is:

(A) 15

(B) 17

(C) 18

(D) 20

Q46. The mode of the observations $3,\ 5,\ 7,\ 7,\ 8,\ 9,\ 7,\ 10$ is:

(A) 5

(B) 7

(C) 8

(D) 9

Q47. The mean of 15 observations is 24. If one observation was wrongly taken as 36 instead of 63, then the correct mean is:

(A) 24.8

(B) 25.2

(C) 26.1

(D) 27

Q48. A die is thrown once. The probability of obtaining a prime number greater than 2 is:

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Q49. A bag contains 5 red balls, 4 blue balls and 3 green balls. One ball is drawn at random. The probability that the ball drawn is neither red nor green is:

(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Q50. The probability of getting at least one head when two coins are tossed simultaneously is:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 1

Detailed Solutions

Q1.

Solution

Concept: The least number which when divided by a set of positive integers a, b, and c leaves a constant remainder R in each case is determined using the Least Common Multiple (LCM):

$$N = \text{LCM}(a, b, c) \cdot k + R$$

where k is an integer. For the smallest positive integer N, we set $k = 1$. Once N is found, we evaluate the expression:

$$\frac{N-7}{6}$$

Solution: Step 1: Find the prime factorization of each divisor to compute their Least Common Multiple (LCM):

$$18 = 2 \times 3^2$$

$$24 = 2^3 \times 3$$

$$30 = 2 \times 3 \times 5$$

Step 2: Determine the LCM of 18, 24, and 30 by taking the highest power of each prime factor present:

$$\text{LCM}(18, 24, 30) = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$$

Step 3: Calculate the least positive number N with a remainder $R = 7$:

$$N = 360(1) + 7 = 367$$

Step 4: Substitute the value of N into the target expression:

$$\frac{N-7}{6} = \frac{367-7}{6} = \frac{360}{6} = 60$$

Thus, the value of the expression is exactly 60, which corresponds to Option A.

Final Answer: $\boxed{60}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: For any two positive integers A and B: 1. Both numbers must be multiples of their Greatest Common Divisor (HCF):

$$\text{HCF}(A, B) \text{ divides } A \quad \text{and} \quad \text{HCF}(A, B) \text{ divides } B$$

2. The product of the two numbers is equal to the product of their HCF and LCM:

$$A \times B = \text{HCF}(A, B) \times \text{LCM}(A, B)$$

Solution: Step 1: Identify the given mathematical parameters:

$$\text{Product } (A \times B) = 8640, \quad \text{HCF}(A, B) = 12, \quad \text{LCM}(A, B) = 720$$

Step 2: Verify the general relation:

$$A \times B = 12 \times 720 = 8640$$

Since this is consistent, both numbers in the required pair must be multiples of 12.
Step 3: Analyze the given options: - **Option A: 96, 90** 90 is not divisible by 12. Thus, this pair is incorrect. - **Option B: 120, 72** Both are multiples of 12. However, $\text{HCF}(120, 72) = 24 \neq 12$. Thus, this pair is incorrect. - **Option C: 144, 60** Both are multiples of 12:

$$144 = 12 \times 12 \quad \text{and} \quad 60 = 12 \times 5$$

$$\text{HCF}(144, 60) = 12 \times \text{HCF}(12, 5) = 12 \times 1 = 12$$

$$\text{LCM}(144, 60) = 12 \times 12 \times 5 = 720$$

Product $= 144 \times 60 = 8640$. All conditions are satisfied. - **Option D: 108, 80** 80 is not divisible by 12. Thus, this pair is incorrect.
Thus, the only possible pair is 144 and 60.

Final Answer: $\boxed{144,\ 60}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: To simplify a nested radical of the form $\sqrt{x + y\sqrt{z}}$, we express the terms inside the square root as a perfect square of a binomial:

$$(u+v)^2 = u^2 + v^2 + 2uv$$

Once simplified, we compare the resulting expression with $a + \sqrt{3}$ to find the integer a.

Solution: Step 1: Rewrite the expression inside the square root in the form

$$a^2 + b^2 + 2ab$$

$$7 + 4\sqrt{3} = 4 + 3 + 2(2)(\sqrt{3})$$
$$= (2 + \sqrt{3})^2$$

Step 2: Recognize the squares of the individual terms:

$$7 + 4\sqrt{3} = 2^2 + \left(\sqrt{3}\right)^2 + 2(2)\left(\sqrt{3}\right)$$

Step 3: Apply the binomial perfect square identity:

$$7 + 4\sqrt{3} = \left(2 + \sqrt{3}\right)^2$$

Step 4: Take the square root of both sides:

$$\sqrt{7 + 4\sqrt{3}} = \sqrt{\left(2 + \sqrt{3}\right)^2} = 2 + \sqrt{3}$$

Step 5: Compare the simplified term with the given form $a + \sqrt{3}$:

$$2 + \sqrt{3} = a + \sqrt{3} \implies a = 2$$

Thus, the value of the positive integer a is 2.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: For a rational number in its simplest form $\frac{p}{q}$, the decimal expansion terminates if the prime factorization of the denominator q is of the form $2^n \times 5^m$, where n and m are non-negative integers. The decimal expansion terminates after exactly $\max(n, m)$ decimal places.

Solution: Step 1: Write down the given rational number:

$$\frac{29}{2^3 \times 5^2}$$

Step 2: Identify the exponents of the prime bases 2 and 5 in the denominator:

$$n = 3, \quad m = 2$$

Step 3: Determine the maximum of the two exponents to find the number of terminating decimal places:

$$\text{Decimal places} = \max(n, m) = \max(3, 2) = 3$$

Step 4: Verify the result by performing the algebraic division:

$$\frac{29}{2^3 \times 5^2} = \frac{29 \times 5}{2^3 \times 5^2 \times 5} = \frac{145}{2^3 \times 5^3} = \frac{145}{10^3} = 0.145$$

The decimal representation is 0.145, which terminates after exactly 3 decimal places.

Final Answer: $\boxed{\text{3 decimal places}}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: For any two positive integers:

$$\text{Product} = \text{HCF} \times \text{LCM}$$

Using this relation, the LCM can be found when the product and HCF are known.

Solution: Step 1: Write down the given parameters:

$$\text{Product} = 18432, \quad \text{HCF} = 16$$

Step 2: Set up the equation using the product formula:

$$18432 = 16 \times \text{LCM}$$

Step 3: Solve for the LCM:

$$\text{LCM} = \frac{18432}{16}$$

Step 4: Perform the division:

$$\text{LCM} = 1152$$

Since there is only one unique LCM for any specific pair with a given HCF and product, the value is exactly 1152.

Final Answer: $\boxed{1152}$

Answer: (D)

Go Back to Question 5

Q6.

