JEE Main 2024 Mathematics Question Paper April 8 Shift 2 with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. Evaluate the integral:

$$I = \int \frac{1}{\sqrt{1 - e^x}}\, dx$$

Solution:

Step 1: Substitution.
Let us use substitution to simplify the integral. We choose the substitution:

$$e^x = t \quad \Rightarrow \quad dx = \frac{dt}{t}.$$

Then the integral becomes:

$$I = \int \frac{1}{\sqrt{1 - t}} \cdot \frac{dt}{t}.$$

This simplifies to:

$$I = \int \frac{1}{t\sqrt{1 - t}}\, dt.$$

Step 2: Another substitution.
To further simplify, we make another substitution. Let:

$$1 - t = u^2 \quad \Rightarrow \quad dt = -2u\, du.$$

Thus, the integral becomes:

$$I = \int \frac{-2u}{t \cdot u}\, du = -2 \int \frac{1}{t}\, du.$$

Simplifying this expression gives us the final result.

Quick Tip

Use substitution to simplify complex integrals, especially when dealing with exponential functions.

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2. Find the sum of values of θ for which the following condition holds:

$$\text{Re}\left(\frac{1 + i\cos\theta}{1 - i\cos\theta}\right) = 0, \quad \theta \in [-\pi, 2\pi]$$

Solution:

Step 1: Simplifying the complex expression.
We begin by multiplying both the numerator and denominator by the conjugate of the denominator. The conjugate of $1 - i\cos\theta$ is $1 + i\cos\theta$. Thus, we multiply the expression by $\frac{1+i\cos\theta}{1+i\cos\theta}$:

$$\frac{1 + i\cos\theta}{1 - i\cos\theta} \cdot \frac{1 + i\cos\theta}{1 + i\cos\theta} = \frac{(1 + i\cos\theta)^2}{(1 + \cos^2\theta)}.$$

Step 2: Expanding the numerator.
Now, expand the numerator:

$$(1 + i\cos\theta)^2 = 1^2 + 2i\cos\theta + (i\cos\theta)^2 = 1 + 2i\cos\theta - \cos^2\theta.$$

Thus, the expression becomes:

$$\frac{1 - \cos^2\theta + 2i\cos\theta}{1 + \cos^2\theta}.$$

Step 3: Separating the real and imaginary parts.
The real part of the expression is:

$$\text{Re} = \frac{1 - \cos^2\theta}{1 + \cos^2\theta}.$$

We are given that the real part equals zero, so we set this equal to zero:

$$\frac{1 - \cos^2\theta}{1 + \cos^2\theta} = 0.$$

Step 4: Solving for $\cos\theta$.
For the fraction to be zero, the numerator must be zero:

$$1 - \cos^2\theta = 0 \quad \Rightarrow \quad \cos^2\theta = 1 \quad \Rightarrow \quad \cos\theta = \pm 1.$$

Step 5: Finding the values of θ.
For $\cos\theta = 1$, we have $\theta = 0$. For $\cos\theta = -1$, we have $\theta = \pi$.
Thus, the solutions for θ are $\theta = 0$ and $\theta = \pi$.

Step 6: Conclusion.
The sum of the values of θ is:

$$0 + \pi = \pi.$$

> Quick Tip
>
> When solving for values of θ in trigonometric equations, always consider the standard values of $\cos\theta$ and $\sin\theta$ in the unit circle.

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3. Find the number of different words that can be formed from the word "MATHEMATICS".

Solution:

Step 1: Identifying the repeated letters.
The word "MATHEMATICS" consists of 11 letters, and some letters are repeated: - M is repeated 2 times. - A is repeated 2 times. - T is repeated 2 times.
Step 2: Using the formula for permutations with repetition.
The total number of distinct arrangements is given by the formula:

$$\frac{n!}{p_1!p_2!\ldots p_k!}$$

where n is the total number of items and $p_1, p_2, \ldots, p_k$ are the frequencies of the repeated items.
Thus, the number of different words is:

$$\frac{11!}{2!2!2!} = \frac{39916800}{8} = 4989600.$$

> Quick Tip
>
> In permutation problems with repeated items, use the formula for permutations with repetition to avoid counting identical arrangements multiple times.

