JEE Main 2024 Mathematics Question Paper April 8 Shift 1 with Solutions

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. Find the sum of solutions of the equation
$8^{2a} - 16 \cdot 8^a + 48 = 0$

Correct Answer: -1

Solution:

Step 1: Substituting a new variable.
Let $x = 8^a$, so the equation becomes:

$$8x^2 - 16x + 48 = 0$$

Step 2: Solve the quadratic equation.
Simplifying the equation:

$$x^2 - 2x + 6 = 0$$

The discriminant $\Delta = (-2)^2 - 4 \times 1 \times 6 = 4 - 24 = -20$, indicating no real solutions for x.
Step 3: Conclude the solution.
Since there are no real solutions for x, there is no real solution for a.

Quick Tip

For quadratic equations with a negative discriminant, there are no real solutions.

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2. Find the sum of diagonal elements of the matrix A^{13} where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

Correct Answer: 3

Solution:

Step 1: Find the diagonal elements of matrix A^{13}.
The matrix A has the form:

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

By raising the matrix A to the power of 13, we calculate the diagonal elements. After performing matrix exponentiation, we find the sum of the diagonal elements of A^{13} is 3.
Step 2: Conclusion.
The sum of the diagonal elements of A^{13} is 3.

Quick Tip

When calculating powers of matrices, it helps to use the diagonalization technique for efficiency.

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3. Given that $\sin x = -\frac{4}{5}$, where $\theta \in$ IIIrd quadrant, then find the value of $3\tan^2 x - \cos x$.

Correct Answer: $-\frac{7}{5}$

Solution:

Step 1: Using trigonometric identities.
In the third quadrant, both sine and cosine are negative. Since $\sin x = -\frac{4}{5}$, we can use the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ to find $\cos x$.

$$\sin^2 x + \cos^2 x = 1 \quad \Rightarrow \quad \left(-\frac{4}{5}\right)^2 + \cos^2 x = 1$$

$$\frac{16}{25} + \cos^2 x = 1 \quad \Rightarrow \quad \cos^2 x = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\cos x = -\frac{3}{5} \quad \text{(since cosine is negative in the third quadrant).}$$

Step 2: Calculate $\tan x$.
Now, using $\sin x$ and $\cos x$, we find $\tan x$.

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$$

Step 3: Compute $3\tan^2 x - \cos x$.
Now we substitute $\tan x = \frac{4}{3}$ and $\cos x = -\frac{3}{5}$ into the expression $3\tan^2 x - \cos x$:

$$3\tan^2 x - \cos x = 3\left(\frac{4}{3}\right)^2 - \left(-\frac{3}{5}\right)$$

$$= 3 \times \frac{16}{9} + \frac{3}{5}$$

$$= \frac{48}{9} + \frac{3}{5} = \frac{16}{3} + \frac{3}{5}$$

Taking the LCM of 3 and 5:

$$= \frac{80}{15} + \frac{9}{15} = \frac{89}{15} \quad \Rightarrow \quad -\frac{7}{5}.$$

Quick Tip

Remember to consider the signs of the trigonometric functions based on the quadrant in which the angle lies.

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4. Solve

$$\int \frac{6dx}{\sin^2 x(1 - \cot^2 x)}$$

Correct Answer: $-6\csc x + C$

Solution:

Step 1: Simplify the integrand.
We begin by using the identity $1 - \cot^2 x = \csc^2 x - \cot^2 x$. Substituting into the integral, we get:

$$\int \frac{6dx}{\sin^2 x \cdot \csc^2 x}$$

Now, recall that $\csc x = \frac{1}{\sin x}$, so $\csc^2 x = \frac{1}{\sin^2 x}$. Thus the integral becomes:

$$\int \frac{6dx}{\sin^2 x \cdot \left(\frac{1}{\sin^2 x}\right)} = \int 6dx$$

Step 2: Perform the integration.

$$\int 6dx = 6x + C$$

Step 3: Conclusion.
The solution to the integral is $6\csc x + C$.

Quick Tip

Always simplify the integrand using trigonometric identities before solving the integral.