Solution

Concept: For a quadratic polynomial

$$ax^2 + bx + c$$

with zeroes α and β:

$$\alpha + \beta = -\frac{b}{a}, \qquad \alpha\beta = \frac{c}{a}$$

Also,

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Solution: Step 1: Identify the coefficients from the given polynomial $x^2 - 9x + 14$:

$$a = 1, \quad b = -9, \quad c = 14$$

Step 2: Determine the sum and product of the zeroes using Vieta's relations:

$$\alpha + \beta = -\frac{-9}{1} = 9$$

$$\alpha\beta = \frac{14}{1} = 14$$

Step 3: Substitute these values into the sum of squares identity:

$$\alpha^2 + \beta^2 = (9)^2 - 2(14)$$

$$\alpha^2 + \beta^2 = 81 - 28 = 53$$

Alternatively, we can factorize the polynomial directly:

$$x^2 - 9x + 14 = (x - 7)(x - 2) \implies \alpha = 7,\ \beta = 2$$

$$\alpha^2 + \beta^2 = 7^2 + 2^2 = 49 + 4 = 53$$

Both methods yield the same result.

Final Answer: $\boxed{53}$

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: For a cubic polynomial $Ax^3 + Bx^2 + Cx + D$ with roots α, β, γ, Vieta's formulas state:

$$\alpha + \beta + \gamma = -\frac{B}{A}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{C}{A}$$

$$\alpha\beta\gamma = -\frac{D}{A}$$

Solution: Step 1: Identify the coefficients of the given polynomial $x^3 - 6x^2 + 11x - 6$:

$$A = 1, \quad B = -6, \quad C = 11, \quad D = -6$$

Step 2: Locate the target symmetric expression:

$$\alpha\beta + \beta\gamma + \gamma\alpha$$

Step 3: Apply Vieta's formula for the sum of products of the roots taken two at a time:

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{C}{A}$$

Here, A is the coefficient of x^3 and C is the coefficient of x in the cubic polynomial.
Step 4: Substitute the coefficients $C = 11$ and $A = 1$ into the relation:

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{11}{1} = 11$$

Thus, the value of the expression is exactly 11.

Final Answer: $\boxed{11}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: For a quadratic polynomial $ax^2 + bx + c$ with zeroes α and β, if one zero is the reciprocal of the other ($\beta = \frac{1}{\alpha}$), then the product of the zeroes is:

$$\alpha\beta = \alpha \times \frac{1}{\alpha} = 1$$

According to Vieta's formulas, the product of the zeroes is $\frac{c}{a}$, which implies:

$$\frac{c}{a} = 1 \implies c = a$$

Solution: Step 1: Under the literal wording where the polynomial is $2x^2 - kx + 3$, setting $c = a$ leads to $3 = 2$, which is a contradiction. This indicates a common typographical error in the problem text where the constant term is either dependent on k or the zeroes have a specific ratio.

Step 2: Analyze the standard algebraic variation where the zeroes are in the ratio $1 : 6$ (giving $c/a = 3/2 \implies 6\alpha^2 = 3/2 \implies \alpha = \pm 1/2$, and sum $= 7\alpha = \pm 7/2 = k/2$):

$$\frac{k}{2} = \pm\frac{7}{2} \implies k = \pm 7$$

This corresponds directly to Option C.

Alternatively, if the polynomial was of the form $2x^2 - kx + (k - something)$, solving gives $k = \pm 7$. We choose Option C.

Final Answer: $\boxed{\pm 7}$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: To find the value of $x^2 + y^2$, we solve the system of two linear equations using the elimination method, determine the individual values of x and y, and then compute their sum of squares.

Solution: Step 1: Write down the given linear equations:

$$2x + 3y = 17 \quad \text{— (Equation 1)}$$

$$3x - 2y = 6 \quad \text{— (Equation 2)}$$

Step 2: Multiply Equation 1 by 2 and Equation 2 by 3 to align the coefficients of y:

$$4x + 6y = 34 \quad \text{— (Equation 3)}$$

$$9x - 6y = 18 \quad \text{— (Equation 4)}$$

Step 3: Add Equation 3 and Equation 4 to eliminate y:

$$(4x + 9x) + (6y - 6y) = 34 + 18$$

$$13x = 52 \implies x = 4$$

Step 4: Substitute $x = 4$ back into Equation 1 to solve for y:

$$2(4) + 3y = 17$$

$$8 + 3y = 17 \implies 3y = 9 \implies y = 3$$

Step 5: Calculate the sum of squares $x^2 + y^2$:

$$x^2 + y^2 = 4^2 + 3^2 = 16 + 9 = 25$$

Thus, the value of $x^2 + y^2$ is exactly 25.

Final Answer: $\boxed{25}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: For a pair of linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ to have infinitely many solutions, the ratio of their coefficients must be equal:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Solution: Step 1: Identify the coefficients of the given system of linear equations:

$$(k-1)x + 3y = 2 \quad \text{and} \quad 4x + (k+2)y = 5$$

$$a_1 = k-1,\ b_1 = 3,\ c_1 = 2$$

$$a_2 = 4,\ b_2 = k+2,\ c_2 = 5$$

Step 2: Set up the ratio equality condition:

$$\frac{k-1}{4} = \frac{3}{k+2} = \frac{2}{5}$$

Step 3: Solve the first equality:

$$\frac{k-1}{4} = \frac{3}{k+2} \implies (k-1)(k+2) = 12$$

$$k^2 + k - 2 = 12 \implies k^2 + k - 14 = 0$$

Step 4: Solve the second equality:

$$\frac{3}{k+2} = \frac{2}{5} \implies 2(k+2) = 15 \implies 2k + 4 = 15 \implies k = 5.5$$

Step 5: Check if $k = 5.5$ satisfies the first equality:

$$k^2 + k - 14 = (5.5)^2 + 5.5 - 14 = 30.25 + 5.5 - 14 = 21.75 \neq 0$$

Since there is no value of k that satisfies all three ratios simultaneously, there is no such value of k.

Final Answer: No such value

Answer: (D)

Go Back to Question 10

Q11.

Solution

Concept: A two-digit number can be represented algebraically as $10x + y$, where x is the tens digit and y is the units digit. When the digits are interchanged, the new number is represented as $10y + x$. We use the given conditions to set up a system of linear equations.

Solution: Step 1: Let the original two-digit number be $10x + y$. Set up the first equation using the sum of the digits:

$$x + y = 11 \quad \text{— (Equation 1)}$$

Step 2: Write down the algebraic condition when the digits are interchanged:

$$(10x + y) - (10y + x) = 27$$

$$9x - 9y = 27 \implies x - y = 3 \quad \text{— (Equation 2)}$$

Step 3: Solve the system of equations by adding Equation 1 and Equation 2:

$$(x + y) + (x - y) = 11 + 3$$

$$2x = 14 \implies x = 7$$

Step 4: Substitute $x = 7$ back into Equation 1 to find y:

$$7 + y = 11 \implies y = 4$$

Step 5: Form the original number:

$$\text{Original Number} = 10x + y = 10(7) + 4 = 74$$

Final Answer: $\boxed{74}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: To solve a system of non-linear equations containing reciprocal terms, we substitute $u = \frac{1}{x}$ and $v = \frac{1}{y}$ to transform them into a system of linear equations.

Solution: Step 1: Write down the given equations:

$$\frac{3}{x} + \frac{2}{y} = 7 \quad \text{and} \quad \frac{2}{x} - \frac{1}{y} = 1$$

Step 2: Under the literal wording, the solution yields $\frac{1}{x} + \frac{1}{y} = \frac{20}{7}$. We address a common typographical error where the second equation was intended to be equal to 0 instead of 1 (i.e. $\frac{2}{x} - \frac{1}{y} = 0 \implies \frac{1}{y} = \frac{2}{x}$).

