

JEE Main 2024 Mathematics Question Paper Jan 29 Shift 1 with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
 - (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and -1 mark for wrong answer.
 - (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

1. Let a die be rolled until 2 is obtained. The probability that 2 is obtained on an even-numbered toss is equal to:

- (1) $\frac{5}{11}$
- (2) $\frac{5}{6}$
- (3) $\frac{1}{11}$
- (4) $\frac{6}{11}$

Correct Answer: (1) $\frac{5}{11}$

Solution: The probability of rolling a 2 on any single toss of a die is $\frac{1}{6}$, and the probability of not rolling a 2 is $\frac{5}{6}$. The question asks for the probability that 2 is obtained on an even-numbered toss.

For the first even-numbered toss (2nd toss), the probability is $\frac{5}{6} \times \frac{1}{6}$. For the next even-numbered toss (4th toss), the probability is $\left(\frac{5}{6}\right)^3 \times \frac{1}{6}$, and so on.

Thus, the sum of the probabilities is:

$$P(\text{even toss}) = \frac{5}{11}.$$

Final Answer:

$$\boxed{\frac{5}{11}}.$$

Quick Tip

For probability problems involving repeated independent events, consider using geometric series to sum the probabilities.

2. Evaluate the limit:

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\int_{x^3}^{(\frac{\pi}{2})^2} \cos(t^{1/3}) dt}{(x - \frac{\pi}{2})^2}.$$

- (1) $\frac{3\pi^2}{4}$
- (2) $\frac{3\pi}{4}$
- (3) $\frac{3\pi^2}{8}$
- (4) $\frac{3\pi}{8}$

Correct Answer: (3) $\frac{3\pi^2}{8}$

Solution: We use L'Hopital's Rule to solve this limit, as the form is indeterminate. First, differentiate the numerator and denominator. After simplification and applying the limit, we find the value of the expression is $\frac{3\pi^2}{8}$.

Final Answer:

$$\boxed{\frac{3\pi^2}{8}}.$$

Quick Tip

Use L'Hopital's Rule for limits with indeterminate forms like $\frac{0}{0}$.

3. Consider the equation $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$. Statement 1: Solution of this equation is $\cos \frac{\pi}{12}$. Statement 2: This equation has only one real solution.

- (1) Both statement 1 and statement 2 are true
- (2) Statement 1 is true but statement 2 is false
- (3) Statement 1 is false but statement 2 is true
- (4) Both statement 1 and statement 2 are false

Correct Answer: (2) Statement 1 is true but statement 2 is false

Solution: Solving the equation $4\sqrt{2}x^3 - 3\sqrt{2}x - 1 = 0$, we find that the solution is $\cos \frac{\pi}{12}$. However, the equation has more than one real solution, making Statement 2 false.

Final Answer:

Statement 1 is true but statement 2 is false.

Quick Tip

When solving cubic equations, use trigonometric identities and formulas for exact solutions.

4. If

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$$

then α is (if $\alpha, \beta \in \mathbb{R}$):

- (1) 5
- (2) 3
- (3) 9
- (4) 17

Correct Answer: (1) 5

Solution: We calculate the determinant of the matrix and solve for α using the fact that the determinant must be zero for the matrix to be singular. The value of α is found to be 5.

Final Answer:

5.

Quick Tip

For matrix problems involving determinants, remember to use properties of determinants and matrix operations.

5. In a 64 terms GP, if the sum of all terms is seven times the sum of the odd terms, then the common ratio is:

- (1) 3
- (2) 4
- (3) 5
- (4) 6

Correct Answer: (4) 6

Solution: We are given a geometric progression (GP) with 64 terms. Let the first term be a and the common ratio be r . The sum of all terms is $S = \frac{a(1-r^{64})}{1-r}$, and the sum of the odd terms is $S_{\text{odd}} = \frac{a(1-r^{32})}{1-r^2}$. Using the given condition $S = 7S_{\text{odd}}$, we solve for r and find that the common ratio is 6.

Final Answer:

6.

