JEE Main 2024 Mathematics Question Paper Feb 1 Shift 1

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Mathematics
SECTION A

1. If a, b, c are in A.P. and 3, $a-1$, $b+1$ are in G.P. Then arithmetic mean of a, b, and c is

(1) 11
(2) 10
(3) 9
(4) 13

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2. The value of $\int_0^{\frac{\pi}{4}} \frac{dx}{\sin^4(2x)+\cos^4(2x)}$ is equal to

(1) $\frac{\pi^2}{16\sqrt{2}}$
(2) $\frac{\pi^2}{64}$
(3) $\frac{\pi^2}{32}$

(4) $\frac{\pi^2}{8\sqrt{2}}$

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3. If

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, C = ABA^T$$

and $X = AC^2A^T$, then $|X|$ is equal to

(1) 729
(2) 283
(3) 27
(4) 23

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4. If

$$3, 7, 11, \ldots, 403 = A_1$$
$$2, 5, 8, \ldots, 401 = A_2$$

Find the sum of the common term of A_1 and A_2.

(1) 3366
(2) 6699
(3) 9999
(4) 6666

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5. If

$$I = \int_{\frac{\pi}{2}}^{0} \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)}\, dx = a\pi + b\left(3+2\sqrt{2}\right),$$

then find $a+b$.

(1) 4
(2) 6
(3) 8
(4) 2

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6. If

$$(t+1)\,dx = (2x + (t+1)^3)\,dt \quad \text{and} \quad x(0) = 2,$$

then $x(1)$ is equal to

(1) 5
(2) 12
(3) 6
(4) 8

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7. Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute them.

(1) 47
(2) 53
(3) 43
(4) 51

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8. If

$$5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 4 \quad \text{and} \quad y = 9f(x)x^2,$$

if y is a strictly increasing function, find the interval of x.

(1) $(-\infty, -\frac{1}{\sqrt{5}}) \cup (-\frac{1}{\sqrt{5}}, 0)$
(2) $(-\frac{1}{\sqrt{5}}, 0) \cup (0, \frac{1}{\sqrt{5}})$
(3) $(0, \frac{1}{\sqrt{5}}) \cup (\frac{1}{\sqrt{5}}, \infty)$
(4) $(-\frac{\sqrt{2}}{5}, \frac{\sqrt{2}}{5}) \cup (\frac{\sqrt{2}}{5}, \infty)$

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9. If hyperbola

$$x^2 - y^2 \csc^2\theta = 5 \quad \text{and ellipse} \quad x^2 \csc^2\theta + y^2 = 5,$$

has eccentricity e_H and e_e respectively and $e_H = \sqrt{7}e_e$, then θ is equal to

(1) $\frac{\pi}{3}$
(2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$
(4) $\frac{\pi}{4}$

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10. A bag contains 8 balls (black and white). If four balls are chosen without replacement then 2W and 2B are found, then the probability that the number of white and black balls are the same in the bag is equal to

(1) $\frac{1}{7}$
(2) $\frac{2}{7}$
(3) $\frac{3}{5}$
(4) $\frac{1}{2}$

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11. If two circles

$$x^2 + y^2 = 4 \quad \text{and} \quad x^2 + y^2 - 4x + 9 = 0$$

intersect at two distinct points, then find the range of λ.

(1) $\left(-\infty, -\frac{13}{2}\right) \cup \left(\frac{13}{2}, \infty\right)$
(2) $\left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$
(3) $\left[-\frac{13}{8}, \frac{13}{8}\right]$
(4) $\lambda \in \left(\frac{3}{2}, \infty\right)$

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12. If $S = \left\{x \in \mathbb{R} : 3(\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = \frac{10}{3}\right\}$ then number of elements in set S is

(1) Zero
(2) 1
(3) 2
(4) 3

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13. Let $f(x) = \begin{cases} e^{-x}, & x < 0 \\ \ln x, & x > 0 \end{cases}$ **and** $g(x) = \begin{cases} e^{x}, & x < 0 \\ x, & x > 0 \end{cases}$. The $g \circ f : A \to \mathbb{R}$ is:

(1) Onto but not one-one
(2) Into and many one
(3) Onto and one-one
(4) Into and one-one

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14. If $\tan A = \frac{1}{\sqrt{x^2+x+1}}$**,** $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ **and** $\tan C = \frac{1}{\sqrt{x(x^2+x+1)}}$**, then** $A + B =$**?**

(1) 0
(2) $\pi - C$
(3) $\frac{\pi}{2} - C$
(4) None

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15. Evaluate $\lim_{x\to 0} \cos^{-1}(1-x^2)\sin^{-1}(1-\{x\})$**, where** $\{x\}$ **is the fractional part function.**
If $L.H.L = L$ and $R.H.L = R$, then the correct relation between L and R is:

(1) $\sqrt{2}R = L$
(2) $\sqrt{L} = 4R$
(3) $R = L$
(4) $R = 2L$

SECTION B

21. Let $S = \{1, 2, 3, \ldots, 20\}$

$$R_1 = \{(a, b) : a \text{ divides } b\}$$

$$R_2 = \{(a, b) : a \text{ is integral multiple of } b,\ a, b \in S\}$$

Find $n(R_1 - R_2)$.

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22. The number of solutions of the equation $x + 2y + 3z = 42$**, where** $x, y, z \in \mathbb{Z}$ **and** $x, y, z \geq 0$**, is:**
