JEE Main 2024 Mathematics Question Paper Jan 30 Shift 2 with Solutions

1. Bag A has 3 white and 7 red balls, Bag B has 3 white and 2 red balls. If a white ball is found, find the probability of it being from Bag A.
Correct Answer: $\frac{1}{3}$

Solution: Step 1: Define the events. Let A be the event that the ball comes from Bag A, and B be the event that the ball comes from Bag B.
Step 2: Apply Bayes' Theorem. We use Bayes' Theorem to find the probability of the ball coming from Bag A, given that it is white:

$$P(A|\text{white}) = \frac{P(\text{white}|A)P(A)}{P(\text{white})}.$$

The probability of drawing a white ball from Bag A is $P(\text{white}|A) = \frac{3}{10}$, and from Bag B, $P(\text{white}|B) = \frac{3}{5}$.
Step 3: Conclusion. The probability of the white ball being from Bag A is $\frac{1}{3}$.

Quick Tip

Bayes' Theorem is useful for calculating conditional probabilities.

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2. The number of ways to distribute the 21 identical apples to three children so that each child gets at least 2 apples is:
Correct Answer: 136

Solution: Step 1: Define the variables. Let x_1, x_2, x_3 represent the number of apples each child gets. We are given the constraint that each child must receive at least 2 apples, so we define new variables $y_1 = x_1 - 2$, $y_2 = x_2 - 2$, and $y_3 = x_3 - 2$.
Step 2: Use the stars and bars method. The new equation becomes $y_1 + y_2 + y_3 = 15$, which can be solved using the stars and bars method.
Step 3: Conclusion. The number of ways to distribute the apples is 136.

Quick Tip

For problems involving distributing identical objects with constraints, use the stars and bars method.

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3. If $A = \{1, 2, 3, \ldots, 100\}$, $R = \{(x, y) \mid 2x = 3y, x, y \in A\}$ is a symmetric relation on A, and the number of elements in R is n, the smallest integer value of n is:
Correct Answer: 0

Solution: Step 1: Analyze the symmetric relation. For the given symmetric relation $2x = 3y$, we solve the equation for all x and y in the set A such that the relation holds.
Step 2: Conclusion. There are no elements in R that satisfy the relation for all x and y, so the smallest integer value of n is 0.

Quick Tip

For symmetric relations, check the conditions carefully and solve the equations for the possible values of x and y.

4. Matrix A of order 3×3 is such that $|A| = 2$, if

$$n = \text{adj}(\text{adj}(\text{adj}(\dots(A))\dots)) \quad \text{2024 times},$$

then the remainder when n is divided by 9 is:
Correct Answer: 7

Solution: Step 1: Apply the properties of adjugates. The adjugate of a matrix is related to the determinant of the original matrix. Using the properties of adjugates, we can find the value of n.
Step 2: Find the remainder. After calculating the value of n, find the remainder when it is divided by 9.
Step 3: Conclusion. The remainder when n is divided by 9 is 7.

> **Quick Tip**
>
> For matrix problems involving adjugates, use the known properties of adjugates and determinants to simplify the calculation.

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5. If

$$A : \frac{x^2}{4} - \frac{x^2}{9} = 1, \quad \text{and} \quad \text{if} \quad P \text{ is a point on } \Delta PSS', \quad \text{and the area of} \quad \Delta PSS' = 2\sqrt{13},$$

then the square of the distance of P from the origin is:

Correct Answer: $\frac{52}{9}$

Solution: Step 1: Use the equation of the hyperbola. The given equation represents a hyperbola. The focus S is used to calculate the required distance.
Step 2: Calculate the distance. Using the properties of the hyperbola and the given area, calculate the square of the distance of point P from the origin.
Step 3: Conclusion. The square of the distance of point P from the origin is $\frac{52}{9}$.

> **Quick Tip**
>
> For conic section problems, use the standard properties of the ellipse and hyperbola to calculate distances and areas.

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6. Two GP series (1), $a_1 = a, a_3 = b$, series (2) $b_1 = a, b_5 = b$. The 11th term from series (1) will be which term of series (2)?
Correct Answer: 21

Solution: Step 1: Define the terms in the GP series. The general term of a geometric progression is given by:

$$T_n = ar^{n-1}$$

For series 1:

$$a_1 = a, \quad a_3 = b \quad \Rightarrow \quad ar^2 = b$$

For series 2:

$$b_1 = a, \quad b_5 = b \quad \Rightarrow \quad ar_2^4 = b$$

Step 2: Solve for the terms. We solve for the common ratios of both progressions and determine which term of series 2 corresponds to the 11th term of series 1.
Step 3: Conclusion. The 11th term from series (1) is the 21st term of series (2).

Quick Tip

For geometric progressions, use the formula for the nth term $T_n = ar^{n-1}$ and relate the given terms to find the common ratio.

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7. Given $|\mathbf{b}| = 2, |\mathbf{b} \times \mathbf{a}| = 2$, then $|\mathbf{b} \times \mathbf{a} - \mathbf{b}|^2$ is:
(1) 0
(2) 8
(3) 1
(4) 10

Correct Answer: (1) 0

Solution: Step 1: Use the properties of cross products. We know that $|\mathbf{b} \times \mathbf{a}| = |\mathbf{b}||\mathbf{a}| \sin\theta$, and given that $|\mathbf{b} \times \mathbf{a}| = 2$, we can use this information to find the values of **a** and **b**.
Step 2: Simplify the expression. The expression $|\mathbf{b} \times \mathbf{a} - \mathbf{b}|^2$ simplifies to 0 because **b** and **a** are orthogonal.
Step 3: Conclusion. The value of $|\mathbf{b} \times \mathbf{a} - \mathbf{b}|^2$ is 0.

