MEMORY BASED QUESTIONS JEE–MAIN EXAMINATION – JANUARY 2026

(HELD ON SATURDAY 24th JANUARY 2026) **TIME : 9:00 AM TO 12:00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

1. $\lim\limits_{x\to 0}\dfrac{e^x\left(e^{\tan x - x}-1\right)+\ell n\left(\sec x+\tan x\right)-x}{(\tan x - x)}$

(1) $\frac{3}{2}$ (2) $\frac{3}{2}e$

(3) $\frac{5}{2}e$ (4) $\frac{5}{2}$

Ans. (1)

Sol. $\lim\limits_{x\to 0}\dfrac{e^{\tan x}-e^x+\ell n\left(\sec x+\tan x\right)-x}{\tan x - x}$

Applying L'hospital rule

$\Rightarrow \lim\limits_{x\to 0}\dfrac{e^{\tan x}.\sec^2 x - e^x + \sec x - 1}{\sec^2 x - 1}$

$\lim\limits_{x\to 0}\dfrac{e^{\tan x}(\sec^2 x - 1)+(e^{\tan x}-e^x)+\sec x - 1}{\tan^2 x}$

$\lim\limits_{x\to 0}\left(e^{\tan x}+\dfrac{e^x\left(e^{\tan x - x}-1\right)}{\tan^2 x}+\dfrac{1}{\sec x + 1}\right)$

$\Rightarrow 1 + 0 + \frac{1}{2} = \frac{3}{2}$

2. z is a complex number satisfying

$\left|\dfrac{z-6i}{z-2i}\right| = 1$ and $\left|\dfrac{z-8+2i}{z+2i}\right| = \dfrac{3}{5}$

then $\Sigma\,|z|^2$ is

(1) 225 (2) 321

(3) 284 (4) 385

Ans. (4)

Sol. Solving $\left|\dfrac{z-6i}{z-2i}\right| = 1 \Rightarrow y = 4$ ….. (1)

(where $z = x + iy$)

Now solving $\left|\dfrac{z-8+2i}{z+2i}\right| = \dfrac{3}{5}$

$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0$ …. (2)

Solving (1) & (2) $\Rightarrow z = 17 + 4i$ & $8 + 4i$

$\Rightarrow \Sigma\,|z|^2 = (17)^2 + (4)^2 + (8)^2 + (4)^2 = 385$

3. There are 10 defective & 90 non-defective balls in a bag. 8 balls are taken one by one with replacement then probability that at least 7 defective balls are selected.

(1) $\left(\dfrac{73}{10^8}\right)$ (2) $\left(\dfrac{37}{10^8}\right)$

(3) $\left(\dfrac{105}{10^8}\right)$ (4) $\left(\dfrac{11}{10^8}\right)$

Ans. (1)

Sol. 10 defective & 90 non-defective

Req. probability = (7 def 1 fair) or (8 defective)

Req. probability $= \dfrac{\left(10^7\times 90\right)\times 8 + 10^8}{100^8}$

$= \dfrac{72\times 10^8 + 10^8}{100^8} = \dfrac{73}{10^8}$

4. Let $f(x) = \displaystyle\int \frac{1-\sin(\ell n t)}{1-\cos(\ell n t)}dt$ and $f\left(e^{\pi/2}\right) = -e^{\frac{\pi}{2}}$

then $f\left(e^{\frac{\pi}{4}}\right)$ is :

(1) $e^{-\frac{\pi}{4}}\left(\sqrt{2}+1\right)$ (2) $-e^{\frac{\pi}{4}}\left(\sqrt{2}+1\right)$

(3) $-e^{\frac{\pi}{4}}\left(\sqrt{2}-1\right)$ (4) $e^{\frac{\pi}{4}}\left(\sqrt{2}-1\right)$

Ans. (2)

Sol. $f(t) = \displaystyle\int \frac{1-\sin(\ln t)}{1-\cos(\ln t)}dt$

Let $\ln t = x \Rightarrow t = e^x \Rightarrow dt = e^x\,dx$

$= \dfrac{1}{2}\displaystyle\int\left(\text{cosec}^2\frac{x}{2} - 2\cot\frac{x}{2}\right)e^x dx - t\cot\left(\frac{\ln t}{2}\right) + C$

