

# JEE Main 2024 Mathematics Question Paper April 8 Shift 2

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90
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## General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
  - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and  $-1$  mark for wrong answer.
  - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and  $-1$  mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

## Mathematics

1. Evaluate the integral:

$$I = \int \frac{1}{\sqrt{1 - e^x}} dx$$

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2. Find the sum of values of  $\theta$  for which the following condition holds:

$$\operatorname{Re} \left( \frac{1 + i \cos \theta}{1 - i \cos \theta} \right) = 0, \quad \theta \in [-\pi, 2\pi]$$

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3. Find the number of different words that can be formed from the word "MATHEMATICS".

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4. Solve the Differential Equation:

$$\sec y \frac{dy}{dx} + 2 \sin y = \cos y, \quad \text{with } y(\sqrt{3}) = 1 \text{ and } x = 0.$$

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5. Find the area bounded by the curves  $x^2 + y^2 = 8$  and  $y^2 = 2x$  in the first quadrant.

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6. The expression  $(2(\sqrt{a}x^2 + \frac{1}{2}x^3))^{10}$  has constant terms 105. The value of  $a^2$  is

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7. Given  $\alpha = \lim_{x \rightarrow 0} \frac{e^{\tan x} - e^{\sqrt{x}}}{\sqrt{\tan x} - \sqrt{x}}$ , and  $\beta = \lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{2} \csc x}$ . Find  $\alpha + \beta$ .

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8. Let  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ ,  $\vec{c} = 2\hat{i} + 3\hat{j} - 5\hat{k}$ . Here,  $\vec{r}$  is parallel to  $\vec{b} + \vec{c}$  and is a unit vector. If  $\vec{r} \cdot \vec{a} = 3$ , find  $\lambda$ .

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9. The sequence of numbers is as follows:

$$\begin{array}{cccccc} 2 & 5 & 8 & 11 & 14 & 17 \\ 20 & 23 & 26 & 29 & \dots & \end{array}$$

Find the sum of all elements in the 10th row.

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10. Solve the equation  $|x + 1| + |x - 3| - |x + 2| + 5 = 0$ . Find the number of solutions.

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11. If  $\alpha \neq a, \beta \neq b, \gamma \neq c$  and

$$\frac{\alpha\beta\gamma}{abc} = 0, \quad \text{then the value of } \frac{\alpha}{\alpha - a} + \frac{\beta}{\beta - b} + \frac{\gamma}{\gamma - c} \text{ is}$$

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12. Find the value of

$$\frac{5 \cos 18^\circ + 3 \sin 36^\circ}{3 \cos 18^\circ - 5 \sin 36^\circ}$$

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13. In a G.P.,  $a_3 = 49$  and  $a_2 + a_4 = \frac{70}{3}$ . Find the value of

$$a_1 + a_6 + a_8$$

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14. Let  $A(5,2)$  &  $B(2,a)$  here  $\angle AOB = \frac{\pi}{4}$  ( $O$  is the origin), find the sum of all absolute values of  $a$ .

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15. Mirror Image of point  $A(-4,5)$  about line  $x + y = 2$  lies on the circle  $(x + 4)^2 + (y - 3)^2 = r^2$ , find  $r$ .

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16. There are 3 bags  $x, y, z$ :

Bag  $x$  contains 5 one rupee coins, 4 five rupee coins. Bag  $y$  contains 4 one rupee coins, 5 five rupee coins.

A bag is selected randomly and a coin is taken out and found to be a one rupee coin. Find the probability.

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17. Let

$$f : [-a, a] \rightarrow [0, 4a], \quad \forall a > 0$$
$$f(x) = \begin{cases} -x & \text{if } x \in [-a, 0], \\ x + a & \text{if } x \in (0, a]. \end{cases}$$

Define

$$g(x) = f(|x|) + |f(x)|$$

Check whether  $g(x)$  is one-one, onto, neither one-one nor onto.

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18. If the system of equations

$$x + y + z = \lambda, \quad 7x + 9y + \mu z = -3, \quad 5x + y + 2z = -1$$

has infinitely many solutions, then the value of  $2\lambda + 3\mu$  is.

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19. Let  $S$  be the region between  $y^2 = 2x$  and  $x = 2y$ . The maximum possible area of a rectangle inscribed in region  $S$  is.

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20. Let  $A = \{2, 3, 5, 8, 9\}$  and  $B = \{1, 4, 6, 10, 11\}$ . A relation  $R$  is defined from  $A \times B \rightarrow A \times B$  such that  $(a, b)R(c, d)$  if  $3ad - 7bc$  is an even integer. Then the relation  $R$  is:

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21. If mean, mean deviation about mean and variance of 5 observations 9, 25,  $a$ ,  $b$ ,  $c$  are 18, 4 and  $\frac{136}{5}$  respectively and  $a < b < c$ , find the value of  $2a + b - c$ .

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