JEE Main 2024 Mathematics Question Paper Jan 29 Shift 1

Time Allowed :3 Hours	**Maximum Marks :**300	**Total Questions :**90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

1. Let a die be rolled until 2 is obtained. The probability that 2 is obtained on an even-numbered toss is equal to:

(1) $\frac{5}{11}$
(2) $\frac{5}{6}$
(3) $\frac{1}{11}$
(4) $\frac{6}{11}$

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2. Evaluate the limit:

$$\lim_{x\to\frac{\pi}{2}} \frac{\int_{x^3}^{\left(\frac{\pi}{2}\right)^2} \cos(t^{1/3})\,dt}{(x-\frac{\pi}{2})^2}.$$

(1) $\frac{3\pi^2}{4}$
(2) $\frac{3\pi}{4}$
(3) $\frac{3\pi^2}{8}$
(4) $\frac{3\pi}{8}$

3. Consider the equation $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$. Statement 1: Solution of this equation is $\cos\frac{\pi}{12}$. Statement 2: This equation has only one real solution.

(1) Both statement 1 and statement 2 are true
(2) Statement 1 is true but statement 2 is false
(3) Statement 1 is false but statement 2 is true
(4) Both statement 1 and statement 2 are false

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4. If

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$$

then α is (if $\alpha, \beta \in \mathbb{R}$):

(1) 5
(2) 3
(3) 9
(4) 17

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5. In a 64 terms GP, if the sum of all terms is seven times the sum of the odd terms, then the common ratio is:

(1) 3
(2) 4
(3) 5
(4) 6

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6. If

$$\frac{dy}{dx}\left(\frac{\sin 2x}{1+\cos^2 x}\right) = \frac{\sin x}{1+\cos^2 x} \quad \text{and} \quad y(0)=0, \text{ then } y\left(\frac{\pi}{2}\right) \text{ is:}$$

(1) -1
(2) 1
(3) 0
(4) 2

7. Given that

$$4\cos\theta + 5\sin\theta = 1, \quad \text{then find } \tan\theta, \text{ where } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

(1) $\frac{10-\sqrt{10}}{6}$
(2) $\frac{10-\sqrt{10}}{12}$
(3) $\frac{\sqrt{10}-10}{6}$
(4) $\frac{\sqrt{10}-10}{12}$

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8. In an increasing arithmetic progression $a_1, a_2, \ldots, a_n$, if $a_6 = 2$ and the product of a_1, a_5, a_4 is greatest, then the value of d is equal to:

(1) 1.6
(2) 1.8
(3) 0.6
(4) 2.0

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9. If relation $R : (a, b)\, R(c, d)$ is defined only if $ad - bc$ is divisible by 5 (where $a, b, c, d \in \mathbb{Z}$), then R is:

(1) Reflexive
(2) Symmetric, Reflexive but not Transitive
(3) Reflexive, Transitive but not symmetric
(4) Equivalence relation

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10. Let

$$f(x) = \begin{cases} 2 + 2x, & \text{for } x \in (-1, 0) \\ 1 - \frac{x}{3}, & \text{for } x \in [0, 3] \end{cases}$$

$$g(x) = \begin{cases} x, & \text{for } x \in [0, 1] \\ -x, & \text{for } x \in (-3, 0) \end{cases}$$

The range of $f \circ g(x)$ is:

(1) $[0, 1]$
(2) $[-1, 1]$

(3) $(0, 1)$
(4) $(-1, 1)$

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11. If

$$\int_{\frac{\pi}{2}}^{\pi} \left(\frac{x^2 \cos x}{1 + \pi x^2} + \frac{1 + \sin^2 x}{1 + e^{(\sin x)^{2023}}} \right) dx = \frac{\pi}{4}(\pi + \alpha) - 2,$$

then the value of α is equal to:

(1) 1
(2) 2
(3) 3
(4) 4

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12. Area under the curve $x^2 + y^2 = 169$ and below the line $5x - y = 13$ is:

(1) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1}\left(\frac{12}{13}\right)$
(2) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1}\left(\frac{13}{13}\right)$
(3) $\frac{169}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1}\left(\frac{13}{14}\right)$
(4) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1}\left(\frac{13}{14}\right)$

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13. If

$$f(x) = \frac{(2^x + 2^{-x})(\tan x)\sqrt{\tan^{-1}(2x^2 - 3x + 1)}}{(7x^2 - 3x + 1)^3},$$

then $f(0)$ is equal to:

(1) $\sqrt{\pi}$
(2) $\frac{\pi}{4}$
(3) π
(4) $2 \cdot \pi^{3/2}$

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14. Evaluate the integral:

$$\int \left((\sin x - \cos x) \sin^2 x \right) dx.$$

(1) $\frac{\ln|\sin x - \cos x|}{3} + c$
(2) $\ln|\sin^3 x + \cos^3 x| + c$
(3) $\frac{\ln|\sin^3 x - \cos^3 x|}{2} + c$
(4) $\frac{\ln|\sin^3 x + \cos^3 x|}{4} + c$

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21. Evaluate the sum

$$\frac{11C_1}{2} + \frac{11C_2}{3} + \cdots + \frac{11C_9}{10} = \frac{m}{n},$$

then find $m+n$.

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22. Rank of the word 'GTWENTY' in the dictionary is:

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23. Curve $y = 2^x - x^2$, $y_1(x)$ and $y_2(x)$ cut the x-axis at M and N points respectively, find $M+N$.

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24. Given the data:

$$60, 60, 44, 58, \alpha, \beta, 68, 56, \quad \text{mean} = 58, \quad \text{variance} = 66.2, \quad \text{find} \quad \alpha^2 + \beta^2.$$

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25. If

$$|z+1| = \alpha z + \beta(i+1) \quad \text{and} \quad z = -\frac{1}{2} - 2i, \quad \text{find}\, \alpha + \beta.$$

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26. If

$$\vec{a}, \vec{b}, \vec{c} \quad \text{are non-zero vectors and} \quad \vec{b} \quad \text{and} \quad \vec{c} \quad \text{are non-collinear,}$$
$$\vec{a} + 5\vec{b} \text{ is collinear with } \vec{c} \quad \text{and} \quad \vec{b} + 6\vec{c} \text{ is collinear with } \vec{a}.$$

If

$$\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0, \quad \text{then find} \quad \alpha + \beta.$$