$\left(\therefore \int (f(x)+f'(x))e^x dx = f(x).e^x + c\right)$

Now $f\left(e^{\pi/2}\right) = -e^{\pi/2}\cot\left(\dfrac{\pi}{4}\right) + C = -e^{\frac{\pi}{2}}$ (given)

$\Rightarrow C = 0$

Now $f\left(e^{\pi/4}\right) = -e^{\pi/4}\cot\left(\dfrac{\pi}{8}\right) + C = -e^{\frac{\pi}{4}}(\sqrt{2}+1)$

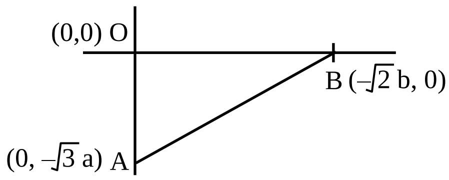
5. Given triangle OAB where O is the origin, $A = \left(0, -\sqrt{3}a\right)$ & $B\left(-\sqrt{2}b, 0\right)$ let the circumradius of ΔOAB is 4 units. If the locus of the centroid of ΔOAB is a circle then its radius is :

(1) $\frac{8}{3}$ (2) $\frac{7}{3}$

(3) $\frac{11}{3}$ (4) $\frac{5}{3}$

Ans. **(1)**

Sol.



$$4 = \frac{\sqrt{2b^2 + 3a^2}}{2}$$

$$2b^2 + 3a^2 = 8^2$$

$$(h, k) = \left(\frac{-\sqrt{2}\,b}{3}, \frac{-\sqrt{3}a}{3}\right)$$

$$b = \frac{3h}{-12}, \quad a = \frac{3k}{-\sqrt{3}}$$

$$2b^2 + 3a^2 = 64$$

$$9h^2 + 9k^2 = 64 \Rightarrow x^2 + y^2 = \left(\frac{8}{3}\right)^2$$

$$r = \frac{8}{3}$$

6. Let the lines $L_2 : \vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$, $\lambda \in R$ and $L_2 : \vec{r} = \left(4\hat{i} + \hat{j}\right) + \mu\left(5\hat{i} + 2\hat{j} + \hat{k}\right)$, $\mu \in R$ intersect at the point R. Let P and Q be the points lying on the line L_1 & L_2 respectively. Such that $|PR| = \sqrt{29}$ and $|PQ| = \sqrt{\frac{47}{3}}$. If the point P lies is the first octant then $27\,(QR)^2$ is :

(1) 340 (2) 360

(3) 320 (4) 348

Ans. **(2)**

Sol. For POI

$2\lambda + 1 = 5\mu + 4$; $3\lambda + 2 = 2\mu + 1$; $4\lambda + 3 = \mu$

$\Rightarrow \lambda = \mu = -1$

$R(-1, -1, -1)$ $P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$

$PR^2 = 29 \Rightarrow (2\lambda + 2)^2 + (3\lambda + 3)^2 + (4\lambda + 4)^2 = 29$

$\Rightarrow \lambda = 0$ or $\lambda = -2$ (Reject)

$\Rightarrow P\ (1, 2, 3)$

$Q\ (5\mu + 4, 2\mu + 1, \mu)$

$$|PQ| = \sqrt{\frac{47}{3}} \Rightarrow PQ^2 = \frac{47}{3}$$

$$\Rightarrow (5\mu + 3)^2 + (2\mu - 1)^2 + (\mu - 3)^2 = \frac{47}{3}$$

$$\Rightarrow \mu = -\frac{1}{3}$$

$$Q = \left(\frac{7}{3}, \frac{1}{3}, -\frac{1}{3}\right)$$

$$(QR)^2 = \left(\frac{7}{3} + 1\right)^2 + \left(\frac{1}{3} + 1\right)^2 + \left(-\frac{1}{3} + 1\right)^2$$

$$= \frac{100 + 16 + 4}{9} = \frac{120}{9}$$

$$\Rightarrow 27 \times (QR)^2 = 27 \times \frac{120}{9} = 360$$

7. If A={1, 2, 3, 4}. A relation from set A to A (a,b) R (c, d) such that 2a + 3b = 3c + 4d, then find the number of element(s) in relation :

(1) 9 (2) 10

(3) 11 (4) 12

Ans. **(3)**

Sol.

