# JEE Main 2024 Mathematics Question Paper April 8 Shift 1

| **Time Allowed :** 3 Hours | **Maximum Marks :** 300 | **Total Questions :** 90 |
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## General Instructions

**Read the following instructions very carefully and strictly follow them:**

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

### Mathematics

**1. Find the sum of solutions of the equation**
$8^{2a} - 16 \cdot 8^a + 48 = 0$

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**2. Find the sum of diagonal elements of the matrix $A^{13}$ where**

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

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**3. Given that** $\sin x = -\frac{4}{5}$, where $\theta \in$ IIIrd quadrant, then find the value of $3\tan^2 x - \cos x$.

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**4. Solve**

$$\int \frac{6dx}{\sin^2 x(1 - \cot^2 x)}$$

**5. Let** $f(x) = \cos x - x + 1$ for all $x \in [0, \pi]$. Let $M$ and $m$ be the maximum and minimum values of $f(x)$.
$m)$.

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**6. Find the shortest distance between the lines**

$$\mathbf{r}_1 = (5+\mu)\hat{\imath} + (1-3\mu)\hat{\jmath} + (1+2\mu)\hat{k} \quad \text{and} \quad \mathbf{r}_2 = (2+\lambda)\hat{\imath} + (3-3\lambda)\hat{\jmath} + (3+4\lambda)\hat{k}$$

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**7. Let $A$ be a 3 × 3 matrix where**

$$A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 1 & b \end{bmatrix} \quad \text{and} \quad A^3 = 3A^2 + 2I, \quad \text{then find the value of } 3a + b.$$

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**8. Find the area where**

$$A = \min(\sin x, \cos x) \quad \text{in} \quad x \in [-\pi, \pi]$$

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**9. If**

$$I_n = \int_0^1 \left(1 - x^k\right)^n dx, \quad \text{if} \quad I_{147,21} = 148 I_{20},$$

then find the value of k.

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**10. Find the value of**

$$\lim_{x \to 0} \left[ \frac{1 - \cos x \cdot \cos 2x \cdot \cos 3x \cdot \ldots \cdot \cos 10x}{x^2} \right]$$

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**11. A Differential Equation is given as**

$$(1 + y^2) e^{\tan x} \, dx + (1 + e^{\tan x}) \cos^2 x \, dy = 0, \quad y(0) = 1,$$

then find the value of $y\left(\frac{\pi}{4}\right)$.

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**12. Find the number of three-digit numbers that can be formed using the digits** $\{2, 3, 4, 5, 7\}$, which are not divisible by 3 and where repetition is not allowed.

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**13. Given**

$$\operatorname{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5} \quad \text{and} \quad |z+2| = 1, \quad \text{then find the value of} \quad |\operatorname{Re}(z+2)|.$$

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**14. There are two natural numbers $A$, $B$ such that their sum is 24. Then find the probability that the product of $A$ and $B$ is not less than $\frac{3}{4}$ of the maximum product of $A$ and $B$.**

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**15. Range of**

$$\frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta} \quad \text{is} \quad [a, b]. \quad \text{If the first term of G.P. is 64 and the common ratio is} \quad \frac{\alpha}{\beta}, \quad \text{find the sum o}$$

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**16. Given that**

$$A + 5B = 42 \quad \text{where} \quad A, B \in \mathbb{N}, \quad \text{and the number of pairs of} \quad (A, B) \quad \text{is} \quad m, \quad \text{then find the value of}$$

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**17. Given a hyperbola**

$$\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and the eccentricity is} \quad \sqrt{3}, \quad \text{length of latus rectum of the given hyperbola is} \quad 4\sqrt{3},$$

and a point $P(\alpha, \beta)$ lies on the hyperbola, where the product of the distance from foci is $\beta$.Find the valu $\beta$.

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**18. There are two circles:**

$$C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2 \quad \text{and} \quad C_2 : (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2,$$

both of them touch each other at $(6, 6)$, and the common point divides the distance between the centres c
$(\alpha^2 + \beta^2) + 4(r_1^2 + r_2^2)$.

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**19. If**

$$f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x, \quad \forall x \in (0, 2\pi),$$

then find the number of local maxima $y \in (0, 2\pi)$.

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**20. Given a triangle ABC such that the equation of AB is** $4x+3y = 14$, and the equation of AC is $3x-2y = 5$, and a point $P\left(2, -\frac{4}{3}\right)$ divides BC in the ratio$2:1$ internally, find the equation of BC.

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