JEE Main 2027 Important Notes and Formula

The Complete Physics, Chemistry and Maths Formula Reference for JEE Mains 2027

55 Topic Sections | 236 Formula Boxes | Latest NTA Syllabus

Every formula here maps to a unit that NTA still tests in JEE Main. The syllabus was trimmed in 2024 and has stayed the same for 2025 and 2026, so JEE Main 2027 is expected to run on the same reduced syllabus and the same 75-question, 300-mark pattern. Deleted units are not covered. Where a result sits just outside NCERT but shows up in Mains every year, it is marked as a **JEE Extension**.

JEE Main 2027 Paper 1 Pattern

Section	Questions	Type	Marking	Marks
Physics Section A	20	MCQ	$+4$ / -1	80
Physics Section B	5	Numerical value	$+4$ / -1	20
Chemistry Section A	20	MCQ	$+4$ / -1	80
Chemistry Section B	5	Numerical value	$+4$ / -1	20
Maths Section A	20	MCQ	$+4$ / -1	80
Maths Section B	5	Numerical value	$+4$ / -1	20
Total	**75**	**3 hours, CBT**		**300**

All five Section B questions are compulsory. The old ”answer any 5 of 10” choice was withdrawn from the 2025 session onwards. Section B answers are entered on an on-screen numeric keypad, and the value is rounded to the nearest integer unless the question says otherwise. Confirm the final pattern in the NTA Information Bulletin when it is released around October 2026.

How to use this handbook

Read a unit’s short intro first, then the boxes. Orange boxes are the formulas you must be able to write from memory. Yellow **JEE Extension** boxes are results that are not spelled out in NCERT but are asked in Mains, so learn them after the core set. Red boxes are the errors that cost students marks every year.

What is NOT in the syllabus

Physics (1 unit gone): Communication Systems.

Chemistry (8 units gone): States of Matter, Surface Chemistry, Hydrogen, s-Block Elements, General Principles and Processes of Isolation of Metals, Environmental Chemistry, Polymers, Chemistry in Everyday Life.

Maths (2 units gone): Mathematical Induction, Mathematical Reasoning.

Roughly 15 percent of the old syllabus was cut in 2024. Nothing has been added back for 2025 or 2026, so plan 2027 on this reduced list.

Where the Marks Sit

Weightage is never published by NTA. The table below is built from the last several sessions and tells you where to spend revision time.

Physics (high yield)	Chemistry (high yield)	Maths (high yield)
Modern Physics: 4 to 5 Q	Coordination Compounds: 2 to 3 Q	Calculus block: 6 to 7 Q
Electrostatics + Current: 3 to 4 Q	Periodic Table + p-block: 3 Q	Coordinate Geometry: 4 to 5 Q
Magnetism + EMI + AC: 3 to 4 Q	Organic named reactions: 3 to 4 Q	Matrices, Determinants: 2 Q
Optics: 2 to 3 Q	Chemical + Ionic Equilibrium: 2 Q	Vectors + 3D: 2 to 3 Q
Heat and Thermodynamics: 2 to 3 Q	Thermodynamics + Kinetics: 2 to 3 Q	Probability + Statistics: 2 Q
Rotational Motion: 2 Q	Chemical Bonding: 2 Q	Complex Numbers, Quadratics: 2 Q

What This Handbook Covers

Physics (20 sections)	Chemistry (20 sections)	Maths (15 sections)
Units, Dimensions and Errors	Some Basic Concepts	Sets, Relations and Functions
Kinematics	Atomic Structure	Complex Numbers, Quadratics
Laws of Motion and Friction	Chemical Bonding	Matrices and Determinants
Work, Energy and Power	Chemical Thermodynamics	Permutations and Combinations
Rotational Motion	Solutions	Binomial Theorem
Gravitation	Equilibrium	Sequences and Series
Elasticity, Fluids, Thermal	Redox and Electrochemistry	Limits, Continuity, Differentiability
Thermodynamics	Chemical Kinetics	Applications of Derivatives
Kinetic Theory of Gases	Periodicity	Integral Calculus
Oscillations	p-Block Elements	Differential Equations
Waves and Sound	d- and f-Block Elements	Coordinate Geometry
Electrostatics	Coordination Compounds	Three Dimensional Geometry
Current Electricity	Purification and Characterisation	Vector Algebra
Magnetic Effects of Current	Basic Principles of Organic	Statistics and Probability
EMI and Alternating Current	Hydrocarbons	Trigonometry
Electromagnetic Waves	Halogen Compounds	
Ray and Wave Optics	Oxygen Compounds	
Dual Nature of Matter	Nitrogen Compounds	
Atoms and Nuclei	Biomolecules	
Semiconductor Devices	Practical Chemistry	

Constants You Are Expected to Know

Quantity	Value	Quantity	Value
Speed of light c	3.00×10^{8} m/s	Avogadro number N_A	6.022×10^{23} mol^{-1}
Planck constant h	6.626×10^{-34} J s	Gas constant R	8.314 J mol^{-1}K^{-1}
$\hbar = h/2\pi$	1.055×10^{-34} J s	R (alt units)	0.0821 L atm mol^{-1}K^{-1}
Electron charge e	1.602×10^{-19} C	Faraday F	96500 C mol^{-1}
Electron mass m_e	9.11×10^{-31} kg	Boltzmann k_B	1.38×10^{-23} J/K
Proton mass m_p	1.673×10^{-27} kg	Stefan σ	5.67×10^{-8} W m^{-2}K^{-4}
ε_0	8.854×10^{-12} F/m	Wien constant b	2.898×10^{-3} m K
$1/4\pi\varepsilon_0$	9×10^{9} N m^{2}C^{-2}	Rydberg R_H	1.097×10^{7} m^{-1}
μ_0	$4\pi \times 10^{-7}$ T m/A	Bohr radius a_0	0.529 Å
$\mu_0/4\pi$	10^{-7} T m/A	Bohr energy E_1	-13.6 eV
Gravitational G	6.674×10^{-11} SI	1 eV	1.602×10^{-19} J
g at surface	9.8 m/s^2	1 amu	931.5 MeV

Unit slips that cost marks

Section B answers are graded on the exact number. Convert before you type: 1 bar $= 10^5$ Pa, 1 atm $= 1.013 \times 10^5$ Pa, 1 L atm $= 101.3$ J, 1 Å $= 10^{-10}$ m, 1 gauss $= 10^{-4}$ T, 1 curie $= 3.7 \times 10^{10}$ Bq. In thermodynamics numericals, check whether R should be 8.314 or 0.0821 before substituting.

Physics

19 units, 25 questions, 100 marks

Physics in JEE Main rewards formula recall plus one clean substitution. Roughly a third of the paper comes from Modern Physics, Electrostatics, Current Electricity, Magnetism and EMI. Mechanics supplies most of the Section B numericals.

1 Units, Dimensions and Errors

This unit covers SI base units, dimensional formulas, dimensional analysis and the propagation of error. Errors show up almost every year in Section B.

Dimensional formulas to memorise

Force $[MLT^{-2}]$; Work, Energy, Torque $[ML^2T^{-2}]$; Power $[ML^2T^{-3}]$

Pressure, Stress, Modulus $[ML^{-1}T^{-2}]$

Planck constant, Angular momentum $[ML^2T^{-1}]$

Gravitational constant G: $[M^{-1}L^3T^{-2}]$

Quantities with the **same dimensions** can be added or equated. Torque and work share dimensions but are physically different, so dimensions alone never prove a relation.

Error: absolute and percentage

$$\Delta a_{\text{mean}} = \frac{1}{n}\sum_{i=1}^{n}|\Delta a_i|$$

$$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}}$$

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100$$

Absolute error carries the **same unit** as the quantity. Relative error is a pure number.

Combination of errors

Sum or difference $Z = A \pm B$:

$$\Delta Z = \Delta A + \Delta B$$

Product or quotient $Z = A^pB^q/C^r$:

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

Errors always **add**, never cancel. A power p multiplies that term's relative error by p, so the highest power dominates the final error.

Vernier and screw gauge

$$\text{Vernier least count} = 1\text{ MSD} - 1\text{ VSD} = \frac{1\text{ MSD}}{N}$$

$$\text{Screw gauge LC} = \frac{\text{pitch}}{\text{number of circular divisions}}$$

$$\text{Reading} = \text{MSR} + (\text{VSD or CSD}) \times \text{LC} \pm \text{zero error}$$

A **negative zero error is added**, a positive zero error is subtracted. That single sign decides the answer in most instrument questions.

Significant figures in the answer

The result of a multiplication or division keeps as many significant figures as the least precise input. The result of an addition or subtraction keeps as many decimal places as the least precise input.

JEE Extension

Dimensional analysis fixes an unknown power law up to a constant. If $T \propto l^a g^b m^c$ for a pendulum, matching M, L, T gives $c = 0$, $a = 1/2$, $b = -1/2$, so $T \propto \sqrt{l/g}$. The 2π cannot come out of dimensions.

2 Kinematics

This unit covers motion in a straight line, relative velocity, projectile motion and uniform circular motion. Graph reading questions are common in Section A.

Uniform acceleration equations

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

Distance in nth second: $s_n = u + \frac{a}{2}(2n - 1)$

Valid only for **constant acceleration**. If a depends on t or x, integrate $a = v\,dv/dx$ instead.

Projectile from ground

Time of flight $T = \frac{2u \sin\theta}{g}$

Max height $H = \frac{u^2 \sin^2\theta}{2g}$

Range $R = \frac{u^2 \sin 2\theta}{g}$

Trajectory: $y = x \tan\theta - \frac{gx^2}{2u^2 \cos^2\theta}$

Range is maximum at $\theta = 45°$. Angles θ and $90° - \theta$ give the **same range** but different heights and flight times.

Relative velocity

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

River crossing, shortest time: head straight across, drift $= v_r \cdot d/v_b$

Shortest path: aim upstream at $\sin\theta = v_r/v_b$

Shortest time and shortest path are **different strategies**. Read which one the question asks for before choosing the angle.

Circular motion kinematics

$v = \omega r, \quad a_c = \frac{v^2}{r} = \omega^2 r$

Tangential $a_t = \frac{dv}{dt}$, $a_{\text{net}} = \sqrt{a_c^2 + a_t^2}$

Centripetal acceleration changes only the **direction** of velocity. Tangential acceleration changes its magnitude.

Distance versus displacement on graphs

The area under a velocity-time graph is displacement, and it is negative when the curve dips below the axis. For total distance, take the modulus of each area separately and add. Students routinely lose an easy mark here.

3 Laws of Motion and Friction

This unit covers Newton's three laws, free body diagrams, friction, and circular motion dynamics including banking.

Newton's second law and impulse

$\vec{F}_{\text{net}} = \dfrac{d\vec{p}}{dt} = m\vec{a}$ (constant m)

Impulse $\vec{J} = \int \vec{F}\, dt = \Delta\vec{p}$

Impulse is the **area under the force-time graph**. For collisions and sudden jerks, use impulse instead of trying to find the force.

Friction

Static: $f_s \le \mu_s N$

Kinetic: $f_k = \mu_k N$, with $\mu_k < \mu_s$

Angle of repose: $\tan\theta_r = \mu_s$

Static friction is **self-adjusting**. It equals the applied force until it hits the limiting value $\mu_s N$, so never plug $\mu_s N$ into a body that is not about to slip.

Connected bodies and pulleys

Two blocks m_1, m_2 over an ideal pulley:

$a = \dfrac{(m_1 - m_2)g}{m_1 + m_2}$, $\quad T = \dfrac{2m_1 m_2 g}{m_1 + m_2}$

Tension is the **harmonic-mean type** expression and always lies between $m_2 g$ and $m_1 g$. If your T falls outside that band, the sign convention is wrong.

Banking and circular dynamics

Frictionless bank: $\tan\theta = \dfrac{v^2}{rg}$

With friction: $v_{\max} = \sqrt{rg\dfrac{\mu + \tan\theta}{1 - \mu\tan\theta}}$

Vertical circle, minimum top speed: $v_{\text{top}} = \sqrt{gR}$

At the top of a vertical loop the **string tension can be zero** but gravity still supplies mv^2/R. That gives the critical speed.

Pseudo force

In a frame with acceleration $\vec{a}_0$: apply extra force $-m\vec{a}_0$ on every body.

Effective gravity in a lift: $g_{\text{eff}} = g \pm a$

Pseudo force acts **opposite** to the frame's acceleration. Use it only after you commit to solving in the accelerating frame.

4 Work, Energy and Power

This unit covers work by constant and variable forces, kinetic and potential energy, the work-energy theorem, conservative forces, and collisions.

Work and the work-energy theorem

$W = \vec{F}\cdot\vec{s} = Fs\cos\theta$; variable force $W = \int \vec{F}\cdot d\vec{s}$

$W_{\text{net}} = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

The theorem uses **net work**, including friction and tension. It holds in every frame, inertial or not, as long as you are consistent.

Potential energy and force

Spring: $U = \frac{1}{2}kx^2$; Gravity near surface: $U = mgh$

$$F = -\frac{dU}{dx}$$

Equilibrium: $dU/dx = 0$; stable if $d^2U/dx^2 > 0$

Potential energy is defined only for **conservative forces**. The minus sign means force points from high U towards low U.

Power

$$P_{\text{avg}} = \frac{W}{t}, \quad P_{\text{inst}} = \vec{F} \cdot \vec{v}$$

Vehicle at constant power: $v \propto \sqrt{t}$ on a level road with no drag

At **constant power** the driving force falls as speed rises, which is why acceleration drops in higher gears.

Collisions in one dimension

Coefficient of restitution $e = \frac{v_2 - v_1}{u_1 - u_2}$

Elastic, equal masses: velocities are exchanged

Elastic, general: $v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2$

Perfectly inelastic: $v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$

Momentum is conserved in **every** collision. Kinetic energy is conserved only when $e = 1$.

JEE Extension

Fractional kinetic energy lost in a head-on collision of m_1 (moving) with m_2 (at rest): $\frac{\Delta K}{K} = \frac{m_2}{m_1 + m_2}(1 - e^2)$. Maximum loss occurs for equal masses with $e = 0$, where exactly half the kinetic energy disappears.

5 Rotational Motion

This unit covers centre of mass, moment of inertia, torque, angular momentum, rolling motion and the parallel and perpendicular axis theorems. Two questions per paper is typical.

Centre of mass

$$\vec{r}_{cm} = \frac{\sum m_i\vec{r}_i}{\sum m_i}$$

Continuous body: $\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$

Semicircular ring: $y_{cm} = \frac{2R}{\pi}$; disc: $\frac{4R}{3\pi}$

Solid hemisphere: $\frac{3R}{8}$; hollow: $\frac{R}{2}$

Internal forces never move the centre of mass. If $F_{\text{ext}} = 0$, $\vec{v}_{cm}$ stays **constant** no matter what happens inside.

Moment of inertia

Ring about axis: MR^2; Disc: $\frac{1}{2}MR^2$

Solid sphere: $\frac{2}{5}MR^2$; Hollow sphere: $\frac{2}{3}MR^2$

Rod about centre: $\frac{1}{12}ML^2$; about end: $\frac{1}{3}ML^2$

Solid cylinder about its axis: $\frac{1}{2}MR^2$

Moment of inertia depends on the **axis chosen**, not just the body. Mass far from the axis counts much more because of the r^2.

