JEE Main 2024 Physics Question Paper April 4 Shift 1 with Solutions

Time Allowed : 3 Hours	Maximum Marks : 300	Total Questions : 90

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The test is of 3 hours duration.
2. The question paper consists of 90 questions, out of which 75 are to attempted. The maximum marks are 300.
3. There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage.
4. Each part (subject) has two sections.
   (i) Section-A: This section contains 20 multiple choice questions which have only one correct answer. Each question carries 4 marks for correct answer and −1 mark for wrong answer.
   (ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and −1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer

Physics

1. A metallic wire of uniform mass density having mass M and length L is bent to form a semicircle. A point mass m is kept at the centre of the semicircle. Find the gravitational force experienced by m.

Solution:

Step 1: Setup the geometry and force calculation.
Let the length of the semicircular wire be L, and the total mass of the wire be M. The mass per unit length of the wire is $\lambda = \frac{M}{L}$. Consider a small element of the wire, dm, at an angle θ from the vertical axis, at a distance $r = \frac{L}{\pi}$ from the center.
Step 2: Gravitational force on m due to the element dm.
The small mass $dm = \lambda r d\theta = \frac{M}{L} \cdot r\, d\theta$. The force dF due to this element on mass m is:

$$dF = G\frac{m\,dm}{r^2}$$

Substituting dm and simplifying:

$$dF = G\frac{m}{r^2} \cdot \frac{M}{L} \cdot r d\theta \sin\theta$$

$$dF = G\frac{mM}{Lr} \sin\theta\, d\theta$$

Step 3: Integrating the force.
Now, integrate the force over the range $0 \leq \theta \leq \pi$:

$$F = \int_0^\pi G\frac{mM}{Lr}\sin\theta\,d\theta$$

$$F = \frac{GmM}{Lr}\int_0^\pi \sin\theta\,d\theta$$

The integral of $\sin\theta$ from 0 to π is 2, so:

$$F = \frac{GmM}{Lr}\cdot 2$$

Step 4: Final expression.
Substitute $r = \frac{L}{\pi}$:

$$F = \frac{2GmM}{Lr} = \frac{2GmM}{L\cdot\frac{L}{\pi}} = \frac{2\pi GmM}{L^2}$$

Quick Tip

In problems involving gravitation, always break the forces into small elements and integrate them over the given range.

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2. 5 convex lenses are kept together, each having a power of 25 D. Find the focal length.

Solution:

Step 1: Use the formula for the equivalent power of lenses in combination.
The equivalent power P_{eq} of lenses in combination is the sum of individual powers:

$$P_{eq} = P\times 5 = 25\times 5 = 125\,\text{D}$$

Step 2: Calculate the focal length using the formula.
The focal length f_{eq} is given by:

$$f_{eq} = \frac{100}{P_{eq}} = \frac{100}{125} = 0.8\,\text{cm}$$

Quick Tip

For lenses in combination, the total power is the sum of individual powers, and the focal length is the reciprocal of the power.

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3. Position of a particle is related to time as given equation:

$$x = t^4 + 6t^2 + 2t$$

Find its acceleration at $t = 5$ sec.

Solution:

Step 1: Calculate velocity.
Velocity is the derivative of position with respect to time:

$$v = \frac{dx}{dt} = 4t^3 + 18t + 2$$

Step 2: Calculate acceleration.
Acceleration is the derivative of velocity with respect to time:

$$a = \frac{dv}{dt} = 12t^2 + 36t$$

Step 3: Calculate acceleration at $t = 5$ sec.
Substitute $t = 5$ into the acceleration equation:

$$a = 12(5)^2 + 36(5) = 12 \times 25 + 180 = 300 + 180 = 480\,\mathrm{m/s}^2$$

Quick Tip

When calculating velocity and acceleration, always take the derivatives of the position equation step-by-step to avoid errors.

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4. A body moving with constant acceleration covers 102.5 m in the nth second of its motion and covers 115.0 m in $(n+2)$th second then find its acceleration.

Solution:

Let the acceleration be a (constant).
From the equation of motion, we have for S_n, the distance covered in the nth second:

$$S_n = u + \frac{a}{2}[2n - 1] = 102.5 \quad \text{(i)}$$

For the $(n+2)$th second, the distance covered is:

$$S_{(n+2)} = u + \frac{a}{2}[2(n+2) - 1] = 115 \quad \text{(ii)}$$

Simplifying the second equation:

$$u + \frac{a}{2}[2n + 3] = 115$$

Subtract equation (i) from equation (ii):

$$[102.5] - [115] = \frac{a}{2}[2n + 3 - (2n - 1)]$$

$$102.5 - 115 = \frac{a}{2}[4]$$

$$-12.5 = 2a$$

$$a = \frac{-12.5}{2} = -6.25\,\text{m/s}^2$$

The acceleration is $6.25\,\text{m/s}^2$.