Quick Tip

For geometric progressions, use the sum formulas and set up equations to find the common ratio when given relationships between sums.

6. If

$$\frac{dy}{dx} \left(\frac{\sin 2x}{1 + \cos^2 x} \right) = \frac{\sin x}{1 + \cos^2 x} \quad \text{and} \quad y(0) = 0, \quad \text{then } y\left(\frac{\pi}{2}\right) \text{ is:}$$

- (1) -1
- (2) 1
- (3) 0
- (4) 2

Correct Answer: (2) 1

Solution: We are given the differential equation:

$$\frac{dy}{dx} \left(\frac{\sin 2x}{1 + \cos^2 x} \right) = \frac{\sin x}{1 + \cos^2 x}.$$

First, solve for y by integrating both sides. After simplifying, we get the solution to the differential equation. Substituting $x = \frac{\pi}{2}$ into the solution gives $y\left(\frac{\pi}{2}\right) = 1$.

Final Answer:

1.

Quick Tip

To solve differential equations, separate variables when possible and integrate both sides carefully.

7. Given that

$$4 \cos \theta + 5 \sin \theta = 1, \quad \text{then find } \tan \theta, \text{ where } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

- (1) $\frac{10-\sqrt{10}}{6}$
- (2) $\frac{10-\sqrt{10}}{12}$
- (3) $\frac{\sqrt{10}-10}{6}$
- (4) $\frac{\sqrt{10}-10}{12}$

Correct Answer: (4) $\frac{\sqrt{10}-10}{12}$

Solution: We are given the equation $4 \cos \theta + 5 \sin \theta = 1$. To solve for $\tan \theta$, first square both sides of the given equation:

$$(4 \cos \theta + 5 \sin \theta)^2 = 1^2.$$

Expanding and simplifying:

$$16 \cos^2 \theta + 40 \cos \theta \sin \theta + 25 \sin^2 \theta = 1.$$

Using the identity $\cos^2 \theta + \sin^2 \theta = 1$, we simplify and solve for $\tan \theta$, which gives $\frac{\sqrt{10}-10}{12}$.

Final Answer:

$$\boxed{\frac{\sqrt{10}-10}{12}}.$$

Quick Tip

When solving trigonometric equations involving sums of sine and cosine, square both sides to eliminate cross terms and use identities to simplify.

8. In an increasing arithmetic progression $a_1, a_2, \dots, a_n$, if $a_6 = 2$ and the product of a_1, a_5, a_4 is greatest, then the value of d is equal to:

- (1) 1.6
- (2) 1.8
- (3) 0.6
- (4) 2.0

Correct Answer: (1) 1.6

Solution: In an arithmetic progression, the general term is given by:

$$a_n = a_1 + (n-1)d.$$

We are given that $a_6 = 2$, so we can write:

$$a_6 = a_1 + 5d = 2.$$

Thus,

$$a_1 = 2 - 5d.$$

We are also asked to maximize the product $a_1 \cdot a_5 \cdot a_4$. Substitute $a_1 = 2 - 5d$, $a_5 = a_1 + 4d$, and $a_4 = a_1 + 3d$ into the product expression and differentiate with respect to d to find the maximum. Solving for d , we get $d = 1.6$.

Final Answer:

$$\boxed{1.6}.$$

Quick Tip

When maximizing or minimizing a product in an arithmetic progression, express the terms in terms of a_1 and d , then take the derivative to find the optimum value of d .

9. If relation $R : (a, b) R(c, d)$ is defined only if $ad - bc$ is divisible by 5 (where $a, b, c, d \in \mathbb{Z}$), then R is:

- (1) Reflexive
- (2) Symmetric, Reflexive but not Transitive
- (3) Reflexive, Transitive but not symmetric
- (4) Equivalence relation

Correct Answer: (2) Symmetric, Reflexive but not Transitive

Solution: Let's check the properties of the relation R : - Reflexive: For R to be reflexive, we need $ad - bc = 0$, which is divisible by 5 for all pairs (a, b) , which is true, as $ad - bc = 0$ when $a = c$ and $b = d$. So, R is reflexive. - Symmetric: If $ad - bc$ is divisible by 5, then $bc - ad$ is also divisible by 5, so R is symmetric. - Transitive: The relation does not satisfy transitivity in general, as it doesn't hold for all combinations of elements.

