# JEE Main 2025 April 3 Shift 1 Question Paper with Solutions

| Time Allowed :3 Hours | Maximum Marks :300 | Total Questions :75 |
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**General Instructions**

**Read the following instructions very carefully and strictly follow them:**

1. Multiple choice questions (MCQs)
2. Questions with numerical values as answers.
3. There are three sections: **Mathematics, Physics, Chemistry**.
4. **Mathematics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
5. **Physics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory..
6. **Chemistry:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
7. Total: 75 Questions (25 questions each).
8. 300 Marks (100 marks for each section).
9. **MCQs:** Four marks will be awarded for each correct answer and there will be a negative marking of one mark on each wrong answer.
10. **Questions with numerical value answers:** Candidates will be given four marks for each correct answer and there will be a negative marking of 1 mark for each wrong answer.

## Mathematics

## Section - A

**1. Let $A$ be a matrix of order $3 \times 3$ and $|A| = 5$. If $|2\,\mathrm{adj}(3A\,\mathrm{adj}(2A))| = 2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}$, $\alpha, \beta, \gamma \in N$ then $\alpha + \beta + \gamma$ is equal to**

(1) 25
(2) 26
(3) 27
(4) 28

**Correct Answer:** (3) 27

**Solution:** We begin with the expression:

$$|2\,\mathrm{adj}(3A\,\mathrm{adj}(2A))|$$

We use the following identities:

- $|\text{adj}(M)| = |M|^{n-1}$ for an $n \times n$ matrix.
- $|kM| = k^n |M|$ for scalar $k$.

Let us simplify step-by-step:

$$\begin{aligned}
|2\,\text{adj}(3A\,\text{adj}(2A))| &= 2^3 \cdot |\text{adj}(3A\,\text{adj}(2A))| \\
&= 2^3 \cdot |3A\,\text{adj}(2A)|^2 \\
&= 2^3 \cdot \left(3^3 \cdot |A| \cdot |\text{adj}(2A)|\right)^2 \\
&= 2^3 \cdot \left(3^3 \cdot |A| \cdot (2^3 \cdot |A|)^2\right)^2 \\
&= 2^3 \cdot \left(2^6 \cdot 3^3 \cdot |A|^3\right)^2 \\
&= 2^3 \cdot 2^{12} \cdot 3^6 \cdot |A|^6 \\
&= 2^{15} \cdot 3^6 \cdot |A|^6
\end{aligned}$$

Given $|A| = 5$, we substitute:

$$= 2^{15} \cdot 3^6 \cdot 5^6$$

Comparing with $2^\alpha \cdot 3^\beta \cdot 5^\gamma$, we get:

$$\alpha = 15, \quad \beta = 6, \quad \gamma = 6$$

Thus:

$$\alpha + \beta + \gamma = 15 + 6 + 6 = 27$$

**Quick Tip**

To solve matrix determinant problems involving adjugates and scalar multiplication, remember the key formulas:

$$|\text{adj}(A)| = |A|^{n-1}, \quad |kA| = k^n |A|$$

Also, the adjugate of a product doesn't distribute over multiplication like regular matrices, so simplify the inner terms first before applying adjugate or determinant rules.

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**2. Let a line passing through the point $(4, 1, 0)$ intersect the line $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point $A(\alpha, \beta, \gamma)$ and the line $L_2 : x - 6 = y = -z + 4$ at the point $B(a, b, c)$. Then**

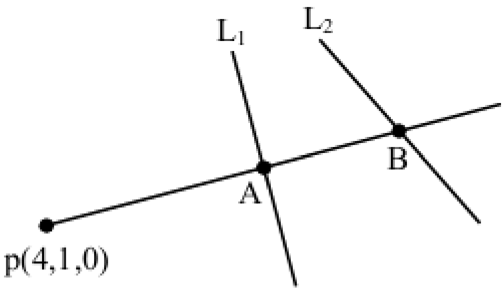
$$\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix} \textbf{ is equal to}$$

(1) 8

(2) 16
(3) 12
(4) 6

**Correct Answer:** (1) 8

**Solution:**




The line passing through point $P(4,1,0)$ intersects line $L_1$ and $L_2$. Let us assume the point $A$ on $L_1$ is given by parameter $p$:

$$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = p \Rightarrow A(2p+1, 3p+2, 4p+3)$$

