JEE Main 2024 Mathematics Question Paper April 5 Shift 1

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. Solve the differential equation:

$$\frac{dy}{dx} + 2y = \sin 2x \quad \text{and} \quad y(0) = \frac{3}{4}$$

Then, the value of $y\left(\frac{\pi}{2}\right)$ is.

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2. Let $f(x) = x^5 + x^4 + x^3 + 3x + 1$ and $f(g(x)) = x$ then value of $\frac{g(7)}{g'(7)}$ is .

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3. Find term independent of x **in** $\left(1 - x + 2x^2\right)\left(3x^2 + \frac{1}{x^3}\right)^9$.

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4. Area bounded by $Y = x^2 - 5x$ **and** $Y = 7x - x^2$**.**

5. Given that $\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{99.100}=n$ **and** $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}=m$**, find** (m,n)**.**

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6. Find the value of

$$\left|AA^{\top}(\text{adj}A)^{\top}(\text{adj}4B)(\text{adj}AB)^{\top}\right|$$

if $|A|=2, |B|=3$. (Given A is a 3×3 matrix)

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7. Find the value of I**, if**

$$I=\int_{-\pi/4}^{\pi/4}\frac{2y\sin y}{1+\cos^2 y}\,dy$$

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8. Evaluate the integral:

$$I=\int_{0}^{\frac{\pi}{2}}\frac{136\sin x}{5\sin x+3\cos x}\,dx$$

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9. If 4 dice are rolled, then find the probability that their sum comes out to be 16.

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10. Let Set $S=\{1,2,3,\ldots,8\}$**, and there are multiple quadratic equations of the form** $ax^2+bx+c=0$**, where** $a,b,c\in S$**. Find the probability such that a randomly chosen quadratic equation has equal roots.**

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11. Solve the equation $||x|-2|-|x-1|-6=0$ **and find the sum of real solutions of** x**.**

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12. Given the function $f(x)=\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}$, and $f(1)=1$, find the value of $2f(2)+3f(3)$.

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13. Let $f : A \to B$, where $A = \{1, 2, 3, \ldots, 8\}$ and $B = \{1, 2, \ldots, 8\}$, find the number of one-to-one functions from A to B such that $f(1) + f(3) = 14$.

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14. If the lines

$$\frac{x-3}{3} = \frac{2y-1}{4} = \frac{z-4}{7} \quad \text{and} \quad \frac{x-3}{3} = \frac{1-2y}{4} = \frac{z-4}{7}$$

are perpendicular, then find the value of $9\mu + 4$.

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15. Given the function

$$f(x) = \sin 2x + c + \frac{2}{\pi}\left(x^2 + x\right), \quad x \in \left[0, \frac{\pi}{2}\right]$$

Find the truth of the following statements:

- **Statement 1:** $f(x)$ is increasing in $\left(0, \frac{\pi}{2}\right)$
- **Statement 2:** $f(x)$ is decreasing in $\left(0, \frac{\pi}{2}\right)$

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16. Given a circle of radius 1 such that it touches the normals drawn from (3, 2) to the coordinate axis. Find minimum distance of circle from point (5, 5)

(1) 4
(2) $7\sqrt{2}$
(3) $4\sqrt{2}$
(4) $5\sqrt{2}$

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17. Suppose $\theta \in [0, \frac{\pi}{4}]$ is a solution of $4\cos\theta - 3\sin\theta = 1$, then $\cos\theta =$

(1) $\frac{6-\sqrt{6}}{3\sqrt{6}+2}$
(2) $\frac{4}{3\sqrt{6}+2}$
(3) $\frac{4}{3\sqrt{6}-2}$
(4) $\frac{4-\sqrt{6}}{3\sqrt{6}+2}$

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18. Given the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, where $x \in \mathbb{R} \setminus \{0\}$, and $f(x)$ is continuous at $x = 0$, find $|\alpha + \beta + f(0)|$.

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19. A rectangle ABCD is inscribed in another rectangle PQRS. Given the length and breadth of the ABCD are 2 and 4 respectively. The length and breadth of rectangle PQRS are a and b respectively. Find $(a+b)^2$ so that the area of PQRS is maximum.

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20. If two lines passing through origin cuts the line $3x + 4y = 12$, at P Q and $\triangle POQ$ is a right angle triangle, then minimum area is:

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21. If the length of the focal chord of $y^2 = 12x$ is ℓ and if the distance of the focal chord from the origin is d, then d^2 is equal to:
