JEE Main 2024 Mathematics Question Paper Jan 27 Shift 2

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

1. If 20^{th} term from the end of progression

$$20, 19, \frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, 129\frac{1}{4}$$

is ___.

(1) -120
(2) -115
(3) -125
(4) -110

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2. Given:

$$P = (1-x)^{2008}\left(1+x+x^2\right)^{2007}, \quad \text{find the coefficient of}\, x^{2012}.$$

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3. The integral

$$\int \frac{x^8 - x^2}{x^{12} + 3x^6 + 1} \tan^{-1}\left(\frac{x^3+1}{x^3}\right) dx$$

is equal to:

(1) $\frac{1}{3}\ln\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)+C$
(2) $\ln\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)+C$
(3) $\frac{1}{6}\ln\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)+C$
(4) $\frac{1}{9}\ln\left(\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|\right)+C$

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4. The given sum is:

$$S_n = 2023\alpha^n + 2024\beta^n, \quad x^2 - x - 1 = 0.$$

Find the value of $2S_{12} = S_{11} + S_{10}$.

(1) $2S_{12} = S_{11} + S_{10}$
(2) $S_{12} = S_{10} + S_{11}$
(3) $S_{12} = S_{10} + S_{11}$
(4) $2S_{12} = S_{10} + S_{11}$

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5. Consider an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad (a > b);$$

eccentricity e_2, and hyperbola

$$\frac{x^2}{16} - \frac{y^2}{9} = 1,$$

with eccentricity e_1, $e_1e_2 = 1$. The ellipse passes through the foci of the hyperbola. Find the length of the ellipse along $y = 2$.

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6. Given:

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1 - 2a\cos x + a^2}$$

is equal to:

(1) $\frac{(1+a^2)\pi}{(1-a^2)^2}$
(2) $\frac{\pi}{(1-a^2)}$
(3) $\frac{(1-a^2)\pi}{(1+a^2)}$
(4) $\frac{(1-a^2)\pi}{(1+a^2)^2}$

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7. Find the limit:

$$\lim_{x\to 0} \frac{3 - a\sin x - b\cos x - \log(1+x)}{3\tan^2 x}$$

is non-zero finite. Find $2b - a$.

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8. The integral:

$$I = \int_0^{\frac{\pi}{2}} \frac{dx}{1 - 2a\cos x + a^2}$$

is equal to:

(1) $\frac{(1+a^2)\pi}{(1-a^2)^2}$
(2) $\frac{\pi}{(1-a^2)}$
(3) $\frac{(1-a^2)\pi}{(1+a^2)}$
(4) $\frac{(1-a^2)\pi}{(1+a^2)^2}$

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9. For the given equation:

$$x^2 - 6x + 3 = 0,$$

if α and β are the roots, then find the value of $\alpha + \beta$.

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10. Find the number of possible equivalence relations for the set

$$R : \{1,2,3,4\} \to \{1,2,3,4\}, \quad \text{if} \quad (1,4),(1,2) \in R.$$

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11. If

$$\begin{vmatrix} 1 & \frac{3}{2} \\ \frac{1}{3} & \alpha + \frac{1}{3} \end{vmatrix} = 0, \text{ then } \alpha \text{ lies in:}$$

(a) $\left[\frac{-3}{2}, \frac{3}{2}\right]$
(b) $(-3, 0)$
(c)
(d)

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12. Find the number of solutions:

$$\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$$

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13. A-m elements, B-n elements, subset of A is 56 more than B, $P(m,n)$ is a point and $Q(-2,-3)$, find the distance between P and Q.

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14. If $\alpha = \frac{(4!)}{(4!)3!}$ and $\beta = \frac{(5!)}{(5!)4!}$, then find:

(a) α is integer, β is not
(b) β is integer, α is not
(c) Both are integers
(d) Both are not integers

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15. Let A be a 2×2 real matrix and roots of equation $|A - x| = 0$ be -1 and 3. Sum of diagonal elements of A^2 is:

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16. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3 - x)$ and $f''(x) > 0$ for $x \in (0, 3)$.