Let point $B$ on $L_2$ be parameterized using $q$:

$$L_2 : x-6 = y = -z+4 = q \Rightarrow B(q+6, q, 4-q)$$

Now find the direction ratios (D.R.) of $\overrightarrow{PA}$ and $\overrightarrow{PB}$:

$$\text{D.R. of } \overrightarrow{PA} = (2p-3, 3p+1, 4p+3)$$

D.R. of $\overrightarrow{PB} = (q+2, q-1, 4-q)$
Since $\overrightarrow{PA}$ and $\overrightarrow{PB}$ are collinear, their components must be proportional:

$$\frac{2p-3}{q+2} = \frac{3p+1}{q-1} = \frac{4p+3}{4-q}$$

Equating pairwise and solving:

$$2pq - 2p - 3q + 3 = 3pq + 6p + q + 2 \Rightarrow pq + rp + 4q - 1 = 0 \quad (1)$$

7pq - 16p + 4q - 7 = 0 (2)

Solving these equations, we find:

$$pq = -3, \quad p = -1, \quad q = 3 \Rightarrow A(-1,-1,-1), \quad B(9,3,1)$$

Now compute the determinant:

$$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & -1 \\ 3 & 1 \end{vmatrix} - 0 + 1 \cdot \begin{vmatrix} -1 & -1 \\ 9 & 3 \end{vmatrix} = 1(-1+3) + 1(-3+9) = 2 + 6 = 8$$

**Quick Tip**

To check whether two vectors are collinear, equating the ratios of their direction ratios is a powerful approach. Also, using parametric representation for lines makes it easier to substitute and calculate points of intersection.

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**3. Let $\alpha$ and $\beta$ be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and $\gamma$ and $\delta$ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q_n = \gamma^n + \delta^n$, then**

$$\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}} \text{ is equal to}$$

(1) 3
(2) 4
(3) 5
(4) 7

**Correct Answer:** (3) 5

**Solution:** We are given:

$$x^2 + \sqrt{3}x - 16 = 0 \quad \text{has roots } \alpha, \beta$$

Define $P_n = \alpha^n + \beta^n$. Since $\alpha, \beta$ are roots, the recurrence relation is:

$$P_n + \sqrt{3}P_{n-1} - 16P_{n-2} = 0$$

Using the recurrence:

$$P_{25} + \sqrt{3}P_{24} - 16P_{23} = 0 \Rightarrow \frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} = \frac{16P_{23}}{2P_{23}} = 8$$

Similarly, for the second quadratic:

$$x^2 + 3x - 1 = 0 \quad \text{has roots } \gamma, \delta$$

Define $Q_n = \gamma^n + \delta^n$
We evaluate:

$$Q_{25} - Q_{23} = \gamma^{25} + \delta^{25} - \gamma^{23} - \delta^{23} = \gamma^{23}(\gamma^2 - 1) + \delta^{23}(\delta^2 - 1)$$

Since $\gamma^2 = -3\gamma + 1$, so:

$$\gamma^2 - 1 = -3\gamma, \quad \delta^2 - 1 = -3\delta \Rightarrow Q_{25} - Q_{23} = -3(\gamma^{24} + \delta^{24}) = -3Q_{24}$$

Therefore:

$$\frac{Q_{25} - Q_{23}}{Q_{24}} = -3$$

Now, summing both terms:

$$\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}} = 8 - 3 = 5$$

Quick Tip

For expressions involving powers of roots of quadratic equations, use recurrence relations derived from the equation itself. If $\alpha, \beta$ are roots of $x^2 + ax + b = 0$, then $P_n = \alpha^n + \beta^n$ satisfies the recurrence $P_n = -aP_{n-1} - bP_{n-2}$.