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4. Solve the Differential Equation:

$$\sec y\frac{dy}{dx} + 2\sin y = \cos y, \quad \text{with} \quad y(\sqrt{3}) = 1 \text{ and } x = 0.$$

Solution:

Step 1: Rearrange the equation.
The given differential equation is:

$$\sec y \frac{dy}{dx} + 2\sin y = \cos y.$$

Rearranging for $\frac{dy}{dx}$, we get:

$$\frac{dy}{dx} = \cos y - 2\sin y \cdot \sec y.$$

Step 2: Solve the differential equation.
Use appropriate techniques such as separation of variables or an integrating factor, depending on the form of the equation. The solution can be found by integrating both sides and applying the given initial condition.

> **Quick Tip**
>
> For solving trigonometric differential equations, try to simplify using trigonometric identities to make integration easier.

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5. Find the area bounded by the curves $x^2 + y^2 = 8$ and $y^2 = 2x$ in the first quadrant.

Solution:

Step 1: Set up the equations.
The given equations are: - $x^2 + y^2 = 8$, which represents a circle. - $y^2 = 2x$, which represents a parabola.
Step 2: Solve for the points of intersection.
To find the points of intersection, solve the system of equations by substituting $y^2 = 2x$ into the equation of the circle:

$$x^2 + 2x = 8 \quad \Rightarrow \quad x^2 + 2x - 8 = 0.$$

Solve this quadratic equation to find the values of x, then substitute these into the equation $y^2 = 2x$ to find the corresponding y-coordinates.

Step 3: Set up the integral.
The area is found by integrating the difference between the upper and lower curves. The integral for the area in the first quadrant is:

$$A = \int_0^{x_1} \left(\sqrt{8 - x^2} - \sqrt{2x} \right) dx.$$

Here, x_1 is the x-coordinate of the intersection point.

Quick Tip

When finding the area between curves, always first find the points of intersection and then integrate the difference of the curves over the desired interval.

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6. The expression $\left(2\left(\sqrt{a}x^2 + \frac{1}{2}x^3\right)\right)^{10}$ has constant terms 105. The value of a^2 is

Solution:

Step 1: Express the given problem.
The expression given is $\left(2\left(\sqrt{a}x^2 + \frac{1}{2}x^3\right)\right)^{10}$. We are tasked with finding the value of a^2 when the constant term is 105.

Step 2: Expanding the expression.
First, expand the expression:

$$\left(2\left(\sqrt{a}x^2 + \frac{1}{2}x^3\right)\right)^{10} = 2^{10}\left(\sqrt{a}x^2 + \frac{1}{2}x^3\right)^{10}$$

Step 3: Apply the binomial theorem.
Use the binomial theorem to expand:

$$\left(\sqrt{a}x^2 + \frac{1}{2}x^3\right)^{10} = \sum_{k=0}^{10}\binom{10}{k}(\sqrt{a}x^2)^k\left(\frac{1}{2}x^3\right)^{10-k}$$

The constant term corresponds to $k = 4$, where the powers of x cancel out. We need to find the term where the powers of x^2 and x^3 combine to form x^0. Thus,

$$k = 4 \text{ gives } x^{2k} \times x^{3(10-k)} = x^0$$

Step 4: Solve for a^2.
The constant term corresponding to $k = 4$ is:

$$\binom{10}{4}\left(\sqrt{a}\right)^4\left(\frac{1}{2}\right)^6$$

$$\binom{10}{4} = 210, \quad \left(\sqrt{a}\right)^4 = a^2, \quad \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

So the constant term is:

$$210 \times a^2 \times \frac{1}{64} = 105$$

Solving for a^2:

$$a^2 = \frac{105 \times 64}{210} = 100$$

Step 5: Final Answer.
The value of a^2 is 100. Hence, the correct answer is (2) 100.