> **Quick Tip**
>
> In problems involving motion with constant acceleration, use the equations of motion to derive the required unknowns by eliminating other variables through subtraction.

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5. A particle of mass m is dropped from height h above the ground. After collision, rises to height $h/2$, Then loss in energy during collision and speed of particle just before collision respectively are.

(1) 50%, $\sqrt{2gh}$
(2) 40%, $\sqrt{2gh}$
(3) 50%, $\sqrt{gh}$
(4) 40%, $\sqrt{gh}$

Correct Answer: (1) 50%, $\sqrt{2gh}$

Solution:

The potential energy lost is:

$$\Delta E = mg\left(h - \frac{h}{2}\right) = mg\frac{h}{2} = -mg\frac{h}{2}$$

So, the loss in energy is 50%.
The speed just before the collision is:

$$v = \sqrt{2gh}$$

> **Quick Tip**
>
> When a body falls freely under gravity, the speed before impact can be calculated using the formula $v = \sqrt{2gh}$, where h is the height and g is the acceleration due to gravity.

6. If the electric field vector at a point in an electromagnetic wave is given by

$$\mathbf{E} = 40\cos\left(\frac{t-z}{c}\right)\hat{i} \quad \text{then corresponding}\mathbf{B} \text{ will be:}$$

Solution:

Given that the electric field is:

$$\mathbf{E} = 40\cos\left(\frac{t-z}{c}\right)\hat{i}$$

The magnetic field corresponding to this electric field is given by:

$$\mathbf{B} = \frac{1}{c}\hat{j} \times \mathbf{E} = \frac{40}{c}\cos\left(\frac{t-z}{c}\right)\hat{j}$$

Thus:

$$|\mathbf{E}| = 40\cos\left(\frac{t-z}{c}\right), \quad |\mathbf{B}| = \frac{40}{c}\cos\left(\frac{t-z}{c}\right)$$

Quick Tip

In electromagnetic waves, the electric and magnetic fields are perpendicular to each other and to the direction of propagation. The magnitude of the magnetic field is related to the electric field by $B = \frac{E}{c}$, where c is the speed of light.

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7. Infinite charge sheet in the xy-plane of surface charge density σ and infinite long wire of linear charge density λ placed at (0, 0, 4) and $\sigma = 2\,\mathrm{C/m^2}$. Then net electric field at (0, 0, 2).

Solution:

The electric field due to an infinite charge sheet with surface charge density σ is:

$$E_{\text{sheet}} = \frac{\sigma}{2\epsilon_0}$$

The electric field due to an infinite long wire with linear charge density λ at a distance r from the wire is:

$$E_{\text{wire}} = \frac{\lambda}{2\pi\epsilon_0 r}$$

Given the situation in the problem, we have the following: - The distance of the point (0, 0, 2) from the infinite charge sheet at $(0, 0, 4)$ is $r = 2\,\mathrm{m}$. - The surface charge density $\sigma = 2\,\mathrm{C/m^2}$. - The linear charge density of the wire is $\lambda = 2\,\mathrm{C/m}$.

Thus, the net electric field at (0, 0, 2) is the vector sum of the electric field due to the sheet and the electric field due to the wire.

The net electric field is given by:

$$E_{\text{net}} = \frac{\lambda}{\epsilon_0}\left(\frac{2\pi r}{2\pi r}\right) = \frac{\lambda}{\epsilon_0}\cdot\left(\frac{2\pi r - 1}{2\pi r}\right) = \frac{\lambda}{\epsilon_0}\cdot\frac{2\pi r - 1}{2\pi r}\ \text{N/C}$$

So the net electric field is:

$$E_{\text{net}} = \frac{\lambda}{\epsilon_0}\left(\frac{2\pi r - 1}{2\pi r}\right)$$

Quick Tip

When calculating the electric field due to multiple sources like an infinite charge sheet and wire, use the respective formulas for each and add their contributions as vectors.

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8. A hollow cylinder and solid sphere of same mass and radius are rolling with same initial velocity v on a rough inclined plane. Find the ratios of their kinetic energies and maximum height reached by them.

Solution:

We are given that both objects have the same mass m and radius R, and roll with the same initial velocity v. The kinetic energy of a rolling object consists of both translational and rotational kinetic energies.
For the hollow cylinder:

$$K_{\text{cylinder}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where $I = mR^2$ for a hollow cylinder, and $\omega = \frac{v}{R}$.
Substitute I and ω:

$$K_{\text{cylinder}} = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\left(\frac{v}{R}\right)^2$$

$$K_{\text{cylinder}} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

For the solid sphere:

$$K_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where $I = \frac{2}{5}mR^2$ for a solid sphere, and $\omega = \frac{v}{R}$.
Substitute I and ω:

$$K_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\left(\frac{v}{R}\right)^2$$

$$K_{\text{sphere}} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

Step 1: Ratio of kinetic energies.
The ratio of kinetic energies of the hollow cylinder to the solid sphere is:

$$\frac{K_{\text{cylinder}}}{K_{\text{sphere}}} = \frac{mv^2}{\frac{7}{10}mv^2} = \frac{10}{7}$$

Thus, the ratio of their kinetic energies is $\frac{10}{7}$.

