

JEE Main 2024 Mathematics Question Paper April 5 Shift 2

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
 - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.
 - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. Find the coefficient of x^0 in the expansion of

$$\left(\frac{3^{1/5}}{x} + \frac{x}{5^{1/3}}\right)^{12}$$

2. Solve the equation $|x| + 2|x - 5| + 2x + 3 = 0$.

3. Minimum value of k so that $4x^{11} + 4x^{12}, \frac{k}{2}, 16x^{16} + 16x^{17}$ are in A.P.

4. Find the word formed by all letters of the word "HBBOJ" whose Rank is 50th when letters are arranged in dictionary order.

5. Let (α, β, γ) be the image of $(3, 5, 8)$ in the line

$$\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-2}{4}, \text{ then find the value of } 3\alpha + 4\beta + 5\gamma.$$

6. Let $f(x) = |x-1|$ and $g(x) = \begin{cases} e^x, & x \geq 0 \\ 3x, & x < 0 \end{cases}$, then $f(g(x))$ is one or many.

7. Find the area enclosed by $y = |x|$ and $y = x - |x|$.

8. Find the minimum value of the expression

$$(x-7)^2 + \left(\sqrt{9 - (x-4)^2} - 7 - 4 \right)^2 \quad \text{where } x \in [1, 7].$$

9. Find the number of points of discontinuity of the function $f(x) = 2x^2 + [x^2] - [x]$, where $x \in [-1, 2]$.

10. Given the circles $C_1 : (x-1)^2 + (y-1)^2 = 1$ and $C_2 : \text{center}(-1, 0), R = 2$, find the number of common chords that pass through the y-axis at P. Also, find the square of the distance from P to the center of C_1 .

11. Let $S = \{2, 2^2, 2^3, 2^4, \dots, 2^9\}$, and $A, B,$ and C are subsets of S having equal elements. Given that $A \cap B = B \cap C = C \cap A = \emptyset$, find the total number of ways $A, B,$ and C can be chosen if $A \cup B \cup C = S$.

12. Let set $A = \{1, 2, 3, 4, 5, 6\}$ and $ax^2 + bx + c = 0$ be a quadratic equation where $(a, b, c \in A)$ has real roots. Then, if the probability that one root is greater than the other is P , find the value of $43P$.

13. Find the number of solutions of the equation:

$$5|x| + 3|x + 3| + 7|x - 7| = 5.$$

14. Find the D.E. of the circle whose center lies on $y = x$ and passes through the origin.

15. A square ABCD of side 4 is given and a square AEFG of side 2 is given where F is the midpoint of AB. Find the radius of the circle which touches BC, CD and passes through F.

16. Area of the region enclosed by

$$S_1 : |z| \leq 5, \quad S_2 : \operatorname{Re}(z) \geq 0 \text{ and } \operatorname{Im}(z) \geq 0.$$

Find the area of the region enclosed by

$$\left| \frac{z + 1 - \sqrt{3}i}{1 - \sqrt{3}i} \right| \geq 0.$$

17. Let

$$\frac{dy}{dx} + \frac{2xy}{(1+x^2)^2} = x \cdot e^{-1+x^2}, \quad y(0) = \frac{1}{e}, \quad y(x) = g(x) \cdot e^{-(1+x^2)},$$

then the area bounded by $y = g(x)$ and $y - x = 4$.

18. The number of real solutions of the equation:

$$x|x| + 5|x| + 2|x| + 7|x| - 2 = 0.$$
