JEE Main 2024 Jan 27 (Shift 1) Mathematics Question Paper

Question 1. ${}^{n-1}C_r = (k^2 - 8)\ {}^{n}C_{r+1}$ **if and only if:**

(1) $2\sqrt{2} < k \leq 3$

(2) $2\sqrt{3} < k \leq 3\sqrt{2}$

(3) $2\sqrt{3} < k < 3\sqrt{3}$

(4) $2\sqrt{2} < k < 2\sqrt{3}$

Question 2. The distance of the point $(7, -2, 11)$ **from the line**

$$\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$$

along the line

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$$

is:

(1) 12

(2) 14

(3) 18

(4) 21

Question 3. Let $x = x(t)$ **and** $y = y(t)$ **be solutions of the differential equations**

$$\frac{dx}{dt} + ax = 0 \quad \text{and} \quad \frac{dy}{dt} + by = 0$$

respectively, $a, b \in \mathbb{R}$. Given that $x(0) = 2$, $y(0) = 1$ and $3y(1) = 2x(1)$, the value of t, for which $x(t) = y(t)$, is:

1. $\log_{\frac{2}{3}} 2$
2. $\log_4 3$
3. $\log_3 4$
4. $\log_4 \frac{2}{3}$

Question 4. If (a, b) be the orthocentre of the triangle whose vertices are $(1, 2)$, $(2, 3)$, and $(3, 1)$, and

$$I_1 = \int_a^b x \sin(4x - x^2)\, dx, \quad I_2 = \int_a^b \sin(4x - x^2)\, dx$$

then $36\frac{I_1}{I_2}$ is equal to:

(1) 72

(2) 88

(3) 80

(4) 66

Question 5. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then:

(1) $A = B^3$

(2) $3A = B$

(3) $B = A^3$

(4) $A = 3B$

Question 6. The number of common terms in the progressions 4, 9, 14, 19, ..., up to 25th term and 3, 6, 9, 12, ..., up to 37th term is:

(1) 9

(2) 5

(3) 7

(4) 8

Question 7. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to:

(1) 16

(2) 24

(3) 20

(4) 36

Question 8. If the shortest distance between the lines

$$\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3} \quad \text{and} \quad \frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$$

is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is:

(1) 5

(2) 8

(3) 7

(4) 10

Question 9. Evaluate the integral

$$\int_0^1 \frac{1}{\sqrt{3+x} + \sqrt{1+x}}\, dx$$

Given that the integral can be expressed in the form $a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, find the value of $2a + 3b - 4c$.

(1) 4

(2) 10

(3) 7

(4) 8

Question 10. Let $S = \{1, 2, 3, \ldots, 10\}$. Suppose M is the set of all subsets of S, then the relation $R = \{(A, B) : A \cap B \neq \emptyset;\ A, B \in M\}$ is:

(1) symmetric and reflexive only

(2) reflexive only

(3) symmetric and transitive only

(4) symmetric only

Question 11. If $S = \{z \in \mathbb{C} : |z - i| = |z + i| = |z - 1|\}$, then $n(S)$ is:

(1) 1

(2) 0

(3) 3

(4) 2

Question 12. Four distinct points $(2k, 3k)$, $(1, 0)$, $(0, 0)$, and $(0, 1)$ lie on a circle for k equal to:

(1) $\frac{2}{13}$

(2) $\frac{3}{13}$

(3) $\frac{5}{13}$

(4) $\frac{1}{13}$

Question 13. Consider the function:

$$f(x) = \begin{cases} \frac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x < 3 \\ \frac{\sin(x-3)}{2^{x-\lfloor x \rfloor}}, & x > 3 \\ b, & x = 3 \end{cases}$$

Where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x = 3$, then the number of elements in S is:

(1) 2

(2) Infinitely many

(3) 4

(4) 1

Question 14. Let $a_1, a_2, \ldots, a_{10}$ be 10 observations such that

$$\sum_{k=1}^{10} a_k = 50 \quad \text{and} \quad \sum_{1 \le k < j \le 10} a_k \cdot a_j = 1100.$$

Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to:

(1) 5

(2) $\sqrt{5}$

(3) 10

(4) $\sqrt{115}$

Question 15. The length of the chord of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, whose midpoint is $\left(1, \frac{2}{5}\right)$, is equal to:

(1) $\frac{\sqrt{1691}}{5}$

(2) $\frac{\sqrt{2009}}{5}$

(3) $\frac{\sqrt{1741}}{5}$

(4) $\frac{\sqrt{1541}}{5}$

Question 16. The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is:

(1) $\frac{8}{5}$

(2) $\frac{25}{41}$

(3) $\frac{2}{5}$

(4) $\frac{30}{41}$

Question 17. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot \left((\vec{c} \times \vec{b}) - \vec{b} \cdot \vec{c}\right)$ is equal to:

(1) 32

(2) 24

(3) 20

(4) 36

Question 18. If $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is:

(1) 36

(2) 32

(3) 25

(4) 30

Question 19. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements:

Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II: $f(x) \cdot f(y) = f(x+y)$.

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Question 20. The function $f : \mathbb{N} - \{1\} \to \mathbb{N}$ defined by $f(n) =$ the highest prime factor of n, is:

(1) both one-one and onto

(2) one-one only

(3) onto only

(4) neither one-one nor onto

Question 21. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is:

(1) 3

(2) 4

(3) 5

(4) 6

Question 22. Let for a differentiable function $f : (0, \infty) \to \mathbb{R}$,

$$f(x) - f(y) \geq \log_e\left(\frac{x}{y}\right) + x - y, \quad \forall\, x, y \in (0, \infty).$$

Then $\sum_{n=1}^{20} f'\left(\frac{1}{n}\right)$ is equal to:

(1) 2890

(2) 2850

(3) 3000

(4) 2750

Question 23. If the solution of the differential equation

$$(2x + 3y - 2)\, dx + (4x + 6y - 7)\, dy = 0, \quad y(0) = 3,$$

is $\alpha x + \beta y + 3\log_e|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to:

(1) 27

(2) 29

(3) 24

(4) 32

Question 24. **Let the area of the region** $\{(x, y) : x - 2y + 4 \geq 0,\ x + 2y^2 \geq 0,\ x + 4y^2 \leq 8,\ y \geq 0\}$ **be** $\frac{m}{n}$**, where** m **and** n **are coprime numbers. Then** $m + n$ **is equal to:**

(1) 100

(2) 119

(3) 137

(4) 145

Question 25. If

$$8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \dots,$$

then the value of p **is:**

(1) 7

(2) 8

(3) 9

(4) 10

Question 26. A fair die is tossed repeatedly until a six is obtained. Let X **denote the number of tosses required and let** $a = P(X = 3)$**,** $b = P(X \geq 3)$**, and** $c = P(X \geq 6 \,|\, X > 3)$**. Then** $\frac{b+c}{a}$ **is equal to:**

(1) 10

(2) 12

(3) 14

(4) 15

Question 27. Let the set of all $a \in \mathbb{R}$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$, and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ + \tan 81^\circ}$. Then pqr is equal to:

(1) 30

(2) 40

(3) 48

(4) 50

Question 28. Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in \mathbb{R}$. Then $f'(10)$ is equal to:

(1) 202

(2) 210

(3) 190

(4) 180

Question 29. Let

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = [B_1, B_2, B_3],$$

where B_1, B_2, B_3 are column matrices, and

$$AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \quad AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to:

(1) 16

(2) 20

(3) 24

(4) 28

Question 30. If α satisfies the equation $x^2+x+1=0$ and $(1+\alpha)^7 = A+B\alpha+C\alpha^2$, $A, B, C \geq 0$, then $5(3A-2B-C)$ is equal to:

(1) 10

(2) 5

(3) 15

(4) 20