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**4. The sum of all rational terms in the expansion of $\left(2+\sqrt{3}\right)^8$ is**

(1) 16923
(2) 3763
(3) 33845
(4) 18817

**Correct Answer:** (4) 18817

**Solution:** Let $S = (2+\sqrt{3})^8$. To find the sum of all **rational terms** in the binomial expansion, we use:

$$(2+\sqrt{3})^8 + (2-\sqrt{3})^8$$

This removes all irrational terms since they cancel out in the symmetric expansion. So the sum of rational terms is:

$$\frac{(2+\sqrt{3})^8 + (2-\sqrt{3})^8}{2}$$

We can also directly select terms where the exponent of $\sqrt{3}$ is even (to ensure the term is rational). From the binomial expansion:

$$= \binom{8}{0}(2)^8 + \binom{8}{2}(2)^6(\sqrt{3})^2 + \binom{8}{4}(2)^4(\sqrt{3})^4 + \binom{8}{6}(2)^2(\sqrt{3})^6 + \binom{8}{8}(\sqrt{3})^8$$

$$= 2^8 + 28 \cdot 2^6 \cdot 3 + 70 \cdot 2^4 \cdot 9 + 28 \cdot 2^2 \cdot 27 + 1 \cdot 81$$

$$= 256 + 5376 + 10080 + 3024 + 81 = 18817$$

Quick Tip

When expanding expressions of the form $(a+\sqrt{b})^n$, rational terms are those in which the exponent of $\sqrt{b}$ is even. Use symmetry by adding $(a+\sqrt{b})^n$ and $(a-\sqrt{b})^n$ to quickly eliminate irrational components and double the rational terms.

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**5. Let A = {-3,-2,-1,0,1,2,3}. Let R be a relation on A defined by xRy if and only if $0 \leq x^2 + 2y \leq 4$. Let $l$ be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. then $l + m$ is equal to**

(1) 19
(2) 20
(3) 17
(4) 18

**Correct Answer:** (4) 18

**Solution:** Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$
Given:

$$0 \leq x^2 + 2y \leq 4 \Rightarrow -2y \leq x^2 \leq 4 - 2y$$

Now for different values of $y \in A$, find the possible $x \in A$ satisfying the condition:

- For $y = -3$, $6 \leq x^2 \leq 10 \Rightarrow x \in \{-3, 3\}$
- For $y = -2$, $4 \leq x^2 \leq 8 \Rightarrow x \in \{-2, 2\}$
- For $y = -1$, $2 \leq x^2 \leq 6 \Rightarrow x \in \{-2, 2\}$
- For $y = 0$, $0 \leq x^2 \leq 4 \Rightarrow x \in \{-2, -1, 0, 1, 2\}$
- For $y = 1$, $-2 \leq x^2 \leq 2 \Rightarrow x \in \{-1, 0, 1\}$
- For $y = 2$, $-4 \leq x^2 \leq 0 \Rightarrow x \in \{0\}$
- For $y = 3$, $-6 \leq x^2 \leq -2 \Rightarrow$ No such $x$

So the relation $R$ consists of the following ordered pairs:

$R = \{(-3, -3), (-3, 3), (-2, -2), (-2, 2), (-1, -2), (-1, 2), (0, -2), (0, -1), (0, 0), (0, 1), (0, 2),$
(1,-1), (1,0), (1,1), (2,0) }

Thus,

$$l = |R| = 15$$

To make $R$ reflexive, we must add the missing self-pairs: From set $A$, reflexive relation requires all $(a, a) \in A \times A$ Already present: $(0, 0)$ Missing: $(-1, -1), (2, 2), (3, 3) \Rightarrow m = 3$

$$l + m = 15 + 3 = \boxed{18}$$

Correct answer: Option (4)

**Quick Tip**

To make a relation reflexive, every element in the set must be related to itself. In other words, for a set A, (a,a) must be in R for all a in A.

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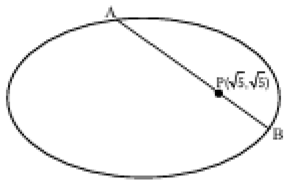
**6. A line passing through the point P($\sqrt{5}, \sqrt{5}$) intersects the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ at A and B such that (PA).(PB) is maximum. Then 5(PA$^2$ + PB$^2$) is equal to :**