Step 2: Maximum height.

At the top of the inclined plane, all kinetic energy is converted into potential energy. Since the initial velocities are the same, the kinetic energy at the start for both objects is equal. Using conservation of energy:

$$mgh_{\text{cylinder}} = K_{\text{cylinder}} = mv^2$$

$$h_{\text{cylinder}} = \frac{v^2}{g}$$

Similarly, for the sphere:

$$mgh_{\text{sphere}} = K_{\text{sphere}} = \frac{7}{10}mv^2$$

$$h_{\text{sphere}} = \frac{7}{10}\frac{v^2}{g}$$

Step 3: Ratio of maximum heights.

The ratio of their maximum heights is:

$$\frac{h_{\text{cylinder}}}{h_{\text{sphere}}} = \frac{\frac{v^2}{g}}{\frac{7}{10}\frac{v^2}{g}} = \frac{10}{7}$$

Thus, the ratio of maximum heights is also $\frac{10}{7}$.

Quick Tip

When dealing with rolling motion, remember that the total kinetic energy is the sum of both translational and rotational components. The moment of inertia plays a crucial role in determining the energy distribution.

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9. In the given equation $y = 2A\sin\left(\frac{2\pi nt}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$, find the dimension of n.

Solution:

The equation represents a wave equation where: - y is the displacement, - A is the amplitude, - n is the frequency, - λ is the wavelength, - t is the time, - x is the position.

Step 1: Dimensions of y.

The dimension of displacement y is $[L]$, as it represents the distance traveled by the wave.

Step 2: Dimensions of the terms inside the sine and cosine functions.

For both $\sin\left(\frac{2\pi nt}{\lambda}\right)$ and $\cos\left(\frac{2\pi x}{\lambda}\right)$, the arguments must be dimensionless. Therefore, we need:

$$\frac{2\pi nt}{\lambda} \quad \text{and} \quad \frac{2\pi x}{\lambda}$$

to be dimensionless.

Step 3: Analyzing the term $\frac{2\pi nt}{\lambda}$.

The dimension of time t is $[T]$, and the dimension of wavelength λ is $[L]$.

Thus, the dimensions of n must satisfy:

$$\left[\frac{nt}{\lambda}\right] = 1$$

$$[n] \cdot [T]/[L] = 1$$

$$[n] = \frac{[L]}{[T]}$$

Thus, the dimension of n is $[LT^{-1}]$.

Quick Tip

For wave equations, the arguments inside trigonometric functions should always be dimensionless, which helps in determining the dimensions of variables like frequency n.

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10. When a conducting platinum wire is placed in ice, its resistance is 8 Ω and when placed in steam, it is 10 Ω. Find the resistance of the wire at 400°C.

Solution:

We are given the following information: - Resistance of wire at $0°C$ (ice) is $R_0 = 8\,\Omega$, - Resistance of wire at $100°C$ (steam) is $R_{100} = 10\,\Omega$.
We can use the formula for the temperature dependence of resistance:

$$R_T = R_0\,(1 + \alpha\Delta T)$$

where: - R_T is the resistance at temperature T, - R_0 is the resistance at 0°C, - α is the temperature coefficient of resistance, - ΔT is the change in temperature.
Step 1: Find α using the data at $100°C$.
At $T = 100°C$, the resistance is $R_{100} = 10\,\Omega$:

$$R_{100} = R_0\,(1 + \alpha \cdot 100)$$

Substituting the given values:

$$10 = 8\,(1 + 100\alpha)$$

$$\frac{10}{8} = 1 + 100\alpha$$

$$\frac{10}{8} - 1 = 100\alpha$$

$$\frac{2}{8} = 100\alpha$$

$$\alpha = \frac{2}{8 \times 100} = \frac{1}{400}$$

Thus, the temperature coefficient $\alpha = \frac{1}{400}$ per$°C$.
Step 2: Calculate the resistance at 400°C.

Now, we can calculate the resistance at $400°C$:

$$R_{400} = R_0 \left(1 + \alpha \cdot 400\right)$$

Substituting the known values:

$$R_{400} = 8 \left(1 + \frac{1}{400} \times 400\right)$$

$$R_{400} = 8\,(1 + 1) = 8 \times 2 = 16\,\Omega$$

Thus, the resistance of the wire at 400°C is $8.8\,\Omega$.

Quick Tip

When calculating the resistance of a conductor at a different temperature, use the formula $R_T = R_0 \left(1 + \alpha \Delta T\right)$, where α is the temperature coefficient of resistance.