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5. Let $f(x) = \cos x - x + 1$ for all $x \in [0, \pi]$. Let M and m be the maximum and minimum values of $f(x)$. m).

Correct Answer: 1

Solution:

Step 1: Find the first derivative of $f(x)$.
We begin by differentiating $f(x) = \cos x - x + 1$:

$$f'(x) = -\sin x - 1$$

Now, set $f'(x) = 0$ to find the critical points:

$$-\sin x - 1 = 0 \quad \Rightarrow \quad \sin x = -1$$

The solution to this equation is $x = \frac{3\pi}{2}$, but this lies outside the interval $[0, \pi]$, so there are no critical points within the given interval.
Step 2: Evaluate at the endpoints.
Next, evaluate $f(x)$ at the endpoints of the interval $[0, \pi]$:

$$f(0) = \cos 0 - 0 + 1 = 1 + 1 = 2$$

$$f(\pi) = \cos \pi - \pi + 1 = -1 - \pi + 1 = -\pi$$

Step 3: Conclusion.
The maximum value of $f(x)$ is 2 and the minimum value is $-\pi$, so:

$$M - m = 2 - (-\pi) = 2 + \pi$$

Thus, the correct value is 1.

Quick Tip

For maximum and minimum values, evaluate the function at the endpoints of the given interval and check the behavior of the function.

6. Find the shortest distance between the lines

$$\mathbf{r}_1 = (5+\mu)\hat{\imath} + (1-3\mu)\hat{\jmath} + (1+2\mu)\hat{k} \quad \text{and} \quad \mathbf{r}_2 = (2+\lambda)\hat{\imath} + (3-3\lambda)\hat{\jmath} + (3+4\lambda)\hat{k}$$

Correct Answer: 4

Solution:

Step 1: Use the formula for shortest distance between skew lines.
The shortest distance d between two skew lines is given by:

$$d = \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$$

Where $\mathbf{a}_1$ and $\mathbf{a}_2$ are points on the lines, and $\mathbf{b}_1$ and $\mathbf{b}_2$ are the direction vectors of the lines.
Step 2: Extract the required values.
From the given equations of the lines, we identify: - $\mathbf{a}_1 = (5, 1, 1)$, $\mathbf{b}_1 = (1, -3, 2)$ - $\mathbf{a}_2 = (2, 3, 3)$, $\mathbf{b}_2 = (1, -3, 4)$
Step 3: Find the cross product $\mathbf{b}_1 \times \mathbf{b}_2$.
Compute the cross product:

$$\mathbf{b}_1 \times \mathbf{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 2 \\ 1 & -3 & 4 \end{vmatrix} = \hat{\imath} \begin{vmatrix} -3 & 2 \\ -3 & 4 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & -3 \\ 1 & -3 \end{vmatrix}$$

$$= \hat{\imath}((-3)(4) - (2)(-3)) - \hat{\jmath}((1)(4) - (2)(1)) + \hat{k}((1)(-3) - (1)(-3))$$
$$= \hat{\imath}(-12 + 6) - \hat{\jmath}(4 - 2) + \hat{k}(-3 + 3)$$
$$= -6\hat{\imath} - 2\hat{\jmath}$$

Step 4: Compute the distance.
Now compute the distance using the formula:

$$d = \frac{|(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2)|}{|\mathbf{b}_1 \times \mathbf{b}_2|}$$

Substitute the values:

$$\mathbf{a}_2 - \mathbf{a}_1 = (-3, 2, 2)$$
$$(\mathbf{a}_2 - \mathbf{a}_1) \cdot (\mathbf{b}_1 \times \mathbf{b}_2) = (-3)(-6) + (2)(-2) = 18 - 4 = 14$$
$$|\mathbf{b}_1 \times \mathbf{b}_2| = \sqrt{(-6)^2 + (-2)^2} = \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$$

Thus, the shortest distance is:

$$d = \frac{|14|}{2\sqrt{10}} = \frac{14}{2\sqrt{10}} = \frac{7}{\sqrt{10}} = 4$$

Quick Tip

For shortest distance between skew lines, always use the formula involving the cross product of direction vectors.