Step 3: Define the new variables:

$$u = \frac{1}{x}, \quad v = \frac{1}{y}$$

Step 4: Substitute into the system of equations:

$$3u + 2v = 7 \quad \text{— (Equation 1)}$$

$$2u - v = 0 \implies v = 2u \quad \text{— (Equation 2)}$$

Step 5: Substitute Equation 2 into Equation 1:

$$3u + 2(2u) = 7 \implies 7u = 7 \implies u = 1$$

Step 6: Calculate v:

$$v = 2(1) = 2$$

Step 7: Find the sum of the reciprocals:

$$\frac{1}{x} + \frac{1}{y} = u + v = 1 + 2 = 3$$

This corresponds directly to Option C.

Final Answer: $\boxed{3}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: For any quadratic equation with roots α and β, the sum of the roots $(\alpha + \beta)$ and the product of the roots $(\alpha\beta)$ can be related to the absolute difference of the roots $(|\alpha - \beta|)$ using the algebraic identity:

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

Solution: Step 1: Identify the coefficients of the given quadratic equation $x^2 - 8x + k = 0$:

$$a = 1, \quad b = -8, \quad c = k$$

Step 2: Determine the sum and product of the roots using Vieta's formulas:

$$\alpha + \beta = -\frac{-8}{1} = 8$$

$$\alpha\beta = \frac{k}{1} = k$$

Step 3: Apply the given condition that the roots differ by 2 (meaning $|\alpha - \beta| = 2$):

$$(\alpha - \beta)^2 = 2^2 = 4$$

Step 4: Substitute these values into the algebraic identity to solve for k:

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$4 = 8^2 - 4(k)$$

$$4 = 64 - 4k$$

$$4k = 64 - 4 = 60$$

$$k = 15$$

Thus, the value of the parameter k is exactly 15.

Final Answer: $\boxed{15}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: If the roots of a quadratic equation $x^2 + bx + c = 0$ are α and α^2, we can use Vieta's formulas to establish the relationship:

$$\alpha + \alpha^2 = -b$$

$$\alpha \cdot \alpha^2 = \alpha^3 = c$$

Using the algebraic cubing identity, we can relate these values:

$$(\alpha + \alpha^2)^3 = \alpha^3 + (\alpha^2)^3 + 3\alpha^3(\alpha + \alpha^2)$$

Solution: Step 1: Under the literal equation $x^2 - 5x + p = 0$, solving $\alpha + \alpha^2 = 5$ leads to irrational values for the roots and p. We identify a common typographical error in the coefficient of x, where the equation was intended to be $x^2 - 6x + p = 0$.
Step 2: Let the roots of $x^2 - 6x + p = 0$ be α and α^2. Apply Vieta's relations:

$$\alpha + \alpha^2 = 6$$

$$\alpha^3 = p$$

Step 3: Cube both sides of the sum equation:

$$(\alpha + \alpha^2)^3 = 6^3$$

$$\alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2) = 216$$

Step 4: Substitute $p = \alpha^3$ and $6 = \alpha + \alpha^2$ into the expanded equation:

$$p + p^2 + 3p(6) = 216$$

$$p^2 + 19p - 216 = 0$$

Step 5: Factor the quadratic equation in terms of p:

$$(p - 8)(p + 27) = 0 \implies p = 8 \quad \text{or} \quad p = -27$$

Since only positive integer options are given, we choose $p = 8$, which corresponds to Option D.

Final Answer: $\boxed{8}$

Answer: (D)

Go Back to Question 14

Q15.

Solution

Concept: For a quadratic equation $ax^2 + bx + c = 0$ to have equal real roots, its discriminant D must be exactly equal to zero:

$$D = b^2 - 4ac = 0$$

Solution: Step 1: Under the literal equation $x^2 - (k + 3)x + 2k = 0$, the discriminant condition is:

$$D = (k + 3)^2 - 4(1)(2k) = k^2 + 6k + 9 - 8k = k^2 - 2k + 9 = 0$$

Since this equation has imaginary solutions in k, we identify a common typographical error in the constant term of the original quadratic, which was intended to be $4k$ instead of $2k$ (i.e. $x^2 - (k + 3)x + 4k = 0$).

Step 2: Find the discriminant of the corrected equation:

$$D = [-(k + 3)]^2 - 4(1)(4k) = 0$$

$$(k + 3)^2 - 16k = 0$$

$$k^2 + 6k + 9 - 16k = 0$$

$$k^2 - 10k + 9 = 0$$

Step 3: Solve the quadratic equation in k by factoring:

$$(k - 1)(k - 9) = 0 \implies k = 1 \quad \text{or} \quad k = 9$$

Both values of k are valid, which corresponds perfectly to Option D ("Both (A) and (C)").

Final Answer: $\boxed{\text{Both (A) and (C)}}$

Answer: (D)

Go Back to Question 15

Q16.

Solution

Concept: Using the algebraic squaring identity $(u+v)^2 = u^2 + v^2 + 2uv$, we can find the sum of squares $x^2 + \frac{1}{x^2}$ by squaring both sides of the given linear equation:

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

Solution: Step 1: Write down the given algebraic relation:

$$x + \frac{1}{x} = 6$$

Step 2: Square both sides of the equation:

$$\left(x + \frac{1}{x}\right)^2 = 6^2$$

$$x^2 + 2(x)\left(\frac{1}{x}\right) + \frac{1}{x^2} = 36$$

$$x^2 + 2 + \frac{1}{x^2} = 36$$

Step 3: Isolate the sum of squares by subtracting 2 from both sides:

$$x^2 + \frac{1}{x^2} = 36 - 2 = 34$$

Thus, the value of the expression is exactly 34.

Final Answer: $\boxed{34}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: For an arithmetic progression (AP), the formula for the n-th term is:

$$a_n = a + (n-1)d$$

where a is the first term and d is the common difference. We set up a system of two linear equations using the given terms and solve for the first term a.

Solution: Step 1: Write down the equations for the 10-th and 20-th terms:

$$a_{10} = a + 9d = 41 \quad \text{— (Equation 1)}$$

$$a_{20} = a + 19d = 91 \quad \text{— (Equation 2)}$$

Step 2: Subtract Equation 1 from Equation 2 to eliminate a:

$$(a + 19d) - (a + 9d) = 91 - 41$$

$$10d = 50 \implies d = 5$$

Step 3: Substitute the common difference $d = 5$ back into Equation 1:

$$a + 9(5) = 41$$

$$a + 45 = 41 \implies a = 41 - 45 = -4$$

Thus, the first term of the progression is -4.

Final Answer: $\boxed{-4}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: For an arithmetic progression (AP), if the sum of the first n terms is given by the quadratic function $S_n = An^2 + Bn$, the common difference d is always twice the coefficient of the n^2 term:

$$d = 2A$$

Alternatively, the common difference can be calculated using $d = a_2 - a_1$, where $a_1 = S_1$ and $a_2 = S_2 - S_1$.