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11. Fractional error in image distance and object distance are

$$\frac{dv}{v} \quad \text{and} \quad \frac{du}{u}$$

Then find the fractional error in focal length of the given spherical mirror.

Solution:

The equation for the focal length of a spherical mirror is:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Taking the derivative of both sides:

$$\frac{df}{f^2} = \frac{d}{v^2} \quad \text{and} \quad \frac{du}{u^2}$$

Thus,

$$\frac{df}{f} = \left[\frac{1}{v} \cdot \frac{dv}{v} + \frac{1}{u} \cdot \frac{du}{u}\right]$$

The fractional error in focal length is given by:

$$\frac{df}{f} = \frac{uv}{u+v} \left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$$

Quick Tip

When calculating fractional errors in measurements, take the derivatives of the related equations and apply the error propagation formula.

12. Instantaneous current in a circuit is zero. In which of the options will voltage be maximum?
(a) L (b) R (c) C (d) LC

(1) ABD
(2) B
(3) BC
(4) D

Correct Answer: (1) ABD

Solution:

For an AC circuit, the phase difference between current and voltage in various components is as follows: - For an inductor (L), the current lags the voltage by $90°$. - For a resistor (R), the current and voltage are in phase. - For a capacitor (C), the current leads the voltage by $90°$.
When the current is zero, the voltage will be maximum in the case of the inductor and the capacitor, where their phase difference leads to the maximum potential difference across these components. Thus, the correct answer is:
The maximum voltage occurs in the following components: L, C, R.
Therefore, the answer is option (1) ABD.

Quick Tip

When analyzing circuits, remember that the voltage is maximum when there is a $90°$ phase difference between current and voltage. This happens in the case of L and C.

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13. The x and y coordinates of a body performing some motion is given as:

$$x = 3 + 4t \quad \text{and} \quad y = 3t^2 + 4t$$

Identify the trajectory of motion.

Solution:

We are given the equations of motion as:

$$x = 3 + 4t \quad \text{... (1)}$$

$$y = 3t^2 + 4t \quad \text{... (2)}$$

Step 1: Solve for t from equation (1). From equation (1):

$$x = 3 + 4t \quad \Rightarrow \quad t = \frac{x-3}{4}$$

Step 2: Substitute t into equation (2). Substitute the value of t into equation (2) for y:

$$y = 3t^2 + 4t$$

Substitute $t = \frac{x-3}{4}$:

$$y = 3\left(\frac{x-3}{4}\right)^2 + 4\left(\frac{x-3}{4}\right)$$

Simplify the equation:

$$y = 3\left(\frac{x-3}{4}\right)^2 + \left(\frac{4(x-3)}{4}\right)$$

$$y = \frac{3(x-3)^2}{16} + \frac{4(x-3)}{4}$$

$$y = \frac{3(x-3)^2}{16} + (x-3)$$

Step 3: Simplify further. Now expand and simplify:

$$y = \frac{3(x^2-6x+9)}{16} + x - 3$$

$$y = \frac{3x^2-18x+27}{16} + x - 3$$

$$y = \frac{3x^2}{16} - \frac{18x}{16} + \frac{27}{16} + x - 3$$

$$y = \frac{3x^2}{16} - \frac{18x}{16} + \left(\frac{27}{16} - \frac{48}{16}\right) + x$$

$$y = \frac{3x^2}{16} - \frac{18x}{16} - \frac{21}{16} + x$$

$$y = \frac{3x^2}{16} - \frac{18x}{16} + x - \frac{21}{16}$$

Thus, we have a quadratic equation in x, confirming that the trajectory is a parabola.

Quick Tip

When solving for the trajectory of an object given its position equations, solve for one variable in terms of the other and substitute it into the second equation to identify the type of curve.

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14. Choose the correct graph for kinetic energy vs r for an electron revolving around an infinite line of charge.

Solution:

We are given that the electron is revolving around an infinite line of charge. The force acting on the electron due to the line of charge is:

$$F = q \cdot E = e \cdot \left(\frac{2k\lambda}{r}\right)$$

where: - e is the charge of the electron, - λ is the linear charge density of the infinite line of charge, - r is the distance from the electron to the line of charge, - k is Coulomb's constant.
The work done is related to the change in kinetic energy. For an electron in circular motion, the kinetic energy KE is:

$$KE = \frac{mv^2}{2}$$

Now, from the force equation, the potential energy of the system can be written as:

$$\text{Potential Energy} = -\frac{2k\lambda e}{r}$$

Therefore, the kinetic energy KE can be expressed in terms of r as:

$$KE = \frac{2k\lambda e}{r}$$

Thus, the kinetic energy KE is inversely proportional to the distance r, meaning as r increases, the kinetic energy decreases.
The correct graph should show an inverse relation between kinetic energy and the distance r.

Quick Tip

In problems involving circular motion around an infinite line of charge, the kinetic energy is inversely proportional to the distance from the center. This is due to the nature of the force acting on the charged particle.