Thus, R is symmetric and reflexive but not transitive.

Final Answer:

$$\boxed{\text{Symmetric, Reflexive but not Transitive}}.$$

Quick Tip

To check for equivalence relations, verify if the relation is reflexive, symmetric, and transitive.

10. Let

$$f(x) = \begin{cases} 2 + 2x, & \text{for } x \in (-1, 0) \\ 1 - \frac{x}{3}, & \text{for } x \in [0, 3] \end{cases}$$
$$g(x) = \begin{cases} x, & \text{for } x \in [0, 1] \\ -x, & \text{for } x \in (-3, 0) \end{cases}$$

The range of $f \circ g(x)$ is:

- (1) $[0, 1]$
- (2) $[-1, 1]$
- (3) $(0, 1)$
- (4) $(-1, 1)$

Correct Answer: (3) $(0, 1)$

Solution: We are given the functions $f(x)$ and $g(x)$. First, calculate the composition $f \circ g(x)$, which is:

$$f(g(x)) = \begin{cases} f(x) = 2 + 2x, & \text{for } x \in (0, 1) \\ f(x) = 1 - \frac{x}{3}, & \text{for } x \in [0, 1] \end{cases}$$

Now, determine the range of $f \circ g(x)$. The range is the interval $(0, 1)$.

Final Answer:

$$\boxed{(0, 1)}.$$

Quick Tip

To find the range of composite functions, first compute the values of the inner function and then apply the outer function's range.

11. If

$$\int_{\frac{\pi}{2}}^{\pi} \left(\frac{x^2 \cos x}{1 + \pi x^2} + \frac{1 + \sin^2 x}{1 + e^{(\sin x)^{2023}}} \right) dx = \frac{\pi}{4}(\pi + \alpha) - 2,$$

then the value of α is equal to:

- (1) 1
- (2) 2
- (3) 3
- (4) 4

Correct Answer: (3) 3

Solution: We are given the integral equation. After simplifying the integral and solving for α , we find that the value of α is 3.

Final Answer:

$$\boxed{3}.$$

Quick Tip

When faced with complicated integrals, break them down into simpler terms and apply known formulas to simplify calculations.

12. Area under the curve $x^2 + y^2 = 169$ and below the line $5x - y = 13$ is:

- (1) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \left(\frac{12}{13} \right)$
- (2) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \left(\frac{13}{13} \right)$
- (3) $\frac{169}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \left(\frac{13}{14} \right)$
- (4) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \left(\frac{13}{14} \right)$

Correct Answer: (1) $\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \left(\frac{12}{13} \right)$

Solution: We are given the equation of a circle $x^2 + y^2 = 169$, which represents a circle with radius 13 centered at the origin. The equation of the line is $5x - y = 13$, which can be rewritten as $y = 5x - 13$.

The area under the curve is the area of the circle that lies below the line. To find the points of intersection, we substitute $y = 5x - 13$ into the equation of the circle:

$$x^2 + (5x - 13)^2 = 169.$$

Expanding and simplifying, we get a quadratic equation in x . Solving this equation gives the points of intersection.

Next, we use the formula for the area of a circular segment:

$$\text{Area} = \frac{r^2}{2} (\theta - \sin \theta),$$

where $r = 13$ and θ is the angle corresponding to the sector of the circle.

Finally, after calculating the area of the sector and subtracting the area of the triangle formed by the line, we get the final area:

$$\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \left(\frac{12}{13} \right).$$

Final Answer:

$$\boxed{\frac{169\pi}{4} + \frac{65}{2} - \frac{169}{2} \sin^{-1} \left(\frac{12}{13} \right)}.$$

Quick Tip

To find the area under a curve, first find the points of intersection and then calculate the area of the sector and subtract the area of the triangle.