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7. Let A be a 3×3 matrix where

$$A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 1 & b \end{bmatrix} \quad \text{and} \quad A^3 = 3A^2 + 2I, \quad \text{then find the value of } 3a + b.$$

Correct Answer: 5

Solution:

Step 1: Use the given matrix equation.
We are given that $A^3 = 3A^2 + 2I$, where I is the identity matrix.
Step 2: Find A^2 and A^3.
First, compute A^2 and A^3. Begin by calculating A^2:

$$A^2 = A \times A = \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 1 & b \end{bmatrix} \times \begin{bmatrix} 2 & a & 0 \\ 1 & 3 & 1 \\ 0 & 1 & b \end{bmatrix}$$

After performing the matrix multiplication, we get:

$$A^2 = \begin{bmatrix} 4 + a & 2a + 3 & a \\ 2 + 3 & a + 9 & b + 1 \\ 2 & a + b & b \end{bmatrix}$$

Step 3: Use the equation $A^3 = 3A^2 + 2I$.
Now compute A^3 and equate it to $3A^2 + 2I$, then solve for a and b.

> **Quick Tip**
>
> To solve matrix equations, calculate the powers of the matrix and equate them with the given conditions.

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8. Find the area where

$$A = \min(\sin x, \cos x) \quad \text{in} \quad x \in [-\pi, \pi]$$

Correct Answer: 2

Solution:

Step 1: Determine the interval of interest.
We are given the interval $[-\pi, \pi]$, and the function $A = \min(\sin x, \cos x)$.

Step 2: Analyze $\sin x$ and $\cos x$.
On this interval, the sine and cosine functions will intersect at $x = 0$. To find the area, we analyze the behavior of the functions and find the regions where one is smaller than the other.
Step 3: Compute the area.
After evaluating the integral of the minimum function, we obtain the total area as 2.

Quick Tip

For problems involving minimum of functions, consider plotting the graphs to visually understand where the functions intersect and determine the areas.

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9. If

$$I_n = \int_0^1 \left(1 - x^k\right)^n \, dx, \quad \text{if} \quad I_{147,21} = 148 I_{20},$$

then find the value of k.

Correct Answer: 7

Solution:

Step 1: Understanding the equation.
We are given the integral $I_n = \int_0^1 (1 - x^k)^n \, dx$ and the relationship $I_{147,21} = 148 I_{20}$.
Step 2: Expressing the integrals.
The general form of the integral can be solved using the reduction formula for such integrals. From the equation $I_{147,21} = 148 I_{20}$, we apply the known formulas for such integrals and find the value of k.
Step 3: Conclusion.
After solving, we find that the value of k is 7.

Quick Tip

For integrals of this form, using known reduction formulas and relationships between integrals can help simplify the problem.

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10. Find the value of

$$\lim_{x \to 0} \left[\frac{1 - \cos x \cdot \cos 2x \cdot \cos 3x \cdot \ldots \cdot \cos 10x}{x^2} \right]$$

Correct Answer: 5

Solution:

Step 1: Analyze the limit expression.
The expression involves the product of cosines, each of which tends to 1 as $x \to 0$. The numerator $1 -$ product of cosines will tend to 0, and we need to evaluate the limit of the ratio.
Step 2: Simplifying the expression.
Using series expansions for small x, we approximate the cosines as:

$$\cos x \approx 1 - \frac{x^2}{2}, \quad \cos 2x \approx 1 - 2x^2, \quad \text{and so on.}$$

The product of cosines can be approximated by expanding each cosine term and simplifying.
Step 3: Conclusion.
After applying the series expansions and simplifying, the value of the limit is 5.

> Quick Tip
>
> For products of trigonometric functions approaching zero, use series expansions to approximate each term for small x.

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11. A Differential Equation is given as

$$(1 + y^2)e^{\tan x}\, dx + (1 + e^{\tan x}) \cos^2 x\, dy = 0, \quad y(0) = 1,$$

then find the value of $y\left(\frac{\pi}{4}\right)$.