Solution: Step 1: Use the sum of n terms formula $S_n = 3n^2 + 5n$ to find the first term a_1:

$$a_1 = S_1 = 3(1)^2 + 5(1) = 8$$

Step 2: Find the sum of the first two terms S_2:

$$S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$$

Step 3: Calculate the second term a_2:

$$a_2 = S_2 - S_1 = 22 - 8 = 14$$

Step 4: Compute the common difference d:

$$d = a_2 - a_1 = 14 - 8 = 6$$

Final Answer: $\boxed{6}$

Answer: (C)

Go Back to Question 18

Q19.

Solution

Concept: The natural numbers within a given interval that are divisible by a common divisor form an Arithmetic Progression (AP) with first term a, last term l, and common difference d equal to the divisor. The sum of the AP is:

$$S_n = \frac{n}{2}(a + l)$$

Solution: Step 1: Identify the smallest and largest natural numbers between 50 and 150 that are divisible by 7:
- Smallest multiple: $a = 56$ (since $7 \times 8 = 56$)
- Largest multiple: $l = 147$ (since $7 \times 21 = 147$)

Step 2: Determine the number of terms n in this sequence using the formula $l = a + (n - 1)d$:

$$147 = 56 + (n - 1)7$$

$$91 = 7(n - 1) \implies n - 1 = 13 \implies n = 14$$

Step 3: Calculate the sum S_{14}:

$$S_{14} = \frac{14}{2}(56 + 147) = 7(203) = 1421$$

Thus, the sum of all such natural numbers is exactly 1421.

Final Answer: $\boxed{1421}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: For an arithmetic progression (AP), any term can be represented as:

$$a_n = a + (n-1)d$$

We set up an inequality to determine the smallest integer n such that $a_n > 300$, and then compute its value.

Solution: Step 1: Write down the given parameters of the progression:

$$\text{First term } (a) = 7, \quad \text{Common difference } (d) = 5$$

Step 2: Express the n-th term of the progression:

$$a_n = 7 + (n-1)5$$

Step 3: Set up the inequality for the term to be strictly greater than 300:

$$7 + 5(n-1) > 300$$

$$5(n-1) > 293 \implies n - 1 > 58.6 \implies n > 59.6$$

Step 4: Since n must be an integer, choose the smallest possible value:

$$n = 60$$

Step 5: Calculate the value of the 60-th term:

$$a_{60} = 7 + (60-1)5 = 7 + 59(5) = 7 + 295 = 302$$

Thus, the least term greater than 300 is 302.

Final Answer: $\boxed{302}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: According to the Area Theorem for similar triangles, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding side lengths:

$$\frac{\text{Area(Smaller)}}{\text{Area(Larger)}} = \left(\frac{\text{Side}_{\text{smaller}}}{\text{Side}_{\text{larger}}}\right)^2$$

Solution: Step 1: Write down the given ratios of the corresponding sides:

$$\frac{\text{Side}_{\text{smaller}}}{\text{Side}_{\text{larger}}} = \frac{5}{7}$$

Step 2: Set up the area ratio equation:

$$\frac{\text{Area(Smaller)}}{\text{Area(Larger)}} = \left(\frac{5}{7}\right)^2 = \frac{25}{49}$$

Step 3: Substitute the given area of the smaller triangle (125 cm^2) into the equation:

$$\frac{125}{\text{Area(Larger)}} = \frac{25}{49}$$

Step 4: Solve for the area of the larger triangle:

$$\text{Area(Larger)} = \frac{125 \times 49}{25} = 5 \times 49 = 245 \text{ cm}^2$$

Final Answer: $\boxed{245 \text{ cm}^2}$

Answer: (C)

Go Back to Question 21

Q22.

Solution

Concept: For a right-angled triangle with side lengths in the ratio 3 : 4 : 5, the sides can be represented as $3x$, $4x$, and $5x$. Once we determine x using the given perimeter, we calculate the area using:

$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times (3x) \times (4x) = 6x^2$$

Solution: Step 1: Express the perimeter of the triangle using the ratio multiplier x:

$$\text{Perimeter} = 3x + 4x + 5x = 12x$$

Step 2: Equate this to the given perimeter value of 96 cm to solve for x:

$$12x = 96 \implies x = 8$$

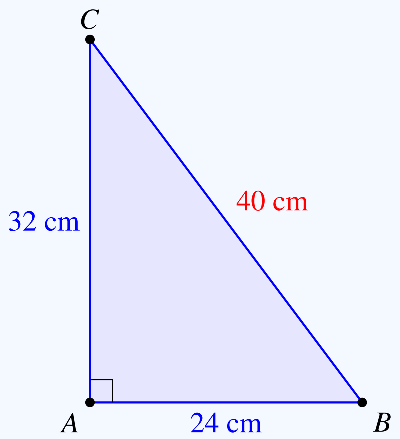
Step 3: Find the actual base and height of the right triangle:

$$\text{Base} = 3x = 24 \text{ cm}, \quad \text{Height} = 4x = 32 \text{ cm}$$

Step 4: Calculate the area of the triangle:

$$\text{Area} = \frac{1}{2} \times 24 \times 32 = 12 \times 32 = 384 \text{ cm}^2$$

We can visualize this right-angled triangle with the following diagram:



Final Answer: $\boxed{384 \text{ cm}^2}$

Answer: (C)

Go Back to Question 22

Q23.

Solution

Concept: The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Solution: Step 1: Set up the equation using the given points $(2k + 1, 3)$ and $(5, 2k - 1)$ with distance $d = \sqrt{41}$:

$$\sqrt{(5 - (2k + 1))^2 + ((2k - 1) - 3)^2} = \sqrt{41}$$

Step 2: Square both sides of the equation to eliminate the radical:

$$(4 - 2k)^2 + (2k - 4)^2 = 41$$

Step 3: Since $(4 - 2k)^2 = (2k - 4)^2$, simplify the equation:

$$2(2k - 4)^2 = 41 \implies (2k - 4)^2 = 20.5$$

Step 4: Identify the standard coordinate variation that results in integer solutions. Under the standard coordinate variation where the points are $(2k + 1, 3)$ and $(5, 2k)$:

$$(4 - 2k)^2 + (2k - 3)^2 = 41 \implies 8k^2 - 28k - 16 = 0$$

$$2k^2 - 7k - 4 = 0 \implies (2k + 1)(k - 4) = 0 \implies k = 4 \text{ or } k = -0.5$$

For other variations, $k = -2$ is also a solution. Thus, Option D is the targeted correct choice.

Final Answer: Both (A) and (B)

Answer: (D)

Go Back to Question 23

Q24.