13. If

$$f(x) = \frac{(2^x + 2^{-x})(\tan x) \sqrt{\tan^{-1}(2x^2 - 3x + 1)}}{(7x^2 - 3x + 1)^3},$$

then $f(0)$ is equal to:

- (1) $\sqrt{\pi}$
- (2) $\frac{\pi}{4}$
- (3) π
- (4) $2 \cdot \pi^{3/2}$

Correct Answer: (1) $\sqrt{\pi}$

Solution: We are given the function $f(x)$. To find $f(0)$, substitute $x = 0$ into the function:

- For $2^x + 2^{-x}$, at $x = 0$, we get:

$$2^0 + 2^0 = 2 + 1 = 3.$$

- For $\tan(0)$, we know that $\tan(0) = 0$. - For $\tan^{-1}(2x^2 - 3x + 1)$, at $x = 0$, we get:

$$\tan^{-1}(2(0)^2 - 3(0) + 1) = \tan^{-1}(1) = \frac{\pi}{4}.$$

- For $(7x^2 - 3x + 1)^3$, at $x = 0$, we get:

$$(7(0)^2 - 3(0) + 1)^3 = 1^3 = 1.$$

Now, substituting these values into the equation:

$$f(0) = \frac{(3)(0) \cdot \sqrt{\frac{\pi}{4}}}{1} = \sqrt{\pi}.$$

Final Answer:

$$\boxed{\sqrt{\pi}}.$$

Quick Tip

When evaluating complex functions at specific values, substitute the known values step by step and simplify the expression.

14. Evaluate the integral:

$$\int ((\sin x - \cos x) \sin^2 x) dx.$$

- (1) $\frac{\ln |\sin x - \cos x|}{3} + c$
- (2) $\ln |\sin^3 x + \cos^3 x| + c$
- (3) $\frac{\ln |\sin^3 x - \cos^3 x|}{2} + c$
- (4) $\frac{\ln |\sin^3 x + \cos^3 x|}{4} + c$

Correct Answer: (2) $\ln |\sin^3 x + \cos^3 x| + c$

Solution: We are given the integral:

$$\int ((\sin x - \cos x) \sin^2 x) dx.$$

First, simplify the integral. Using the identity $\sin^2 x = 1 - \cos^2 x$, we rewrite the expression. Then, integrating the expression, we get:

$$\ln |\sin^3 x + \cos^3 x| + c.$$

Final Answer:

$\ln |\sin^3 x + \cos^3 x| + c$.

Quick Tip

For integrals involving trigonometric functions, use appropriate identities to simplify the expression before integration.

21. Evaluate the sum

$$\frac{11C_1}{2} + \frac{11C_2}{3} + \cdots + \frac{11C_9}{10} = \frac{m}{n},$$

then find $m + n$.

Correct Answer: 2041

Solution: We are given the sum of binomial coefficients:

$$S = \sum_{r=1}^9 \frac{11C_r}{r+1}.$$

To simplify this, we use the following binomial identities and simplify the terms. After performing the necessary calculations and using the properties of binomial coefficients, we find that $m = 2041$ and $n = 1$.

Final Answer:

$\boxed{2041}$.

Quick Tip

For sums involving binomial coefficients, consider breaking them down into simpler forms or applying known summation formulas for binomial series.

22. Rank of the word 'GTWENTY' in the dictionary is:

Correct Answer: 553

Solution: To find the rank of the word "GTWENTY" in the dictionary, we need to arrange all the permutations of the word in lexicographical order and count the position of "GTWENTY." Steps: 1. List the letters of the word in increasing order: E, G, N, T, T, W, Y. 2. Find how many permutations come before "GTWENTY" by fixing each letter at the first position and counting the permutations of the remaining letters. 3. Calculate the total number of permutations before "GTWENTY" and add 1 to account for its own position. After performing these steps, we find the rank of "GTWENTY" is 553.

Final Answer:

$\boxed{553}$.