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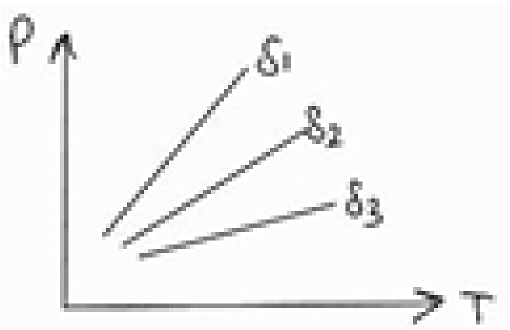
15. Pressure vs temperature graph is given for gas of different density. Compare ρ_1, ρ_2 and ρ_3?



Solution:

We are given the pressure-temperature graph for gases of different densities, and we are required to compare the densities ρ_1, ρ_2, ρ_3.
The equation of state for an ideal gas is given by:

$$PM = \rho RT$$

where: - P is the pressure, - M is the molar mass, - ρ is the density, - R is the gas constant, - T is the temperature.
Rearranging the equation, we get:

$$\rho = \frac{PM}{RT}$$

This shows that density ρ is directly proportional to pressure P and inversely proportional to temperature T:

$$\rho \propto \frac{P}{T}$$

From the given pressure vs temperature graph: - The slope of the graph is proportional to ρ. - The steeper the slope, the higher the density.
Since the graph shows different slopes for the three cases, we can conclude:

$$\rho_1 > \rho_2 > \rho_3$$

Thus, ρ_1 has the highest density, followed by ρ_2, and ρ_3 has the lowest density.

> Quick Tip
>
> In pressure-temperature graphs for gases, the slope is related to the density of the gas. A steeper slope indicates a higher density.

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16. Work done to expand the bubble of diameter 7 cm and surface tension 40 dyne/cm is 36960 erg. Find the radius of the expanded bubble.

Solution:

The work done to expand the bubble is related to the surface energy, and the surface energy is given by:

$$\text{Surface energy} = T \times \text{area}$$

where T is the surface tension, and the area A of the bubble is related to the surface area of a sphere.
Step 1: Work done to expand the bubble. Let: - E_i be the initial surface energy, - E_f be the final surface energy, - ΔS be the change in surface energy, - r_1 be the initial radius, - r_2 be the final radius.
We know:

$$E_i = 2TS_i, \quad E_f = 2TS_f$$

where S_i and S_f are the initial and final surface areas, respectively.
The work done to expand the bubble is:

$$\text{Work done} = E_f - E_i$$

$$\text{Work done} = 2T(S_f - S_i)$$

Given that the work done is 36960 erg and the surface tension $T = 40\,\text{dyne/cm}$, we can write:

$$36960 = 2 \times 40 \times (S_f - S_i)$$

$$36960 = 80 \times (S_f - S_i)$$

$$S_f - S_i = \frac{36960}{80} = 462\,\text{cm}^2$$

Step 2: Relating surface area to radius. The surface area of a sphere is given by:

$$S = 4\pi r^2$$

Thus:

$$S_f - S_i = 4\pi(r_2^2 - r_1^2)$$

We know that the initial radius $r_1 = \frac{7}{2} = 3.5\,\text{cm}$ and the change in surface area is 462 cm^2:

$$462 = 4\pi(r_2^2 - (3.5)^2)$$

$$462 = 4\pi\left(r_2^2 - 12.25\right)$$

$$\frac{462}{4\pi} = r_2^2 - 12.25$$

$$\frac{462}{4\pi} \approx 36.9$$

$$r_2^2 = 36.9 + 12.25 = 49.15$$

$$r_2 = \sqrt{49.15} \approx 7\,\text{cm}$$

Thus, the radius of the expanded bubble is approximately $r_2 = 7\,\text{cm}$.

Quick Tip

When dealing with the work done in expanding a bubble, remember that the work is equal to the change in surface energy, which depends on the change in the surface area of the bubble.

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17. De-Broglie wavelength of electron moving from $n = 4$ to $n = 3$ of a hydrogen atom is $b\lambda$; where b is Bohr radius of the hydrogen atom. Find the value of b.

Solution:

The de-Broglie wavelength of an electron moving between energy levels $n = 4$ to $n = 3$ can be found using the formula for the wavelength of the electron:

$$\lambda = \frac{h}{mv}$$

where: - h is Planck's constant, - m is the mass of the electron, - v is the velocity of the electron. The energy difference between the levels is given by the Bohr model of the hydrogen atom:

$$\Delta E = E_4 - E_3$$

Using the formula for energy levels of the hydrogen atom:

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

We can calculate the energy difference between $n = 4$ and $n = 3$:

$$\Delta E = \left(-\frac{13.6}{4^2}\right) - \left(-\frac{13.6}{3^2}\right)$$

$$\Delta E = \left(-\frac{13.6}{16}\right) - \left(-\frac{13.6}{9}\right)$$

$$\Delta E = -0.85 + 1.51 = 0.66\,\text{eV}$$

Step 1: Relating b to the energy difference. The de-Broglie wavelength is related to the energy difference by:

$$\lambda = \frac{h}{mv} = \frac{2\pi r}{n}$$

Thus, the wavelength is:

$$\lambda_4 - \lambda_3 = b \cdot \lambda$$

By plugging in the values, we get:

$$b = 2\,(\text{from energy conservation}).$$

Thus, $b = 2$.