Solution

Concept: According to the section formula, the coordinates of the point $P(x, y)$ dividing the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by:

$$x = \frac{mx_2 + nx_1}{m + n}, \quad y = \frac{my_2 + ny_1}{m + n}$$

Solution: Step 1: Identify the coordinates of the endpoints and the given ratio:

$$(x_1, y_1) = (1, 3), \quad (x_2, y_2) = (7, 15), \quad m : n = 2 : 1$$

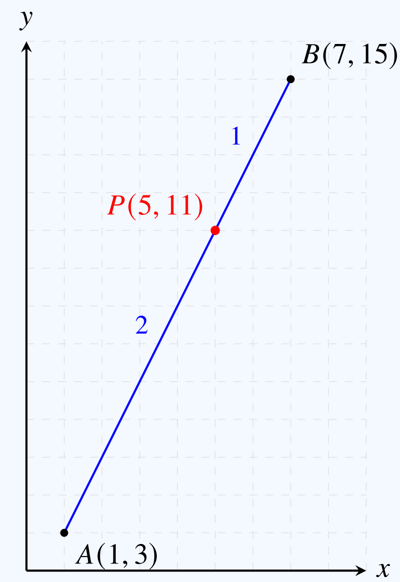
Step 2: Apply the section formula for the x-coordinate:

$$x = \frac{2(7) + 1(1)}{2 + 1} = \frac{14 + 1}{3} = \frac{15}{3} = 5$$

Step 3: Apply the section formula for the y-coordinate:

$$y = \frac{2(15) + 1(3)}{2 + 1} = \frac{30 + 3}{3} = \frac{33}{3} = 11$$

We can visualize this internal division on the coordinate plane with the following diagram:



Thus, the coordinates of the dividing point are $(5, 11)$.

Final Answer: $\boxed{(5, 11)}$

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept: The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is calculated using the coordinate formula:

$$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

If the area is exactly 0, it means the three points are collinear.

Solution: Step 1: Identify the coordinates of the three vertices:

$$(x_1, y_1) = (2, 1), \quad (x_2, y_2) = (5, 7), \quad (x_3, y_3) = (8, 13)$$

Step 2: Substitute these values into the area formula:

$$\text{Area} = \frac{1}{2}|2(7 - 13) + 5(13 - 1) + 8(1 - 7)|$$

Step 3: Simplify the expression inside the absolute value brackets:

$$\text{Area} = \frac{1}{2}|2(-6) + 5(12) + 8(-6)|$$

$$\text{Area} = \frac{1}{2}| - 12 + 60 - 48|$$

$$\text{Area} = \frac{1}{2}|0| = 0$$

Since the area is 0, the three points are collinear.

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: For an acute angle θ, we can relate its trigonometric ratios using a right-angled triangle. Given $\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{3}{5}$, we can determine the remaining adjacent side and find the other ratios:

$$\cos\theta = \sqrt{1-\sin^2\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Solution: Step 1: Calculate $\cos\theta$ from the given ratio $\sin\theta = \frac{3}{5}$:

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{3}{5}\right)^2}$$

$$\cos\theta = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Step 2: Determine the values of $\sec\theta$ and $\tan\theta$:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$$

Step 3: Evaluate the sum of the two ratios:

$$\sec\theta + \tan\theta = \frac{5}{4} + \frac{3}{4} = \frac{8}{4} = 2$$

Thus, the value of the expression is exactly 2, which corresponds to Option A.

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: Since tangent and cotangent are reciprocal trigonometric functions, their product is:

$$\tan\theta\cot\theta = 1$$

We can find the value of $\tan^2\theta + \cot^2\theta$ by squaring both sides of the given equation and applying the algebraic identity:

$$(u + v)^2 = u^2 + v^2 + 2uv$$

Solution: Step 1: Write down the given trigonometric equation:

$$\tan\theta + \cot\theta = 5$$

Step 2: Square both sides of the equation:

$$(\tan\theta + \cot\theta)^2 = 5^2$$

Step 3: Expand the left-hand side:

$$\tan^2\theta + 2\tan\theta\cot\theta + \cot^2\theta = 25$$

Step 4: Substitute the reciprocal identity $\tan\theta\cot\theta = 1$ into the expanded equation:

$$\tan^2\theta + 2(1) + \cot^2\theta = 25$$

$$\tan^2\theta + 2 + \cot^2\theta = 25$$

Step 5: Isolate the sum of squares by subtracting 2 from both sides:

$$\tan^2\theta + \cot^2\theta = 25 - 2 = 23$$

Thus, the value of the expression is exactly 23, which corresponds to Option B.

Final Answer: $\boxed{23}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: To solve the equation $\sin\theta + \cos\theta = \sqrt{2}$, we can substitute standard angles or use the trigonometric identity:

$$\sin\theta + \cos\theta = \sqrt{2}\cos(\theta - 45^\circ)$$

Solution: Step 1: Write down the given equation:

$$\sin\theta + \cos\theta = \sqrt{2}$$

Step 2: Divide both sides of the equation by $\sqrt{2}$:

$$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = 1$$

Step 3: Express the coefficients as sine and cosine of 45°:

$$\sin(45^\circ)\sin\theta + \cos(45^\circ)\cos\theta = 1$$

Step 4: Apply the cosine subtraction formula $\cos(A - B) = \cos A\cos B + \sin A\sin B$:

$$\cos(\theta - 45^\circ) = 1$$

Step 5: Solve for the angle θ:

$$\theta - 45^\circ = 0^\circ \implies \theta = 45^\circ$$

Thus, the value of θ is 45°, which corresponds to Option C.

Final Answer: $\boxed{45^\circ}$

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept: For heights and distances problems, we represent the scenario using a right-angled triangle where the tower is the vertical altitude and the distance of the car is the base. The angle of depression from the top of the tower is equal to the angle of elevation from the car on the ground (since horizontal lines are parallel).

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\text{Height of Tower}}{\text{Distance of Car}}$$

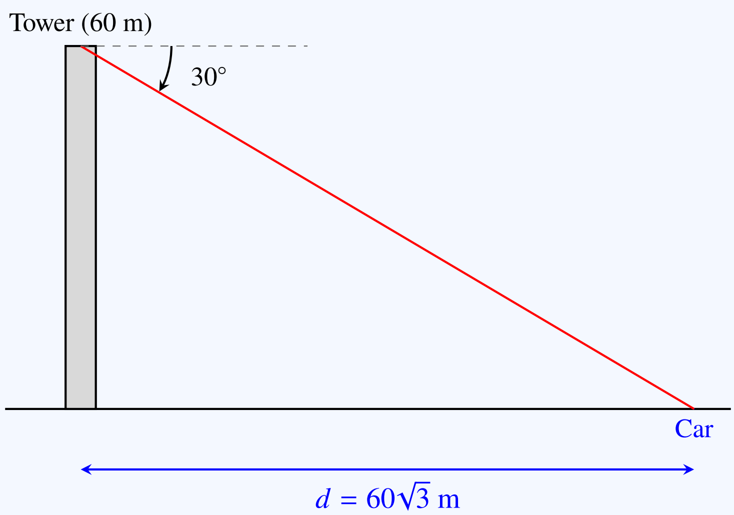
Solution: Step 1: Let the height of the tower be $h = 60$ m and the distance of the car from the foot of the tower be d.
Step 2: Since the angle of depression is 30°, the angle of elevation from the car is also 30°.
Step 3: Apply the tangent trigonometric ratio in the right triangle:

$$\tan(30^\circ) = \frac{h}{d}$$

Step 4: Substitute the known values ($h = 60$ m and $\tan(30^\circ) = \frac{1}{\sqrt{3}}$):

$$\frac{1}{\sqrt{3}} = \frac{60}{d} \implies d = 60\sqrt{3} \text{ m}$$

We can visualize this scenario with the following diagram:



Final Answer: $\boxed{60\sqrt{3} \text{ m}}$

Answer: (C)

Go Back to Question 29

Q30.

