JEE Main 2024 Mathematics Question Paper Jan 31 Shift 1

1. Solve the differential equation:

$$\frac{dx}{dy} = x\left(\ln x - \ln y + 1\right)$$

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2. Limit:

$$\lim_{x\to 0} \frac{e^{2\sin x} - 2\sin x - 1}{x^2}$$

(1) Does not exist
(2) 2
(3) 1
(4) -1

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3. Let S be the set of positive integral values of a for which

$$\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 + 8x + 32} < 0, \quad \forall x \in \mathbb{R}.$$

Then, the number of elements in S is:
(1) 1
(2) 2
(3) 3
(4) 4

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4. Area of the region is:

$$\begin{cases} y^2 < 4x, & 0 < x < 4, \\ \frac{x(x-1)(x-2)}{(x-3)(x-4)} < 0, & \text{for } x \in (0,4) \end{cases}$$

(1) $\frac{16}{3}$
(2) $\frac{32}{3}$
(3) $\frac{20}{3}$
(4) $\frac{25}{3}$

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5. If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$**, find** $2f(0) + f'(0)$**.**
(1) 42.00
(2) 50.00
(3) 30.00
(4) 20.00

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6. If $f(x) = \frac{4x+3}{6x-4}$**, find** $(f \circ f)(x)$**, where** $g : r \to \left[\frac{2}{3} \to \frac{2}{3}\right]$**, then** $(g(g(g(4))))$ **is equal to:**
(1) 1
(2) 2
(3) 3

(4) 4

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7. Find the sum:

$$\sum_{r=1}^{10} \frac{r}{1-3r^2+r^4} = S. \quad \text{Find } S.$$

(1) $\frac{-55}{109}$
(2) $\frac{100}{109}$
(3) $\frac{-65}{109}$
(4) $\frac{50}{109}$

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8. If the system of linear equations $x-2y+z=-4; 2x+\alpha y+3z=5$ and $3x-y+\beta z=3$ has infinitely many solutions, then $12\alpha+13\beta$ is equal to:
(1) 58
(2) 42
(3) 36
(4) 50

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9. If $f(x)=\begin{cases} g(x), & x\le 0 \\ \frac{x+1}{x+2}, & x>0 \end{cases}$, where $g(x)$ is a linear function and $f(x)$ is continuous at $x=0$, also $f'(1)=g(-1), g(0)=f(0)$, then find the value of $g(3)$?
(1) 5.00
(2) 15.00
(3) 10.00
(4) 12.00

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10. 3 rotten apples are mixed with 15 normal apples. Let the random variable be defined as the number of rotten apples on picking 3 apples with replacement. Find the variance of x.
(1) $\frac{3}{4}$
(2) $\frac{5}{12}$
(3) $\frac{7}{9}$
(4) $\frac{9}{16}$

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11. Using the word "DISTRIBUTION", find the number of ways of selecting 4 letters.
(1) 160.00
(2) 191.00
(3) 202.00
(4) 150.00

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12. Find n if:

$$\sum_{r=0}^{n} \binom{n}{r+1} = \alpha, \quad \sum_{r=0}^{n} \binom{n}{r+1} = \beta. \quad \text{If } 4\beta = 7\alpha, \text{ find } n.$$

(1) 2
(2) 4
(3) 6

(4) 5