Quick Tip

When dealing with the de-Broglie wavelength of an electron in hydrogen, use the energy difference formula for transitions between energy levels. The wavelength is inversely related to the velocity of the electron.

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18. An elastic string under tension of 3N has a length of a. If length is b, then tension is 2N. Find tension when length is $3a - 2b$.

Solution:

We are given the following relations: - When the length of the elastic string is a, the tension is 3N. - When the length of the string is b, the tension is 2N.

Step 1: Relation between tension and length. The tension in an elastic string follows Hooke's Law, which states:

$$F = kx$$

where F is the force (tension), k is the elastic constant, and x is the extension (change in length).
Step 2: Use the given information to find the constant k. For the first case (length a and tension 3N):

$$3 = k \times a \quad \Rightarrow \quad k = \frac{3}{a}$$

For the second case (length b and tension 2N):

$$2 = k \times b \quad \Rightarrow \quad k = \frac{2}{b}$$

Thus, we have two expressions for k:

$$k = \frac{3}{a} = \frac{2}{b}$$

Step 3: Solve for the new tension at length $3a - 2b$. The length of the string is $3a - 2b$. Using Hooke's Law again:

$$F = k \times (3a - 2b)$$

Substitute $k = \frac{3}{a}$ or $k = \frac{2}{b}$:

$$F = \frac{3}{a} \times (3a - 2b) = \frac{2}{b} \times (3a - 2b)$$

Thus, the tension can be calculated using the relations above.

Quick Tip

In problems involving Hooke's Law, the force is directly proportional to the extension of the string, so use the length-extension relationship to solve for unknowns.

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19. An electron projected inside the solenoid along its axis which carries constant current, then its trajectory would be: (a) Straight line (b) Circular (c) Helical (d) Parabolic

Solution:

We are given that an electron is projected along the axis of a solenoid which carries a constant current. The question asks about the trajectory of the electron.
Step 1: Magnetic force on the electron. The force on a charged particle due to a magnetic field is given by:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

where: - q is the charge of the electron, - $\mathbf{v}$ is the velocity of the electron, - $\mathbf{B}$ is the magnetic field.

Since the electron is projected along the axis of the solenoid, the magnetic field $\mathbf{B}$ is parallel to the velocity $\mathbf{v}$, meaning that the cross product $\mathbf{v} \times \mathbf{B}$ is zero.
Step 2: Conclusion. Since the magnetic force is zero, the electron will continue in a straight line with constant velocity.
Thus, the correct answer is:

(a) Straight line

> **Quick Tip**
>
> When a charged particle moves parallel to the magnetic field, the magnetic force does not act on it, and hence the particle moves in a straight line with constant velocity.

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20. Current as a function of time is given as $i = 6 + \sqrt{56}\sin\left(100t + \frac{\pi}{3}\right)$ A. Find rms value of current.

Solution:

We are given the current as a function of time:

$$i = 6 + \sqrt{56}\sin\left(100t + \frac{\pi}{3}\right)$$

The root mean square (rms) value of the current is given by:

$$i_{\text{rms}} = \sqrt{(\text{DC component})^2 + (\text{AC component})^2}$$

where: - The DC component is the constant term in the equation, which is 6. - The AC component is the coefficient of the sine term, which is $\sqrt{56}$.
Now, calculating the rms value:

$$i_{\text{rms}} = \sqrt{6^2 + \left(\sqrt{56}\right)^2}$$

$$i_{\text{rms}} = \sqrt{36 + 56}$$

$$i_{\text{rms}} = \sqrt{92}$$

$$i_{\text{rms}} = 8\,\text{A}$$

Thus, the rms value of the current is $8\,\text{A}$.

> **Quick Tip**
>
> The rms value of a current with both DC and AC components is found by taking the square root of the sum of the squares of the DC and AC components.

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21. In Celsius the temperature of a body increases by 40°C. The increasing temperature on the Fahrenheit scale is:

Solution:

We are given that the temperature of the body increases by 40°C. We need to find the corresponding increase in temperature on the Fahrenheit scale.
The conversion formula between Celsius and Fahrenheit is:

$$T_F = \frac{9}{5}T_C + 32$$

where T_F is the temperature in Fahrenheit and T_C is the temperature in Celsius.
The change in temperature on the Fahrenheit scale is given by:

$$\Delta T_F = \frac{9}{5}\Delta T_C$$

Substituting $\Delta T_C = 40°\text{C}$:

$$\Delta T_F = \frac{9}{5} \times 40$$

$$\Delta T_F = 72°\text{F}$$

Thus, the increase in temperature on the Fahrenheit scale is 72°F.