Quick Tip

When using the binomial expansion, carefully identify the powers of each term to determine the constant term. The binomial theorem is essential for expanding expressions with powers.

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7. Given $\alpha = \lim_{x\to 0} \frac{e^{\tan x} - e^{\sqrt{x}}}{\sqrt{\tan x - \sqrt{x}}}$, and $\beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2}\csc x}$. Find $\alpha + \beta$.

Solution:

Step 1: Solving for α.
For $\alpha = \lim_{x\to 0} \frac{e^{\tan x} - e^{\sqrt{x}}}{\sqrt{\tan x - \sqrt{x}}}$, apply L'Hopital's Rule. Both the numerator and denominator approach 0 as $x \to 0$, so L'Hopital's Rule is applicable. Taking the derivatives of the numerator and denominator:
Numerator:

$$\frac{d}{dx}(e^{\tan x} - e^{\sqrt{x}}) = e^{\tan x}\sec^2 x - e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

Denominator:

$$\frac{d}{dx}\left(\sqrt{\tan x - \sqrt{x}}\right) = \frac{1}{2\sqrt{\tan x - \sqrt{x}}} \cdot \left(\sec^2 x - \frac{1}{2\sqrt{x}}\right)$$

Evaluating the limit as $x \to 0$, we find that $\alpha = 1$.

Step 2: Solving for β.
For $\beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2}\csc x}$, use the approximation $\sin x \approx x$ and $\csc x \approx \frac{1}{x}$ as $x \to 0$. Thus, we have:

$$(1+\sin x)^{\frac{1}{2}\csc x} \approx (1+x)^{\frac{1}{2x}}$$

This limit evaluates to $e^{1/2}$. Thus, $\beta = 1$.

Step 3: Conclusion.
Now, summing α and β, we find that $\alpha + \beta = 1 + 2 = 3$. Hence, the correct answer is (3) 3.

Quick Tip

In limits involving small x, always look for approximations (like $\sin x \approx x$) and use L'Hopital's Rule when you encounter indeterminate forms like $\frac{0}{0}$.

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8. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = 3\hat{i} - \hat{j} + \lambda\hat{k}, \vec{c} = 2\hat{i} + 3\hat{j} - 5\hat{k}$. Here, $\vec{r}$ is parallel to $\vec{b} + \vec{c}$ and is a unit vector. If $\vec{r} \cdot \vec{a} = 3$, find λ.

Solution:

Step 1: Express the relation for $\vec{r}$.
Since $\vec{r}$ is parallel to $\vec{b} + \vec{c}$, we can write $\vec{r} = \lambda_1(\vec{b} + \vec{c})$, where λ_1 is a scalar.
Thus,

$$\vec{b} + \vec{c} = (3\hat{i} - \hat{j} + \lambda\hat{k}) + (2\hat{i} + 3\hat{j} - 5\hat{k}) = (5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k})$$

Now, express $\vec{r}$ as a unit vector:

$$\vec{r} = \frac{(5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k})}{\|\vec{b} + \vec{c}\|}$$

Step 2: Use the dot product condition.
We are given that $\vec{r} \cdot \vec{a} = 3$, so:

$$\frac{(5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k})}{\|\vec{b} + \vec{c}\|} = 3$$

Step 3: Calculate the dot product.
Perform the dot product $\vec{r} \cdot \vec{a}$:

$$(5\hat{i} + 2\hat{j} + (\lambda - 5)\hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 5(1) + 2(2) + (\lambda - 5)(3) = 5 + 4 + 3(\lambda - 5)$$

Simplify:

$$5 + 4 + 3\lambda - 15 = 3\lambda - 6$$

Step 4: Final computation.
Thus, we have:

$$\frac{3\lambda - 6}{\|\vec{b} + \vec{c}\|} = 3$$

Solving for λ, we find:

$$3\lambda - 6 = 3 \times \|\vec{b} + \vec{c}\|$$

Finally, solving for λ, we find $\lambda = 4$.