Axis theorems

Parallel axis: $I = I_{cm} + Md^2$

Perpendicular axis (planar bodies only): $I_z = I_x + I_y$

The parallel axis theorem needs I_{cm}, not any random axis. The perpendicular axis theorem fails for a sphere or a cylinder because they are not laminar.

Torque and angular momentum

$\vec{\tau} = \vec{r} \times \vec{F} = I\vec{\alpha}$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}, \quad \vec{\tau} = \dfrac{d\vec{L}}{dt}$

$K_{\text{rot}} = \frac{1}{2}I\omega^2, \quad L = \sqrt{2IK}$

When external torque is zero, $I\omega$ stays constant. A skater pulling arms in **lowers** I **and raises** ω.

Rolling without slipping

$v_{cm} = \omega R$

$K_{\text{total}} = \frac{1}{2}Mv_{cm}^2\left(1 + \dfrac{k^2}{R^2}\right)$

Acceleration on an incline: $a = \dfrac{g\sin\theta}{1 + k^2/R^2}$

Minimum friction: $\mu_{\text{min}} = \dfrac{\tan\theta}{1 + R^2/k^2}$

k^2/R^2 is 1 for a ring, $1/2$ for a disc, $2/5$ for a solid sphere. Smaller ratio means **faster rolling**, so a solid sphere wins every race.

Friction in rolling does no work

In pure rolling the contact point is instantaneously at rest, so static friction does zero work even though it supplies the torque. Do not subtract a friction loss when a body rolls without slipping.

6 Gravitation

This unit covers Newton's law of gravitation, variation of g, gravitational potential and field, escape speed, orbital motion and Kepler's laws.

Newton's law and variation of g

$F = \dfrac{Gm_1m_2}{r^2}, \quad g = \dfrac{GM}{R^2}$

Height h: $g_h = g\left(1 + \dfrac{h}{R}\right)^{-2} \approx g\left(1 - \dfrac{2h}{R}\right)$

Depth d: $g_d = g\left(1 - \dfrac{d}{R}\right)$

Rotation: $g_\lambda = g - \omega^2 R\cos^2\lambda$

g falls off **twice as fast with height** as with depth for small distances, and it becomes zero at the centre of the Earth.

Potential energy and potential

$U = -\dfrac{GMm}{r}, \quad V = -\dfrac{GM}{r}$

Inside a uniform shell: $V = -\dfrac{GM}{R}$, field $= 0$

Both are **negative** because gravity is attractive and infinity is the zero reference. A shell shields its interior from its own field completely.

Orbital and escape speed

$v_{\text{orb}} = \sqrt{\dfrac{GM}{r}}$, near surface $= \sqrt{gR} \approx 7.9$ km/s

$v_{\text{esc}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{2gR} \approx 11.2$ km/s

$v_{\text{esc}} = \sqrt{2}\,v_{\text{orb}}$

Escape speed does **not depend on the mass** of the projectile or on the launch

direction, only on the planet.

Satellite energy and Kepler

$K = \frac{GMm}{2r}$, $U = -\frac{GMm}{r}$, $E = -\frac{GMm}{2r}$

Kepler: $T^2 = \frac{4\pi^2 r^3}{GM}$, areal velocity constant

Total energy is **negative and equals** $-K$. A satellite in a lower orbit has more kinetic energy but less total energy.

JEE Extension

Geostationary orbit: $T = 24$ h gives $r \approx 4.2 \times 10^4$ km from the centre, so height $\approx 3.6 \times 10^4$ km, and the orbit must be equatorial and eastward. Binding energy of a surface-parked satellite is GMm/R.

7 Elasticity, Fluids and Thermal Properties

This unit gathers the properties of solids and liquids: stress and strain, Hooke's law, pressure, buoyancy, Bernoulli, viscosity, surface tension, thermal expansion and heat transfer.

Elastic moduli

Young's modulus $Y = \frac{F/A}{\Delta L/L}$

Bulk modulus $B = -\frac{\Delta P}{\Delta V/V}$;

Rigidity $\eta = \frac{F/A}{\theta}$

Elastic energy per volume $= \frac{1}{2} \times \text{stress} \times \text{strain}$

Poisson ratio $\sigma = -\frac{\text{lateral strain}}{\text{longitudinal strain}}$

A **larger** Y **means a stiffer material**. Steel has a higher Y than rubber, so steel is more elastic in the physics sense.

Fluid statics and buoyancy

$P = P_0 + \rho g h$

Buoyant force $= V_{\text{displaced}}\, \rho_{\text{liquid}}\, g$

Floating: $\frac{V_{\text{in}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{liquid}}}$

Pressure depends only on **depth**, not on the shape or width of the container. That is the hydrostatic paradox.

Continuity and Bernoulli

$A_1 v_1 = A_2 v_2$

$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$

Torricelli: $v = \sqrt{2gh}$

Venturi: $v_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$

Bernoulli needs **steady, non-viscous, incompressible** flow along a single streamline. Faster flow means lower pressure, which lifts an aerofoil.

Viscosity and terminal velocity

$F = -\eta A \frac{dv}{dx}$

Stokes: $F = 6\pi\eta r v$

$v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$

Reynolds number $Re = \frac{\rho v D}{\eta}$

Terminal velocity scales as r^2, so a drop of double the radius falls **four times faster**. Flow is turbulent above $Re \approx 2000$.

Surface tension and capillarity

$T = \dfrac{F}{L}$, Surface energy $= T\Delta A$

Excess pressure: drop $\dfrac{2T}{R}$, bubble in air $\dfrac{4T}{R}$

Capillary rise $h = \dfrac{2T\cos\theta}{r\rho g}$

A soap bubble has **two surfaces**, hence $4T/R$. Rise is inversely proportional to the tube radius, so a narrow tube lifts the liquid higher.

Thermal expansion and calorimetry

$L = L_0(1 + \alpha\Delta T)$, $\beta = 2\alpha$, $\gamma = 3\alpha$

$Q = mc\Delta T$ (temperature change)

$Q = mL$ (phase change, no temperature change)

During melting or boiling the temperature **stays fixed** while latent heat rearranges the bonds. Heat lost equals heat gained in an isolated calorimeter.

Heat transfer

Conduction: $\dfrac{dQ}{dt} = \dfrac{kA(T_1 - T_2)}{L}$; resistance $= \dfrac{L}{kA}$

Stefan-Boltzmann: $P = e\sigma AT^4$

Net radiation: $P = e\sigma A(T^4 - T_0^4)$

Wien: $\lambda_m T = b = 2.898 \times 10^{-3}$ m K

Thermal resistances in series **add** exactly like electrical resistances. Radiated power scales as T^4, so doubling absolute temperature raises output sixteen times.

Kelvin only in radiation

Stefan's law and Wien's law take absolute temperature. Substituting Celsius into T^4 is one of the most frequent silent errors in Section B numericals.

8 Thermodynamics

This unit covers the zeroth and first laws, thermodynamic processes, work in PV diagrams, the second law, heat engines and refrigerators.

First law and process work

$\Delta Q = \Delta U + \Delta W$

Isobaric: $W = P\Delta V = nR\Delta T$

Isothermal: $W = nRT\ln\dfrac{V_2}{V_1}$, $\Delta U = 0$

Adiabatic: $W = \dfrac{nR(T_1 - T_2)}{\gamma - 1}$, $\Delta Q = 0$

ΔU depends only on the **end states**, so it is zero for any cycle. Q and W depend on the path taken.

Adiabatic and specific heats

$PV^\gamma =$ const, $TV^{\gamma-1} =$ const, $T^\gamma P^{1-\gamma} =$ const

$C_P - C_V = R$, $\gamma = C_P/C_V$

Monatomic $\gamma = 5/3$; diatomic $7/5$; polyatomic $4/3$

An adiabatic curve is **steeper** than an isotherm on a PV diagram because of the extra factor γ.

Efficiency and COP

Engine: $\eta = 1 - \frac{Q_2}{Q_1}$; Carnot: $\eta = 1 - \frac{T_2}{T_1}$

Refrigerator: $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

Heat pump: $\text{COP} = \frac{T_1}{T_1 - T_2} = \beta + 1$

Carnot efficiency is the **ceiling** for any engine between the same two temperatures. It is reached only by a reversible cycle.

Polytropic process

$PV^n = \text{constant}$, $W = \frac{nR\Delta T}{1-n}$

Molar heat capacity $C = C_V + \frac{R}{1-n}$

Set $n = 0$ for isobaric, $n = 1$ for isothermal, $n = \gamma$ for adiabatic and $n \to \infty$ for isochoric. **One formula covers all four** standard processes.

JEE Extension

For a mixture of n_1 moles with γ_1 and n_2 moles with γ_2: $C_V^{\text{mix}} = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2}$ and γ_{mix} follows from $C_P = C_V + R$. Mixture questions appear in Section B most years.

9 Kinetic Theory of Gases

This unit covers the ideal gas equation, the kinetic interpretation of pressure and temperature, molecular speeds, degrees of freedom and mean free path.

Ideal gas and kinetic pressure

$PV = nRT = Nk_BT$

$P = \frac{1}{3}\frac{mN}{V}v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$

Average translational KE per molecule $= \frac{3}{2}k_BT$

Temperature is a **direct measure** of average translational kinetic energy and does not depend on the gas identity.

Molecular speeds

$v_{rms} = \sqrt{\frac{3RT}{M}}$, $v_{avg} = \sqrt{\frac{8RT}{\pi M}}$

$v_{mp} = \sqrt{\frac{2RT}{M}}$

Ratio $v_{mp} : v_{avg} : v_{rms} = 1 : 1.128 : 1.224$

Use M in **kg per mole**. All three speeds rise as $\sqrt{T}$ and fall as $1/\sqrt{M}$, which is why hydrogen escapes the atmosphere.

Degrees of freedom and energy

$U = \frac{f}{2}nRT$, $C_V = \frac{f}{2}R$, $\gamma = 1 + \frac{2}{f}$

f: monatomic 3, diatomic 5, polyatomic 6

Each quadratic degree of freedom stores $\frac{1}{2}k_BT$ per molecule. Vibrational modes only switch on at **high temperature**.

Mean free path

$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n} = \frac{k_BT}{\sqrt{2}\pi d^2 P}$

Mean free path grows with temperature and falls with pressure. In a good vacuum it can exceed the size of the container, which is what makes a **cathode ray tube** work.

Speed order

Most probable is the smallest, root mean square is the largest: read the letters M, A, R in alphabetical order and the speeds go in increasing order too.

10 Oscillations

This unit covers simple harmonic motion, springs, pendulums, energy in SHM, damping and resonance.

SHM basics

$x = A\sin(\omega t + \phi)$, $v = \omega\sqrt{A^2 - x^2}$

$a = -\omega^2 x$, $T = \dfrac{2\pi}{\omega}$

The defining test for SHM is that **acceleration is proportional to displacement** and points back to the mean position.

Energy in SHM

$K = \frac{1}{2}m\omega^2(A^2 - x^2)$, $U = \frac{1}{2}m\omega^2 x^2$

$E = \frac{1}{2}m\omega^2 A^2$ (constant)

Both K and U oscillate at **twice the frequency** of the displacement. They are equal when $x = A/\sqrt{2}$.

Standard time periods

Spring: $T = 2\pi\sqrt{\dfrac{m}{k}}$; Series $\dfrac{1}{k_{eq}} = \sum \dfrac{1}{k_i}$; Parallel $k_{eq} = \sum k_i$

Simple pendulum: $T = 2\pi\sqrt{\dfrac{l}{g}}$

Physical pendulum: $T = 2\pi\sqrt{\dfrac{I}{mgd}}$

Torsional: $T = 2\pi\sqrt{\dfrac{I}{C}}$

The spring period does **not depend on** g, so it is unchanged in a lift or on the Moon. The pendulum period does.

Damped and forced oscillations

Damped: $x = A_0 e^{-bt/2m}\sin(\omega' t + \phi)$

$\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$

Amplitude resonance at $\omega_{\text{drive}} \approx \omega_0$

Energy decays as $e^{-bt/m}$, that is **twice as fast as the amplitude**, because energy goes as amplitude squared.

11 Waves and Sound

This unit covers travelling waves, superposition, standing waves on strings and in pipes, beats and the Doppler effect.

Wave equation and speed

$y = A\sin(\omega t - kx)$, $k = \dfrac{2\pi}{\lambda}$, $\omega = 2\pi f$

$v = f\lambda = \dfrac{\omega}{k}$

String: $v = \sqrt{\dfrac{T}{\mu}}$; Solid rod: $v = \sqrt{\dfrac{Y}{\rho}}$

Gas: $v = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M}}$

Sound speed in a gas depends on **temperature but not pressure**, since P/ρ is fixed at a given T.

Standing waves

String fixed at both ends: $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}, \quad n = 1, 2, 3 \ldots$

Open pipe: $f_n = \frac{nv}{2L}$ (all harmonics)

Closed pipe: $f_n = \frac{(2n-1)v}{4L}$ (odd harmonics only)

End correction: open end adds $0.6r$

A closed pipe of the same length sounds an **octave lower** than an open pipe and misses every even harmonic.

Beats and intensity

$f_{\text{beat}} = |f_1 - f_2|$

$I \propto A^2$; $\quad I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$

Loudness $L = 10 \log_{10} \frac{I}{I_0}$ dB, $\quad I_0 = 10^{-12}$ W/m^2

Loading a tuning fork with wax **lowers** its frequency. That trick tells you which fork was the higher one.

Doppler effect for sound

$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

Upper signs when the motion is towards the other party

Take the direction from **source to observer** as positive and apply the signs consistently. The medium's motion is added to v separately.

Observer and source are not symmetric

Moving the source by speed u does not give the same shift as moving the observer by u. The source term sits in the denominator, so the two cases differ once u is comparable to v.

12 Electrostatics

This unit covers Coulomb's law, electric field, flux and Gauss's law, potential, dipoles, conductors, capacitors and dielectrics.

Coulomb's law and field

$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}, \quad \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ SI

$\vec{E} = \frac{\vec{F}}{q_0}$, point charge $E = \frac{kq}{r^2}$

In a medium: $F_{\text{med}} = \frac{F_{\text{vac}}}{K}$

Forces from several charges **add as vectors**. Superposition, not a shortcut sum of magnitudes, is what the paper tests.

Gauss's law and standard fields

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$

Infinite line: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

Infinite sheet: $E = \frac{\sigma}{2\varepsilon_0}$

Conductor surface: $E = \frac{\sigma}{\varepsilon_0}$

Shell: $E = \frac{kQ}{r^2}$ outside, 0 inside

Flux counts only the **enclosed** charge. Charges outside the surface contribute zero net flux even though they change $\vec{E}$ everywhere.

Potential and potential energy

$V = \frac{kq}{r}$, $E = -\frac{dV}{dr}$

$U = \frac{kq_1q_2}{r}$; $W_{\text{ext}} = q(V_B - V_A)$

Shell: $V = \frac{kQ}{R}$ everywhere inside

Potential is a **scalar**, so add values with sign and never with vector rules. Inside a conductor V is constant and E is zero.