(a, b)	(c, d)
(1, 1)	x
(1, 2)	x
(1, 3)	(1, 2)
(1, 4)	(2, 2)
(2, 1)	(1, 1)
(2, 2)	(2, 1)
(2, 3)	(3, 1)
(2, 4)	(4, 1)
(3, 1)	x
(3, 2)	x

(3, 3)	(1, 3)
(3, 4)	(2, 3)
(4, 1)	(1, 2)
(4, 2)	(2, 2)
(4, 3)	x
(4, 4)	(4, 2)

8. Find the number of real solutions of

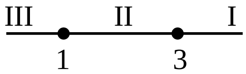
$x|x-3| + |x-1| + 3 = 0$:

(1) 1 (2) 2

(3) 3 (4) 4

Ans. (1)

Sol.



(I) if $x \geq 3$

$\therefore\ x^2 - 3x + x - 1 + 3 = 0$

$\therefore\ x^2 - 2x + 2 = 0$

No real solution

(ii) If $1 < x < 3$

$\therefore\ 3x - x^2 + x - 1 + 3 = 0$

$\Rightarrow x^2 - 4x - 2 = 0$

$$\rightarrow x = \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm \sqrt{24}}{2} = 2 \pm \sqrt{6}$$

(III) If $x \leq 1$

$\therefore\ 3x - x^2 + 1 - x + 3 = 0$

$$\Rightarrow x^2 - 2x - 4 = 0 \rightarrow x = \frac{2 \pm \sqrt{4+16}}{2} = 1 \pm \sqrt{5}$$

So, total 1 real solution

9. If $\cot x = \frac{5}{12}$ for some $x \in \left(\pi, \frac{3\pi}{2}\right)$ then

$$\sin 7x\left(\cos\frac{13x}{2} + \sin\frac{13x}{2}\right) + \cos 7x\left(\cos\frac{13x}{2} - \sin\frac{13x}{2}\right)$$ is

equal to :

(1) $\frac{1}{\sqrt{13}}$ (2) $\frac{5}{\sqrt{13}}$

(3) $-\frac{1}{\sqrt{13}}$ (4) $\frac{8}{\sqrt{13}}$

Ans. (1)

Sol. $\cot x = \frac{5}{12} \Rightarrow \cos x = \frac{-5}{13} = 2\cos^2\frac{x}{2} - 1$

$$\cos\left(\frac{x}{2}\right) = -\frac{2}{\sqrt{13}} \text{ or } \frac{2}{\sqrt{13}} (\text{rejected})$$

$$\left\{\because \frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)\right\}$$

$$\left(\sin 7x\frac{\sin 13x}{2} + \cos 7x\frac{\cos 13x}{2}\right) + \left(\sin 7x\frac{\cos 13x}{2} - \cos 7x\frac{\sin 13x}{2}\right)$$

$$\cos\left(7x - \frac{13x}{2}\right) + \sin\left(7x - \frac{13x}{2}\right)$$

$$\cos\frac{x}{2} + \sin\left(\frac{x}{2}\right)$$

$$\frac{3}{\sqrt{13}} - \frac{2}{\sqrt{13}} = \frac{1}{\sqrt{13}}$$

10. Consider an ellipse $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ (B > A) where $e = \frac{4}{5}$ for both the curves and ℓ_1 is the length of latus rectum of E_1 and ℓ_2 is the length of latus rectum of E_2. Let the distance between the focii of the first curve is 8. Find the distance between the focii of the second curve (Given $\left(2\ell_1^2 = 9\ell_2\right)$) :

(1) $\frac{64}{5}$ (2) $\frac{8}{5}$

(3) $\frac{32}{5}$ (4) $\frac{16}{5}$

Ans. (3)

Sol. $2ae = 8 \Rightarrow a = 5$

$b^2 = a^2\ (1 - e^2)$

$$b^2 = a^2 \times \frac{9}{25} \quad b^2 = 9$$

$$E_1 : \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\ell_1 : \frac{2b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$$

$$A^2 = B^2\left(1 - e^2\right) \Rightarrow A^2 = \frac{9}{25}B^2 \Rightarrow A = \frac{3}{5}B$$

$$2\ell^2 = 9\ell_2 \Rightarrow 2\left(\frac{18}{5}\right)^2 = 9\ell_2 \Rightarrow \ell_2 = \frac{4\times 18}{25}$$

$$\frac{2A^2}{B} = \frac{72}{25} \Rightarrow A^2 \frac{36}{25}B$$

$$\frac{9}{25}B^2 = \frac{36B}{25} \Rightarrow B = 4,$$

Distance between focii 2Be = $2\times\frac{4}{5}\times 4 = \frac{32}{5}$

11. Let mean & variance of 10 numbers are 10 & 2 respectively. If one number α is replaced by another number β, then new mean & variance are 10.1 & 1.99. Find $(\alpha + \beta)$