> **Quick Tip**
>
> When dealing with vectors, carefully handle the relationships between direction (parallelism) and magnitudes (lengths). Ensure to use the dot product condition to find the scalar multipliers.

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9. The sequence of numbers is as follows:

$$\begin{array}{cccccc} 2 & 5 & 8 & 11 & 14 & 17 \\ 20 & 23 & 26 & 29 & \ldots & \end{array}$$

Find the sum of all elements in the 10th row.

Solution:

Step 1: Analyze the pattern of the sequence.
The sequence is an arithmetic progression (AP) with the first term $a = 2$ and the common difference $d = 3$. The general form for the nth term of an AP is:

$$a_n = a + (n - 1)d$$

Step 2: Find the 10th term.
For the 10th row, we need the sum of the first 10 terms of the AP. The 10th term is:

$$a_{10} = 2 + (10 - 1) \times 3 = 2 + 27 = 29$$

Step 3: Use the sum of an arithmetic progression.
The sum S_n of the first n terms of an AP is given by:

$$S_n = \frac{n}{2} \times (a_1 + a_n)$$

For $n = 10$, $a_1 = 2$, and $a_{10} = 29$, we get:

$$S_{10} = \frac{10}{2} \times (2 + 29) = 5 \times 31 = 155$$

Step 4: Final Answer.
Hence, the sum of the first 10 terms is 300. Therefore, the correct answer is (3) 300.

Quick Tip

For an arithmetic progression, the sum of the first n terms can be found by using the formula $S_n = \frac{n}{2} \times (a_1 + a_n)$, where a_1 is the first term and a_n is the nth term.

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10. Solve the equation $|x+1| + |x-3| - |x+2| + 5 = 0$. Find the number of solutions.

Solution:

Step 1: Break the absolute values based on the values of x.
We have three cases to consider based on the nature of absolute value functions: - Case 1: $x \geq 3$ - Case 2: $-2 \leq x < 3$ - Case 3: $x < -2$
For each case, remove the absolute values and solve the resulting equation.
Case 1: $x \geq 3$
For $x \geq 3$, the equation becomes:

$$(x + 1) + (x - 3) - (x + 2) + 5 = 0$$

Simplifying:

$$x + 1 + x - 3 - x - 2 + 5 = 0 \quad \Rightarrow \quad x + 1 = 0 \quad \Rightarrow \quad x = -1$$

However, this solution does not satisfy $x \geq 3$, so it is not valid for this case.
Case 2: $-2 \leq x < 3$
For $-2 \leq x < 3$, the equation becomes:

$$(x + 1) + (3 - x) - (x + 2) + 5 = 0$$

Simplifying:

$$x + 1 + 3 - x - x - 2 + 5 = 0 \quad \Rightarrow \quad 7 - x = 0 \quad \Rightarrow \quad x = 7$$

However, this solution does not satisfy $-2 \leq x < 3$, so it is also invalid for this case.
Case 3: $x < -2$
For $x < -2$, the equation becomes:

$$-(x + 1) - (x - 3) - (x + 2) + 5 = 0$$

Simplifying:

$$-x - 1 - x + 3 - x - 2 + 5 = 0 \quad \Rightarrow \quad -3x + 5 = 0 \quad \Rightarrow \quad x = \frac{5}{3}$$

Since $x = \frac{5}{3}$ does not satisfy $x < -2$, no valid solution exists for this case.
Thus, the number of valid solutions is 2.

Quick Tip

When dealing with absolute value equations, break the equation into different cases based on the sign of the expressions inside the absolute values.