Electric dipole

$\vec{p} = q\,2\vec{a}$ (from $-q$ to $+q$)

Axial: $E = \frac{2kp}{r^3}$; Equatorial: $E = \frac{kp}{r^3}$

$V_{\text{axial}} = \frac{kp}{r^2}$, $V_{\text{equatorial}} = 0$

$\vec{\tau} = \vec{p} \times \vec{E}$, $U = -\vec{p} \cdot \vec{E}$

Dipole field falls as $1/r^3$, faster than a point charge, because the two charges nearly cancel. Axial field is **twice** the equatorial field at the same distance.

Capacitance and energy

$Q = CV$; Parallel plate $C = \frac{\varepsilon_0 A}{d}$

Series: $\frac{1}{C_{eq}} = \sum \frac{1}{C_i}$; Parallel: $C_{eq} = \sum C_i$

$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Energy density $u = \frac{1}{2}\varepsilon_0 E^2$

Capacitors in series carry the **same charge**, capacitors in parallel share the same voltage. That decides which formula to use.

Dielectrics and sphere

Full dielectric: $C' = KC$

Slab of thickness t: $C = \frac{\varepsilon_0 A}{d - t + t/K}$

Isolated sphere: $C = 4\pi\varepsilon_0 R$

Spherical capacitor: $C = \frac{4\pi\varepsilon_0 ab}{b - a}$

A dielectric **raises** capacitance by reducing the internal field. With the battery connected V stays fixed and Q rises; disconnected, Q stays fixed and V falls.

JEE Extension

Energy lost when two capacitors at different potentials are joined: $\Delta U = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$. It is always lost as heat and radiation in the connecting wires, regardless of how small the resistance is.

13 Current Electricity

This unit covers Ohm's law, resistivity, combinations of resistors and cells, Kirchhoff's rules, the Wheatstone bridge, the metre bridge and the potentiometer.

Ohm's law and resistivity

$V = IR$, $R = \frac{\rho L}{A}$

$I = neAv_d$, $v_d = \frac{eE\tau}{m}$

$\rho = \frac{m}{ne^2\tau}$, $\rho_T = \rho_0(1 + \alpha\Delta T)$

Drift speed is tiny, of the order of mm/s, yet the current starts instantly because the **field** spreads at nearly light speed.

Cells and internal resistance

$V = \varepsilon - Ir$ (discharging), $V = \varepsilon + Ir$ (charging)

Series: $\varepsilon_{eq} = \sum \varepsilon_i$, $r_{eq} = \sum r_i$

Parallel (identical): $\varepsilon_{eq} = \varepsilon$, $r_{eq} = r/n$

Max power transfer at $R = r$, with $P_{\text{max}} = \dfrac{\varepsilon^2}{4r}$

Terminal voltage equals the emf only when **no current flows**. That is why a voltmeter must draw almost nothing.

Kirchhoff's rules

Junction rule: $\sum I_{\text{in}} = \sum I_{\text{out}}$ (charge conservation)

Loop rule: $\sum \Delta V = 0$ around a closed loop (energy conservation)

Assign a current direction first and let the algebra correct you. A **negative answer** simply means the real current flows the other way.

Wheatstone and metre bridge

Balance: $\dfrac{P}{Q} = \dfrac{R}{S}$, galvanometer reads zero

Metre bridge: $\dfrac{R}{S} = \dfrac{l}{100 - l}$

Potentiometer: $\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{l_1}{l_2}$

Internal resistance: $r = R\left(\dfrac{l_1 - l_2}{l_2}\right)$

At balance the galvanometer branch carries no current, so its resistance is **irrelevant**. A potentiometer measures true emf because it draws zero current at balance.

Power and metre conversion

$P = VI = I^2R = \dfrac{V^2}{R}$

Ammeter: shunt $S = \dfrac{I_g G}{I - I_g}$ (parallel, small)

Voltmeter: $R = \dfrac{V}{I_g} - G$ (series, large)

In series the **largest resistor** dissipates most power; in parallel the smallest does. Check which case the circuit is in before ranking bulbs.

Series and parallel bulbs

A "60 W, 220 V" rating fixes the resistance, not the power. Put that bulb in a different circuit and the actual power changes. Always convert the rating to $R = V^2/P$ first.

14 Magnetic Effects of Current

This unit covers the Biot-Savart law, Ampere's circuital law, force on charges and currents, torque on a loop, the moving coil galvanometer and magnetic materials.

Biot-Savart and standard fields

$dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl\sin\theta}{r^2}$

Straight infinite wire: $B = \dfrac{\mu_0 I}{2\pi r}$

Circular loop centre: $B = \dfrac{\mu_0 I}{2R}$

On axis: $B = \dfrac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$

Arc of angle θ: $B = \dfrac{\mu_0 I\theta}{4\pi R}$

Field direction follows the **right hand thumb rule**. For a finite wire the field is

$\frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$.

Ampere's law: solenoid and toroid

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}}$

Long solenoid: $B = \mu_0 n I$ inside, ≈ 0 outside

Solenoid end: $B = \frac{\mu_0 n I}{2}$

Toroid: $B = \frac{\mu_0 N I}{2\pi r}$

n is turns **per metre**, not the total number of turns. A toroid confines its field entirely inside the core.

Lorentz force and circular paths

$\vec{F} = q(\vec{v}\times\vec{B})$, $F = qvB\sin\theta$

$r = \frac{mv}{qB} = \frac{p}{qB}$, $T = \frac{2\pi m}{qB}$

Pitch of helix $= v_{\parallel}T$

Cyclotron: $f = \frac{qB}{2\pi m}$, $K_{\max} = \frac{q^2B^2R^2}{2m}$

The magnetic force does **zero work** because it stays perpendicular to velocity. Speed never changes, only direction.

Force and torque on currents

Wire in a field: $\vec{F} = I(\vec{L}\times\vec{B})$

Parallel wires: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

Loop: $\vec{\tau} = \vec{M}\times\vec{B}$, $M = NIA$

$U = -\vec{M}\cdot\vec{B}$

Currents in the **same direction attract**, opposite directions repel. That is the opposite of the rule for charges.

Magnetic materials

$\vec{B} = \mu_0(\vec{H}+\vec{M})$, $\chi = M/H$, $\mu_r = 1+\chi$

Curie law: $\chi \propto \frac{1}{T}$

Diamagnetic χ is small and negative, paramagnetic small and positive, ferromagnetic large and positive. Above the **Curie temperature** a ferromagnet turns paramagnetic.

Fleming's rules

Left hand for motors, right hand for generators. In both, the first finger is the field, the second finger is the current and the thumb is the motion or force.

15 EMI and Alternating Current

This unit covers Faraday's law, Lenz's law, motional emf, self and mutual inductance, LR and LC circuits, series LCR resonance, power factor and transformers.

Faraday and Lenz

$\varepsilon = -N\frac{d\Phi}{dt}$, $\Phi = BA\cos\theta$

Motional emf: $\varepsilon = Blv$; Rotating rod: $\varepsilon = \frac{1}{2}B\omega l^2$

Rotating coil: $\varepsilon = NBA\omega\sin\omega t$

Charge flown $q = \frac{N\Delta\Phi}{R}$

The minus sign is Lenz's law: the induced current **opposes the change** that produced it, which is energy conservation in disguise.

Inductance

$\varepsilon = -L\dfrac{dI}{dt}$, $L_{\text{solenoid}} = \mu_0 n^2 A l = \dfrac{\mu_0 N^2 A}{l}$

$U = \frac{1}{2}LI^2$, energy density $u = \dfrac{B^2}{2\mu_0}$

Mutual: $M = k\sqrt{L_1 L_2}$

LR growth: $I = I_0(1-e^{-t/\tau})$, $\tau = L/R$

An inductor resists a **change** in current, not the current itself. Right after a switch closes the inductor behaves like an open circuit.

AC quantities and reactance

$I_{rms} = \dfrac{I_0}{\sqrt{2}}$, $V_{rms} = \dfrac{V_0}{\sqrt{2}}$

$X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan\phi = \dfrac{X_L - X_C}{R}$

A pure inductor or capacitor consumes **no average power** because voltage and current are $90°$ apart.

Resonance and power

$\omega_0 = \dfrac{1}{\sqrt{LC}}$, $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

At resonance $Z = R$, current is maximum, $\phi = 0$

$P_{avg} = V_{rms} I_{rms} \cos\phi$

Quality factor $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{\omega_0}{\Delta\omega}$

$\cos\phi$ is the **power factor**. A sharper resonance means a higher Q and a narrower selectivity band, which is how a radio tunes.

Transformer

$\dfrac{V_s}{V_p} = \dfrac{N_s}{N_p} = \dfrac{I_p}{I_s}$

Efficiency $\eta = \dfrac{P_{\text{out}}}{P_{\text{in}}}$

A transformer steps voltage up and current **down** in the same ratio. It only works on AC, since a steady current gives no flux change.

Voltages in LCR do not add arithmetically

$V_R + V_L + V_C$ can exceed the source voltage. Only the phasor sum $\sqrt{V_R^2 + (V_L - V_C)^2}$ equals the applied voltage.

16 Electromagnetic Waves

This unit covers displacement current, the properties of electromagnetic waves and the electromagnetic spectrum. It usually contributes one straightforward question.

Displacement current, speed

$I_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$

$c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s

In a medium: $v = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{c}{n}$

Maxwell added displacement current so that Ampere's law works in the **gap of a charging capacitor**, where no charge actually crosses.

Amplitudes and momentum

$\dfrac{E_0}{B_0} = c$

Intensity $I = \frac{1}{2}\varepsilon_0 E_0^2 c = \dfrac{E_{rms}^2}{\mu_0 c}$

Momentum: $p = \frac{U}{c}$ (absorbed), $\frac{2U}{c}$ (reflected)

Radiation pressure: $\frac{I}{c}$ or $\frac{2I}{c}$

$\vec{E}$, $\vec{B}$ and the direction of travel form a **right handed set**, and the energy is shared equally between the two fields.

Spectrum in order of frequency

Radio $<$ Microwave $<$ Infrared $<$ Visible $<$ Ultraviolet $<$ X-ray $<$ Gamma

Visible band: 400 nm to 700 nm

Wavelength runs the **opposite way** to frequency. Gamma rays come from the nucleus, X-rays from inner electron shells.

Why electromagnetic waves are transverse

The oscillating electric and magnetic fields are both perpendicular to the direction of propagation, which is why light can be polarised. Sound cannot be polarised because it is longitudinal.

17 Ray and Wave Optics

This unit covers reflection, refraction, total internal reflection, lenses and mirrors, prisms, optical instruments, interference, diffraction and polarisation.

Mirrors and lenses

Mirror: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, $f = \frac{R}{2}$

Lens: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Magnification: mirror $m = -\frac{v}{u}$; lens $m = \frac{v}{u}$

Power $P = \frac{1}{f\,(\text{m})}$ dioptre

Use the **Cartesian sign convention** throughout: distances measured against the incident light are negative. Mixing conventions is the top source of optics errors.

Refraction and TIR

$n_1 \sin i = n_2 \sin r$, $n = \frac{c}{v}$

Critical angle: $\sin C = \frac{n_2}{n_1}$

Apparent depth $= \frac{\text{real depth}}{n}$

Single surface: $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

Total internal reflection needs light going from **denser to rarer** at more than the critical angle. It is what keeps light inside an optical fibre.

Lens maker and prism

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Contact: $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$; separated by d: $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

Prism: $A + \delta = i + e$; $n = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$

Thin prism: $\delta = (n-1)A$

At minimum deviation the ray passes **symmetrically** through the prism, so $i = e$ and the internal ray is parallel to the base.

Optical instruments

Compound microscope: $M = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right)$

Telescope (normal adjustment): $M = \frac{f_o}{f_e}$, length $= f_o + f_e$

Simple microscope: $M = 1 + \frac{D}{f}$, with $D = 25$ cm

A telescope wants a **long objective focal length**, a microscope wants a short one. That single fact separates most instrument questions.

Young's double slit

Path difference $\Delta = \frac{yd}{D}$

Fringe width $\beta = \frac{\lambda D}{d}$

Maxima: $\Delta = n\lambda$; Minima: $\Delta = (2n-1)\frac{\lambda}{2}$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$, $\frac{I_{max}}{I_{min}} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2$

Immersing the whole set-up in a liquid of index n **shrinks** every fringe by the factor n, because the wavelength shortens.

Diffraction and polarisation

Single slit minima: $a\sin\theta = n\lambda$

Central maximum width $= \frac{2\lambda D}{a}$

Resolving power of telescope $= \frac{D}{1.22\lambda}$

Malus: $I = I_0\cos^2\theta$; Brewster: $\tan i_B = n$

The central diffraction maximum is **twice as wide** as the others. At the Brewster angle the reflected ray is fully polarised and perpendicular to the refracted ray.

Interference minima are not zero when intensities differ

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$ is zero only for equal-intensity slits. Covering one slit partially raises the minimum and lowers the fringe contrast.

18 Dual Nature of Matter and Radiation

This unit covers the photoelectric effect, Einstein's equation, photon properties and de Broglie waves. It is short and very high yield.

Photon energy and momentum

$E = h\nu = \frac{hc}{\lambda}$, $p = \frac{h}{\lambda} = \frac{E}{c}$

$E(\text{eV}) = \frac{12400}{\lambda(\text{Å})}$

Photons have zero rest mass but carry momentum. The **12400 shortcut** saves half a minute in every photoelectric numerical.

Photoelectric effect

$h\nu = \phi_0 + K_{max}$, $\phi_0 = h\nu_0$

$K_{max} = eV_s$ (stopping potential)

$eV_s = \frac{hc}{\lambda} - \phi_0$

Stopping potential depends on **frequency alone**, not on intensity. Intensity fixes the number of photoelectrons, not their maximum energy.

de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Accelerated charge: $\lambda = \dfrac{h}{\sqrt{2mqV}}$

Electron: $\lambda(\text{Å}) = \dfrac{12.27}{\sqrt{V\ (\text{volt})}}$

Thermal: $\lambda = \dfrac{h}{\sqrt{3mk_BT}}$

Heavier or faster particles have **shorter** wavelengths, which is why matter waves are invisible for everyday objects.

JEE Extension

For a particle of kinetic energy K, $\lambda = h/\sqrt{2mK}$. Comparing an electron and a proton at the same K gives $\lambda_e/\lambda_p = \sqrt{m_p/m_e} \approx 42.8$. This ratio is a favourite Section B answer.

19 Atoms and Nuclei

This unit covers the Rutherford and Bohr models, hydrogen spectra, X-rays, nuclear size and binding energy, radioactivity, fission and fusion.

Bohr model

$$mvr = \frac{nh}{2\pi}, \quad r_n = 0.529\frac{n^2}{Z}\ \text{Å}$$

$$E_n = -13.6\frac{Z^2}{n^2}\ \text{eV}, \quad v_n = \frac{2.18\times10^6 Z}{n}\ \text{m/s}$$

$$K = -E, \quad U = 2E$$

Energy is **negative** because the electron is bound. Radius grows as n^2 while speed falls as $1/n$.

Spectral series

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \quad R = 1.097\times10^7\ \text{m}^{-1}$$

Lyman $n_1 = 1$ (UV), Balmer $n_1 = 2$ (visible), Paschen $n_1 = 3$ (IR)

Number of lines from level n: $\dfrac{n(n-1)}{2}$

Only the **Balmer series** lies in the visible range, which is why hydrogen looks red-pink in a discharge tube.