(1) 20 (2) 19

(3) 18 (4) 17

Ans. (1)

Sol. Let first 10 numbers are $x_1, x_2, \ldots\ldots x_9, \alpha$

$$\Rightarrow \alpha + \sum_{i=1}^{9} x_i = 100 \Rightarrow \sum_{i=1}^{9} x_i = 100 - \alpha$$

$$\text{Variance} = \left(\frac{\sum x_i^2}{n}\right) - \left(\frac{\sum x_i}{n}\right)^2$$

$$\Rightarrow \frac{\sum x_i^2}{n} = 98$$

$$\Rightarrow x_1^2 + x_2^2 + \ldots\ldots x_9^2 + \alpha^2 = 1020 \Rightarrow \sum x_i^2 = 1020 - \alpha^2$$

In second case, let number are

$x_1, x_2, \ldots\ldots x_9, \beta$

$100 - \alpha + \beta = 101 \ \alpha - \beta + 1 = 0$

$$\frac{\sum x_i^2 + \beta^2}{10} - (10.1)^2 = 1.99$$

$$\beta^2 - \alpha^2 = 20$$

$$\alpha = \frac{19}{2}$$

$$\beta = \frac{21}{2}$$

$$\alpha + \beta = \frac{19+21}{2} = 20$$

12. The value of $\dfrac{\sqrt{3}\ \text{cosec}20^\circ - \sec 20^\circ}{\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}$ is :

(1) 64 (2) 48

(3) 46 (4) 40

Ans. (1)

Sol. $$E = \frac{\dfrac{\sqrt{3}}{\sin 20^\circ} - \dfrac{1}{\cos 20^\circ}}{\dfrac{1}{2}.\dfrac{1}{4}.\cos 60^\circ}$$

$$= \frac{\left(\sqrt{3}\cos 20^\circ - \sin 20^\circ\right)}{\cos 20^\circ . \sin 20^\circ}16$$

$$= \frac{\left(\dfrac{\sqrt{3}}{2}\cos 20^\circ - \dfrac{1}{2}\sin 20^\circ\right)32\times 2}{2\cos 20^\circ . \sin 20^\circ}$$

$$= \frac{\sin 40^\circ}{\sin 40^\circ}\times 64 = 64$$

13. Find number of matrices A whose order is 3 × 2 has elements from the set $\{\pm 2, \pm 1, 0\}$ if $\text{Tr}(A^TA) = 5$:

(1) 310 (2) 312

(3) 320 (4) 325

Ans. **(2)**

Sol. $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times 2}$

$$A^TA = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}_{2\times 3} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3\times 2}$$

$$= \begin{pmatrix} a_1^2+a_2^2+a_3^2 & \underline{\qquad} \\ \underline{\qquad} & b_1^2+b_2^2+b_3^2 \end{pmatrix}$$

$\text{Tr}(A^TA) = a_1^2+a_2^2+a_3^2+b_1^2+b_2^2+b_3^2 = 5$

{2, 1, 0, 0, 0, 0}
{2, –1, 0, 0, 0, 0}
{–2, 1, 0, 0, 0, 0}
{–2, –1, 0, 0, 0, 0}
{1, 1, 1, 1, 1, 0}

$$\text{No. of ways} = \frac{6!}{4!}\times 4 + 2\times\frac{6!}{5!} + 2\times\frac{6!}{4!} + 2\times\frac{6!}{3!2!}$$

$$= \frac{6!}{3!} + 2\times 6 + 2\times 15\times 2\times\frac{6!}{3!}$$

= 120 + 120 + 12 + 60 = 312

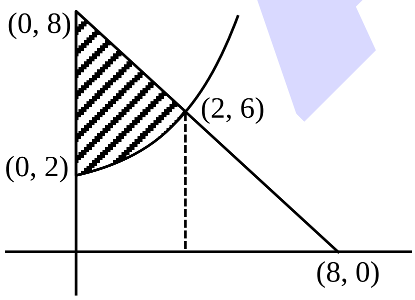
14. A_1 is the area bounded by $y = x^2 + 2$, $x + y = 8$, y-axis in the I[st] quadrant and A_2 is the area bounded by $y = x^2 + 2$, $y^2 = x$, $x = 0$ and $x = 2$ in the 1[st] quadrant find $(A_1 - A_2)$:

(1) $\frac{2}{3}+\frac{4\sqrt{2}}{3}$ (2) $\frac{3}{2}+\frac{4\sqrt{2}}{3}$

(3) $\frac{3}{5}+\frac{4\sqrt{2}}{3}$ (4) None of these

Ans. **(1)**

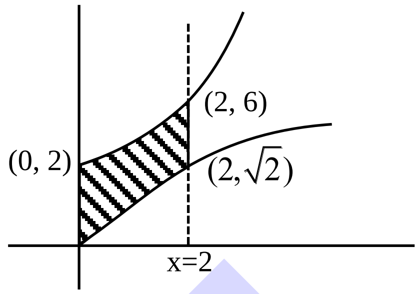
Sol.



$$A_1 = \int_0^2 ((8-x)-(x^2+2))\,dx$$

$$= A_1 = \int_0^2 (6-x-x^2)\,dx$$

$$A_1\left(6x-\frac{x^2}{2}-\frac{x^3}{3}\right)_0^2 = 12-2-\frac{8}{3} = 10-\frac{8}{3} = \frac{22}{3}$$



$$A_2 = \int_0^2 (x^2+2)dx - \frac{2}{3}(2\sqrt{2})$$

$$A_2 = \left(\frac{x^3}{3}+2x\right)_0^2 - \frac{4\sqrt{2}}{3}$$

$$A_2 = \frac{8}{3}+4-\frac{4\sqrt{2}}{3} = \frac{20}{3}-\frac{4\sqrt{2}}{3}$$

$$A_1 - A_2 = \frac{2}{3}+\frac{4\sqrt{2}}{3}$$

15. Evaluate the series

$$\frac{1}{25!}+\frac{1}{3!23!}+\frac{1}{5!21!}+\ldots\ldots..+\text{upto 13 terms} \ :$$

(1) $\frac{2^{26}}{26!}$ (2) $\frac{2^{25}}{26!}$

(3) $\frac{2^{26}}{25!}$ (4) $\frac{2^{25}}{25!}$

Ans. **(2)**

Sol. $\frac{1}{26!}\left(\frac{26!}{25!1!}+\frac{26!}{3!23!}+\frac{26!}{5!21!}+\ldots..+13\text{ terms}\right)$

$$\frac{1}{26!}\left({}^{26}C_1+{}^{26}C_3+{}^{26}C_5+\ldots.+13\,\text{terms}\right)$$

$$\frac{1}{26!}\left({}^{26}C_1+{}^{26}C_5+\ldots.+{}^{26}C_{25}\right)$$

$$\frac{1}{26!}\times 2^{25}$$

16. Consider a geometric sequence 729, 81, 9, 1, ……
If P_n denotes the product of Ist n terms of G.P. such that $\sum_{n=1}^{40}(P_n)^{\frac{1}{n}}=\frac{3^{\alpha}-1}{2\times3^{\beta}}$, then value of $(\alpha+\beta)$ is :

(1) 72 (2) 74
(3) 73 (4) 75

Ans. **(3)**

Sol. $P_n = 729.81.9$ …….(n terms)

$= 3^6.3^4.3^2$ …….. 3^{-2n+8}

$P_n = 3^{6+4+2+\ldots..+(-2n+8)} = 3^{n(7-n)}$

$P_n^{\ 1/n} = 3^{7-n}$

$$\Rightarrow \sum_{n=1}^{40}(P_n)^{\frac{1}{n}} = 3^6+3^5+\ldots..+(40\,\text{terms})$$

$$=3^6\left[\frac{1-\left(\frac{1}{3}\right)^{40}}{1-\frac{1}{3}}\right]$$

$$=\frac{3^6\left[3^{40}-1\right]\times3^1}{3^{40}\times2}$$

$$\sum(P_n)^{\frac{1}{n}}=\frac{(3^{40}-1)}{2\times3^{33}}, \qquad \alpha=40$$

$\beta = 33$

$\alpha+\beta = 73$

17. Consider an A.P. $a_1, a_2,$ ……… a_n ; $a_1 > 0$, $a_2 - a_1 = \frac{-3}{4}$, $a_n = \frac{a_1}{4}$, $\sum_{i=1}^{n}a_i=\frac{525}{2}$ then $\sum_{i=1}^{17}a_i$ is equal to :