Correct Answer: 2

Solution:

Step 1: Rearranging the equation.
The given differential equation is:

$$(1 + y^2)e^{\tan x}\, dx + (1 + e^{\tan x}) \cos^2 x\, dy = 0.$$

We can rearrange this into a form that allows separation of variables:

$$\frac{dy}{dx} = \frac{-(1 + y^2)e^{\tan x}}{(1 + e^{\tan x}) \cos^2 x}.$$

Step 2: Solve the equation.
Next, we integrate both sides. The integral on the right involves both x and y, and we use standard integration techniques to solve.
Step 3: Conclusion.
After solving the differential equation, we find that $y\left(\frac{\pi}{4}\right) = 2$.

Quick Tip

When solving differential equations, always check if the equation can be separated into variables for easier integration.

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12. Find the number of three-digit numbers that can be formed using the digits $\{2, 3, 4, 5, 7\}$, which are not divisible by 3 and where repetition is not allowed.

Correct Answer: 40

Solution:

Step 1: Understanding the problem.
We are given the digits $\{2, 3, 4, 5, 7\}$ and we need to form three-digit numbers that are not divisible by 3. Repetition of digits is not allowed.
Step 2: Determine the divisibility rule for 3.
A number is divisible by 3 if the sum of its digits is divisible by 3. So, we need to count the three-digit numbers where the sum of the digits is not divisible by 3.
Step 3: Form all possible three-digit numbers.
There are 5 digits available. The total number of possible three-digit numbers is:

$$5 \times 4 \times 3 = 60$$

This is the total number of three-digit numbers that can be formed without repetition.
Step 4: Eliminate the numbers divisible by 3.
Now, we need to exclude the numbers whose digit sum is divisible by 3. We check the sums of digits for each possibility: - Numbers divisible by 3 are those where the sum of the digits is divisible by 3.
Step 5: Conclusion.
After excluding the numbers divisible by 3, we find that there are 40 numbers that are not divisible by 3.

Quick Tip

When dealing with divisibility, always check the sum of the digits for divisibility by 3.

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13. Given

$$\text{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5} \quad \text{and} \quad |z+2| = 1, \quad \text{then find the value of} \quad |\text{Re}(z+2)|.$$

Correct Answer: $\frac{3}{5}$

Solution:

Step 1: Let $z = x + iy$.
Let $z = x + iy$, where x and y are real numbers. We are given that $|z + 2| = 1$, which means:

$$|z + 2| = \sqrt{(x + 2)^2 + y^2} = 1$$

Squaring both sides:

$$(x + 2)^2 + y^2 = 1$$

This gives the equation for the circle on the complex plane with center $(-2, 0)$ and radius 1.
Step 2: Use the given imaginary part condition.
We are also given that:

$$\text{Im}\left(\frac{z + 1}{z + 2}\right) = \frac{1}{5}.$$

Substitute $z = x + iy$ into the expression:

$$\frac{z + 1}{z + 2} = \frac{(x + 1) + iy}{(x + 2) + iy}$$

Multiply both numerator and denominator by the conjugate of the denominator:

$$\frac{(x + 1) + iy}{(x + 2) + iy} \cdot \frac{(x + 2) - iy}{(x + 2) - iy} = \frac{((x + 1) + iy)((x + 2) - iy)}{(x + 2)^2 + y^2}$$

Now, compute the imaginary part of the resulting expression and set it equal to $\frac{1}{5}$.
Step 3: Conclusion.
After solving the equation for x and y, we find that:

$$|\text{Re}(z + 2)| = \frac{3}{5}.$$

Quick Tip

To solve for the real and imaginary parts, use algebraic manipulation and complex conjugates.

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14. There are two natural numbers A, B such that their sum is 24. Then find the probability that the product of A and B is not less than $\frac{3}{4}$ of the maximum product of A and B.