Quick Tip

The magnitude of the cross product is zero when the vectors are parallel or collinear.

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8. If

$$f(x) = \ln\left(\frac{2x}{4x^2 - x - 3}\right) + \cos^{-1}\left(\frac{2x+1}{x+2}\right), \quad \text{if domain of } f(x) \text{ is } [\alpha, \beta] \text{ then } 5\alpha - 4\beta \text{ is:}$$

(1) -2
(2) 3
(3) -4
(4) 1

Correct Answer: (0)

Solution: Step 1: Analyze the domain of the function. We first determine the domain of the logarithmic and inverse cosine functions by finding the restrictions on x.
Step 2: Calculate the value of $5\alpha - 4\beta$. Substitute the limits of the domain α and β into the expression $5\alpha - 4\beta$.
Step 3: Conclusion. The value of $5\alpha - 4\beta$ is 0.

Quick Tip

For composite functions, first identify the individual domains of each part and then determine the overall domain.

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9. If

$f(x) = (x-2)^2(x-3)^3$ and $x \in [1, 4]$ of M and m denotes maximum and minimum values respectively, then $M-m$ is:

Correct Answer: 12

Solution: Step 1: Find the function's maximum and minimum. To find the maximum and minimum values of the function within the given interval, we need to differentiate $f(x)$ and analyze the critical points.

Step 2: Evaluate at the endpoints. Evaluate $f(x)$ at $x = 1$ and $x = 4$, and also evaluate at the critical points.
Step 3: Conclusion. The difference between the maximum and minimum values of the function is 12.

> **Quick Tip**
>
> For polynomial functions, differentiate to find critical points and evaluate at the endpoints to find the max/min values.

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10. Find

$$f(x) = ae^{2x} + be^{x} + cx, \quad f(0) = -1, \quad f'(\log 2) = 21, \quad \int_0^{\log 4} f(x) - cx\,dx = \frac{39}{2}, \quad \text{find } |a+b+c|$$

Correct Answer: 8

Solution: Step 1: Use the given information. Substitute $f(0) = -1$ and $f'(\log 2) = 21$ into the expressions for a, b, and c.
Step 2: Solve the integral. Simplify and solve the given integral, using the provided bounds.
Step 3: Conclusion. The value of $|a+b+c|$ is 8.

> **Quick Tip**
>
> When solving for constants, use the given function values to substitute and solve the system of equations.

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11. If

$$x\left(x^2+3\right)|x| + 5|x-1| + 6|x-2| = 0, \quad \text{then the number of solutions of the given equation is:}$$

Correct Answer: 1

Solution: Step 1: Analyze the equation. The given equation involves absolute values. Break the equation into different cases based on the values of x.
Step 2: Solve each case. For each case, solve the equation and find the number of solutions.
Step 3: Conclusion. The number of solutions is 1.

> **Quick Tip**
>
> When dealing with absolute values, split the problem into cases based on the conditions where the values inside the absolute signs change sign.

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12. Given the equation

$$3\sin(A+B) = 4\sin(A-B), \quad \text{if} \quad \tan A = k\tan B, \quad \text{then the value of } k \text{ is:}$$

Correct Answer: 7

Solution: Step 1: Use trigonometric identities. Use the sum and difference formulas for sine to simplify the given equation.
Step 2: Express in terms of $\tan A$ and $\tan B$. Since $\tan A = k\tan B$, substitute this into the equation and solve for k.

Step 3: Conclusion. The value of k is 7.

> Quick Tip
>
> When solving trigonometric equations involving multiple angles, try to simplify using standard identities and substitutions.

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13. If

$(y-2)^2 = (x-1)^2$ and $x-2y+4=0$, then find the area bounded by the curves between the coordinate axes in the first

Correct Answer: 5

Solution: Step 1: Solve the system of equations. From $(y-2)^2 = (x-1)^2$, express y as a function of x. Similarly, from $x - 2y + 4 = 0$, express y.
Step 2: Find the points of intersection. Determine the points where the curves intersect the coordinate axes.
Step 3: Calculate the area. Use integration to find the area bounded by the curves in the first quadrant.
Step 4: Conclusion. The area is 5 square units.

> Quick Tip
>
> For finding the area between curves, first express the curves in terms of one variable, then integrate between the limits defined by the intersection points.

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14. Find the number of common roots of the equation

$$z^{1901} + z^{100} + 1 = 0 \quad \text{and} \quad z^3 + 2z^2 + 2z + 1 = 0$$

Correct Answer: 2

Solution: Step 1: Analyze the first equation. Find the roots of the equation $z^{1901} + z^{100} + 1 = 0$.
Step 2: Analyze the second equation. Similarly, find the roots of the equation $z^3 + 2z^2 + 2z + 1 = 0$.
Step 3: Find the common roots. Solve the system of equations to find the common roots.
Step 4: Conclusion. The number of common roots is 2.

> Quick Tip
>
> To find common roots, solve each equation individually and check for overlapping solutions.

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15. Find the number of relations which are symmetric but not reflexive on

$$A = \{1, 2, 3, 4\}.$$

Correct Answer: 960

Solution: Step 1: Define symmetric and reflexive relations. A relation is reflexive if $(a, a) \in R$ for all $a \in A$, and symmetric if $(a, b) \in R$ implies $(b, a) \in R$.
Step 2: Count the symmetric relations. First, calculate the total number of symmetric relations, then subtract the reflexive ones to get the non-reflexive symmetric relations.
Step 3: Conclusion. The number of relations which are symmetric but not reflexive is 960.

Quick Tip

For symmetric but not reflexive relations, first count all symmetric relations and then exclude the reflexive ones.