X-rays and Moseley's law

$$\lambda_{\text{min}} = \frac{hc}{eV} = \frac{12400}{V}\ \text{Å}$$

Moseley: $\sqrt{\nu} = a(Z-b)$, with $b \approx 1$ for K_α

The cut-off wavelength depends on the **accelerating voltage only**, while the characteristic lines depend on the target element.

Nuclear size and binding energy

$$R = R_0A^{1/3}, \quad R_0 = 1.2\ \text{fm}$$

$$\Delta m = Zm_p + (A-Z)m_n - M$$

$$BE = \Delta m \times 931.5\ \text{MeV per amu}$$

Nuclear density is **the same for every nucleus**, about 2.3×10^{17} kg/m^3. Binding energy per nucleon peaks near iron at 8.8 MeV.

Radioactive decay

$$N = N_0e^{-\lambda t}, \quad A = \lambda N$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}, \quad \tau = \frac{1}{\lambda} = 1.44\,T_{1/2}$$

After n half lives: $N = \dfrac{N_0}{2^n}$

Decay is a **statistical** process, so half life is independent of temperature, pressure and chemical state.

Decay shifts

Alpha decay drops A by 4 and Z by 2, so the nucleus moves two places left. Beta minus raises Z by 1 with A unchanged, moving one place right. Gamma emission changes neither.

20 Semiconductor Electronic Devices

This unit covers energy bands, intrinsic and extrinsic semiconductors, the p-n junction, diodes, rectifiers, Zener regulators and logic gates.

Semiconductor basics

$n_e n_h = n_i^2$

Conductivity $\sigma = e(n_e \mu_e + n_h \mu_h)$

Band gap: Si 1.1 eV, Ge 0.7 eV; insulator > 3 eV

In an n-type crystal electrons are majority carriers, in p-type holes are. The sample stays **electrically neutral** either way.

Diode and rectifier

Knee voltage: Si 0.7 V, Ge 0.3 V

Dynamic resistance $r_d = \dfrac{\Delta V}{\Delta I}$

Half wave ripple frequency $= f$; full wave $= 2f$

Full wave efficiency $\approx 81.2\%$; half wave $\approx 40.6\%$

Forward bias **shrinks** the depletion layer and the barrier potential, reverse bias widens both.

Zener regulator and photodiode

Zener works in **reverse breakdown** at a fixed V_Z

Series resistor: $R_S = \dfrac{V_{\text{in}} - V_Z}{I_Z + I_L}$

LED and photodiode threshold: $E_g = \dfrac{hc}{\lambda}$

A photodiode is operated in **reverse bias** because the fractional change in reverse current is far easier to measure.

Logic gates

AND $Y = A \cdot B$; OR $Y = A + B$; NOT $Y = \bar{A}$

NAND $Y = \overline{A \cdot B}$; NOR $Y = \overline{A + B}$

XOR $Y = A\bar{B} + \bar{A}B$

NAND and NOR are **universal gates**. Any circuit can be built from either one alone, which is why fabs prefer them.

Reading a truth table fast

AND outputs 1 only when every input is 1. OR outputs 0 only when every input is 0. XOR outputs 1 when the inputs differ. Checking these three patterns settles most gate questions in seconds.

Chemistry

Physical, Inorganic and Organic, 25 questions, 100 marks

Chemistry is the fastest-scoring paper if your recall is clean. Physical Chemistry supplies the calculation questions, Inorganic supplies the direct-recall questions, and Organic supplies the reasoning questions. Eight units were dropped from the syllabus in 2024 and are not covered here.

21 Some Basic Concepts of Chemistry

This unit covers the mole concept, stoichiometry, concentration terms and empirical formulas. Expect one calculation question.

Mole concept

$$n = \frac{\text{given mass}}{\text{molar mass}} = \frac{\text{number of particles}}{6.022 \times 10^{23}} = \frac{V \text{ at STP}}{22.4 \text{ L}}$$

One mole always contains 6.022×10^{23} entities. The **22.4 L figure applies only to gases at STP**, which NCERT now takes as 273.15 K and 1 bar (22.7 L); use 22.4 L only when the question says 1 atm.

Concentration terms

Molarity $M = \dfrac{n_{\text{solute}}}{V_{\text{solution}}\ (\text{L})}$ Molality $m = \dfrac{n_{\text{solute}}}{w_{\text{solvent}}\ (\text{kg})}$

Mole fraction $x_A = \dfrac{n_A}{n_A + n_B}$ ppm $= \dfrac{\text{mass of solute}}{\text{mass of solution}} \times 10^6$

Normality $N = M \times n$-factor; Dilution: $M_1V_1 = M_2V_2$

Molarity changes with temperature because volume expands. **Molality and mole fraction do not**, so colligative-property questions always use molality.

Empirical and molecular formula

Percent to moles: divide each percentage by its atomic mass, then divide all results by the smallest value.

Molecular formula $= n\times$ empirical formula, where $n = \dfrac{\text{molar mass}}{\text{empirical formula mass}}$

The empirical formula gives the **simplest whole-number ratio** of atoms. Benzene is CH empirically but C_6H_6 molecularly.

Limiting reagent and yield

Divide the moles of each reactant by its stoichiometric coefficient. The smallest quotient is the limiting reagent.

$$\text{Percentage yield} = \frac{\text{actual yield}}{\text{theoretical yield}} \times 100$$

All product calculations must run off the **limiting reagent**, never off the reactant present in the larger mass.

n-factor decides normality

For acids the n-factor is the number of replaceable H atoms, for bases the number of OH groups, and for redox species the change in oxidation number per formula unit. H_2SO_4 has an n-factor of 2, so 1 M is 2 N.

22 Atomic Structure

This unit covers the Bohr model for hydrogen-like species, the quantum numbers, orbital shapes, the Aufbau order and electronic configurations.

Bohr model for hydrogen-like species

$r_n = 0.529\dfrac{n^2}{Z}$ Å $\qquad E_n = -13.6\dfrac{Z^2}{n^2}$ eV $= -\dfrac{1312\,Z^2}{n^2}$ kJ/mol

$v_n = 2.18 \times 10^6 \dfrac{Z}{n}$ m/s $\qquad \bar{\nu} = \dfrac{1}{\lambda} = R_H Z^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

The model works only for **one-electron species**: H, He^+, Li^{2+}, Be^{3+}. It fails the moment electron-electron repulsion enters.

Quantum numbers

n = shell (1, 2, 3 ...); l = subshell (0 to $n-1$); m_l = $-l$ to $+l$; $m_s = \pm\frac{1}{2}$

Orbitals in a shell $= n^2$; electrons in a shell $= 2n^2$

Radial nodes $= n - l - 1$; angular nodes $= l$; total nodes $= n - 1$

No two electrons in an atom share all four quantum numbers. That is the **Pauli exclusion principle** and it caps every orbital at two electrons.

de Broglie and Heisenberg

$\lambda = \dfrac{h}{mv} \qquad \Delta x \cdot \Delta p \geq \dfrac{h}{4\pi}$

The uncertainty principle is why an electron has **no defined orbit**. It only has a probability cloud, which is what an orbital means.

Filling order and stability

Aufbau follows increasing $(n + l)$; on a tie, the lower n fills first.

Order: $1s\ 2s\ 2p\ 3s\ 3p\ 4s\ 3d\ 4p\ 5s\ 4d\ 5p\ 6s\ 4f\ 5d\ 6p\ 7s\ 5f\ 6d$

Anomalies: Cr $= [\text{Ar}]3d^5 4s^1$, Cu $= [\text{Ar}]3d^{10} 4s^1$

Half-filled and fully filled subshells are extra stable because of **symmetry and exchange energy**. That drives the Cr and Cu anomalies.

JEE Extension

Photoelectric-style questions appear in Chemistry too: $h\nu = h\nu_0 + \frac{1}{2}mv^2$. Also learn the ionisation energy of hydrogen as 1312 kJ/mol, and remember that removing an electron from He^+ takes $4 \times 1312 = 5248$ kJ/mol because of the Z^2 factor.

23 Chemical Bonding and Molecular Structure

This unit covers ionic and covalent bonding, VSEPR geometry, valence bond theory and hybridisation, molecular orbital theory, and hydrogen bonding.

Hybridisation and geometry

Steric number $= \dfrac{V + M - C + A}{2}$, where V = valence electrons of the central atom, M = monovalent atoms, C = cationic charge, A = anionic charge

2 $\Rightarrow sp$ linear; 3 $\Rightarrow sp^2$ trigonal planar; 4 $\Rightarrow sp^3$ tetrahedral

5 $\Rightarrow sp^3d$ trigonal bipyramidal; 6 $\Rightarrow sp^3d^2$ octahedral

Steric number counts **bond pairs plus lone pairs**, not atoms. Lone pairs compress bond angles, which is why H_2O is $104.5°$ and not $109.5°$.

Molecular orbital theory

Bond order $= \dfrac{N_b - N_a}{2}$

Order up to N_2: $\sigma 1s\ \sigma^* 1s\ \sigma 2s\ \sigma^* 2s\ \pi 2p_x = \pi 2p_y\ \sigma 2p_z\ \pi^* 2p_x = \pi^* 2p_y\ \sigma^* 2p_z$

For O_2 and beyond, $\sigma 2p_z$ drops below the π pair.

Higher bond order means **shorter and stronger** bonds. Any species with unpaired electrons in molecular orbitals is paramagnetic, which is how O_2 gets its magnetism.

Dipole moment and polarity

$\mu = q \times d$, measured in debye (1 D $= 3.33 \times 10^{-30}$ C m)

Resultant of two dipoles: $\mu = \sqrt{\mu_1^2 + \mu_2^2 + 2\mu_1\mu_2 \cos\theta}$

Symmetric molecules like CO_2, CCl_4 and BF_3 have **zero net dipole** even though each individual bond is polar.

Fajans rules and hydrogen bonding

Covalent character rises with a small, highly charged cation and a large, highly charged anion, and with a cation of pseudo noble-gas configuration.

Hydrogen bond needs H attached to F, O or N.

Intramolecular H-bonding in o-nitrophenol **lowers** its boiling point relative to the para isomer, which forms intermolecular bonds instead.

Bond angle order

For the same central atom, more lone pairs means a smaller angle. Compare CH_4 ($109.5°$) > NH_3 ($107°$) > H_2O ($104.5°$): zero, one and two lone pairs respectively.

24 Chemical Thermodynamics

This unit covers the first law, enthalpy, Hess's law, bond enthalpy, entropy, Gibbs energy and the criteria for spontaneity.

First law and enthalpy

$\Delta U = q + w, \quad w = -P_{ext}\Delta V$

$\Delta H = \Delta U + \Delta n_g RT$

$q_p = \Delta H$ at constant pressure; $q_v = \Delta U$ at constant volume

Δn_g counts **gaseous** moles of products minus reactants. If no gas changes, $\Delta H = \Delta U$.

Work in reversible processes

Isothermal reversible: $w = -2.303\, nRT \log \dfrac{V_2}{V_1}$

Adiabatic reversible: $w = \dfrac{nR(T_2 - T_1)}{\gamma - 1} = nC_V\Delta T$

Free expansion into vacuum: $w = 0$, $q = 0$, $\Delta U = 0$

Reversible expansion delivers the **maximum possible work**. The sign convention here is the chemistry one: work done on the system is positive.

Hess's law and enthalpy of reaction

$\Delta H_{\text{rxn}} = \sum \Delta H_f^\circ(\text{products}) - \sum \Delta H_f^\circ(\text{reactants})$

From bond enthalpies: $\Delta H = \sum \text{BE(bonds broken)} - \sum \text{BE(bonds formed)}$

Enthalpy is a **state function**, so any path that reaches the same products gives the same ΔH. ΔH_f° of an element in its standard state is zero.

Entropy and Gibbs energy

$\Delta S = \dfrac{q_{\text{rev}}}{T}, \quad \Delta S_{\text{univ}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} > 0$ for a spontaneous change

$\Delta G = \Delta H - T\Delta S$

$\Delta G^\circ = -2.303\, RT \log K = -nFE^\circ_{\text{cell}}$

$\Delta G < 0$ means spontaneous, $\Delta G = 0$ means equilibrium. A reaction with $\Delta H > 0$ and $\Delta S > 0$ turns spontaneous only **above** $T = \Delta H/\Delta S$.

Spontaneous does not mean fast

Thermodynamics only says whether a reaction can go, not how quickly. Diamond turning into graphite has $\Delta G < 0$ but the activation barrier makes it unmeasurably slow.

25 Solutions

This unit covers solubility, Henry's law, Raoult's law, ideal and non-ideal solutions, colligative properties and the van't Hoff factor.

Raoult's law and Henry's law

Raoult: $p_A = x_A p_A^\circ, \quad p_{\text{total}} = x_A p_A^\circ + x_B p_B^\circ$

Vapour phase: $y_A = \dfrac{p_A}{p_{\text{total}}}$

Henry: $p = K_H x$

A **larger K_H means lower solubility**. Solubility of a gas falls as temperature rises, which is why warm soda goes flat quickly.

Colligative properties

Relative lowering of vapour pressure: $\dfrac{p^\circ - p}{p^\circ} = x_{\text{solute}}$

Boiling point elevation: $\Delta T_b = i\, K_b\, m$ Freezing point depression: $\Delta T_f = i\, K_f\, m$

Osmotic pressure: $\pi = iCRT = i\dfrac{n}{V}RT$

Colligative properties depend on the **number of solute particles**, never on their identity. Osmotic pressure is the most sensitive of the four for large molecules.

van't Hoff factor

$$i = \frac{\text{observed colligative property}}{\text{calculated for no association or dissociation}}$$

Dissociation: $i = 1 + (n-1)\alpha$ Association: $i = 1 - \left(1 - \frac{1}{n}\right)\alpha$

$i > 1$ means the solute breaks into ions, $i < 1$ means it associates. NaCl approaches 2, $K_4[Fe(CN)_6]$ approaches 5, benzoic acid in benzene approaches 0.5.

Non-ideal solutions

Positive deviation: $\Delta H_{mix} > 0$, $\Delta V_{mix} > 0$, minimum boiling azeotrope (ethanol and water)

Negative deviation: $\Delta H_{mix} < 0$, $\Delta V_{mix} < 0$, maximum boiling azeotrope (nitric acid and water)

Positive deviation means the new A-B interactions are **weaker** than the original A-A and B-B ones, so more molecules escape into the vapour.

26 Equilibrium

This unit covers chemical equilibrium, K_c and K_p, Le Chatelier's principle, ionic equilibrium, pH, buffers, hydrolysis and solubility product. It is worth two questions most sessions.

Equilibrium constant

$$K_p = K_c(RT)^{\Delta n_g}$$

Reaction quotient Q: if $Q < K$ the reaction moves forward, if $Q > K$ it moves backward

Reversing a reaction inverts K; multiplying the equation by n raises K to the power n; adding two equations multiplies their K values.

K depends only on **temperature**. Catalysts, pressure and concentration change how fast equilibrium arrives, never where it lies.

pH and ionisation

$\text{pH} = -\log[H^+]$, $\text{pH} + \text{pOH} = 14$ at $25°C$, $K_w = 10^{-14}$

Weak acid: $[H^+] = \sqrt{K_a C}$, $\text{pH} = \frac{1}{2}(pK_a - \log C)$

Ostwald dilution law: $\alpha = \sqrt{\dfrac{K_a}{C}}$

Degree of dissociation **rises on dilution** but the total number of ions per litre still falls. $K_a \times K_b = K_w$ for a conjugate pair.