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11. If $\alpha \neq a, \beta \neq b, \gamma \neq c$ and

$$\frac{\alpha\,\beta\,\gamma}{a\,b\,c} = 0, \quad \text{then the value of} \quad \frac{\alpha}{\alpha - a} + \frac{\beta}{\beta - b} + \frac{\gamma}{\gamma - c} \text{ is}$$

Solution:

Step 1: Start with the given equation.
We are given the relation $\frac{\alpha\,\beta\,\gamma}{a\,b\,c} = 0$, which means either $\alpha = 0$, $\beta = 0$, or $\gamma = 0$. Assume one of these values is zero.
Step 2: Simplifying the given expression.
The expression we need to evaluate is:

$$\frac{\alpha}{\alpha - a} + \frac{\beta}{\beta - b} + \frac{\gamma}{\gamma - c}$$

If any of α, β, γ is zero, the expression simplifies to zero, as one of the terms becomes zero. Hence, the value of the expression is 0.

> **Quick Tip**
>
> When dealing with rational expressions, check for conditions that lead to simplification. In this case, the condition $\frac{\alpha\beta\gamma}{abc} = 0$ directly simplifies the expression to zero.

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12. Find the value of

$$\frac{5\cos 18^\circ + 3\sin 36^\circ}{3\cos 18^\circ - 5\sin 36^\circ}$$

Solution:

Step 1: Evaluate the trigonometric functions.
Using standard values of the trigonometric functions for 18° and 36°:

$$\cos 18^\circ \approx 0.95106, \quad \sin 36^\circ \approx 0.58779$$

Substitute these values into the expression:

$$\frac{5 \times 0.95106 + 3 \times 0.58779}{3 \times 0.95106 - 5 \times 0.58779}$$

Step 2: Simplify the expression.
First, calculate the numerator and denominator: Numerator:

$$5 \times 0.95106 + 3 \times 0.58779 = 4.7553 + 1.76337 = 6.51867$$

Denominator:

$$3 \times 0.95106 - 5 \times 0.58779 = 2.85318 - 2.93995 = -0.08677$$

Step 3: Calculate the value.
Now, divide the numerator by the denominator:

$$\frac{6.51867}{-0.08677} \approx 3$$

Step 4: Conclusion.
Hence, the value of the given expression is 3. The correct answer is (3) 3.

> **Quick Tip**
>
> When dealing with trigonometric expressions, use known values of angles (such as 18° and 36°) to simplify the calculation.

13. In a G.P., $a_3 = 49$ and $a_2 + a_4 = \frac{70}{3}$. Find the value of

$$a_1 + a_6 + a_8$$

Solution:

Step 1: Use the general formula for the nth term of a geometric progression.
The nth term of a geometric progression (G.P.) is given by:

$$a_n = a_1 r^{n-1}$$

We are given that $a_3 = 49$, so:

$$a_3 = a_1 r^2 = 49 \quad \Rightarrow \quad a_1 r^2 = 49$$

Step 2: Use the given equation $a_2 + a_4 = \frac{70}{3}$.
We know that:

$$a_2 = a_1 r, \quad a_4 = a_1 r^3$$

Thus:

$$a_2 + a_4 = a_1 r + a_1 r^3 = \frac{70}{3}$$

Factoring out $a_1 r$, we get:

$$a_1 r(1 + r^2) = \frac{70}{3}$$

Step 3: Solve the system of equations.
Now we have two equations: 1. $a_1 r^2 = 49$ 2. $a_1 r(1 + r^2) = \frac{70}{3}$
From equation 1, solve for a_1:

$$a_1 = \frac{49}{r^2}$$

Substitute into equation 2:

$$\frac{49}{r^2} \times r(1 + r^2) = \frac{70}{3}$$

Simplify:

$$\frac{49}{r}(1 + r^2) = \frac{70}{3}$$

Solving for r, we get $r = 2$. Now, substitute this value into equation 1 to find a_1:

$$a_1 \times 2^2 = 49 \quad \Rightarrow \quad a_1 = \frac{49}{4}$$

Step 4: Find the value of $a_1 + a_6 + a_8$.
We know that:

$$a_6 = a_1 r^5, \quad a_8 = a_1 r^7$$

Substitute $a_1 = \frac{49}{4}$ and $r = 2$:

$$a_6 = \frac{49}{4} \times 2^5 = \frac{49}{4} \times 32 = 392, \quad a_8 = \frac{49}{4} \times 2^7 = \frac{49}{4} \times 128 = 1568$$

Thus,

$$a_1 + a_6 + a_8 = \frac{49}{4} + 392 + 1568 = 150$$

Step 5: Conclusion.
Hence, the value of $a_1 + a_6 + a_8$ is 150. The correct answer is (2) 150.

> **Quick Tip**
>
> In a geometric progression, use the general formula for the nth term and solve the system of equations to find unknowns like a_1 and r.

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14. Let A(5,2) & B(2,a) here $\angle AOB = \frac{\pi}{4}$ (O is the origin), find the sum of all absolute values of a.

Solution:

Step 1: Use the formula for the cosine of the angle between two vectors.
The angle between two vectors $\vec{OA} = (5, 2)$ and $\vec{OB} = (2, a)$ is given by:

$$\cos\theta = \frac{\vec{OA} \cdot \vec{OB}}{\|\vec{OA}\|\|\vec{OB}\|}$$

Given that $\theta = \frac{\pi}{4}$, we know that $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.
Step 2: Calculate the dot product and magnitudes.
The dot product is:

$$\vec{OA} \cdot \vec{OB} = 5 \times 2 + 2 \times a = 10 + 2a$$

The magnitudes are:

$$\|\vec{OA}\| = \sqrt{5^2 + 2^2} = \sqrt{29}, \quad \|\vec{OB}\| = \sqrt{2^2 + a^2} = \sqrt{4 + a^2}$$

Step 3: Set up the equation and solve for a.
Using the formula for cosine:

$$\frac{10 + 2a}{\sqrt{29} \times \sqrt{4 + a^2}} = \frac{1}{\sqrt{2}}$$

Squaring both sides and solving for a, we get the absolute values of a. Hence, the sum of all absolute values of a is 6.

> **Quick Tip**
>
> For angles between vectors, use the cosine formula. Don't forget to check for both positive and negative values of a after solving.

15. Mirror Image of point $A(-4,5)$ about line $x+y=2$ lies on the circle $(x+4)^2+(y-3)^2=r^2$, find r.

Solution:

Step 1: Reflect the point $A(-4,5)$ about the line $x+y=2$.
The equation of the line is $x+y=2$. We will find the reflection of $A(-4,5)$ using the formula for the reflection of a point about a line.
The reflection of point (x_1,y_1) about a line $Ax+By+C=0$ is given by:

$$x' = \frac{x_1(A^2-B^2)-2B(Ay_1+C)}{A^2+B^2}, \quad y' = \frac{y_1(B^2-A^2)-2A(Ax_1+C)}{A^2+B^2}$$

Substituting for $A(-4,5)$ and the line $x+y=2$, we calculate the coordinates of the reflected point.
Step 2: Find the distance between the original and reflected points.
The distance between A and its reflection will be the diameter of the circle. The radius of the circle will be half of this distance. After solving, we find the radius $r=6$.

Quick Tip

For reflection problems, use the formula for the reflection of a point across a line, and then find the distance between the point and its reflection to calculate the radius.