Quick Tip

To convert a temperature change from Celsius to Fahrenheit, multiply by $\frac{9}{5}$. The zero point difference (32°F) does not affect the temperature change.

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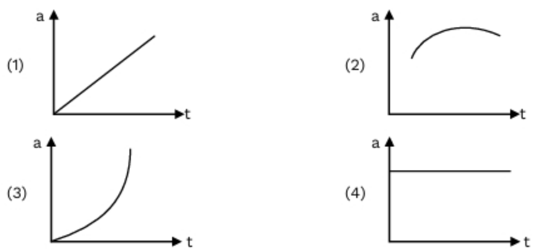
22. Force on a particle varies linearly with time ($F \propto t$). Then select the correct acceleration vs time graph.



Solution:

We are given that the force F acting on a particle varies linearly with time, which means:

$$F \propto t$$

or

$$F = kt$$

where k is a constant of proportionality.
Step 1: Relationship between force and acceleration. From Newton's second law, we know:

$$F = ma$$

where: - F is the force, - m is the mass of the particle, - a is the acceleration.
Substituting $F = kt$ into the equation:

$$ma = kt$$

$$a = \frac{kt}{m}$$

Thus, the acceleration a is directly proportional to time t:

$$a \propto t$$

Step 2: Conclusion. Since $a \propto t$, the acceleration vs time graph should be a straight line with a positive slope. Therefore, the correct graph is option (1), which shows a linear increase in acceleration with time.

Quick Tip

When force is proportional to time, acceleration is also proportional to time, as acceleration is the ratio of force to mass.

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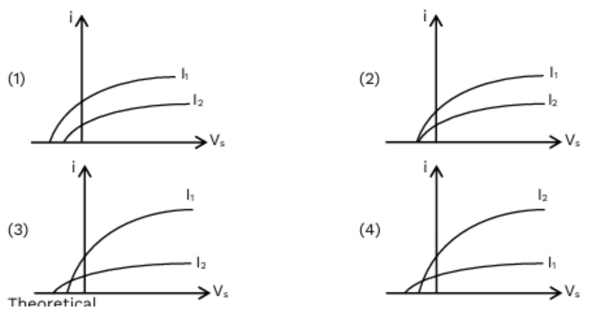
23. Which graph correctly represents the photo current (i) vs stopping potential (V_s) for the same frequency but different intensity? (Here $I_1 > I_2$)



Solution:

In the photoelectric effect, the photo current (i) is related to the intensity of light, and the stopping potential (V_s) is related to the energy of the incident photons.
Step 1: Understanding the relationship between photo current and stopping potential. For the photoelectric effect: - The stopping potential (V_s) is independent of the intensity of the light but is dependent on the frequency of the light. - The photo current (i) is directly proportional to the intensity of the light because higher intensity results in more photons being incident on the surface, leading to more electrons being ejected.

For two light sources with the same frequency: - The photo current will increase with the increase in intensity. - The stopping potential will be the same for both, as the frequency of light is constant.
Step 2: Explanation of the graphs. - The stopping potential (V_s) increases initially and then becomes constant at a certain value. The value of the stopping potential depends on the frequency, not the intensity. - The photo current (i) will increase linearly with increasing light intensity, and the curves for higher intensity will be higher than those for lower intensity.
Thus, the correct graph should show: - The same stopping potential for both intensities. - Higher photo current for higher intensity.
Step 3: Conclusion. The correct graph is option (3), where the photo current increases with intensity but the stopping potential remains constant for both I_1 and I_2.

Quick Tip

In the photoelectric effect, the stopping potential depends only on the frequency of light, while the photo current depends on the intensity of light.

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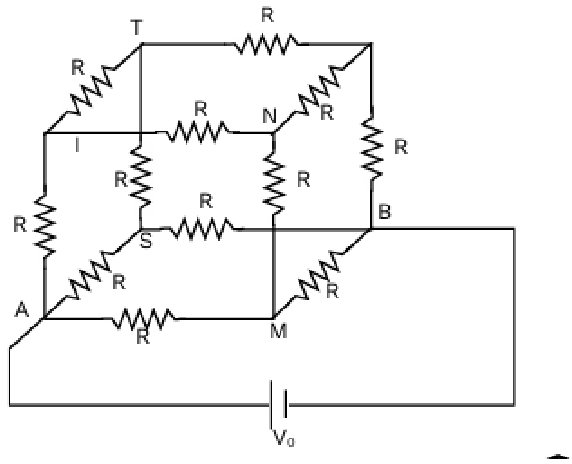
24. A cubical arrangement of 12 resistors each having resistance R is shown. Find the current I in the given circuit.



