JEE Main 2024 Mathematics Question Paper April 4 Shift 2

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. If a, b, and c are in Arithmetic Progression (A.P.), and $a+1$, b, $c+3$ are in Geometric Progression (G.P.) with $a > 10$ and the Arithmetic Mean (A.M.) of a, b, c is 8, then find the value of $(G.M.)^3$ of a, b, and c.

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2. Find the area bounded by the curves $y^2 \leq 2x$ and $y \geq 4x - 1$.

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3. Let $f(x) = \int_0^x \left(t + \sin(1 - e^t)\right) dt$, and $f(0) = 0$. Then, find $\lim_{x\to 0} \frac{f(x)}{x^3}$.

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4. If $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$, the maximum value is α and the minimum value is β, then find $\alpha^2 + \beta^2$.

5. Given that $\sin^{-1} x + \cos^{-1} y = \alpha$, where $\alpha \in \left(-\frac{\pi}{2}, \pi\right)$, find the value of $x^2 + y^2 - 2xy \sin\alpha$.

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6. Given the function

$$F(x) = \frac{72x^2 - 9x - 8x + 1}{\sqrt{2 - \sqrt{1 + \cos 2x}}}, \quad x \neq 0,$$

and

$$F(x) = a \ln 2 \cdot \ln 3, \quad x = 0.$$

If $f(x)$ is continuous at $x = 0$, find the value of a.

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7. Let $f(x) = 4/\sqrt{x-2} + \sqrt{4-x}$, find the maximum and minimum value of $f(x)$.

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8. Find the value of $p - q$, where

$$1^2 + 2^2 + 3^2 + \cdots + 100 \cdot 101^2 = \frac{p}{q}.$$

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9. A relation is $(x_1, y_1)R(x_2, y_2)$ is defined as $\{(x, y) \in \mathbb{N}, x_1 \leq x_2, y_1 \leq y_2\}$, then the relation is:

(1) Reflexive and symmetric
(2) Symmetric and transitive
(3) Transitive and reflexive
(4) None

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10. If

$$\int \csc^6 \theta \, d\theta = \alpha(f(x))^4 + \beta(f(x))^2 + \gamma|f(x)| + C,$$

where C is the constant of integration, find

$$|2\alpha + \beta + \gamma|.$$

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11. Coefficients of x^4, x^5, x^6 are in AP in $(1+x)^n$. Find n.

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12. In group A, there are 4 men and 5 women, and in group B, there are 5 men and 4 women. If 4 people are selected from each group, find the number of ways to select 4 men and 4 women.

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13. A circle C_1 centered at $(0,0)$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the vertex. Another circle C_2 centered at the focus of the hyperbola touches circle C_1. If the areas of C_1 and C_2 are 36π and 4π respectively, then find the latus rectum of the hyperbola.

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14. If

$$\frac{dy}{dx} = \frac{1}{(x+y+2)^2} \quad \text{and} \quad f(0) = 0, \text{ then} \quad f(x) = \tan^{-1}\left(\frac{x+y}{x+y+\lambda}\right),$$

then find λ.

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15. Let

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad B = I + (\text{adj } A) + (\text{adj } A)^2 + \cdots + n \text{ terms. Then find B.}$$

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16. A team plays 10 games. In every game, the team wins with probability $\frac{2}{3}$ and loses with probability $\frac{1}{3}$. Let X be the number of wins of this team in these 10 games, while Y be the number of losses of this team in these 10 games. The probability that $|X - Y| \leq 2$ is:

17. Let

$$\mathbf{a} = i\hat{i} + j\hat{j} + k\hat{k} = 2i + 4j - 5k, \quad \mathbf{b} = i\hat{i} + 2j\hat{j} + 3k\hat{k}, \quad \mathbf{c} = x\hat{i} + 2j\hat{j} + 3k\hat{k},$$

where d is an unit vector in the direction of

$$\mathbf{b} + \mathbf{c}, \quad \text{such that} \quad \mathbf{a} \cdot \mathbf{d} = 1. \text{ Find } (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}.$$

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18. The equation $y^2 = 12x$ has a chord PQ with midpoint $(4, 1)$. The equation of PQ passes through:

(1) $(-4, 0)$
(2) $(4, 0)$
(3) $(4, 8)$
(4) None of these

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19. Solve the differential equation $(x^2 + 4^2)\frac{dy}{dx} + (2x^3y + 8xy - 2) = 0$, if $y = y(x)$; If $y(0) = 0$, then $y(2)$ is equal to:

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20. Two lines L_1 and L_2 are given and they intersect at point P. A and B are two points, $A(8, 7, -1)$ and $B(5, 1, 17)$. Find the minimum distance of point P from the line joining A and B.

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21. Centre of a circle is $(0, 0)$ and radius is $\sqrt{10}$. The line $x + y = 2$ is a chord of this circle. Another chord of slope -1 has length 2. Find the least possible distance between $x + y = 2$ and this chord.