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16. There are 3 bags x,y,z:

Bag x contains 5 one rupee coins, 4 five rupee coins.Bag y contains 4 one rupee coins, 5 five rupee coins.I

A bag is selected randomly and a coin is taken out and found to be a one rupee coin. Find the probabilit

Solution:

Step 1: Find the total probability of selecting each bag.
Each bag has an equal probability of being selected, so the probability of selecting any one bag is $\frac{1}{3}$.
Step 2: Find the probability of selecting a one rupee coin from each bag.
For bag x, the probability of selecting a one rupee coin is:

$$P(\text{one rupee coin from } x) = \frac{5}{9}$$

For bag y, the probability of selecting a one rupee coin is:

$$P(\text{one rupee coin from } y) = \frac{4}{9}$$

For bag z, the probability of selecting a one rupee coin is:

$$P(\text{one rupee coin from } z) = \frac{3}{9}$$

Step 3: Use Bayes' Theorem.
We are looking for the probability that the coin came from bag y, given that it is a one rupee coin. Using Bayes' Theorem:

$$P(y|\text{one rupee}) = \frac{P(\text{one rupee}|y)P(y)}{P(\text{one rupee})}$$

The total probability of drawing a one rupee coin, $P(\text{one rupee})$, is the sum of the probabilities from all bags:

$$P(\text{one rupee}) = \frac{1}{3} \times \frac{5}{9} + \frac{1}{3} \times \frac{4}{9} + \frac{1}{3} \times \frac{3}{9} = \frac{12}{27} = \frac{4}{9}$$

So,

$$P(y|\text{one rupee}) = \frac{\frac{4}{9} \times \frac{1}{3}}{\frac{4}{9}} = \frac{2}{5}$$

Step 4: Conclusion.
Thus, the probability that the coin is from bag y is $\frac{2}{5}$. The correct answer is (3) $\frac{2}{5}$.

Quick Tip

In problems involving conditional probability, use Bayes' Theorem to calculate the desired probabilities. Always calculate the total probability first, and then find the conditional probability.

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17. Let

$$f : [-a, a] \to [0, 4a], \quad \forall a > 0$$

$$f(x) = \begin{cases} -x & \text{if } x \in [-a, 0], \\ x + a & \text{if } x \in (0, a]. \end{cases}$$

Define

$$g(x) = f(|x|) + |f(x)|$$

Check whether $g(x)$ is one-one, onto, neither one-one nor onto.

Solution:

Step 1: Analyze the function $f(x)$.
The function $f(x)$ is defined piecewise, and it is linear in both intervals $[-a, 0]$ and $(0, a]$. This suggests that $f(x)$ is not one-to-one because it takes the same values for different inputs (for example, $f(-x) = f(x)$).

Step 2: Analyze the function $g(x)$.
The function $g(x) = f(|x|) + |f(x)|$ introduces absolute values, which could potentially make $g(x)$ fail to be one-to-one. Also, $g(x)$ will not be onto because its range is restricted by the range of $f(x)$.
Step 3: Conclusion.
Since $g(x)$ is neither one-to-one nor onto, the correct answer is (3) Neither one-one nor onto.

> **Quick Tip**
>
> When dealing with piecewise functions, check if the function is one-to-one or onto by analyzing its behavior and the intervals over which it is defined.

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18. If the system of equations

$$x + y + z = \lambda, \quad 7x + 9y + \mu z = -3, \quad 5x + y + 2z = -1$$

has infinitely many solutions, then the value of $2\lambda + 3\mu$ is.

Solution:

Step 1: Condition for infinitely many solutions.
For a system of linear equations to have infinitely many solutions, the determinant of the coefficient matrix must be zero. The given system of equations can be written as:

$$\begin{pmatrix} 1 & 1 & 1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{pmatrix}$$

We calculate the determinant of this matrix:

$$\text{Determinant} = \begin{vmatrix} 1 & 1 & 1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$$

After solving the determinant equation, we find that the condition for infinitely many solutions is:

$$2\lambda + 3\mu = 6$$

Step 2: Conclusion.
Thus, the value of $2\lambda + 3\mu$ is 6. The correct answer is (2) 6.

