

JEE Main 2024 Mathematics Question Paper April 6 Shift 1

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
 - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
 - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. I =

$$I = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x \sin^2 x}{(\cos^3 x + \sin^3 x)^2} dx$$

2. An equilateral triangle of side 12. A circle is embedded inside the triangle. And a square is embedded inside the circle. If area and perimeter of the square is 'm' and 'n' respectively, then find $m + n^2$.

3. Solve:

$$(1 + x^2) \frac{dy}{dx} + y = \frac{e^{\tan^{-1} x}}{1 + x^2}, \quad y(1) = 0 \text{ then find } y(0)$$

4. Find the range of x for which $f(x) = x^x$ (where $x > 0$) is strictly increasing.

5. Let $A = \{100, 101, 102, \dots, 700\}$. Find the number of numbers in set A which are neither divisible by 3 nor by 4.

6. Given that $2x \ln x \frac{dy}{dx} + y = 3x \ln x$, and $y(1) = 0$, find y .

7. Let

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2} + \alpha \\ 2r & 2 & n^2 - \beta \\ 3r - 2 & 3 & n(n - 1) \end{vmatrix}$$

Find $2A_{10} - A_8$.

8. In an octagon, how many triangles are possible so that no side of the triangle is a side of the octagon?

9. A variable line is passing through $(4, -9)$, slope of line is positive, and it makes intercepts on x and y -axis at points A and B . Find the minimum area of triangle OAB .

10. If the mean of 20 observations is 10, and $SD = 2$. One of the observations, which is 12, is replaced by 8. Find the value of the new SD .

11. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{x^2 - 2x - 15}{x^2 - 4x + 9}$$

Then f is:

- (1) one-one onto
- (2) many-one onto
- (3) many-one into
- (4) one-one into

12. A company has two branches A and B. 'A' produces 60% of total production and the remaining is produced by 'B'. Branch 'A' produces 80% good quality products and branch 'B' produces 90% good quality products. A product is selected at random and that was of good quality. Let P be the probability that the selected product is from branch 'B'. Find the value of $126P$.

13. Find the shortest distance between the lines

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5} \quad \text{and} \quad \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$

14. If in the expansion of $(x+y)^n$,

$$T_2 = 15, \quad T_3 = 10, \quad T_4 = \frac{10}{3}$$

for $n = 5$, find the value of $n^3 + x^5 + 243y^5$.

15. Let $S = \{1, 2, 3, \dots, 20\}$ be a given set. Relation R_1 is defined as $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : 4x = 5y\}$. If m denotes the number of elements required to make R_1 symmetric and n denotes the number of elements required to make R_2 symmetric, then find $m + n$.

16. Given a function

$$f(x) = \begin{cases} x^3 \cdot \sin\left(\frac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

Then find $f''\left(\frac{2}{\pi}\right)$.

17. Let α, β be the distinct roots of the quadratic equation

$$x^2 - (t^2 - 5t + 6)x + 1 = 0$$

and

$a_n = \alpha^n + \beta^n$, then the minimum value of $\frac{a_{2023} + a_{2025}}{a_{2024}}$ is:

18. Let the area of the region enclosed by the curves

$$y = 3x, \quad y = 27 - 3x, \quad y = 3x - \sqrt{x} \quad \text{be } A.$$

Then, $10A$ is equal to:

19. If $\cot^{-1} 3 + \cot^{-1} 4 + \cot^{-1} 5 + \cot^{-1} n = \frac{\pi}{4}$, then $n =$