(1) 231 (2) 234
(3) 236 (4) 238

Ans. **(4)**

Sol. $S_n = \frac{n}{2}[a_1+a_n] = \frac{525}{2}$, $d=\frac{-3}{4}$

$$\frac{n}{2}\left[a_1+\frac{a_1}{4}\right]=\frac{525}{2}$$

$$\frac{5a_1n}{4}=525$$

$a_1n = 420$

$$a_n = a_1+(n-1)\left(\frac{-3}{4}\right)$$

$$\Rightarrow \frac{-3}{4}a_1=\left(\frac{-3}{4}\right)(n-1) \Rightarrow a_1 = n-1$$

$n(n-1) = 420$

$n^2-n-420=0$

$(n-21)(n+20)=0$

$n = 21,\ a_1 = 20$

$$\sum_{i=1}^{17}a_i=\frac{17}{2}[2a_1+16d]$$

$$=\frac{17}{2}\left[40+16\left(\frac{-3}{4}\right)\right]$$

$$=\frac{17}{2}[40-12]$$

$= 17 \times 14 = 238$

18. If the domain of $f(x)=\log_{(10x^2-17x+7)}(18x^2-11x+1)$ is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then find 90 $(a + b + c + d + e)$:

(1) 316 (2) 320
(3) 163 (4) 631

Ans. **(1)**

Sol. $18x^2 - 11x + 1 > 0$

$(9x-1)(2x-1) > 0 \Rightarrow \boxed{x>\frac{1}{2}\text{ or } x<\frac{1}{9}}$ …..(1)

$10x^2 - 17x + 7 > 0$

$(10x-7)(x-1) > 0$

$\boxed{x>1 \text{ or } x<\frac{7}{10}}$ ….(2)

$10x^2 - 17x + 7 > 0$

$10x^2 - 17x + 6 \neq 0$

$(5x-6)(2x-1) \neq 0$

$\boxed{x\neq\frac{6}{5},\frac{1}{2}}$ ………(3)

Eq. (1) & (2) & (3)

$$x\in\left(-\infty,\frac{1}{9}\right)\cup\left(\frac{1}{2},\frac{7}{10}\right)\cup(1,\infty)-\left\{\frac{6}{5}\right\}$$

$a=\frac{1}{9}, b=\frac{1}{2}, c=\frac{7}{10}$, $d = e$, 1, $e=\frac{6}{5}$

$90(9 + b + c + d + e)$

$= 316$

19. Given that $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \vec{a} \times \vec{b}$

$|\vec{d} \times \vec{c}| = 3$ & $\vec{d} \wedge \vec{c} = \frac{\pi}{4}$ & $|\vec{a} - \vec{d}| = \sqrt{11}$ find $\vec{a}.\vec{d}$

(1) 2 (2) $\frac{3}{2}$

(3) $\frac{1}{2}$ (4) $-\frac{1}{4}$

Ans. (3)

Sol. $\vec{c} = \vec{a} \times \vec{d} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = i - j + k$ …….(1)

$$\left|\vec{d} \times \vec{c}\right| = \left|\vec{d}\right|\left|\vec{c}\right| \sin\theta = \left|\vec{d}\right| \sqrt{3}.\frac{1}{\sqrt{2}} = 3 \quad \text{(given)}$$

$$\Rightarrow \left|\vec{d}\right| = \sqrt{6} \qquad \text{..... (2)}$$

Now

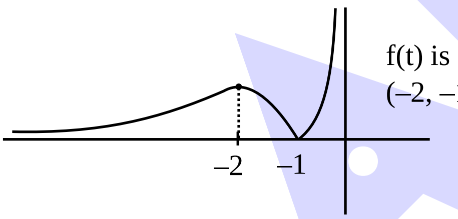
$$\left|\vec{a} - \vec{d}\right| = \sqrt{11} \Rightarrow \left|\vec{a}\right|^2 + \left|\vec{d}\right|^2 - 2\vec{a}.\vec{d} = 11$$

$$\Rightarrow 6 + 6 - 2\vec{a}.\vec{d} = 11 \Rightarrow \vec{a}.\vec{d} = \frac{1}{2}$$