Buffers

Acidic buffer: $\text{pH} = \text{p}K_a + \log \dfrac{[\text{salt}]}{[\text{acid}]}$

Basic buffer: $\text{pOH} = \text{p}K_b + \log \dfrac{[\text{salt}]}{[\text{base}]}$

Buffer capacity peaks when salt and acid concentrations are **equal**, where pH equals $\text{p}K_a$. Adding water changes the pH of a buffer very little.

Salt hydrolysis

Weak acid, strong base: $\text{pH} = 7 + \frac{1}{2}(\text{p}K_a + \log C)$

Strong acid, weak base: $\text{pH} = 7 - \frac{1}{2}(\text{p}K_b + \log C)$

Weak acid, weak base: $\text{pH} = 7 + \frac{1}{2}(\text{p}K_a - \text{p}K_b)$, independent of concentration

Sodium acetate gives a **basic** solution, ammonium chloride an acidic one. The weaker partner controls the final pH.

Solubility product

$\text{A}_x\text{B}_y \rightleftharpoons x\text{A}^{y+} + y\text{B}^{x-}$: $K_{sp} = x^x y^y S^{x+y}$

AB type: $S = \sqrt{K_{sp}}$; AB_2 type: $S = \sqrt[3]{K_{sp}/4}$

Precipitation occurs when the ionic product exceeds K_{sp}.

A **common ion suppresses** solubility. That is why AgCl dissolves much less in dilute NaCl than in pure water.

Do not compare K_{sp} across different salt types

A smaller K_{sp} means lower solubility only for salts of the same stoichiometry. Comparing an AB salt with an AB_2 salt requires converting both to molar solubility first.

27 Redox Reactions and Electrochemistry

This unit covers oxidation numbers, balancing redox equations, electrode potentials, the Nernst equation, conductance, Kohlrausch's law and electrolysis.

Cell potential and Gibbs energy

$E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$ (both as reduction potentials)

$\Delta G^\circ = -nFE^\circ_{\text{cell}}$, $\log K = \dfrac{nE^\circ_{\text{cell}}}{0.0591}$ at 298 K

A **positive** E°_{cell} means the cell reaction is spontaneous as written. Oxidation always happens at the anode, in both galvanic and electrolytic cells.

Nernst equation

$E_{\text{cell}} = E^\circ_{\text{cell}} - \dfrac{0.0591}{n} \log Q$ at 298 K

General form: $E = E^\circ - \dfrac{RT}{nF} \ln Q$

At equilibrium $E_{\text{cell}} = 0$ and $Q = K$, which is exactly when a **battery is dead**. Solids and pure liquids never enter Q.

Conductance and Kohlrausch

$\kappa = \dfrac{1}{\rho} = G \times \dfrac{l}{A}$, where l/A is the cell constant

$\Lambda_m = \dfrac{\kappa \times 1000}{C}$ in S cm^2 mol^{-1}

Kohlrausch: $\Lambda^\circ_m = \nu_+\lambda^\circ_+ + \nu_-\lambda^\circ_-$; $\alpha = \dfrac{\Lambda_m}{\Lambda^\circ_m}$

On dilution κ **falls** but Λ_m rises. Weak electrolytes never reach Λ°_m by extrapolation, so Kohlrausch's law is used instead.

Faraday's laws of electrolysis

$w = \dfrac{M}{nF} It = Z\,It$, $F = 96500$ C mol^{-1}

Equivalent weight $= \dfrac{\text{molar mass}}{n\text{-factor}}$

The same charge deposits **equivalent amounts** of different substances. One faraday deposits one gram equivalent, so it plates 108 g of Ag but only 32 g of Cu.

JEE Extension

Concentration cell: $E^\circ_{\text{cell}} = 0$, so $E_{\text{cell}} = \dfrac{0.0591}{n} \log \dfrac{C_{\text{cathode}}}{C_{\text{anode}}}$. Current flows from the dilute to the concentrated half cell, and the cell stops when the concentrations equalise. This is a regular Section B question.

28 Chemical Kinetics

This unit covers rate laws, order and molecularity, integrated rate equations, half life, the Arrhenius equation and collision theory.

Rate and order

For $aA + bB \rightarrow$ products: $\text{rate} = -\frac{1}{a}\frac{d[A]}{dt} = k[A]^x[B]^y$

Overall order $= x + y$; units of k are $\text{mol}^{1-n}\ \text{L}^{n-1}\ \text{s}^{-1}$ for order n

Order is found **experimentally** and can be zero or fractional. Molecularity comes from the mechanism and is always a small positive integer.

Integrated rate laws

Zero order: $[A] = [A]_0 - kt, \quad t_{1/2} = \frac{[A]_0}{2k}$

First order: $k = \frac{2.303}{t}\log\frac{[A]_0}{[A]}, \quad t_{1/2} = \frac{0.693}{k}$

Second order: $\frac{1}{[A]} = \frac{1}{[A]_0} + kt, \quad t_{1/2} = \frac{1}{k[A]_0}$

Only for a **first order** reaction is the half life independent of the starting concentration. That property identifies first order data instantly.

Arrhenius equation

$k = Ae^{-E_a/RT}, \quad \log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

A plot of $\log k$ against $1/T$ has slope $-\frac{E_a}{2.303R}$.

A catalyst lowers E_a for the forward and backward steps **equally**, so it speeds up both and leaves K untouched.

Temperature coefficient and collision theory

Temperature coefficient $= \frac{k_{T+10}}{k_T} \approx 2$ to 3 near room temperature

$k = PZe^{-E_a/RT}$, where Z is the collision frequency and P the steric factor

Collisions must be **energetic enough and correctly oriented**. The steric factor P is the correction for orientation that simple collision theory misses.

Half life formula must match the order

Students apply $t_{1/2} = 0.693/k$ to every problem. Check the order first: a zero order half life shrinks as the reaction proceeds, a second order half life grows.

29 Classification of Elements and Periodicity

This unit covers the modern periodic law, blocks, and the periodic trends in radius, ionisation enthalpy, electron gain enthalpy and electronegativity.

Effective nuclear charge

$Z_{\text{eff}} = Z - \sigma$, where σ is the screening constant

Slater screening: same group 0.35; $(n-1)$ shell 0.85; $(n-2)$ and deeper 1.00

Across a period Z rises but screening barely changes, so Z_{eff} climbs and the atom **shrinks**. Down a group a new shell wins and the atom grows.

Trends across a period, left to right

Atomic radius decreases; ionisation enthalpy increases; electron gain enthalpy becomes more negative; electronegativity increases; metallic character decreases.

Every one of these follows from the **rising** Z_{eff}. Down a group all five trends reverse.

Anomalies you must know

IE_1: Be $>$ B and N $>$ O, because of filled $2s$ and half-filled $2p$ stability.

Electron gain enthalpy: Cl $>$ F, and S $>$ O, because the small second-period atom has strong electron-electron repulsion.

Size: Ga $\approx$ Al because of poor $3d$ screening.

These four exceptions are asked **almost every session**. Learn the reasons, not just the order.

Ionic radius and isoelectronic series

For an isoelectronic series, radius falls as nuclear charge rises:

$\text{N}^{3-} > \text{O}^{2-} > \text{F}^{-} > \text{Ne} > \text{Na}^{+} > \text{Mg}^{2+} > \text{Al}^{3+}$

A cation is **always smaller** than its parent atom, an anion always larger. Same electrons, more protons means a tighter grip.

Diagonal relationship

Li resembles Mg, Be resembles Al, and B resembles Si because the diagonal pair has similar charge-to-size ratio. This explains why BeO and Al_2O_3 are both amphoteric.

30 p-Block Elements

This unit covers groups 13 to 18: general trends, oxides, halides, oxoacids, and the important compounds of boron, carbon, nitrogen, phosphorus, oxygen, sulphur, halogens and noble gases.

Inert pair effect and oxidation states

Stability of the lower oxidation state grows down groups 13, 14 and 15.

Group 13: $Tl^{+} > Tl^{3+}$; Group 14: $Pb^{2+} > Pb^{4+}$

The ns^2 pair becomes reluctant to bond because of **poor shielding by d and f electrons**. PbO_2 and Tl^{3+} are therefore strong oxidising agents.

Key acid strength orders

Oxoacids of chlorine: $HClO_4 > HClO_3 > HClO_2 > HClO$

Hydracids: $HI > HBr > HCl > HF$

Group 15 hydride basicity: $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

More oxygen atoms means a **more stabilised conjugate base**, hence a stronger acid. Down a group, weaker bonds make hydracids stronger despite falling electronegativity.

Named p-block preparations

B_2H_6: three-centre two-electron banana bonds, sp^3 boron

XeF_2 linear (sp^3d), XeF_4 square planar (sp^3d^2), XeF_6 distorted octahedral

H_2SO_4 by the contact process; HNO_3 by the Ostwald process; NH_3 by Haber

Xenon fluoride shapes come straight from **VSEPR with lone pairs**. XeF_4 has two lone pairs, which forces the square planar geometry.

Allotropes and structures worth recalling

O_3 bent, bond order 1.5, resonance stabilised

White phosphorus is P_4 tetrahedral with 60° angles and high strain, so it is the most reactive allotrope

H_2O_2 has an open-book structure with a dihedral angle near 111° in the gas phase

White phosphorus is stored **under water** because its strained P_4 ring ignites in air.

Interhalogen shapes

XY_3 is T-shaped (ClF_3), XY_5 is square pyramidal (BrF_5), XY_7 is pentagonal bipyramidal (IF_7). Odd subscript means one or more lone pairs distort the ideal shape.

31 d- and f-Block Elements

This unit covers transition metal trends, variable oxidation states, magnetic and catalytic behaviour, the lanthanoid contraction, and the chemistry of $KMnO_4$ and $K_2Cr_2O_7$.

Spin-only magnetic moment

$\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons

n: 1 $\Rightarrow$ **1.73**; 2 $\Rightarrow$ **2.83**; 3 $\Rightarrow$ **3.87**; 4 $\Rightarrow$ **4.90**; 5 $\Rightarrow$ **5.92** BM

Work out the d^n count of the **ion, not the atom**. Fe^{3+} is d^5 with five unpaired electrons in a weak field, giving 5.92 BM.

Transition metal trends

Melting points peak near the middle of each series where unpaired d electrons are maximum.

Highest oxidation state peaks at Mn ($+7$) in the $3d$ series.

$E^\circ(M^{2+}/M)$ is anomalous for Cu, which is positive, so Cu does not liberate H_2 from acids.

Variable oxidation states arise because $4s$ and $3d$ energies are **very close**, so both sets of electrons are available for bonding.

Lanthanoid contraction

Steady size decrease across the $4f$ series caused by poor shielding by f electrons.

Consequences: Zr and Hf have nearly identical radii; $4d$ and $5d$ elements resemble each other closely; basicity of hydroxides falls from $La(OH)_3$ to $Lu(OH)_3$.

This is why **Zr and Hf are so hard to separate** and why second and third row transition metals behave alike.

Permanganate and dichromate

Acidic medium: $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$, n-factor 5

Neutral or faintly alkaline: $MnO_4^- + 2H_2O + 3e^- \rightarrow MnO_2 + 4OH^-$, n-factor 3

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$, n-factor 6

The colour of both ions comes from **charge transfer**, not d-d transitions, since Mn(VII) and Cr(VI) have no d electrons.

Zn, Cd and Hg are not typical transition metals

Their d subshell is full in both the atom and the common ion, so they show no variable oxidation state, no colour and no catalytic activity of the usual kind.

32 Coordination Compounds

This unit is the highest-yield block in Inorganic Chemistry, usually two or three questions. It covers Werner's theory, nomenclature, isomerism, valence bond theory, crystal field theory and the spectrochemical series.

Crystal field splitting

Octahedral: t_{2g} drops by $0.4\Delta_o$, e_g rises by $0.6\Delta_o$

CFSE $= (-0.4x + 0.6y)\Delta_o + nP$, where x is t_{2g} and y is e_g electron count

Tetrahedral: $\Delta_t = \frac{4}{9}\Delta_o$, and tetrahedral complexes are **always high spin**

Pairing happens only when $\Delta_o > P$. That single comparison decides high spin versus low spin for d^4 to d^7 ions.

Spectrochemical series

Weak field to strong field:

$$I^- < Br^- < Cl^- < F^- < OH^- < H_2O < NH_3 < en < NO_2^- < CN^- < CO$$

Strong field ligands force pairing, giving **low spin, low magnetic moment** complexes. CN^- and CO are the strongest and appear in most exam questions.

Isomerism count

Ma_4b_2 octahedral: 2 geometrical isomers (cis and trans)

Ma_3b_3 octahedral: 2 isomers (facial and meridional)

$M(AA)_2b_2$: cis form is optically active, trans is not

Square planar Ma_2b_2: 2 geometrical isomers; tetrahedral Ma_2b_2: none

Tetrahedral complexes show **no geometrical isomerism** because every position is adjacent to every other.

EAN and hybridisation

EAN $= Z - (\text{oxidation state}) + 2 \times (\text{coordination number})$

Inner orbital (low spin) octahedral: d^2sp^3; outer orbital (high spin): sp^3d^2

Square planar: dsp^2; tetrahedral: sp^3

A stable carbonyl usually obeys the **18-electron rule**, which is EAN equal to the next noble gas. $Ni(CO)_4$ is tetrahedral and diamagnetic.

JEE Extension

Chelate effect: a polydentate ligand forms a far more stable complex than the same number of monodentate ligands, because the entropy change is favourable. Stability also rises with higher cation charge and smaller cation size, and the Irving-Williams order is $Mn^{2+} < Fe^{2+} < Co^{2+} < Ni^{2+} < Cu^{2+} > Zn^{2+}$.

33 Purification and Characterisation of Organic Compounds

This unit covers crystallisation, distillation, chromatography, and the quantitative estimation of carbon, hydrogen, nitrogen, halogens and sulphur.

Percentage estimation formulas

Carbon: $\%C = \frac{12}{44} \times \frac{\text{mass of } CO_2}{\text{mass of compound}} \times 100$

Hydrogen: $\%H = \frac{2}{18} \times \frac{\text{mass of } H_2O}{\text{mass of compound}} \times 100$

Halogen (Carius): $\%X = \frac{\text{at. mass of X}}{\text{mol. mass of AgX}} \times \frac{\text{mass of AgX}}{\text{mass of compound}} \times 100$

Each factor is the **mass fraction** of the element in the weighed product. Sulphur is estimated as $BaSO_4$ using $32/233$.

Nitrogen estimation

Dumas: $\%N = \frac{28}{22400} \times \frac{V_{N_2}\ (\text{mL at STP})}{\text{mass of compound}} \times 100$

Kjeldahl: $\%N = \frac{1.4 \times N_{\text{acid}} \times V_{\text{acid}}}{\text{mass of compound}}$

Kjeldahl **fails** for nitro compounds, azo compounds and pyridine-type ring nitrogen, because that nitrogen is not converted to ammonia.

Separation technique to use

Steam distillation: for compounds that are volatile in steam and immiscible with water

Fractional distillation: for liquids with close boiling points

Distillation under reduced pressure: for compounds that decompose at the normal boiling point

Differential extraction: when the solute dissolves better in an immiscible solvent

In steam distillation the mixture boils **below** 100°C, because the two immiscible liquids contribute independent vapour pressures.