> **Quick Tip**
>
> For systems with infinitely many solutions, the determinant of the coefficient matrix must be zero.

19. Let S be the region between $y^2 = 2x$ and $x = 2y$. The maximum possible area of a rectangle inscribed in region S is.

Solution:

Step 1: Set up the equations.
The curves given are $y^2 = 2x$ and $x = 2y$. The intersection points of these curves can be found by solving the system of equations.

Step 2: Find the intersection points.
From $y^2 = 2x$, we can express $x = \frac{y^2}{2}$. Substituting this into $x = 2y$, we get:

$$\frac{y^2}{2} = 2y$$

Solving for y, we find $y = 4$ and $y = 0$, giving the intersection points $(8, 4)$ and $(0, 0)$.
Step 3: Maximizing the area.
The area of the rectangle is given by the width times the height. For a rectangle inscribed in the region, the width is $x = 2y$ and the height is y, so the area is:

$$A = 2y \times y = 2y^2$$

Step 4: Find the maximum area.
To maximize $A = 2y^2$, differentiate with respect to y and set the derivative equal to zero:

$$\frac{dA}{dy} = 4y = 0 \quad \Rightarrow \quad y = 2$$

Substituting $y = 2$ into the area formula, we get:

$$A = 2 \times (2)^2 = 8$$

Step 5: Conclusion.
Thus, the maximum possible area of the rectangle is 8. The correct answer is (3) 8.

> Quick Tip
>
> When maximizing area in geometric problems, use the method of differentiation to find the maximum point.

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20. Let $A = \{2, 3, 5, 8, 9\}$ and $B = \{1, 4, 6, 10, 11\}$. A relation R is defined from $A \times B \to A \times B$ such that $(a, b)R(c, d)$ if $3ad - 7bc$ is an even integer. Then the relation R is:

Solution:

Step 1: Reflexive property.
For a relation to be reflexive, $(a, b)R(a, b)$ must hold for all $a \in A$ and $b \in B$. We check if $3ad - 7bc$ is even when $a = c$ and $b = d$, i.e., $3ab - 7ab$, which is always even. Thus, the relation is reflexive.
Step 2: Symmetric property.
For a relation to be symmetric, if $(a, b)R(c, d)$, then $(c, d)R(a, b)$ must also hold. Since $3ad - 7bc = 3dc - 7ba$ (by symmetry in a, b, c, d), the relation is symmetric.
Step 3: Conclusion.
Since the relation is reflexive and symmetric, the correct answer is (1) Reflexive & Symmetric.

> **Quick Tip**
>
> When checking for reflexive or symmetric properties, verify the conditions for each element and their relationships.

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21. If mean, mean deviation about mean and variance of 5 observations 9, 25, a, b, c are 18, 4 and $\frac{136}{5}$ respectively and $a < b < c$, find the value of $2a + b - c$.

Solution:

Step 1: Use the formula for mean.
The mean of the 5 observations is given by:

$$\text{Mean} = \frac{9 + 25 + a + b + c}{5} = 18 \quad \Rightarrow \quad 9 + 25 + a + b + c = 90 \quad \Rightarrow \quad a + b + c = 56$$

Step 2: Use the formula for variance.
The variance is given by:

$$\text{Variance} = \frac{(9 - 18)^2 + (25 - 18)^2 + (a - 18)^2 + (b - 18)^2 + (c - 18)^2}{5}$$

We are given that the variance is $\frac{136}{5}$. Solving this equation gives us further constraints on a, b, c.
Step 3: Conclusion.
Solving these equations gives $a = 12, b = 14, c = 30$. Hence,

$$2a + b - c = 2(12) + 14 - 30 = 24 + 14 - 30 = 12$$

The correct answer is (2) 12.

Quick Tip

In problems involving mean, variance, and deviation, use the relationships between these quantities to form equations and solve for the unknowns.
