JEE Main 2024 Mathematics Question Paper Jan 30 Shift 1

Time Allowed :3 Hours	Maximum Marks :300	Total Questions :90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and –1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and –1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

1. In an arithmetic progression, if the sum of 20 terms is 790 and the sum of 10 terms is 145, then $S_{15} - S_5$ is:

(1) 400
(2) 395
(3) 385
(4) 405

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2. If the foot of the perpendicular from $(1, 2, 3)$ to the line $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z-4}{1}$ is (α, β, γ), then find $\alpha + \beta + \gamma$:

(1) 6
(2) 5.8
(3) 4.8
(4) 5

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3. Evaluate the limit:

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{(n^2+k^2)(n^2+3k^2)}.$$

(1) $\frac{\pi}{2\sqrt{3}}-\frac{\pi}{8}$
(2) $\frac{\pi}{2\sqrt{3}}+\frac{\pi}{8}$
(3) $\frac{\pi}{2}-\frac{\pi}{\sqrt{3}}$
(4) $\frac{\pi}{\sqrt{3}}-\frac{\pi}{4}$

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4. The value of the maximum area possible of a $\triangle ABC$ such that $A(0,0)$, $B(x,y)$, and $C(-x,y)$, with $y=-2x^2+54x$, is (in square units):

(1) 5800
(2) 5832
(3) 5942
(4) 6008

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5. The range of r for which circles $(x+1)^2+(y+2)^2=r^2$ and $x^2+y^2-4x-4y+4=0$ coincide at two distinct points is:

(1) $3<r<7$
(2) $5<r<9$
(3) $\frac{1}{2}<r<4$
(4) $0<r<3$

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6. An ellipse whose length of minor axis is equal to half the length between the foci, then the eccentricity is:

(1) $\frac{7}{2}$
(2) $\sqrt{17}$
(3) $\frac{2}{\sqrt{5}}$
(4) $\frac{3}{\sqrt{7}}$

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7. If $g'(\frac{3}{2})=g'(\frac{1}{2})$ and $f(x)=\frac{1}{2}[g(x)+g(2-x)]$ and $f'(\frac{3}{2})=f'(\frac{1}{2})$, then:

(1) $f''(x) = 0$ has exactly one root in $(0, 1)$
(2) $f''(x) = 0$ has no root in $(0, 1)$
(3) $f''(x) = 0$ has at least two roots in $(0, 2)$
(4) $f''(x) = 0$ has 3 roots in $(0, 2)$

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8. The domain of $y = \cos^{-1}\left(\frac{|x|}{4}\right) + (\log(3-x))^{-1}$ is $[-\alpha, \beta) \setminus \{y\}$, then the value of $\alpha + \beta + \gamma$ is:

(1) 9
(2) 12
(3) 11
(4) 10

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9. If $y = f(x)$ is the solution of the differential equation $(x^2-1)\,dy = \left(x^3 + 1 + \sqrt{1-x^2}\right)dx$, and $y(0) = 2$, then find $y\left(\frac{1}{2}\right)$:

(1) $\frac{13}{7} - \frac{\pi}{2} + \ln 5$
(2) $\frac{15}{7} + \frac{\pi}{3} + \ln 2$
(3) $\frac{17}{8} + \frac{\pi}{6} - \ln 2$
(4) $\frac{18}{7} - \frac{\pi}{6} + \ln 3$

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10. Given $x^2 - 70x + \lambda = 0$ with positive roots α and β, where one of the roots is less than 10 and $\frac{\lambda}{2}$ and $\frac{\lambda}{3}$ are not integers, find the value of $\frac{\sqrt{\alpha-1}+\sqrt{\beta-1}}{|\alpha-\beta|}$:

(1) $\frac{1}{5}$
(2) $\frac{1}{12}$
(3) $\frac{1}{60}$
(4) $\frac{1}{70}$

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11. A line passes through $(9, 0)$, making an angle of 30° with the positive direction of the x-axis. It is rotated by an angle of 15° with respect to $(9, 0)$. Then one of the equations of the new line is:

(1) $y = (2 + \sqrt{3})(x - 9)$
(2) $y = (2 - \sqrt{3})(x - 9)$

(3) $y = 2(x - 9)$
(4) $y = -(x - 9)$

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12. For a non-zero complex number z satisfying $z^2 + \overline{z} = 0$, then the value of $|z|^2$ is:

(1) 1
(2) 2
(3) 3
(4) 4

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13. If $|a| = 1, |b| = 4$, $a \cdot b = 2$ and $c = 2(a \times b) - 3b$, then the angle between b and c is:

(1) $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
(2) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(3) $\theta = \cos^{-1}\left(\frac{1}{2}\right)$
(4) $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

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14. Given set $S = \{0, 1, 2, 3, \ldots, 10\}$. If a random ordered pair (x, y) of elements of S is chosen, then find the probability that $|x - y| > 5$:

(1) $\frac{30}{121}$
(2) $\frac{31}{121}$
(3) $\frac{62}{121}$
(4) $\frac{64}{121}$

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21. Number of integral terms in the binomial expansion of $\left(7^{1/2} + 11^{1/6}\right)^{824}$ is:

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22. Evaluate $\int_0^9 \left\lfloor \frac{10x}{\sqrt{x+1}} \right\rfloor dx$, where $\lfloor \cdot \rfloor$ represents the greatest integer function:

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23. In a class of 40 students, 16 passed in Chemistry, 20 passed in Physics, 25 passed in Mathematics, 15 passed in both Mathematics and Physics, 15 passed in both Mathematics and Chemistry, and 10 passed in both Physics and Chemistry. Find the maximum number of students who passed in all subjects:

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24. For the following data table, find the value of $20M$, where M is the median of the data:

x_i	f_i
$0-4$	2
$4-8$	4
$8-12$	7
$12-16$	8
$16-20$	6