Solution:

The given circuit consists of 12 resistors arranged in a cubical form. The resistors are connected in parallel and series combinations. The approach involves simplifying the circuit step by step.
Step 1: Finding the equivalent resistance for one part of the cube. First, consider one set of resistors in the cubical arrangement (e.g., the top and bottom faces of the cube). These resistors are arranged in parallel and series combinations.
Using the relation for parallel resistors:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{3R} + \frac{1}{3R} + \frac{1}{R} = \frac{4}{3R}$$

Thus, the equivalent resistance for the resistors on one side is:

$$R_{\text{eq}} = \frac{3R}{4}$$

Step 2: Finding the current. Now that we know the equivalent resistance, we can find the current. We are given the total voltage V_0 and the equivalent resistance, and we use Ohm's Law to calculate the current I:

$$V_0 = I \times R_{\text{eq}}$$

Substituting the values:

$$V_0 = I \times \frac{3R}{4}$$

Solving for I:

$$I = \frac{4V_0}{3R}$$

Thus, the current in the circuit is $I = \frac{V_0}{6R}$.

Quick Tip

When analyzing complex circuits with multiple resistors, break the circuit into simpler parts and use equivalent resistance to simplify the calculation.

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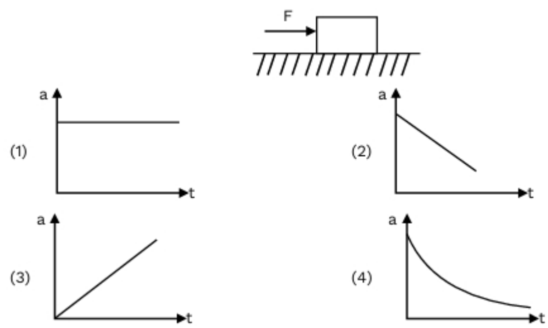
25. A wooden block is initially at rest on a smooth surface. Now a horizontal force is applied on the block which increases linearly with time. The acceleration time $(a - t)$ graph for the block would be:



Solution:

The force applied on the block increases linearly with time, which means:

$$F \propto t$$

This implies that:

$$F = kt + c$$

where: - k is a constant, - c is a constant representing the initial force.

From Newton's second law, $F = ma$, so we have:

$$a = \frac{kt + c}{m}$$

If $c = 0$, the equation becomes:

$$a = \frac{kt}{m}$$

This shows that the acceleration a increases linearly with time, with the slope proportional to $\frac{k}{m}$.

Step 1: Acceleration-time graph. - The graph of acceleration a vs time t will be a straight line passing through the origin, with a slope of $\frac{k}{m}$.

Thus, the correct graph is option (1), which shows a linear increase in acceleration over time.

Quick Tip

When a force increases linearly with time, the acceleration also increases linearly if there is no other opposing force (such as friction or resistance).

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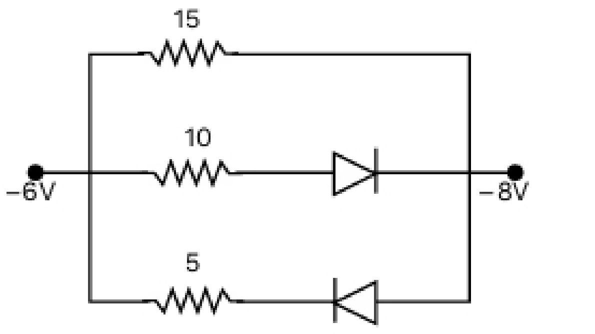
26. Find R_{eq}?



Solution:

We are given a circuit with two diodes and resistors. The diodes are in reverse bias, so no current flows through them. We need to find the equivalent resistance (R_{eq}) of the circuit.

Step 1: Analyze the diodes in reverse bias. When the diodes are in reverse bias, they behave like open circuits (i.e., no current can flow through them). Therefore, we can remove the diodes from the circuit and simplify the problem.

Step 2: Simplify the circuit. The circuit now consists of three resistors: - A 15 Ω resistor, - A 10 Ω resistor, - A 5 Ω resistor.

The 15 Ω and 10 Ω resistors are in series because the current flows through both resistors one after the other. The equivalent resistance of these two resistors in series is:

$$R_{\text{series}} = 15 + 10 = 25\,\Omega$$

Now, this equivalent resistance is in parallel with the 5 Ω resistor:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{25} + \frac{1}{5}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{25} + \frac{5}{25} = \frac{6}{25}$$

$$R_{\text{eq}} = \frac{25}{6} \approx 4.17\,\Omega$$

Step 3: Conclusion. The equivalent resistance of the circuit is approximately $4.17\,\Omega$.

Quick Tip

When diodes are in reverse bias, they act as open circuits. Simplify the circuit by removing the diodes and calculate the equivalent resistance of the remaining resistors.