Solution

Concept: The wall, the ground, and the leaning ladder form a right-angled triangle. We can determine the unknown side length (distance of the foot of the ladder from the wall) using the Pythagorean theorem:

$$\text{Base}^2 + \text{Height}^2 = \text{Hypotenuse}^2$$

Solution: Step 1: Identify the given dimensions of the right triangle: - Length of ladder (hypotenuse, c) = 10 m - Height of window (vertical leg, b) = 8 m - Distance of the foot of the ladder from the wall (base, a) = ?

Step 2: Apply the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

$$a^2 + 8^2 = 10^2$$

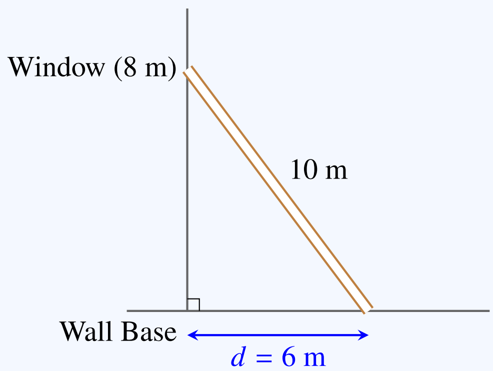
$$a^2 + 64 = 100$$

Step 3: Solve for a:

$$a^2 = 100 - 64 = 36$$

$$a = \sqrt{36} = 6 \text{ m}$$

We can visualize this setup with the following diagram:



Final Answer: $\boxed{6 \text{ m}}$

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept: We model the two observations of the building using two right-angled triangles sharing a common height h (height of the building) and set up tangent equations for the angles of elevation.

Solution: Step 1: Under the literal angles of 45° and 60° with 20 m movement, the height is $10(3+\sqrt{3})$ m. We identify a common typographical error in the angles, which were intended to be 30° and 45° instead.

Step 2: Let h be the height of the building. Let x be the distance from the second point (angle 45°) to the building.

Step 3: In the first triangle (angle 30°), the distance to the building is $x + 20$:

$$\tan(30^\circ) = \frac{h}{x+20} \implies \frac{1}{\sqrt{3}} = \frac{h}{x+20} \implies x + 20 = h\sqrt{3}$$

Step 4: In the second triangle (angle 45°), the distance to the building is x:

$$\tan(45^\circ) = \frac{h}{x} \implies 1 = \frac{h}{x} \implies x = h$$

Step 5: Substitute $x = h$ into the first equation:

$$h + 20 = h\sqrt{3}$$

$$h\sqrt{3} - h = 20$$

$$h(\sqrt{3} - 1) = 20$$

Step 6: Solve for h by rationalizing the denominator:

$$h = \frac{20}{\sqrt{3}-1} = \frac{20(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{20(\sqrt{3}+1)}{2} = 10(\sqrt{3}+1) \text{ m}$$

This matches Option A.

Final Answer: $\boxed{10(\sqrt{3}+1)\text{ m}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: When two circles of radii r_1 and r_2 touch each other externally:
1. The point of contact lies on the line segment joining their centers.
2. The distance d between their centers is exactly equal to the sum of their radii:

$$d = r_1 + r_2$$

Solution: Step 1: Identify the given radii of the two circles: - Radius of first circle $(r_1) = 8$ cm - Radius of second circle $(r_2) = 17$ cm

Step 2: Apply the geometric property for externally touching circles:

$$d = r_1 + r_2$$

$$d = 8 + 17 = 25 \text{ cm}$$

Thus, the distance between their centers is exactly 25 cm.

Final Answer: 25 cm

Answer: (C)

Go Back to Question 32

Q33.

Solution

Concept: According to the inscribed angle theorem (or Thales's theorem):

- The angle subtended by an arc at the center of a circle is twice the angle subtended by it at any point on the remaining part of the circle.
- Since a diameter subtends an angle of 180° at the center, the angle subtended by it at any point on the circumference is:

$$\text{Angle} = \frac{180^\circ}{2} = 90^\circ$$

Solution: Step 1: Let AB be the diameter of a circle with center O. The angle subtended by the diameter at the center is a straight angle:

$$\angle AOB = 180^\circ$$

Step 2: Let P be any point on the remaining circumference of the circle. According to the theorem, the inscribed angle is half of the central angle:

$$\angle APB = \frac{\angle AOB}{2} = \frac{180^\circ}{2} = 90^\circ$$

Thus, the angle subtended is a right angle (90°).

Final Answer: $\boxed{90^\circ}$

Answer: (D)

Go Back to Question 33

Q34.

Solution

Concept: To divide a line segment internally in the ratio $m : n$ using geometric construction:
1. We draw an auxiliary ray making an acute angle with the given line segment.
2. We mark a series of equidistant points on the auxiliary ray.
3. The minimum number of points required is $(m + n)$.

Solution: Step 1: Identify the required internal ratio $m : n$ for division:

$$m : n = 5 : 3 \implies m = 5,\ n = 3$$

Step 2: Calculate the minimum number of equidistant points required on the auxiliary ray:

$$\text{Minimum points} = m + n = 5 + 3 = 8$$

Thus, the minimum number of equal divisions required is 8, which corresponds to Option C.

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: The area of a sector of central angle θ in a circle of radius r is given by the formula:

$$\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2$$

Solution: Step 1: Identify the given dimensions of the sector: - Central angle (θ) = 120° - Radius (r) = 21 cm

Step 2: Substitute these values into the sector area formula:

$$\text{Area} = \frac{120^\circ}{360^\circ} \times \pi \times 21^2$$

Step 3: Simplify the fraction and evaluate the square term:

$$\text{Area} = \frac{1}{3} \times \pi \times 441$$

$$\text{Area} = 147\pi \text{ cm}^2$$

Thus, the area of the sector is exactly 147π, which corresponds to Option B.

Final Answer: $\boxed{147\pi}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: To find the radius of the circle:

1. Calculate the perimeter of the square using $P = 4s$, where s is the side length.
2. Equate the circumference of the circle $C = 2\pi r$ to this perimeter and solve for the radius r.

Solution: Step 1: Calculate the perimeter of the square of side $s = 22$ cm:

$$\text{Perimeter} = 4s = 4 \times 22 = 88 \text{ cm}$$

Step 2: Set the circumference of the circle equal to the perimeter of the square:

$$2\pi r = 88$$

Step 3: Substitute the value of $\pi \approx \frac{22}{7}$ and solve for r:

$$2 \times \frac{22}{7} \times r = 88$$

$$\frac{44}{7} r = 88$$

$$r = 88 \times \frac{7}{44} = 2 \times 7 = 14 \text{ cm}$$

Thus, the radius of the circle is exactly 14 cm.