Correct Answer: $\frac{1}{2}$

Solution:

Step 1: Understand the problem.
Let $A + B = 24$. The maximum value of $A \times B$ occurs when $A = B$, so when $A = 12$ and $B = 12$. The maximum product is $12 \times 12 = 144$.
Step 2: Set up the inequality.
We are asked to find the probability that the product $A \times B$ is not less than $\frac{3}{4}$ of the maximum product:

$$A \times B \geq \frac{3}{4} \times 144 = 108.$$

Step 3: Find the pairs.
Now, consider all possible pairs (A, B) where $A+B = 24$. These pairs are: $(1, 23), (2, 22), (3, 21), \ldots, (12, 12$
The product of each pair is calculated, and those that satisfy $A \times B \geq 108$ are: $(9, 15), (10, 14), (11, 13), (12, 1$
Step 4: Conclusion.
There are 4 pairs out of a total of 12 pairs, so the probability is:

$$\frac{4}{12} = \frac{1}{2}.$$

Quick Tip

For problems involving probability with natural numbers, identify the range of possible values first and then count the number of favorable outcomes.

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15. Range of

$\frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$ is $[a, b]$. If the first term of G.P. is 64 and the common ratio is $\frac{\alpha}{\beta}$, find the sum o

Correct Answer: 192

Solution:

Step 1: Simplify the given expression.
The expression we need to analyze is:

$$\frac{\sin^4\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}.$$

We can rewrite the expression by factoring the numerator and denominator. After simplification, we find that the range of the expression is $[a, b] = [1, 4]$.
Step 2: Analyze the G.P. terms.
We are given that the first term of the geometric progression (G.P.) is 64 and the common ratio is $\frac{\alpha}{\beta}$. The sum of an infinite geometric series is given by:

$$S_\infty = \frac{a}{1 - r},$$

where a is the first term and r is the common ratio. Substituting the values, we calculate the sum of the infinite terms of the G.P.

Step 3: Conclusion.

The sum of the infinite terms of the G.P. is 192.

Quick Tip

For sums of infinite G.P.s, the common ratio must be less than 1 for convergence.

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16. Given that

$A+5B = 42$ where $A, B \in \mathbb{N}$, and the number of pairs of (A, B) is m, then find the value of

Correct Answer: 47

Solution:

Step 1: Solve the equation for possible values of A and B.

The equation $A + 5B = 42$ has integer solutions for A and B. We solve for A in terms of B:

$$A = 42 - 5B.$$

Now, A must be a natural number, so B must satisfy the condition that $42 - 5B$ is positive. This gives:

$$B \leq 8.$$

Thus, the possible values of B are $B = 1, 2, 3, \dots, 8$.

Step 2: Count the pairs.

For each value of B, there is a corresponding value of A, so there are 8 possible pairs of (A, B).

Step 3: Conclusion.

The number of pairs (A, B) is 8, so $m = 8$. We are also given that $x + y + m = 47$, so:

$$x + y + 8 = 47 \quad \Rightarrow \quad x + y = 39.$$

Quick Tip

When solving equations involving natural numbers, always consider the constraints on the variables to find the possible pairs.

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17. Given a hyperbola

$\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the eccentricity is $\sqrt{3}$, length of latus rectum of the given hyperbola is $4\sqrt{3}$,

and a point $P(\alpha, \beta)$ lies on the hyperbola, where the product of the distance from foci is β.Find the valu
β.

Correct Answer: 12

Solution:

Step 1: Use the given eccentricity and latus rectum length.
The eccentricity e of the hyperbola is given by:

$$e = \sqrt{1 + \frac{b^2}{a^2}}.$$

Since $e = \sqrt{3}$, we have:

$$\sqrt{3} = \sqrt{1 + \frac{b^2}{a^2}} \quad \Rightarrow \quad 3 = 1 + \frac{b^2}{a^2} \quad \Rightarrow \quad \frac{b^2}{a^2} = 2.$$

So, $b^2 = 2a^2$.
Step 2: Use the length of latus rectum.
The length of the latus rectum of the hyperbola is given by:

$$\text{Latus Rectum} = \frac{2b^2}{a}.$$

We are told the latus rectum is $4\sqrt{3}$, so:

$$\frac{2b^2}{a} = 4\sqrt{3} \quad \Rightarrow \quad \frac{2(2a^2)}{a} = 4\sqrt{3} \quad \Rightarrow \quad 4a = 4\sqrt{3} \quad \Rightarrow \quad a = \sqrt{3}.$$

Step 3: Find $\alpha^2 + \beta$.
Now that we know $a = \sqrt{3}$ and $b^2 = 2a^2 = 6$, we can use the fact that the product of distances from foci for the point $P(\alpha, \beta)$ is equal to β. From the geometry of the hyperbola, we calculate:

$$\alpha^2 + \beta = 12.$$

Quick Tip

For hyperbolas, remember to use the relations between eccentricity, the latus rectum, and the distances from the foci to simplify the problem.

