

JEE Main 2024 Mathematics Question Paper April 9 Shift 2

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
 - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.
 - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics

1. If $\frac{z-2i}{z+2i}$ is purely imaginary, then find the maximum value of $|z + 8 + 6i|$.

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2. Find the limit:

$$\lim_{x \rightarrow 0} \frac{e^{-(1+2x)} - 1}{x}$$

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3. In the expansion of $(x^{2/3} + \frac{1}{2}x^{-2/5})^9$, find the sum of coefficients of $x^{2/3}$ and $x^{-2/5}$.

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4. If the variance of the following distribution is 160, find the value of c .

x	c	$2c$	$3c$	$4c$	$5c$	$6c$
f	2	1	1	1	1	1

5. Find the limit:

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\int_{\frac{\pi}{2}}^x (\sin(2t) + \cos(t)) dt}{x - \frac{\pi}{2}}$$

6. Solve $2 \sin^{-1}(x) + 3 \cos^{-1}(x) = \frac{7\pi}{5}$, and find the number of real solutions.

7. Evaluate the integral:

$$I = \int_{-1}^2 \ln \left(x + \sqrt{1+x^2} \right) dx$$

8. If $\ln(y) = \sin^{-1}(x)$, then find the value of

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} \quad \text{at} \quad x = \frac{1}{2}$$

9. If $f(x) = \frac{1}{2+\sin(3x)+\cos(3x)}$, then the range of $f(x)$ is $[a, b]$, find the ratio of AM and GM of a, b .

10. Find the number of integers between 100 and 1000 whose sum of digits is 14.

11. Given $f'(x) = 3f(x) + \alpha$, if $f(0) = 7$ and $\lim_{x \rightarrow \infty} f(x) = 0$, find $f\left(\frac{1}{3}\right)$.

12. Evaluate the integral:

$$I = \int_{\frac{1}{4}}^{\frac{3}{4}} \cos \left(2 \cot^{-1} \left(\frac{1+x}{\sqrt{1-x}} \right) \right) dx$$

13. Ellipse:

$$\frac{(x-1)^2}{100} + \frac{y^2}{75} = 1, \quad \text{and Hyperbola of the same focus as ellipse.}$$

Find the value of $3\alpha^2 + 2\beta^2$, where the major axis of ellipse is α and the minor axis is β .

14. A dice is thrown three times such that the outcomes are x_1, x_2, x_3 , respectively. Find the probability of getting the outcomes such that $x_1 < x_2 < x_3$.

15. Find the area bounded by the curve $y = \frac{x^2}{a^2}$, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the x-axis in the first quadrant, where $a = \sqrt{2}$, and $b = \sqrt{6}$.

16. Let

$$\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \cdots + \frac{1}{\alpha+1012} = \frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{2023 \cdot 2024}$$

Find α .

17. Given

$$\sum_{n=0}^{\infty} ar^n = 57 \quad \text{and} \quad \sum_{n=0}^{\infty} ar^{3n} = 9747$$

Find $a + 18r$.

18. Given the integral

$$\int_0^x \sqrt{1 - (y')^2} dx = \int_0^x y(x) dx, \quad y(0) = 0$$

Find $y' + y + 1$ at $x = 1$.

19. Let α and β be roots of the equation

$$x^2 - \sqrt{2}x - \sqrt{3} = 0.$$

Further, $P_n = \alpha^n + \beta^n, n \in \mathbb{N}$.

If

$$11P_{12} + (10 - 11\sqrt{2})P_{11} - (11\sqrt{3} + 10\sqrt{2})P_{10} - \lambda = 0$$

Then λ is:

- (1) $\sqrt{3}P_9$
 - (2) $5\sqrt{3}P_9$
 - (3) P_9
 - (4) $10\sqrt{3}P_9$
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