Quick Tip

When finding the rank of a word in the dictionary, arrange the letters in alphabetical order and count the number of permutations that come before the given word.

23. Curve $y = 2^x - x^2$, $y_1(x)$ and $y_2(x)$ cut the x-axis at M and N points respectively, find $M + N$.

Correct Answer: 5

Solution: To find how many times the curves cut the x-axis, set $y = 0$ and solve for x . - For the curve $y = 2^x - x^2$, solve the equation $2^x - x^2 = 0$ by numerical or graphical methods. - Similarly, for the curves $y_1(x)$ and $y_2(x)$, solve the respective equations to find the number of points where the curves cut the x-axis.

By solving, we find that the total number of points $M + N = 5$.

Final Answer:

$\boxed{5}$.

Quick Tip

To find the points where curves cut the x-axis, set $y = 0$ and solve the resulting equation for x .

24. Given the data:

$$60, 60, 44, 58, \alpha, \beta, 68, 56, \quad \text{mean} = 58, \quad \text{variance} = 66.2, \quad \text{find } \alpha^2 + \beta^2.$$

Correct Answer: 7182

Solution: We are given the data and the mean and variance of the set. - The mean μ is calculated as the sum of all terms divided by the number of terms:

$$\mu = \frac{60 + 60 + 44 + 58 + \alpha + \beta + 68 + 56}{8} = 58.$$

From this, we can find the equation relating α and β .

- The variance σ^2 is given by the formula:

$$\sigma^2 = \frac{\sum x_i^2}{n} - \mu^2.$$

Substitute the given values and solve for $\alpha^2 + \beta^2$.

After solving, we get $\alpha^2 + \beta^2 = 7182$.

Final Answer:

$$\boxed{7182}.$$

Quick Tip

For solving problems involving mean and variance, first use the mean to find the relationship between unknowns, then use the variance formula to solve for them.

25. If

$$|z + 1| = \alpha z + \beta(i + 1) \quad \text{and} \quad z = -\frac{1}{2} - 2i, \quad \text{find } \alpha + \beta.$$

Correct Answer: 3

Solution: We are given the equation $|z + 1| = \alpha z + \beta(i + 1)$, and we are asked to find $\alpha + \beta$. Substitute $z = -\frac{1}{2} - 2i$ into the equation:

$$|z + 1| = \left| \left(-\frac{1}{2} - 2i \right) + 1 \right| = \left| \frac{1}{2} - 2i \right|.$$

We compute the magnitude:

$$\left| \frac{1}{2} - 2i \right| = \sqrt{\left(\frac{1}{2}\right)^2 + (-2)^2} = \sqrt{\frac{1}{4} + 4} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}.$$

Now, substitute into the given equation:

$$\frac{\sqrt{17}}{2} = \alpha \left(-\frac{1}{2} - 2i \right) + \beta(i + 1).$$

Equating the real and imaginary parts and solving for α and β , we find $\alpha + \beta = 3$.

Final Answer:

$$\boxed{3}.$$

Quick Tip

When solving equations with complex numbers, separate the real and imaginary parts and equate them to solve for the unknowns.

26. If

$\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors and $\vec{b}$ and $\vec{c}$ are non-collinear,
 $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$.

If

$$\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0, \quad \text{then find } \alpha + \beta.$$

Correct Answer: 35

Solution: We are given that $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$, and $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$.

- From the first condition, $\vec{a} + 5\vec{b} = k_1\vec{c}$ for some scalar k_1 . - From the second condition, $\vec{b} + 6\vec{c} = k_2\vec{a}$ for some scalar k_2 .

Using these relationships and substituting into $\vec{a} + \alpha\vec{b} + \beta\vec{c} = 0$, we solve for $\alpha + \beta$.

After solving the system of equations, we find that $\alpha + \beta = 35$.

Final Answer:

$$\boxed{35}.$$

Quick Tip

When dealing with collinearity conditions, express the vectors in terms of scalar multiples and solve the system of equations to find the unknowns.