JEE Main 2024 Mathematics Question Paper Jan 29 Shift 2

Time Allowed : 3 Hours	**Maximum Marks :** 300	**Total Questions :** 90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

1. Given the integral:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$$

Find $r = 3\beta$, and then find $\alpha\beta\gamma$.

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2. Given the points $P(3,2,3)$, $Q(4,6,2)$, and $R(7,3,2)$ as the vertices of triangle PQR, find $\angle QPR$.

(1) $\cos^{-1}\left(\frac{1}{18}\right)$
(2) $\frac{\pi}{6}$
(3) $\frac{\pi}{3}$
(4) $\cos^{-1}\left(\frac{7}{18}\right)$

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3. Find the area bounded by

$$0 \le y \le \min\left(x^2+2, 2x+2\right), \quad x \in [0,3].$$

4. Find the remainder when

$$64^{32^{12}} \text{ is divided by } 9.$$

5. If

$\cos\left(2\sin^{-1} x\right) = \dfrac{1}{9}$ holds for $x = \dfrac{m}{n}$, and $\alpha\beta$ are the roots of the equation $mx^2 - nx - m + n = 0$, find $\alpha+$

(1) $5x - 8y = 9$
(2) $5x + 8y = 9$
(3) $8x + 5y = 9$
(4) $8x - 5y = 9$

6. If α and β are roots of the equation

$$x^2 - \sqrt{6}x + 3 = 0 \quad \text{and} \quad \frac{\alpha^{99}}{\beta} + \beta^{98} = 3^n(a + ib),$$

where $\text{Im}(\beta) < 0$, **then find** a, b, n.

7. Find the distance of point $(2, 4)$ **from the line** $2x + y + 2 = 0$, **measured parallel to the line** $\sqrt{3}x + y + 2 = 0$.

8. Given that the sequence $a_1, a_2, \dots$ **is in geometric progression such that**

$$\frac{a_1}{a_2} = \frac{1}{8} \quad \text{and} \quad a_1 \neq a_2,$$

and every term is equal to the arithmetic mean of its two successive terms, find $S_{20} - S_{18}$.

9. Given the function

$$f(x) = 2x + 3x^{\frac{2}{3}}, \quad x \in \mathbb{R}.$$

Determine the nature of critical points and select the correct option.

(1) It has one maxima, no minima.
(2) It has one minima, no maxima.
(3) It has 2 maxima and 1 minima.
(4) It has 1 maxima and 1 minima.

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10. Given the relation $R : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ such that $(1, 4)$, $(1, 2) \in R$, find the number of possible equivalence relations.

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11. There are 8 identical books and 4 identical shelves. Find the number of ways to arrange the books such that any shelf may be empty, and every shelf can accommodate all the books.

(1) 13
(2) 14
(3) 15
(4) 16

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12. Find the probability that a number selected from 1 to 50 is divisible by at least one of 4, 6, or 7.

(1) $\frac{21}{25}$
(2) $\frac{18}{50}$
(3) $\frac{8}{25}$
(4) $\frac{21}{25}$

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13. Let A and B be points of the lines L_1 and L_2, respectively, such that OA and OB are the shortest distance points from the origin.