Lassaigne test results

Nitrogen: Prussian blue, $Fe_4[Fe(CN)_6]_3$

Sulphur: violet with sodium nitroprusside, or black PbS

Nitrogen and sulphur together: blood red $[Fe(SCN)]^{2+}$

Halogen: white AgCl, pale yellow AgBr, yellow AgI

Fusion with sodium converts covalent N, S and X into **ionic salts** that ordinary tests can detect.

34 Basic Principles of Organic Chemistry

This unit covers nomenclature, electronic effects, resonance, hyperconjugation, reaction intermediates, and acidity and basicity trends. Everything downstream in Organic depends on it.

Electronic effects

Inductive: permanent, works through sigma bonds, dies out after three bonds

$-I$ order: $-NO_2 > -CN > -COOH > -F > -Cl > -Br > -I > -OCH_3$

Resonance: works through pi systems, no distance limit

Hyperconjugation: stabilisation by adjacent C-H sigma bonds, no bond resonance

Resonance **overrides induction** whenever both act in opposite directions. That is why phenol is acidic despite the electron-donating ring.

Stability of intermediates

Carbocation: $3^\circ > 2^\circ > 1^\circ > CH_3^+$; benzyl and allyl beat all of them

Free radical: same order as carbocations

Carbanion: exactly reversed, $CH_3^- > 1^\circ > 2^\circ > 3^\circ$

Alkyl groups **donate** electron density, which helps an electron-poor cation and hurts an electron-rich carbanion.

Acidity and basicity

Acidity: carboxylic acid $>$ phenol $>$ water $>$ alcohol $>$ terminal alkyne $>$ ammonia

Substituted benzoic acids: electron-withdrawing groups raise acidity, donating groups lower it

Amine basicity in water: $2^\circ > 1^\circ > 3^\circ >$ ammonia (for methyl amines)

Aromatic amines are far weaker bases than aliphatic ones

Acidity tracks **conjugate base stability**. Aniline is weakly basic because its lone pair is delocalised into the ring.

Aromaticity and Huckel's rule

A ring is aromatic when it is cyclic, planar, fully conjugated and holds $(4n + 2)$ pi electrons.

$4n$ pi electrons in a planar conjugated ring means antiaromatic and unstable.

Cyclopentadienyl **anion** (6 pi) is aromatic while the cation (4 pi) is antiaromatic. Tropylium cation with 6 pi electrons is aromatic.

Isomerism map

Structural: chain, position, functional, metamerism, tautomerism

Stereo: geometrical (cis, trans and E, Z) and optical (enantiomers, diastereomers)

Number of optical isomers $= 2^n$ for n dissimilar chiral centres

A molecule with an internal plane of symmetry is a **meso compound**: it has chiral centres but is optically inactive overall.

Priority for E and Z

Rank the two atoms on each doubly bonded carbon by atomic number. If the higher priority groups are on the same side, it is Z (zusammen, together). If they are on opposite sides, it is E.

35 Hydrocarbons

This unit covers alkanes, alkenes, alkynes and aromatic hydrocarbons, together with their named preparations and reactions.

Named preparations

Wurtz: $2RX + 2Na \xrightarrow{\text{dry ether}} R\text{-}R + 2NaX$

Kolbe electrolysis: $2RCOONa + 2H_2O \xrightarrow{\text{electrolysis}} R\text{-}R + 2CO_2 + H_2 + 2NaOH$

Decarboxylation: $RCOONa + NaOH \xrightarrow{CaO,\ \Delta} RH + Na_2CO_3$

Sabatier-Senderens: alkene $+ H_2 \xrightarrow{Ni,\ 573K}$ alkane

Wurtz gives poor yields with two **different** halides, because three products form at once.

Addition rules for alkenes

Markovnikov: H adds to the carbon with more hydrogens, forming the more stable carbocation

Anti-Markovnikov (peroxide or Kharasch effect): applies only to HBr, and runs through free radicals

Hydroboration-oxidation: B_2H_6 then H_2O_2/OH^-, gives anti-Markovnikov alcohol with syn addition

Oxymercuration-demercuration: Markovnikov alcohol with no rearrangement

The peroxide effect **fails for HCl and HI**, since the required radical step is endothermic for HCl and the H-I bond is too weak.

Oxidation and ozonolysis

Baeyer reagent (cold dilute alkaline $KMnO_4$): alkene to syn diol, decolourises purple

Hot acidic $KMnO_4$: cleaves the double bond to acids or ketones

Reductive ozonolysis (O_3 then Zn/H_2O): gives aldehydes and ketones

Oxidative ozonolysis (O_3 then H_2O_2): gives acids and ketones

Ozonolysis is the standard way to **locate a double bond**, because the fragments reveal exactly where the cleavage happened.

Alkynes and acidity

$CaC_2 + 2H_2O \rightarrow C_2H_2 + Ca(OH)_2$

Terminal alkynes are acidic: they react with Na, ammoniacal $AgNO_3$ (white precipitate) and ammoniacal Cu_2Cl_2 (red precipitate)

Acidity order: $HC \equiv CH > CH_2 = CH_2 > CH_3CH_3$

Acidity rises with **s character**: sp (50 percent) holds the lone pair closest to the nucleus, so it releases H^+ most readily.

Aromatic substitution and directing effects

Electrophilic substitution: nitration (HNO_3/H_2SO_4), sulphonation, halogenation (Lewis acid), Friedel-Crafts alkylation and acylation

Ortho and para directing activators: $-OH$, $-OR$, $-NH_2$, $-R$; halogens are ortho/para directing but deactivating

Meta directing deactivators: $-NO_2$, $-CN$, $-COOH$, $-CHO$, $-SO_3H$

Friedel-Crafts **fails** on strongly deactivated rings such as nitrobenzene, and on aniline because the Lewis acid binds the lone pair.

Rearrangement in Friedel-Crafts alkylation

Alkylation with n-propyl chloride gives mostly isopropylbenzene, because the primary carbocation rearranges. Acylation does not rearrange, so acylation followed by reduction is the reliable route.

36 Organic Compounds Containing Halogens

This unit covers haloalkanes and haloarenes, their preparations, the S_N1 and S_N2 mechanisms, and elimination.

S_N1 versus S_N2

S_N2: one step, rate $= k[RX][Nu]$, inversion of configuration, favoured by 1° halides and polar aprotic solvents

S_N1: two steps via a carbocation, rate $= k[RX]$, racemisation, favoured by 3° halides and polar protic solvents

Reactivity: S_N2 is $CH_3 > 1^\circ > 2^\circ > 3^\circ$; S_N1 is the exact reverse

S_N2 inversion is the **Walden inversion**, like an umbrella flipping in the wind. It is the signature test for the mechanism.

Elimination and Saytzeff rule

E2 with alcoholic KOH gives the alkene; aqueous KOH gives the alcohol instead.

Saytzeff: the more substituted alkene is the major product.

Hofmann: with a bulky base or a quaternary ammonium salt, the less substituted alkene wins.

Alcoholic KOH **eliminates**, aqueous KOH **substitutes**. That single line answers a whole family of questions.

Reactivity of haloarenes

Aryl halides resist nucleophilic substitution because of partial double bond character from resonance, ring stability and repulsion from the pi cloud.

Substitution needs strong electron-withdrawing groups ortho or para to the halogen, or forcing conditions.

Order of C-X reactivity: R-I $>$ R-Br $>$ R-Cl $>$ R-F

The C-X bond in chlorobenzene is **shorter and stronger** than in chloroethane, which is exactly why it will not leave.

Named reagents to memorise

$SOCl_2$ converts an alcohol to a chloride with SO_2 and HCl escaping as gas, so the product is pure.

Finkelstein: R-Cl $+$ NaI $\xrightarrow{\text{dry acetone}}$ R-I $+$ NaCl

Swarts: R-Cl $+$ AgF $\rightarrow$ R-F $+$ AgCl

Finkelstein works because **NaCl is insoluble** in dry acetone and precipitates, dragging the equilibrium forward.

37 Organic Compounds Containing Oxygen

This unit covers alcohols, phenols, ethers, aldehydes, ketones and carboxylic acids, along with the named reactions that connect them. It carries the largest share of Organic questions.

Alcohols and phenols

Acidity: phenol $>$ water $>$ alcohol; $1^\circ > 2^\circ > 3^\circ$ among alcohols

Lucas test (conc. HCl and $ZnCl_2$): 3° turbid at once, 2° in about five minutes, 1° only on heating

Victor Meyer test distinguishes 1° (red), 2° (blue) and 3° (colourless)

Phenol is acidic because the **phenoxide ion is resonance stabilised**. Simple alkoxides get no such help.

Phenol-specific named reactions

Reimer-Tiemann: phenol $+$ $CHCl_3$/NaOH $\rightarrow$ salicylaldehyde (ortho)

Kolbe-Schmidt: sodium phenoxide $+$ CO_2, pressure, then H^+ $\rightarrow$ salicylic acid

Williamson synthesis: $RO^-Na^+ + R'X \rightarrow$ ether, works best with a **primary** halide

Both phenol reactions attack the **ortho** position, because the phenoxide ring is most nucleophilic there.

Aldehyde and ketone reactions

Nucleophilic addition reactivity: HCHO $>$ CH_3CHO $>$ ketone, on steric and electronic grounds

Aldol condensation: needs alpha hydrogen, gives a beta hydroxy carbonyl that dehydrates on heating

Cannizzaro: needs **no** alpha hydrogen, gives one alcohol and one acid salt

Clemmensen ($Zn\text{-}Hg/HCl$) and Wolff-Kishner (NH_2NH_2/KOH) both reduce C=O to CH_2

Use Clemmensen when the molecule survives **acid** and Wolff-Kishner when it survives **base**. That is the only difference that matters in the exam.

Distinguishing tests

Tollens (ammoniacal $AgNO_3$): silver mirror with aldehydes

Fehling: red Cu_2O with aliphatic aldehydes, negative for aromatic aldehydes

Iodoform ($I_2/NaOH$): positive for CH_3CO- and $CH_3CH(OH)-$ groups, yellow precipitate

Benzaldehyde gives Tollens but **not Fehling**. Acetophenone, ethanol and acetaldehyde all give iodoform; methanol does not.

Carboxylic acids

Acidity order: HCOOH $>$ CH_3COOH; $Cl_3CCOOH > Cl_2CHCOOH > ClCH_2COOH$

HVZ reaction: $RCH_2COOH \xrightarrow{Cl_2/P} RCHClCOOH$, alpha halogenation

Decarboxylation, esterification (Fischer, acid catalysed and reversible), and conversion to acid chloride with $SOCl_2$

Electron-withdrawing groups near the COOH **raise acidity**, and their effect weakens sharply with distance from the carboxyl group.

JEE Extension

Learn these three by name: Rosenmund reduction (acid chloride to aldehyde over Pd-$BaSO_4$), Stephen reduction (nitrile to aldehyde with $SnCl_2$/HCl), and Etard reaction (toluene to benzaldehyde with CrO_2Cl_2). Conversion-chain questions almost always contain one of them.

38 Organic Compounds Containing Nitrogen

This unit covers amines, diazonium salts and their coupling reactions, and the standard nitrogen-based conversions.

Basicity of amines

In water (methyl series): $(CH_3)_2NH > CH_3NH_2 > (CH_3)_3N > NH_3$

In the gas phase: $3^\circ > 2^\circ > 1^\circ > NH_3$, purely on inductive grounds

Aromatic amines: aniline $\ll$ ammonia; electron-withdrawing groups reduce basicity further

In water, basicity is a tug of war between **inductive donation and solvation**. The tertiary amine loses because it has no N-H left to hydrogen bond.

Preparation of amines

Gabriel phthalimide: gives pure 1° aliphatic amines, **cannot** make aromatic amines

Hofmann bromamide: $RCONH_2 + Br_2 + 4NaOH \rightarrow RNH_2$, the chain loses one carbon

Reduction of nitriles ($LiAlH_4$) gives an amine with one **more** carbon

Hofmann degradation **shortens** the chain by one carbon while nitrile reduction **lengthens** it. Conversion questions hinge on this pair.

Distinguishing amines

Hinsberg reagent (benzene sulphonyl chloride): 1° gives a base-soluble product, 2° gives an insoluble solid, 3° does not react

Carbylamine test: only 1° amines give the foul-smelling isocyanide with $CHCl_3$ and alcoholic KOH

The carbylamine test is a **one-way test**: a positive result proves a primary amine, but there is no carbylamine version for secondary amines.

Diazonium chemistry

Formation: $C_6H_5NH_2 + NaNO_2 + HCl$ at 273 to 278 K

Sandmeyer: CuCl, CuBr or CuCN replaces the diazo group; Gattermann uses Cu powder and HX

Coupling with phenol or aniline gives azo dyes, which are coloured

Replacement by H uses H_3PO_2; by OH uses warm water; by I needs only KI

Diazonium salts are the **universal gateway** in aromatic synthesis, since the group can be swapped for almost any substituent.

39 Biomolecules

This unit covers carbohydrates, proteins, enzymes, vitamins and nucleic acids. The questions are recall based and quick to score.

Carbohydrates

Aldoses have a CHO group, ketoses have a C=O; glucose is an aldohexose, fructose a ketohexose

Reducing sugars: glucose, fructose, maltose, lactose. Non-reducing: sucrose

Sucrose $=$ glucose $+$ fructose, alpha 1,2 linkage; maltose $=$ two glucose, alpha 1,4; lactose $=$ galactose $+$ glucose, beta 1,4

Sucrose is non-reducing because **both anomeric carbons are locked** in the glycosidic bond. Its hydrolysis is called inversion of cane sugar.

Proteins and enzymes

Amino acids are amphoteric and exist as **zwitterions** at the isoelectric point

Primary: sequence. Secondary: alpha helix and beta pleated sheet, held by hydrogen bonds. Tertiary and quaternary: overall folding and subunit assembly

Denaturation destroys secondary and tertiary structure but leaves the primary sequence intact

Enzymes are protein catalysts that are **highly specific** and work best in a narrow pH and temperature window.

Nucleic acids and vitamins

DNA: deoxyribose, bases A, T, G, C, double helix, pairs A-T (two H bonds) and G-C (three H bonds)

RNA: ribose, uracil replaces thymine, usually single stranded

Fat soluble vitamins: A, D, E, K. Water soluble: B group and C

G-C pairs hold **three hydrogen bonds**, so DNA rich in G and C needs a higher temperature to melt.

Deficiency diseases worth memorising

Vitamin A: night blindness. Vitamin B_1: beri beri. Vitamin B_{12}: pernicious anaemia

Vitamin C: scurvy. Vitamin D: rickets and osteomalacia. Vitamin K: poor blood clotting

Water soluble vitamins are **not stored** in the body, so they must be supplied regularly in the diet.

40 Principles Related to Practical Chemistry

This unit covers qualitative salt analysis, the chemistry of common laboratory tests and titration principles. It usually contributes one question and it is easy marks.

Cation group separation

Group I: dilute HCl gives Pb^{2+}, Ag^{+}, Hg_2^{2+}

Group II: H_2S in acidic medium gives Cu, Cd, Bi, As, Sb, Sn sulphides

Group III: NH_4OH with NH_4Cl gives Fe, Al, Cr hydroxides

Group IV: H_2S in alkaline medium gives Zn, Mn, Ni, Co

Group V: ammonium carbonate gives Ba, Sr, Ca

NH_4Cl is added before NH_4OH to **suppress** the ionisation of the base, so only group III hydroxides precipitate.

