

JEE Main 2024 Mathematics Question Paper April 4 Shift 1

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
 - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.
 - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. If $f(x) = \begin{cases} x - 2, & 0 \leq x \leq 2 \\ -2, & -2 \leq x \leq 0 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$, then $\int_0^k h(x) dx$ is equal to
($k > 0$)

- (A) 0
(B) $\frac{k}{2}$
(C) $2k$
(D) k

2. There are three bags A, B and C. Bag A contains 7 black balls and 5 red balls, Bag B contains 5 red and 7 black balls and Bag C contains 7 red and 7 black balls. A ball is drawn and found to be black. Find the probability that it is drawn from Bag A.

3. Find the number of rational numbers in the expansion of $\left(\frac{1}{2^5} + \frac{1}{5^3}\right)^{15}$.

4. Find the value of $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + \sin x \cos x} dx$

5. If $x^2 - ax + b = 0$ has roots 2, 6 and $\alpha = \frac{1}{2a+1}$, $\beta = \frac{1}{2b-a}$. Find the equation having roots α, β .

6. $\lim_{x \rightarrow 4} \frac{(5+x)^{1/3} - (1+2x)^{1/3}}{(5+x)^{1/2} - (1+2x)^{1/2}}$

7. AB, BC, CA are sides of a triangle having 5, 6, 7 points respectively. How many triangles are possible using these points?

8. 2, p and q are in G.P. In an A.P., 2 is the 3rd term, p is the 7th term and q is the 8th term. Find p and q.

9. If the domain of the function $\sin^{-1} \left(\frac{3x-22}{2x-19}\right) + \log_e \left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $[\alpha, \beta]$ then $3\alpha + 10\beta$ is equal to

- (1) 100
 - (2) 95
 - (3) 97
 - (4) 98
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10.

$$x + (2 \sin 2\theta) y + 2 \cos 2\theta = 0$$

$$x + (\sin \theta) y + \cos \theta = 0$$

$$x + (\cos \theta) y - \sin \theta = 0$$

Find the nontrivial solution.

11. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in \mathbb{R}$. Consider a function $(g \circ f)(x) = x$ for all $x \in \mathbb{R}$. Then the value of $8g'(2)$ is

- (1) 4
 - (2) 16
 - (3) 8
 - (4) 2
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12. Let $f(x) = \frac{2x^2 - 3x + 9}{2x^2 + 3x + 4}$. If maximum value of $f(x)$ is m and minimum value is n , then find $m + n$.

14. Let α and β be the sum and the product of all the nonzero solutions of the equation

$$(\bar{z})^2 + |z| = 0, z \in \mathbb{C}.$$

Then $4(\alpha^2 + \beta^2)$ is equal to

- (1) 6
 - (2) 2
 - (3) 4
 - (4) 8
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15. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum(x_i^2 + y_i^2)$ is equal to:

- (1) 148
 - (2) 156
 - (3) 152
 - (4) 160
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16. If the differential equation satisfies

$$\frac{dy}{dx} - y = \cos x \text{ at } x = 0, y = -\frac{1}{2}, \text{ find } y\left(\frac{\pi}{4}\right).$$

17. Let $\alpha, \beta, \epsilon \in \mathbb{R}$. Let the mean and the variance of 6 observations $-3, 4, 7, -6, \alpha, \beta$ be 2 and 23 respectively. The mean deviation about the mean of these 6 observations is

- (1) $\frac{11}{3}$
 - (2) $\frac{16}{3}$
 - (3) $\frac{13}{3}$
 - (4) $\frac{14}{3}$
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18.

$\mathbf{a} = 2\hat{i} + 2\hat{j} - \hat{k}$, $\mathbf{b} = \hat{i} - \hat{k}$, \mathbf{c} is a unit vector making angle 60° with \mathbf{a} and 45° with \mathbf{b} .

Find \mathbf{c} .

19. If the length of the focal chord of $y^2 = 12x$ is 15 and if the distance of the focal chord from the origin is p , then $10p^2$ is equal to

- (1) 36
- (2) 25
- (3) 72
- (4) 144

20. Shortest distance between lines

$$\frac{x+1}{-2} = \frac{y}{1} = \frac{z-1}{1} \quad \text{and} \quad \frac{x-5}{2} = \frac{y-2}{-3} = \frac{z-1}{1}$$

is $\frac{38k}{6\sqrt{5}}$, find $\int_0^k [x^2] dx$.

21. If $y = y(x)$ is a solution of the differential equation

$$(x^4 + 2x^3 + 3x^2 + 2x + 2) \frac{dy}{dx} - (2x^2 + 2x + 3)y = 0. \quad \text{If } y(0) = \frac{\pi}{4}, \text{ find } y(-1)$$

22. Curve $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ intersects at point $(\frac{1}{2}, 2)$. Then the area enclosed between the curves is

$$\frac{1}{24} (\sqrt{5} + m) - \ln_e (1 + \sqrt{5})$$

Then find the value of $\ell + m + n$ is.
