JEE Main 2024 Mathematics Question Paper April 6 Shift 2

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. Let $f(x) = \frac{1}{7-\sin x}$, then Range of $f(x)$ is

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2. Given $|A| = 3$, order = 3, find $m+n$ given that $|\mathbf{adj}(4\cdot\mathbf{adj}(-3\cdot\mathbf{adj}(3\cdot\mathbf{adj}(2A)^{-1})))| = 2^m \cdot 3^n$

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3. Let $x^2 + \sqrt{2}x - 8 = 0$ and $S_n = \alpha^n + \beta^n$, where α and β are roots of the given equation, $(n \geq 1)$. Find the value of

$$\frac{S_{10} + \sqrt{2}S_9}{8S_8}$$

4.

$$\int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}\, dx = \frac{1}{12} \tan^{-1}(3 \tan x) + c$$

Find the maximum and minimum values of $a\sin x + b\cos x$.

5.

$$1(1+x) + 2(1+x)^2 + 3(1+x)^3 + \cdots + 60(1+x)^{60}$$

Let $S = (1+x)^{61}$, then $S = \frac{60}{x} - \frac{1}{x^2} + (1+x)$.

6.

$$\vec{A} = \hat{i} + \hat{j} - 2\hat{k}, \quad \vec{B} = (\vec{A} \times (\hat{i} + \hat{j})) \times \hat{i}$$

Then find the projection of $\vec{A}$ on $\vec{B}$.

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7. In $\triangle ABC$, vertices $A(2,5)$, $B(8,3)$, and $C(h,k)$ and orthocenter is $(6,1)$. Then find the value of $2h+k$.

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8. Sides of a triangle are $AB = 9$, $BC = 7$, $AC = 8$. Find $\cos C$.

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9. Find the locus of P such that the ratio of distance of P from $A(3,1)$ and $B(1,2)$ is $5:4$.

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10. If the area enclosed by the region $\frac{a}{x^2} \le y \le \frac{1}{x}$ between $x = 1$ and $x = 2$ (where $a \in (0,1)$) is $\ln 2 - \frac{1}{7}$, then find $7a - 3$.

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11. Let $A = \{1,2,3,4,5\}$, a relation is defined as $4x \ge 5y$, $x \in A$, $y \in A$. The number of elements in $R = m$ and the number of elements in $A \times A$ is n. Then find $m+n$.

12.

$$\binom{n+1}{r+1} \cdot \binom{n}{r-1} = 55 : 35 : 35 : 21 \quad \textbf{then} \quad 2n + 5r \textbf{ is equal to}$$

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13. If $f(x) = 3 + \left[\frac{x}{2}\right] - [\sqrt{x}]$ and $x \in [0, 8]$, then find the sum of points of discontinuity of $f(x)$.

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14. If the function $f(x) = \left(\frac{1}{x}\right)^x$ for $x > 0$ attains the maximum value at $x = \frac{1}{e}$, then:

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15. NAGPUR, rearrange it and find the 315th word in the dictionary.

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16. A curve $e^y \sin x + \cos x(e^{y+1})\, dx = 0$ passes through $\left(\frac{\pi}{2}, 0\right)$. Then find $e^{y(\frac{\pi}{6})}$.

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17. M computers complete a work in 17 days. If 4 computers decrease every day, then it takes 8 more days to complete the same work. Find the value of M.

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18. Let $g(x) = h(e^x) \cdot e^{h(x)}$, and it is given that $h(0) = 0$, $h(1) = 1$, $h'(0) = 2$, then find $g'(0)$.

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19. If $\int_0^2 \left[x^2 + \left(\frac{x^2}{2}\right)^2\right] dx = a + b\sqrt{2} + c\sqrt{3}$, then find the value of $a + b + c$.

20.

$$\frac{x-\lambda}{-1} = \frac{y-2}{1} = \frac{z-3}{2} \quad \text{and} \quad \frac{x-1}{2} = \frac{y-4}{3} = \frac{z-0}{1}$$

The shortest distance between these two given lines is $\frac{44}{\sqrt{3}}$, find λ.