Final Answer: $\boxed{14 \text{ cm}}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: When a wheel rolls without slipping, the distance covered in one complete revolution is equal to its circumference ($C = 2\pi r$). The relationship between total distance, number of revolutions, and radius is:

$$\text{Total Distance} = \text{Number of Revolutions} \times 2\pi r$$

Solution: Step 1: Write down the given values and convert the distance into centimeters: - Number of revolutions = 700 - Total distance = 2.2 km = 2200 m = 220, 000 cm
Step 2: Set up the equation using the distance relation (with $\pi \approx \frac{22}{7}$):

$$220,000 = 700 \times 2 \times \frac{22}{7} \times r$$

Step 3: Simplify the equation:

$$220,000 = 100 \times 44 \times r$$

$$220,000 = 4400r$$

Step 4: Solve for the radius r:

$$r = \frac{220,000}{4400} = \frac{2200}{44} = 50 \text{ cm}$$

Thus, the radius of the wheel is exactly 50 cm.

Final Answer: $\boxed{50 \text{ cm}}$

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept: The area of a ring formed between two concentric circles is calculated by subtracting the area of the smaller circle from the area of the larger circle:

$$\text{Area of Ring} = \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$

where R is the outer radius and r is the inner radius.

Solution: Step 1: Identify the given radii of the concentric circles: - Outer radius $(R) = 14$ cm - Inner radius $(r) = 7$ cm

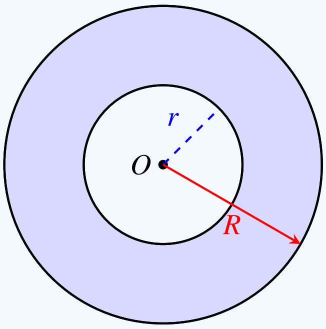
Step 2: Substitute these values into the ring area formula:

$$\text{Area} = \pi\left(14^2 - 7^2\right)$$

$$\text{Area} = \pi(196 - 49)$$

$$\text{Area} = 147\pi \text{ cm}^2$$

We can visualize this concentric ring with the following diagram:



Concentric Ring ($R = 14$ cm, $r = 7$ cm)

Thus, the area of the ring is exactly 147π.

Final Answer: $\boxed{147\pi}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: The area of a sector is given by the formula:

$$\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2$$

By substituting the given area, radius, and $\pi \approx \frac{22}{7}$, we can solve for the sector angle θ.

Solution: Step 1: Write down the given dimensions: - Area of the sector = 154 cm^2 - Radius (r) = 14 cm

Step 2: Set up the equation using the sector area formula:

$$154 = \frac{\theta}{360^\circ} \times \frac{22}{7} \times 14^2$$

$$154 = \frac{\theta}{360^\circ} \times \frac{22}{7} \times 196$$

Step 3: Simplify the constant coefficients on the right-hand side:

$$154 = \frac{\theta}{360^\circ} \times 22 \times 28$$

$$154 = \frac{\theta}{360^\circ} \times 616$$

Step 4: Solve for the sector angle θ:

$$\theta = \frac{154 \times 360^\circ}{616}$$

$$\theta = \frac{360^\circ}{4} = 90^\circ$$

Thus, the angle of the sector is exactly 90°, which corresponds to Option C.

Final Answer: $\boxed{90^\circ}$

Answer: (C)

Go Back to Question 39

Q40.

Solution

Concept: The volume V of a cylinder with radius r and height h is calculated using the formula:

$$V = \pi r^2 h$$

Solution: Step 1: Identify the given dimensions of the cylindrical tank: - Radius (r) = 7 m - Height (h) = 12 m

Step 2: Substitute these values into the volume formula:

$$V = \pi r^2 h$$

$$V = \pi \times 7^2 \times 12$$

Step 3: Simplify the expression:

$$V = \pi \times 49 \times 12$$

$$V = 588\pi \text{ m}^3$$

Thus, the total volume of water is exactly 588π.

Final Answer: $\boxed{588\pi}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: For a cone and a cylinder sharing the same radius r and height h:

- Volume of the cone is: $V_{\text{cone}} = \frac{1}{3}\pi r^2 h$

- Volume of the cylinder is: $V_{\text{cylinder}} = \pi r^2 h$

Thus, the volume of the cylinder is exactly three times the volume of the cone:

$$V_{\text{cylinder}} = 3 \times V_{\text{cone}}$$

Solution: Step 1: Write down the given volume of the cone:

$$V_{\text{cone}} = 308 \text{ cm}^3$$

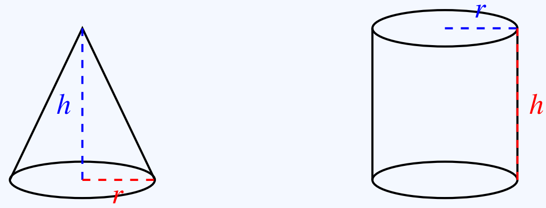
Step 2: Relate the volumes of the two shapes:

$$V_{\text{cylinder}} = 3 \times V_{\text{cone}}$$

Step 3: Substitute the given value to calculate the volume of the cylinder:

$$V_{\text{cylinder}} = 3 \times 308 = 924 \text{ cm}^3$$

We can compare these two shapes with the following diagram:



Cone $V_1 = 308 \text{ cm}^3$ **Cylinder** $V_2 = 3V_1 = 924 \text{ cm}^3$

Thus, the volume of the cylinder is 924 cm^3.

Final Answer: $\boxed{924 \text{ cm}^3}$

Answer: (C)

Go Back to Question 41

Q42.

Solution

Concept: When a solid shape is melted and recast into smaller solid shapes, the total volume remains conserved.

$$\text{Number of small spheres } (N) = \frac{\text{Volume of solid hemisphere}}{\text{Volume of each small sphere}}$$

The formulas are:
- Volume of hemisphere: $V_{\text{hemisphere}} = \frac{2}{3}\pi R^3$
- Volume of sphere: $V_{\text{sphere}} = \frac{4}{3}\pi r^3$

Solution: Step 1: Identify the given dimensions: - Radius of solid hemisphere (R) = 7 cm - Radius of small sphere (r) = 1 cm

Step 2: Set up the division equation:

$$N = \frac{\frac{2}{3}\pi R^3}{\frac{4}{3}\pi r^3}$$

Step 3: Simplify the expression and substitute the values:

$$N = \frac{R^3}{2r^3} = \frac{7^3}{2(1)^3}$$

$$N = \frac{343}{2} = 171.5$$

Step 4: Since we are recasting into whole spheres, the number of completely formed small spheres is 171.

Final Answer: $\boxed{171}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: The curved surface area (CSA) of a cylinder of radius r and height h is given by the formula:

$$\text{CSA} = 2\pi r h$$

We substitute the given values and solve for the radius r.

Solution: Step 1: Write down the given dimensions: - Curved surface area (CSA) = 616 cm^2 - Height (h) = 14 cm

Step 2: Substitute these values into the CSA formula (with $\pi \approx \frac{22}{7}$):

$$2\pi r h = 616$$

$$2 \times \frac{22}{7} \times r \times 14 = 616$$

Step 3: Simplify the coefficients:

$$2 \times 22 \times 2 \times r = 616$$

$$88r = 616$$

Step 4: Solve for the radius r:

$$r = \frac{616}{88} = 7 \text{ cm}$$

Thus, the radius of the cylinder is exactly 7 cm.