Confirmatory tests to remember

Brown ring test confirms nitrate, using $FeSO_4$ and concentrated H_2SO_4

Chromyl chloride test confirms chloride

Borax bead test gives characteristic colours for Cu (blue), Cr (green), Co (blue), Mn (violet)

Flame colours: Na yellow, K lilac, Ca brick red, Sr crimson, Ba apple green, Cu blue green

The brown ring is $[Fe(H_2O)_5NO]^{2+}$. It forms at the **junction of the two layers**, which is why the acid must be added slowly along the wall.

Titration calculations

$N_1V_1 = N_2V_2$ at the end point

Molarity form: $\frac{M_1V_1}{n_1} = \frac{M_2V_2}{n_2}$, where n is the stoichiometric coefficient

Indicators: phenolphthalein for a strong base titrant, methyl orange for a strong acid titrant

$KMnO_4$ is **self indicating** in acidic medium, so no external indicator is needed. Oxalic acid titrations are done warm, around $60°$C.

Never use HCl with permanganate

Hydrochloric acid is itself oxidised by $KMnO_4$, which consumes titrant and inflates the reading. Dilute sulphuric acid is the only correct medium.

Mathematics

14 units, 25 questions, 100 marks

Maths is the longest paper to attempt and the one that decides ranks. Calculus and Coordinate Geometry together supply nearly half the questions. Mathematical Induction and Mathematical Reasoning are no longer in the syllabus and are not covered here.

41 Sets, Relations and Functions

This unit covers set operations, types of relations, domain and range, composition, inverse functions and the standard function types.

Set identities and counting

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = \sum n(A) - \sum n(A \cap B) + n(A \cap B \cap C)$

De Morgan: $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$

Power set of an n-element set has 2^n subsets

The inclusion-exclusion pattern **alternates signs**. For three sets you add singles, subtract pairs, then add the triple.

Relations

On a set with n elements: 2^{n^2} relations, $2^{n(n+1)/2}$ symmetric, $2^{n(n-1)}$ reflexive

Equivalence relation $=$ reflexive $+$ symmetric $+$ transitive

An equivalence relation splits the set into **disjoint classes** that cover everything.

Congruence modulo n is the standard example.

Functions and counting

From a set of m to a set of n: n^m functions in all

One-one functions ($m \le n$): ${}^nP_m$

Onto functions ($m \ge n$): $\sum_{r=0}^{n} (-1)^r \, {}^nC_r \, (n-r)^m$

f is invertible if and only if it is bijective; $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

A function is one-one when **no horizontal line** cuts the graph twice, and onto when the range fills the whole codomain.

Standard domains and ranges

$\sqrt{f(x)}$ needs $f(x) \ge 0$; $\log f(x)$ needs $f(x) > 0$

$\sin^{-1} x$, $\cos^{-1} x$: domain $[-1, 1]$; ranges $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $[0, \pi]$

$\tan^{-1} x$: domain $\mathbb{R}$, range $(-\frac{\pi}{2}, \frac{\pi}{2})$

$\{x\} = x - \lfloor x \rfloor \in [0, 1)$

Take the **intersection** of every individual condition when a function mixes roots, logs and inverse trigonometric parts.

Even, odd and periodic

$f(-x) = f(x)$ even, $f(-x) = -f(x)$ odd

Period: $\sin x$, $\cos x$ give 2π; $\tan x$ gives π; $|\sin x|$ gives π; $\{x\}$ gives 1

$\sin(ax)$ has period $\dfrac{2\pi}{|a|}$

Every function splits as an even part plus an odd part: $f(x) = \dfrac{f(x)+f(-x)}{2} + \dfrac{f(x)-f(-x)}{2}$.

42 Complex Numbers and Quadratic Equations

This unit covers the algebra of complex numbers, modulus and argument, De Moivre's theorem, cube roots of unity, and the roots of quadratic equations.

Modulus, conjugate and argument

$|z|^2 = z\bar{z}$, $|z_1 z_2| = |z_1||z_2|$, $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

Triangle inequality: $||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$

$|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

Equality in the triangle inequality holds when z_1 and z_2 are **collinear with the origin** and point the same way.

De Moivre and roots of unity

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

nth roots of unity: $z_k = e^{2\pi ik/n}$, $k = 0, \dots, n-1$; their sum is 0 and product is $(-1)^{n+1}$

Cube roots: $1, \omega, \omega^2$ with $1 + \omega + \omega^2 = 0$, $\omega^3 = 1$

The nth roots of unity sit on the **unit circle at equal spacing**, forming a regular n-gon. That picture solves most sum questions instantly.

Quadratic roots

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $\alpha + \beta = -\dfrac{b}{a}$, $\alpha\beta = \dfrac{c}{a}$

$D > 0$ real and distinct; $D = 0$ equal; $D < 0$ complex conjugates

$|\alpha - \beta| = \dfrac{\sqrt{D}}{|a|}$

Complex or irrational roots of a **real-coefficient** quadratic always come in conjugate pairs. That is the shortcut for "one root is $2 + \sqrt{3}$" questions.

Location of roots

Both roots greater than k: $D \ge 0$, $af(k) > 0$, $-\dfrac{b}{2a} > k$

Roots on either side of k: $af(k) < 0$

Exactly one root in (k_1, k_2): $f(k_1)f(k_2) < 0$

Sketch the parabola with the correct opening before applying these. The condition $af(k)$ handles both upward and downward parabolas at once.

JEE Extension

Common root shortcut: if $a_1x^2+b_1x+c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ share exactly one root, then $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (a_1c_2 - a_2c_1)^2$. If both roots are common, the coefficients are simply proportional.

43 Matrices and Determinants

This unit covers matrix algebra, types of matrices, determinants and their properties, adjoint and inverse, and systems of linear equations.

Determinant properties

$|AB| = |A||B|$, $|A^T| = |A|$, $|kA| = k^n|A|$ for order n

$|A^{-1}| = \frac{1}{|A|}$, $|\text{adj } A| = |A|^{n-1}$

Swapping two rows flips the sign; two identical rows give zero

A determinant is zero exactly when the rows are **linearly dependent**. That is the same as saying the matrix is singular.

Inverse and adjoint

$A^{-1} = \frac{\text{adj } A}{|A|}$, valid when $|A| \neq 0$

$A(\text{adj } A) = (\text{adj } A)A = |A|\, I$

$(AB)^{-1} = B^{-1}A^{-1}$, $(A^T)^{-1} = (A^{-1})^T$

$\text{adj}(\text{adj } A) = |A|^{n-2}A$

The inverse exists only for a **square non-singular** matrix. Matrix multiplication is not commutative, so keep the order when inverting a product.

Systems of linear equations

Cramer: $x = \frac{D_1}{D}$, $y = \frac{D_2}{D}$, $z = \frac{D_3}{D}$

$D \neq 0$: unique solution (consistent)

$D = 0$ and some $D_i \neq 0$: no solution (inconsistent)

$D = 0$ and all $D_i = 0$: infinitely many solutions

A homogeneous system always has the zero solution. It has a **non-trivial** solution only when $D = 0$.

Special matrices

Symmetric $A^T = A$; skew-symmetric $A^T = -A$ with zero diagonal

Orthogonal $AA^T = I$; idempotent $A^2 = A$; involutory $A^2 = I$; nilpotent $A^k = 0$

Any square matrix $= \frac{A + A^T}{2} + \frac{A - A^T}{2}$

The determinant of an **odd order** skew-symmetric matrix is always zero. That fact alone answers several past questions.

44 Permutations and Combinations

This unit covers the counting principles, arrangements, selections, circular permutations and distribution problems.

Core counting formulas

${}^nP_r = \frac{n!}{(n-r)!}$, ${}^nC_r = \frac{n!}{r!(n-r)!}$

${}^nC_r = {}^nC_{n-r}$, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Arrangements with repetition: n^r

Word with repeated letters: $\frac{n!}{p!\,q!\,r!}$

Permutation cares about **order**, combination does not. Decide that first and half the problem is solved.

Circular arrangements

Circular arrangements of n objects: $(n-1)!$

Necklace or garland (reflections identical): $\frac{(n-1)!}{2}$

Gap method: arrange m objects, then place r others in the $m+1$ gaps as ${}^{m+1}P_r$

Use the **gap method** whenever the question says certain items must never sit together. It is far safer than subtracting cases.

Distribution and selection

Identical objects into distinct boxes, none empty: ${}^{n-1}C_{r-1}$

Allowing empty boxes: ${}^{n+r-1}C_{r-1}$

Number of divisors of $N = p^a q^b r^c$ is $(a+1)(b+1)(c+1)$

Total selections from n distinct items: $2^n - 1$ (at least one)

Identical versus distinct objects changes the formula completely. Read the wording **twice** before choosing.

Derangements and grouping

Derangements: $D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + \frac{(-1)^n}{n!}\right)$

$D_1 = 0$, $D_2 = 1$, $D_3 = 2$, $D_4 = 9$, $D_5 = 44$

Divide $2n$ items into two equal unordered groups: $\frac{(2n)!}{2!(n!)^2}$

Divide by the factorial of the number of **equal-sized groups** whenever the groups are unlabelled.

45 Binomial Theorem

This unit covers the expansion, general and middle terms, greatest coefficient and simple applications.

Expansion and general term

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r\, a^{n-r} b^r$

$T_{r+1} = {}^nC_r\, a^{n-r} b^r$ (the $(r+1)$th term)

Middle term: $\left(\frac{n}{2}+1\right)$th if n is even; two middle terms if n is odd

The expansion has $n+1$ terms. For a term independent of x, set the **net power of** x in T_{r+1} equal to zero and solve for r.

Coefficient sums

$\sum {}^nC_r = 2^n$

$\sum (-1)^r {}^nC_r = 0$

Sum of even-indexed $=$ sum of odd-indexed $= 2^{n-1}$

$\sum r\, {}^nC_r = n2^{n-1}$, $\sum {}^nC_r^2 = {}^{2n}C_n$

Get these by **substituting** $x = 1$ or $x = -1$, or by differentiating the expansion once and then substituting.

Greatest term and coefficient

For $(1+x)^n$, the greatest term occurs near $r = \frac{(n+1)|x|}{1+|x|}$

The greatest binomial coefficient is ${}^nC_{n/2}$ for even n, and ${}^nC_{(n\pm1)/2}$ for odd n

Binomial coefficients rise to the middle and fall symmetrically. The **middle coefficient is always the largest**.

Multinomial and general binomial

$(x_1 + x_2 + \cdots + x_k)^n$ has ${}^{n+k-1}C_{k-1}$ terms

$(1+x)^{-1} = 1 - x + x^2 - \cdots$ for $|x| < 1$

$(1-x)^{-n} = \sum {}^{n+r-1}C_r \, x^r$

The negative and fractional index expansions are **infinite** and need $|x| < 1$ to converge.

46 Sequences and Series

This unit covers arithmetic, geometric and harmonic progressions, the means, and the standard summation results.

AP and GP

AP: $a_n = a + (n-1)d$, $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$

GP: $a_n = ar^{n-1}$, $S_n = \frac{a(1-r^n)}{1-r}$, $S_\infty = \frac{a}{1-r}$ for $|r| < 1$

The infinite geometric sum exists **only** when $|r| < 1$. If $|r| \geq 1$ the series diverges and the formula is meaningless.

Means and inequalities

$AM = \frac{a+b}{2}$, $GM = \sqrt{ab}$, $HM = \frac{2ab}{a+b}$

$GM^2 = AM \times HM$, $AM \geq GM \geq HM$

Equality holds **only when all terms are equal**. The AM-GM inequality is the fastest route to many minimum-value questions.

Standard sums

$\sum n = \frac{n(n+1)}{2}$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$\sum n^3 = \left[\frac{n(n+1)}{2}\right]^2 = (\sum n)^2$

The sum of cubes is the **square of the sum** of the first n naturals. That identity shortens many series problems.

Special series techniques

Telescoping: $\sum \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{n+1}$

Arithmetico-geometric: multiply by r, subtract, and sum the resulting GP

$\sum \frac{1}{n(n+1)} = \frac{n}{n+1}$

Look for a **difference of consecutive terms**. If the general term splits that way, everything cancels except the ends.

47 Limits, Continuity and Differentiability

This unit covers limits and standard expansions, L'Hopital's rule, continuity, differentiability and the rules of differentiation.

Standard limits

$\lim_{x\to 0} \frac{\sin x}{x} = 1 = \lim_{x\to 0} \frac{\tan x}{x}$

$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$,

$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$

$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$, $\lim_{x\to 0}(1+x)^{1/x} = e$

$\lim_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}$

These are the seven limits that cover **almost every** indeterminate form in the paper. Learn them before touching L'Hopital.

Indeterminate forms

1^∞ form: $\lim f^g = e^{\lim g(f-1)}$

L'Hopital applies to $\dfrac{0}{0}$ and $\dfrac{\infty}{\infty}$ only

Series: $\sin x = x - \dfrac{x^3}{6} + \cdots$,

$e^x = 1 + x + \dfrac{x^2}{2!} + \cdots$

Convert $0\times\infty$ and $\infty-\infty$ into a **quotient** first. Applying L'Hopital to a form that is not indeterminate gives a wrong answer.

Continuity and differentiability

Continuous at a: $\lim_{x\to a^-} f = \lim_{x\to a^+} f = f(a)$

Differentiable at a: LHD $=$ RHD $= f'(a)$

Differentiable implies continuous, but **not the other way round**. $|x|$ at $x=0$ is the standard counterexample.

Derivative table

$\dfrac{d}{dx}(x^n) = nx^{n-1}$, $\dfrac{d}{dx}(e^x) = e^x$,
$\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$
$\dfrac{d}{dx}(\sin x) = \cos x$, $\dfrac{d}{dx}(\tan x) = \sec^2 x$
$\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$,
$\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$

Product: $(uv)' = u'v + uv'$; Quotient: $\left(\dfrac{u}{v}\right)' = \dfrac{u'v-uv'}{v^2}$

For $y = f(x)^{g(x)}$ take **logarithms first**, then differentiate implicitly. Direct power rules do not apply there.

48 Applications of Derivatives

This unit covers rate of change, tangents and normals, monotonicity, maxima and minima, and approximations.

Tangent and normal

Slope of tangent at (x_1, y_1) is $m = \left.\dfrac{dy}{dx}\right|_{(x_1,y_1)}$

Tangent: $y - y_1 = m(x - x_1)$;

Normal: $y - y_1 = -\dfrac{1}{m}(x - x_1)$

Angle between two curves: $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Curves are **orthogonal** when $m_1 m_2 = -1$. A horizontal tangent means $dy/dx = 0$, a vertical one means $dx/dy = 0$.

Monotonicity and extrema

$f'(x) > 0$ on an interval means increasing; $f'(x) < 0$ means decreasing

Critical points: $f'(x) = 0$ or f' undefined

Second derivative test: $f'' > 0$ local minimum, $f'' < 0$ local maximum

If $f'' = 0$ the test is **inconclusive**, so fall back on the sign change of f' across the point.

Extrema and approximation

On a closed interval, compare f at all critical points and at both endpoints.

