JEE Main 2024 Mathematics Question Paper April 9 Shift 1

Time Allowed : 3 Hours	**Maximum Marks :** 300	**Total Questions :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. Given the equation

$$\int \frac{2-\tan x}{3+\tan x}dx = \alpha x + \beta \ln(3\cos x + \sin x) + \gamma$$

Where γ is the constant of integration. Find $\alpha + \beta$.

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2. Given the function

$$f(x) = 3ax^3 + bx^2 + cx + 41$$

with the conditions $f(1) = 41$, $f'(1) = 2$, and $f''(1) = 4$, find $a^2 + b^2 + c^2$.

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3. The remainder when

$$(428)^{2024}$$

is divided by 21 is:

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4. If the domain of function

$$f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$$

is $\mathbb{R} - (\alpha, \beta)$. Then find $12\alpha\beta$.

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5. Given the function

$$f(x) = \begin{cases} \frac{\tan 8x}{\tan 7x}, & x < \frac{\pi}{2} \\ a - 8, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^b, & x > \frac{\pi}{2} \end{cases}$$

where $f(x)$ is continuous at $x = \frac{\pi}{2}$, find $a^2 + b^2$.

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6. Find the value of

$$\frac{1}{1+d} + \frac{1}{(1+d)(1+2d)} + \frac{1}{(1+2d)(1+3d)} + \cdots + \frac{1}{(1+9d)(1+10d)} = 5$$

Find the value of $50d$.

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7. Given

$$\cos\theta \cos(60^\circ - \theta)\cos(60^\circ + \theta) \leq \frac{1}{8}, \quad \theta \in [0, 2\pi]$$

Find the sum of values of θ for which $\cos 3\theta$ is maximum.

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8. A variable line passing through (3, 5) cuts the positive x and y axes. Find the minimum area made between the axes and the line.

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9. If the roots of the equation

$$x^2 + 2\sqrt{2}x - 1 = 0$$

are α and β, find the equation whose roots are

$$\alpha^4 + \beta^4 \quad \text{and} \quad \frac{1}{10}\left(\alpha^6 + \beta^6\right)$$

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10. Given the system of equations:

$$3x + 4y + \lambda z = 4$$

$$5x + 7y + 2z = 8$$

$$97x + 197y + 83z = \mu$$

Find $\lambda + 3\mu$ if the system has infinite solutions.

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11. A triangle ABC is made of three vectors

$$\vec{a} = (\alpha\hat{\imath} + 5\hat{\jmath} + 4\hat{k}), \quad \vec{b} = (3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}), \quad \vec{c} = \vec{a} - \vec{b}$$

respectively. The area of $\triangle ABC$ is given as $5\sqrt{6}$. Find $|\vec{c}|^2$.

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12. A circle with centre (α, β) passes through the points $(0, 0)$ and $(0, 1)$ and touches the circle $x^2 + y^2 = 9$ for all possible values of (α, β). Find the value of $4(\alpha^4 + \beta^4)$.

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13. Find the coefficient of

$$x^2(1+x)^{98} + x^3(1+x)^{97} + \cdots + x^{46}(1+x)^{54}$$

if the coefficient of x^{70} is $99C_p - 54C_q$, find $p + q$.

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14. Given

$$f(x) = x^2 + 9 \quad \text{and} \quad g(x) = \frac{x}{x-9}$$

And a curve:

$$\frac{x^2}{a} + \frac{y^2}{b} = 1$$

where $a = f \circ g(10)$, $b = g \circ f(3)$. Then find $8e^2 + \ell^2$, where e is the eccentricity, and ℓ is the latus rectum length.

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15. A circle $x^2+y^2 = 5$ and a parabola $y^2 = 4x$ intersecting each other. Then find the area of the smalles

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16. A tetrahedral die written 1, 2, 3, 4 on their faces is thrown. Find the probability such that the quadratic equation

$$ax^2 + bx + c = 0$$

has real roots.

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17. If

$$f(m+n) = f(m) + f(n) \quad \text{and} \quad f(1) = 1,$$

$$\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$$

Find the maximum value of λ.

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18. The solution of the differential equation

$$(x^2 + y^2)dx - 5xy\,dy = 0, \quad y(1) = 0$$

Find the solution.

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19. For a quadrilateral OABC, given that

$$\overrightarrow{OA} = 2\alpha, \quad \overrightarrow{OB} = 6\alpha + 2\beta, \quad \overrightarrow{OC} = 3\beta$$

It is also given that the area of the parallelogram with adjacent sides OA and OC is 15. Then find the ar

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20. If $\sqrt{2}\,|\mathbf{a} - \mathbf{b}| = |\mathbf{a} + \mathbf{b}|$, $|\mathbf{a}| = n$, and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\cos^{-1}\left(\frac{5}{9}\right)$, then find $n =$?

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21. If set $A = \{z : |z-1| \leq 1\}$ **and set** $B = \{z : |z-5| \leq |z-5|\}$**, if** $z = a + ib$**, where** $a, b \in \mathbb{I}$**. The sum of modulus squares of** $A \cap B$ **is**

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22. A ray of light passing through $(1, 2)$ **after reflecting on the x-axis at point Q passes through** $R(4, 3)$**. If** $S(h, k)$ **is such that PQRS is a parallelogram, then find** (h, k)**.**

23. If A is a 3×3 matrix, det$(3 \cdot \mathbf{adj}(2 \cdot A)) = 2^{-13} \cdot 3^{-10}$ and det$(3 \cdot \mathbf{adj}(3 \cdot A)) = 2^{-m} \cdot 2^{-n}$, then $2m + 2n$ is equal to