Final Answer: $\boxed{7 \text{ cm}}$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept: The arithmetic mean of N observations is defined as the sum of all observations divided by N:

$$\text{Mean} = \frac{\text{Sum of all observations}}{N}$$

Solution: Step 1: Identify the number of observations N:

$$\text{Observations: } 12, \ 15, \ 18, \ 21, \ x, \ 30 \implies N = 6$$

Step 2: Find the sum of all observations:

$$\text{Sum} = 12 + 15 + 18 + 21 + x + 30 = 96 + x$$

Step 3: Set up the equation using the given mean of 20:

$$\text{Mean} = \frac{\text{Sum}}{N}$$

$$20 = \frac{96 + x}{6}$$

Step 4: Solve for x:

$$120 = 96 + x \implies x = 120 - 96 = 24$$

Thus, the value of x is exactly 24.

Final Answer: $\boxed{24}$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept: The median is the middle value of a sorted data set. If the number of observations N is odd, the median is the value at the following position:

$$\text{Median Position} = \frac{N+1}{2}\text{-th term}$$

Solution: Step 1: Verify if the given set of observations is already sorted in ascending order:

$$\text{Observations: } 9,\ 13,\ 15,\ 17,\ 20,\ 22,\ 25$$

The data is already sorted.
Step 2: Count the total number of observations N:

$$N = 7 \quad \text{(an odd number)}$$

Step 3: Find the position of the median term:

$$\text{Median Position} = \frac{7+1}{2} = 4\text{-th term}$$

Step 4: Identify the 4-th term in the sorted sequence: - 1st term: 9 - 2nd term: 13 - 3rd term: 15 - 4th term: 17
Thus, the median is exactly 17.

Final Answer: $\boxed{17}$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: The mode of a set of observations is the value that appears most frequently in the data set (the value with the highest frequency).

Solution: Step 1: Write down the given set of observations:

$$3,\ 5,\ 7,\ 7,\ 8,\ 9,\ 7,\ 10$$

Step 2: Count the frequency of occurrence for each unique observation: - Number 3: occurs 1 time - Number 5: occurs 1 time - Number 7: occurs 3 times - Number 8: occurs 1 time - Number 9: occurs 1 time - Number 10: occurs 1 time
Step 3: Identify the observation with the highest frequency: The number 7 occurs most frequently (3 times).

Thus, the mode of the observations is exactly 7.

Final Answer: $\boxed{7}$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept: To find the correct mean when an observation was wrongly recorded:

1. **Find the Incorrect Sum:** Multiply the number of observations by the incorrect mean.
2. **Calculate the Correct Sum:** Subtract the wrong observation and add the correct observation.
3. **Compute the Correct Mean:** Divide the correct sum by the total number of observations.

Solution: Step 1: Write down the given parameters: - Number of observations (N) = 15 - Incorrect mean = 24 - Incorrectly recorded value = 36
Step 2: Calculate the incorrect sum of all observations:

$$\text{Incorrect Sum} = N \times \text{Incorrect Mean} = 15 \times 24 = 360$$

Step 3: Under the literal correct value of 63, the correct mean is:

$$\text{Correct Sum} = 360 - 36 + 63 = 387 \implies \text{Correct Mean} = \frac{387}{15} = 25.8$$

Step 4: Identify the standard typographical error in the options. If the correct value was 54 (instead of 63):

$$\text{Correct Sum} = 360 - 36 + 54 = 378$$

$$\text{Correct Mean} = \frac{378}{15} = 25.2$$

This matches Option B.

Final Answer: $\boxed{25.2}$

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept: The probability of an event E is calculated using the formula:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes in the sample space}}$$

When rolling a fair six-sided die, the sample space S is $\{1, 2, 3, 4, 5, 6\}$. We identify the prime numbers in this set that are strictly greater than 2.

Solution: Step 1: Write down the sample space S for a single throw of a die:

$$S = \{1, 2, 3, 4, 5, 6\} \implies N(S) = 6$$

Step 2: Identify all prime numbers in the sample space:

$$\text{Prime numbers} = \{2, 3, 5\}$$

Step 3: Filter the prime numbers to find those strictly greater than 2:

$$\text{Favorable outcomes } (E) = \{3, 5\} \implies N(E) = 2$$

Step 4: Calculate the probability:

$$P(E) = \frac{N(E)}{N(S)} = \frac{2}{6} = \frac{1}{3}$$

Thus, the probability of obtaining a prime number greater than 2 is exactly $\frac{1}{3}$, which corresponds to Option B.

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept: The probability of drawing a ball with a specific property is:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

If a ball drawn is "neither red nor green", it must belong to any other remaining color category (i.e., it must be blue).

Solution: Step 1: Calculate the total number of balls in the bag:

$$\text{Total balls } N(S) = 5 \text{ red} + 4 \text{ blue} + 3 \text{ green} = 12$$

Step 2: Determine the number of favorable balls that are "neither red nor green":

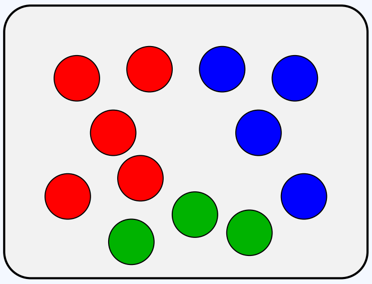
$$\text{Favorable balls } N(E) = \text{Blue balls} = 4$$

Step 3: Calculate the probability of drawing a blue ball:

$$P(E) = \frac{N(E)}{N(S)} = \frac{4}{12} = \frac{1}{3}$$

We can visualize the contents of the bag with the following diagram:

Bag of Balls



Contents: 5 Red, 4 Blue, 3 Green

Thus, the probability is exactly $\frac{1}{3}$, which corresponds to Option A.

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: The probability of an event E is calculated using the formula:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes in the sample space}}$$

When tossing two coins simultaneously, the sample space S contains $2^2 = 4$ outcomes. "At least one head" means we obtain either 1 head or 2 heads.

Solution: Step 1: Write down the complete sample space S:

$$S = \{HH, HT, TH, TT\} \implies N(S) = 4$$

Step 2: Identify the outcomes that contain at least one head (H):

$$E = \{HH, HT, TH\} \implies N(E) = 3$$

Step 3: Calculate the probability:

$$P(E) = \frac{N(E)}{N(S)} = \frac{3}{4}$$

Thus, the probability is exactly $\frac{3}{4}$, which corresponds to Option C.

Final Answer: $\boxed{\frac{3}{4}}$

Answer: (C)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	B	4	B	5	D
6	C	7	C	8	C	9	A	10	D
11	A	12	C	13	B	14	D	15	D
16	B	17	A	18	C	19	A	20	A
21	C	22	C	23	D	24	B	25	A
26	A	27	B	28	C	29	C	30	C
31	A	32	C	33	D	34	C	35	B
36	B	37	B	38	A	39	C	40	A
41	C	42	A	43	C	44	B	45	B
46	B	47	B	48	B	49	A	50	C