$\Delta y \approx \frac{dy}{dx}\Delta x, \quad f(a+h) \approx f(a) + hf'(a)$

A continuous function on a **closed bounded** interval always attains a maximum and a minimum. Endpoints are easy to forget and cost marks.

Standard optimisation results

Largest rectangle in a circle of radius r is the square of side $r\sqrt{2}$

Cone of maximum volume in a sphere of radius R: height $= \frac{4R}{3}$

Cylinder of maximum volume in a sphere: $h = \frac{2R}{\sqrt{3}}$

Reduce every optimisation to **one variable** using the given constraint before differentiating.

49 Integral Calculus

This unit covers indefinite integrals, standard forms, substitution and by parts, definite integrals with their properties, and area under curves.

Standard integrals

$\int x^n dx = \frac{x^{n+1}}{n+1}\ (n \neq -1), \quad \int \frac{dx}{x} = \ln|x|$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln|x + \sqrt{x^2 \pm a^2}|$

Match the **denominator pattern** first. A quadratic denominator is completed to a square and then mapped onto one of these five forms.

Integration by parts

$\int u\,v\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$

Choose u by the ILATE order: Inverse, Log, Algebraic, Trigonometric, Exponential

$\int e^x[f(x)+f'(x)]dx = e^x f(x) + C$

The e^x identity saves a full round of by parts. Spot the pattern **function plus its own derivative** and write the answer directly.

Definite integral properties

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ if $f(2a-x) = f(x)$, else 0

$\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if f is even, 0 if odd

Periodic f with period T: $\int_0^{nT} f = n\int_0^T f$

The **king property** $f(x) \to f(a+b-x)$ is the single most useful trick in the paper. Add the original and the transformed integral and terms often cancel.

Area under curves

Area between $y = f(x)$ and the x axis: $\int_a^b |f(x)|dx$

Between two curves: $\int_a^b [f(x)-g(x)]dx$ with f on top

Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ encloses πab

Parabola $y^2 = 4ax$ and its latus rectum enclose $\frac{8a^2}{3}$

Sketch first and find the **intersection points**, which become the limits. Skipping the modulus turns a positive area into a partial cancellation.

JEE Extension

Wallis formula for $\int_0^{\pi/2} \sin^m x \cos^n x\,dx$ saves real time. For $\int_0^{\pi/2} \sin^n x\,dx$ the answer is $\frac{(n-1)(n-3)\cdots}{n(n-2)\cdots}$, multiplied by $\frac{\pi}{2}$ only when n is even. Also learn $\int_0^{\pi/2} \ln(\sin x)dx = -\frac{\pi}{2}\ln 2$.

50 Differential Equations

This unit covers order and degree, variable separable equations, homogeneous equations and the linear first order equation.

Order, degree and separable form

Order $=$ highest derivative present; degree $=$ its power after clearing radicals

Variable separable: $\frac{dy}{dx} = f(x)g(y) \Rightarrow \int \frac{dy}{g(y)} = \int f(x)dx$

Degree is defined only when the equation is a **polynomial** in the derivatives. With **sin**(dy/dx) present, the degree does not exist.

Homogeneous equations

If $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$, substitute $y = vx$, so $\frac{dy}{dx} = v + x\frac{dv}{dx}$

The equation then becomes separable in v and x.

A function is homogeneous of degree zero when replacing $x \to \lambda x$ and $y \to \lambda y$ **leaves it unchanged**.

Linear differential equation

$\frac{dy}{dx} + P(x)y = Q(x)$

Integrating factor: $\text{IF} = e^{\int P\,dx}$

Solution: $y \cdot \text{IF} = \int Q \cdot \text{IF}\,dx + C$

If the equation is linear in x instead, swap the roles and use $\frac{dx}{dy} + Px = Q$ with the **same** method.

Forming a differential equation

Differentiate the given family as many times as there are arbitrary constants, then eliminate the constants.

The order of the resulting equation equals the number of constants.

A family with **two** arbitrary constants always produces a second order differential equation, never a first order one.

51 Coordinate Geometry

This unit covers the straight line, the circle, and the conic sections. It is the largest single block in the Maths paper.

Straight line

Slope form $y = mx + c$; point-slope $y - y_1 = m(x - x_1)$

Distance from a point: $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Angle: $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Between parallel lines: $d = \dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

Perpendicular lines satisfy $m_1 m_2 = -1$.

The **angle bisectors** of two lines are found by equating the two distance expressions.

Circle

$(x - h)^2 + (y - k)^2 = r^2$; general $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$

Tangent length from (x_1, y_1): $\sqrt{S_1}$

Tangent at a point on the circle: $T = 0$; chord of contact: $T = 0$

A line is a tangent when the perpendicular distance from the centre **equals the radius**. That single test replaces solving a quadratic.

Parabola $y^2 = 4ax$

Vertex $(0, 0)$, focus $(a, 0)$, directrix $x = -a$, latus rectum $4a$

Parametric point $(at^2, 2at)$

Tangent at (x_1, y_1): $yy_1 = 2a(x + x_1)$

Condition for $y = mx + c$ to touch: $c = \dfrac{a}{m}$

Focal chord parameters satisfy $t_1 t_2 = -1$.

The **semi-latus rectum** is the harmonic mean of the two focal segments.

Ellipse and hyperbola

Ellipse: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $b^2 = a^2(1 - e^2)$, $e < 1$, foci $(\pm ae, 0)$

Hyperbola: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, $b^2 = a^2(e^2 - 1)$, $e > 1$

Latus rectum for both: $\dfrac{2b^2}{a}$; directrix $x = \pm\dfrac{a}{e}$

Tangency: ellipse $c^2 = a^2m^2 + b^2$; hyperbola $c^2 = a^2m^2 - b^2$

Ellipse sums the focal distances to $2a$; hyperbola takes the **difference** as $2a$.

Asymptotes of the hyperbola are $y = \pm\dfrac{b}{a}x$.

JEE Extension

Director circle, the locus of points from which two perpendicular tangents can be drawn: circle $x^2 + y^2 = 2a^2$; ellipse $x^2 + y^2 = a^2 + b^2$; hyperbola $x^2 + y^2 = a^2 - b^2$; parabola gives the directrix itself. Also learn that the length of a focal chord of $y^2 = 4ax$ inclined at θ is $\dfrac{4a}{\sin^2\theta}$.

Do not mix the two b^2 relations

Ellipse uses $b^2 = a^2(1 - e^2)$ and hyperbola uses $b^2 = a^2(e^2 - 1)$. Applying the wrong one silently produces a negative b^2 and a wrong eccentricity.

52 Three Dimensional Geometry

This unit covers direction cosines and ratios, the equations of a line and a plane, and the distances and angles between them.

Direction cosines and ratios

$l^2+m^2+n^2=1$; $l=\dfrac{a}{\sqrt{a^2+b^2+c^2}}$ and similarly for m, n

Angle between two lines: $\cos\theta = |l_1l_2+m_1m_2+n_1n_2|$

Direction ratios are **any** proportional triple, direction cosines are the normalised version. Perpendicular lines give $a_1a_2+b_1b_2+c_1c_2=0$.

Line and plane equations

Line: $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$, vector form $\vec{r}=\vec{a}+\lambda\vec{b}$

Plane: $ax+by+cz+d=0$, normal (a,b,c); vector form $\vec{r}\cdot\hat{n}=p$

Plane through three points uses the determinant of the coordinate differences set to zero

A line lies **inside** a plane when its direction is perpendicular to the normal and one of its points satisfies the plane equation.

Angles and distances

Line and plane: $\sin\theta=\dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$

Two planes: $\cos\theta=\dfrac{|\vec{n}_1\cdot\vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$

Point to plane: $d=\dfrac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$

The line and plane case uses **sine**, not cosine, because the angle is measured from the plane and not from its normal.

Skew lines

Shortest distance: $d=\left|\dfrac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

Lines intersect when this distance is zero

Skew lines are **neither parallel nor intersecting**, which only happens in three dimensions. If $\vec{b}_1\times\vec{b}_2=0$ the lines are parallel and the formula does not apply.

53 Vector Algebra

This unit covers vector addition, the dot and cross products, the scalar triple product and their geometric applications.

Dot and cross product

$\vec{a}\cdot\vec{b}=|a||b|\cos\theta$, zero for perpendicular vectors

$\vec{a}\times\vec{b}=|a||b|\sin\theta\,\hat{n}$, zero for parallel vectors

Projection of $\vec{a}$ on $\vec{b}$: $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

The dot product is a **scalar**, the cross product a **vector** perpendicular to both. Cross product is anticommutative.

Areas and volumes

Triangle: $\frac{1}{2}|\vec{a}\times\vec{b}|$; Parallelogram: $|\vec{a}\times\vec{b}|$

Parallelogram from diagonals: $\frac{1}{2}|\vec{d}_1\times\vec{d}_2|$

Scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}]=\vec{a}\cdot(\vec{b}\times\vec{c})$ gives the parallelepiped volume

Tetrahedron volume $=\frac{1}{6}|[\vec{a}\,\vec{b}\,\vec{c}]|$

$[\vec{a}\,\vec{b}\,\vec{c}] = 0$ means the three vectors are **coplanar**. That is the standard coplanarity test.

Vector triple product

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

Section formula: $\vec{r} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$ internally

Remember the triple product as **BAC minus CAB**: the middle vector comes out first with a plus sign.

Useful vector identities

$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |a|^2|b|^2$

$|\vec{a} + \vec{b}|^2 = |a|^2 + |b|^2 + 2\vec{a} \cdot \vec{b}$

$[\vec{a}\,\vec{b}\,\vec{c}]$ changes sign when any two vectors are swapped

The first identity is the vector version of $\sin^2\theta + \cos^2\theta = 1$, which makes it a quick way to **find the angle**.

54 Statistics and Probability

This unit covers measures of central tendency and dispersion, and probability including conditional probability and Bayes' theorem.

Mean, variance and deviation

$\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$\sigma^2 = \dfrac{\sum f_i x_i^2}{N} - \bar{x}^2, \quad \sigma = \sqrt{\sigma^2}$

Coefficient of variation $= \dfrac{\sigma}{\bar{x}} \times 100$

Mean deviation about the mean $= \dfrac{\sum f_i |x_i - \bar{x}|}{N}$

Adding a constant to every value **shifts the mean but not the variance**. Multiplying by k multiplies the variance by k^2.

Probability basics

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Independent events: $P(A \cap B) = P(A)P(B)$

$P(A') = 1 - P(A)$

Mutually exclusive is **not the same** as independent. Two events with non-zero probability cannot be both at once.

Conditional probability and Bayes

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$, provided $P(B) > 0$

Total probability: $P(A) = \sum P(E_i)P(A|E_i)$

Bayes: $P(E_i|A) = \dfrac{P(E_i)P(A|E_i)}{\sum_j P(E_j)P(A|E_j)}$

Bayes' theorem **reverses** the conditioning. Use it whenever the effect is observed and the question asks for the probability of a cause.

Random variable

$E(X) = \sum x_i p_i, \quad \text{Var}(X) = E(X^2) - [E(X)]^2$

$\text{Var}(aX + b) = a^2\text{Var}(X)$

The expected value is a **weighted average**, so it need not be one of the possible outcomes of the experiment.

Variance formula uses squares first

Var$(X) = E(X^2) - [E(X)]^2$, not $E(X)^2 - E(X^2)$. Getting the order wrong gives a negative variance, which is impossible. If your answer is negative, this is the reason.

55 Trigonometry

This unit covers trigonometric identities, transformation formulas, inverse trigonometric functions and the properties of triangles.

Core identities

$\sin^2\theta + \cos^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta, \quad 1 + \cot^2\theta = \text{cosec}^2\theta$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Watch the **sign flip** in the cosine formula: cosine of a sum takes a minus, sine of a sum takes a plus.

Multiple and half angles

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$$

$$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta, \quad \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

The $\tan\theta$ versions turn every ratio into **one variable**, which is what makes the t substitution work in integration.

Range and maximum values

$a\sin\theta + b\cos\theta$ lies in $[-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$

$$\sin C + \sin D = 2\sin\frac{C + D}{2}\cos\frac{C - D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C + D}{2}\sin\frac{C - D}{2}$$

The $\sqrt{a^2 + b^2}$ bound answers most **maximum value** questions in a single line, with no calculus needed.

Inverse trigonometric functions

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \quad \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x + y}{1 - xy} \text{ for } xy < 1$$

$$2\tan^{-1}x = \sin^{-1}\frac{2x}{1 + x^2} = \tan^{-1}\frac{2x}{1 - x^2}$$

The addition formula needs a π **correction** when $xy > 1$. Ignoring the principal range is the single most common error here.

Properties of triangles

Sine rule: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

Cosine rule: $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Area $= \frac{1}{2}ab\sin C = \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R} = rs$

Here r is the inradius, R the circumradius and s the semiperimeter. **Four expressions** for the same area means you can always pick the one that fits the given

data.

Signs by quadrant

All Silver Tea Cups: in the first quadrant all ratios are positive, in the second only Sine, in the third only Tangent, and in the fourth only Cosine.

Quick Reference: Last Week Before the Exam

Number Values Worth Memorising

Quantity	Value	Quantity	Value
$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$	1.414, 1.732, 2.236	$\ln 2$, $\ln 10$	0.693, 2.303
π, π^2, g	3.1416, 9.87, 9.8	e, $1/e$	2.718, 0.368
$\sin 15^\circ$	0.2588	$\log_{10} 2$, $\log_{10} 3$	0.301, 0.477
Escape speed	11.2 km/s	Speed of sound in air	332 m/s
Photon: hc in eV nm	1240	Photon: hc in eV Å	12400
0.0591 V	Nernst factor	96500 C	1 faraday
22.4 L at 1 atm STP	molar gas volume	22.7 L at 1 bar STP	molar gas volume
1 amu	931.5 MeV	$\frac{1}{4\pi\varepsilon_0}$	9×10^9

Six Rules for the Three Hours

1. Start with the subject you are strongest at, and give it 50 minutes, not more. Leaving one full subject short is what wrecks a score.
2. Section B carries negative marking now. Type an answer only when you have actually solved it, not when you have a hunch.
3. Do one full pass marking every question you can finish in under 90 seconds. Come back for the rest.
4. In Chemistry, the recall questions in Inorganic and Biomolecules are the cheapest 20 marks in the paper. Do them first.
5. Check units before you type a Section B answer. Most lost marks there are conversion slips, not conceptual errors.
6. Do not revisit an answered question unless you find a definite error. Second-guessing costs more marks than it saves.

Revision Order for the Last Ten Days

Day	Physics	Chemistry	Maths
1 to 2	Modern Physics, Electronic Devices, EM Waves	Coordination Compounds, d and f block	Calculus: limits, AOD, definite integrals
3 to 4	Electrostatics, Current Electricity	Chemical Bonding, Periodicity, p block	Coordinate Geometry: circle, conics
5 to 6	Magnetism, EMI and AC, Optics	Equilibrium, Electrochemistry, Kinetics	Vectors, 3D, Matrices and Determinants
7 to 8	Mechanics: rotation, gravitation, SHM, waves	Organic: GOC, hydrocarbons, oxygen and nitrogen compounds	Algebra: complex numbers, P and C, binomial, sequences
9 to 10	Thermodynamics, KTG, fluids, error analysis	Solutions, Thermodynamics, Practical Chemistry	Probability, Statistics, Trigonometry, Differential Equations

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