20. Given $f(t) = \left|\frac{t+1}{t^2}\right|; (t<0)$ is strictly decreasing in the interval $(2\alpha, \alpha)$ then maximum value of $g(x) = 2\log_e(x-2) + \alpha x^2 + 4x - \alpha$ is

Ans. (4)

Sol. Drawing graph of f(t) for $t < 0$



f(t) is decreasing in $(-2, -1) \Rightarrow \alpha = -1$

$g(x) = \log_e(x-2) - x^2 + 4x + 1;\quad x > 2$

$$g'(x) = \frac{2}{x-2} - (2(x-2)\,;\ x > 2$$

$$g'(x) = \frac{1-(x-2)^2}{(x-2)} = \frac{-(x-3)(x-1)}{(x-2)}$$

$\underset{2}{\vert}\overset{+}{\text{——}}\underset{3}{\vert}\overset{-}{\text{——}}$ as $x > 2$

maxima occur at $x = 3$

$g(3) = 2\log_e 1 - 9 + 12 + 1 = 4$

21. Let $5000 < N < 9000$ and N has digits from {0, 1, 2, 5, 9} and digits can be repeated then find the number of N divisible by 3.

Ans. (84)

Sol. 5 9 9 9 ×

0 1 = 3! = 6

2 2 = $\frac{3!}{2!}$ = 3

5 5 = $\frac{3!}{2!}$ = 3

5 2 = 3! = 6

5 5 0 2 = 3! = 6

0 5 = $\frac{3!}{2!}$ = 3

9 2 = 3! = 6

9 5 = $\frac{3!}{2!}$ = 3

5 2 0 2 = 3

0 5 = 6

9 2 = 3

9 5 = 6

5 1 0 0 = 3

2 1 = 3

5 1 = 3

9 0 = 6

5 0 0 1 = 3

2 2 = 3

5 2 = 3! = 6

5 5 = 3

Total = 8 × 6 + 12 × 3

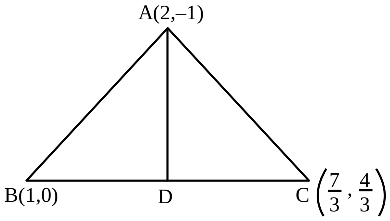
= 48 + 36

= 84

22. Let the vertices of the triangle are (1, 0), (2, –1), $\left(\frac{7}{3}, \frac{4}{3}\right)$. If the equation of internal angle bisector through (2, –1) is $\alpha x + \beta y = 5$ then value of $(\alpha^2 + \beta^2)$ is:

Ans. (10)

Sol.



$$\frac{BD}{DC}=\frac{AB}{AC}=\frac{\sqrt{2}\times 3}{5\sqrt{2}}=\frac{3}{5}$$

$$D=\left(\frac{12}{8},\frac{4}{8}\right)=\left(\frac{3}{2},\frac{1}{2}\right)$$

$$\text{Slope of AD} = \frac{-3/2}{\frac{1}{2}}=-3$$

$3x + y = 5$

$\alpha = 3, \beta = 1;\ \alpha^2 + \beta^2 = 10$

23. If $\int_0^{36} f\left(\frac{tx}{36}\right)dt$ at $= 4\alpha\ f(x)$, $y = f(x)$ is standard parabola passing through $(2, 1)$ and $(-4, \beta)$. Then value of β^{α} is :

Ans. **(64)**

Sol. $\int_0^{36} f\left(\frac{tx}{36}\right)dt = 4\alpha f(x)$, Put $\frac{tx}{36}=y$

$$\frac{dy}{dt}=\frac{x}{36}$$

$$\int_0^{x}\frac{f(y)36dy}{x}=4\alpha f(x)$$

$$\int_0^{x} f(y)dy=\frac{\alpha f(x)x}{9}$$

$$f(x)=\frac{\alpha}{9}\left(f(x)+xf'(x)\right)$$

$$\left(1-\frac{\alpha}{9}\right)f(x)=\frac{\alpha x}{9}f'(x)\Rightarrow(9-\alpha)f(x)=\alpha x f'(x)$$