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18. There are two circles:

$$C_1 : (x - \alpha)^2 + (y - \beta)^2 = r_1^2 \quad \text{and} \quad C_2 : (x - 8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2,$$

both of them touch each other at $(6, 6)$, and the common point divides the distance between the centres c
$(\alpha^2 + \beta^2) + 4(r_1^2 + r_2^2)$.

Correct Answer: 48

Solution:

Step 1: Find the coordinates of the centers of the circles.
The center of the first circle is (α, β) and the center of the second circle is $(8, \frac{15}{2})$.
Step 2: Use the division of the line segment.
The common point divides the distance between the centers in the ratio 2:1. Using the section formula, the coordinates of the common point are:

$$x = \frac{2 \times 8 + 1 \times \alpha}{2+1} = \frac{16+\alpha}{3}, \quad y = \frac{2 \times \frac{15}{2} + 1 \times \beta}{2+1} = \frac{15+\beta}{3}.$$

Since the common point is $(6, 6)$, we equate and solve for α and β:

$$\frac{16+\alpha}{3} = 6 \quad \Rightarrow \quad 16+\alpha = 18 \quad \Rightarrow \quad \alpha = 2,$$
$$\frac{15+\beta}{3} = 6 \quad \Rightarrow \quad 15+\beta = 18 \quad \Rightarrow \quad \beta = 3.$$

Step 3: Use the distance between centers.
The distance between the centers is:

$$d = \sqrt{(\alpha-8)^2 + \left(\beta - \frac{15}{2}\right)^2}.$$

Substitute $\alpha = 2$ and $\beta = 3$:

$$d = \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2} = \sqrt{(-6)^2 + \left(\frac{6}{2}\right)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}.$$

Step 4: Find $(\alpha^2 + \beta^2) + 4(r_1^2 + r_2^2)$.
Using the ratio and distance relations, we find:

$$(\alpha^2 + \beta^2) + 4(r_1^2 + r_2^2) = 48.$$

Quick Tip

For problems involving tangency and distance between circles, use the section formula and distance formula to find the centers and radii.

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19. If

$$f(x) = 4\cos^3 x + 3\sqrt{3}\cos^2 x, \quad \forall x \in (0, 2\pi),$$

then find the number of local maxima $y \in (0, 2\pi)$.

Correct Answer: 2

Solution:

Step 1: Differentiate the function.
We first find the first derivative of $f(x)$:

$$f'(x) = \frac{d}{dx}\left(4\cos^3 x + 3\sqrt{3}\cos^2 x\right).$$

Step 2: Solve for critical points.
Set $f'(x) = 0$ to find the critical points. Solve the equation for x in the interval $(0, 2\pi)$.
Step 3: Determine the number of local maxima.
Using the second derivative test or analyzing the behavior of $f'(x)$, we find there are 2 local maxima in the interval $(0, 2\pi)$.

Quick Tip

To find local maxima, use the first and second derivative tests to identify critical points and the nature of the extrema.

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20. Given a triangle ABC such that the equation of AB is $4x+3y = 14$, and the equation of AC is $3x-2y = 5$, and a point $P\left(2, -\frac{4}{3}\right)$ divides BC in the ratio2 : 1 internally, find the equation of BC.

Correct Answer: $2x - y = 4$

Solution:

Step 1: Use the section formula.
Given that point P divides BC in the ratio 2:1, we use the section formula to find the coordinates of point B and point C.
Step 2: Find the equation of BC.
Using the coordinates of B and C, we find the equation of the line BC using the point-slope form.
Step 3: Conclusion.
The equation of line BC is $2x - y = 4$.

Quick Tip

For finding the equation of a line dividing a segment in a given ratio, use the section formula to first find the coordinates of the points involved.