$$\frac{f'(x)}{f(x)}=\left(\frac{9}{\alpha}-1\right)\frac{1}{x}$$

$$\log_e f(x)=\left(\frac{9}{\alpha}-1\right)\log_e x+\log_e c$$

$f(x)=cx^{\left(\frac{9}{\alpha}-1\right)}$ for standard parabola

$$\frac{9}{\alpha}-1=2$$

$\alpha = 3$

$f(x) = cx^2$

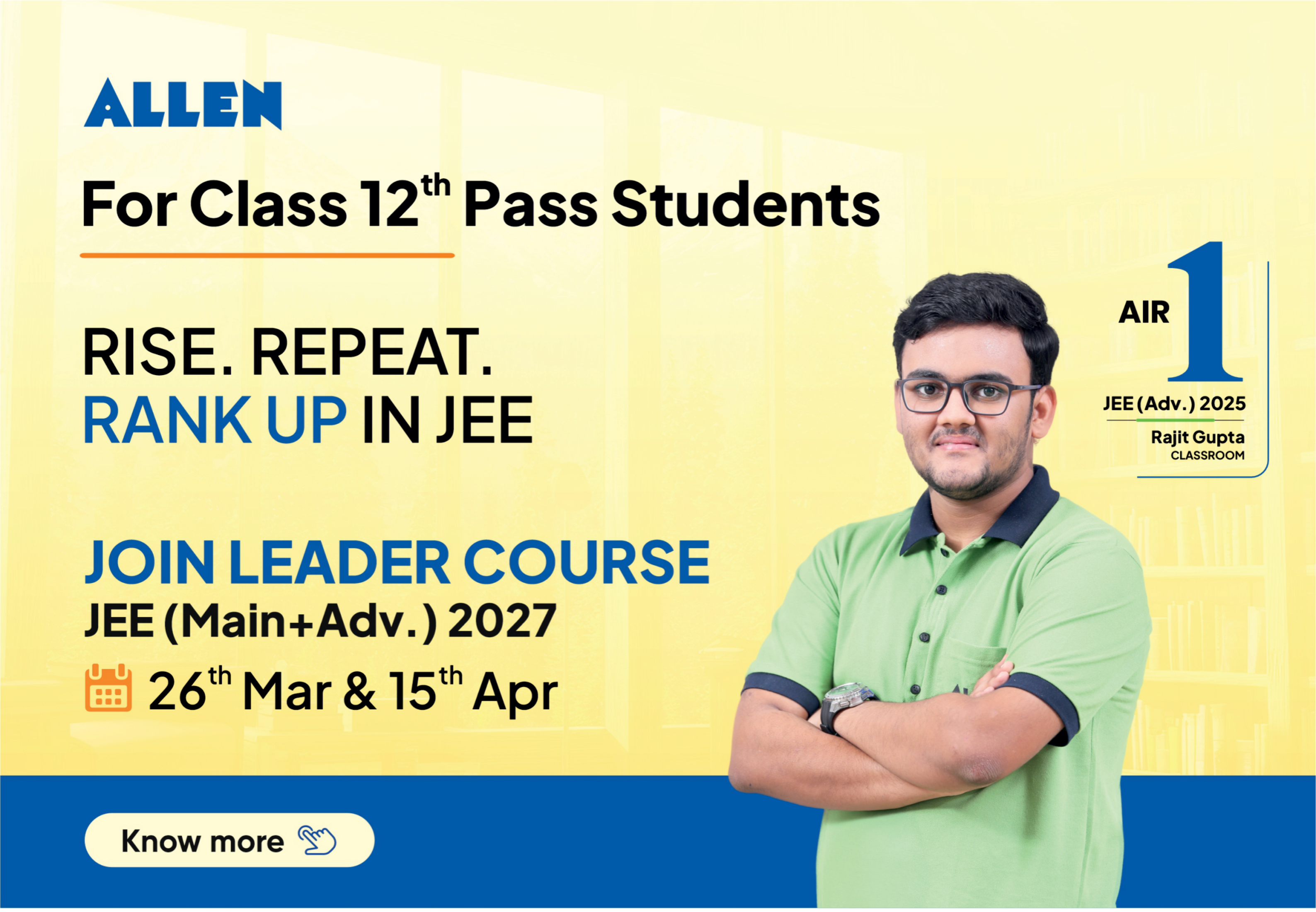
passing through $(2,1)$

$1 = 4c \Rightarrow c = 1/4$

$y=\frac{x^2}{4}$ passing through $(-4, \beta)$

$\beta = 4$

$\beta^{x} = 4^3 = 64$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