(1) 218
(2) 377
(3) 290
(4) 338

**Correct Answer:** (4) 338

**Solution:** Given ellipse is

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$




Any point on line $AB$ can be assumed as

$$Q(\sqrt{5} + r\cos\theta, \sqrt{5} + r\sin\theta)$$

Substituting this into the equation of the ellipse:

$$25(\sqrt{5} + r\cos\theta)^2 + 36(\sqrt{5} + r\sin\theta)^2 = 900$$

Expanding and simplifying:

$$r^2(25\cos^2\theta + 36\sin^2\theta) + 2\sqrt{5}r(25\cos\theta + 36\sin\theta) + 25 \cdot 5 + 36 \cdot 5 = 900$$
$$r^2(25\cos^2\theta + 36\sin^2\theta) + 2\sqrt{5}r(25\cos\theta + 36\sin\theta) = 900 - 305 = 595$$
$$\Rightarrow r^2(25\cos^2\theta + 36\sin^2\theta) + 2\sqrt{5}r(25\cos\theta + 36\sin\theta) - 595 = 0$$

Let the roots of this quadratic in $r$ be $PA$ and $PB$, then

$$PA \cdot PB = \frac{595}{25\cos^2\theta + 36\sin^2\theta}$$

To maximize $PA \cdot PB$, the denominator should be minimized:

$$25\cos^2\theta + 36\sin^2\theta = 25 + 11\sin^2\theta$$

Maximum value of $PA \cdot PB$ occurs when $\sin^2\theta = 0$, i.e., $\theta = 0$ or $\pi$

$$\Rightarrow \text{Line } AB \text{ must be parallel to the x-axis} \Rightarrow y_A = y_B = \sqrt{5}$$

Putting $y = \sqrt{5}$ in the equation of the ellipse:

$$\frac{x^2}{36} + \frac{5}{25} = 1 \Rightarrow \frac{x^2}{36} = \frac{4}{5} \Rightarrow x^2 = \frac{4}{5} \cdot 36 = \frac{144}{5}$$

Hence, coordinates of $A$ and $B$ are:

$$A = \left(-\frac{12}{\sqrt{5}}, \sqrt{5}\right), \quad B = \left(\frac{12}{\sqrt{5}}, \sqrt{5}\right)$$

Now,

$$PA^2 + PB^2 = \left(\sqrt{5} - \frac{12}{\sqrt{5}}\right)^2 + \left(\sqrt{5} + \frac{12}{\sqrt{5}}\right)^2$$

$$= 2\left(5 + \frac{144}{5}\right) = 2 \cdot \frac{169}{5} = \frac{338}{5}$$

$$\Rightarrow 5(PA^2 + PB^2) = 338$$

**Quick Tip**

To maximize the product of two distances from a point to the intersection points of a line and an ellipse, the line should be parallel to the major or minor axis of the ellipse.

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**7. The sum 1 + 3 + 11 + 25 + 45 + 71 + ... upto 20 terms, is equal to**

(1) 7240
(2) 7130
(3) 6982
(4) 8124

**Correct Answer:** (1) 7240

**Solution:** Given sum is $S_n = 1 + 3 + 11 + 25 + 45 + 71 + ... + T_n$ First order differences are in A.P. Thus, we can assume that $T_n = an^2 + bn + c$

Solving $\begin{cases} T_1 = 1 = a + b + c \\ T_2 = 3 = 4a + 2b + c \\ T_3 = 11 = 9a + 3b + c \end{cases}$

we get a = 3, b = -7, c = 5 Hence, general term of given series is $T_n = 3n^2 - 7n + 5$ Hence, required sum equals $\sum_{n=1}^{20}(3n^2 - 7n + 5) = 3\frac{20\cdot21\cdot41}{6} - 7\frac{20\cdot21}{2} + 5(20) = 7240$

**Quick Tip**

If the first differences of a series form an arithmetic progression (AP), then the general term of the series can be represented by a quadratic equation $T_n = an^2 + bn + c$.

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**8. If the domain of the function $f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is $[\alpha, \beta]$, then $\alpha^2 + 4\beta$ is equal to**

(1) 5
(2) 4
(3) 3

(4) 7

**Correct Answer:** (2) 4

**Solution:** Given function is

$$f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$$

For the domain of $f(x)$, we require:

$$\frac{2x-3}{5+4x} > 0 \quad \text{and} \quad \left|\frac{4+3x}{2-x}\right| \leq 1$$

Start with the logarithmic condition:

$$\frac{2x-3}{5+4x} > 0 \Rightarrow x \in \left(-\infty, -\frac{5}{4}\right) \cup \left(\frac{3}{2}, \infty\right)$$

Now consider the inverse sine condition:

$$-1 \leq \frac{4+3x}{2-x} \leq 1$$

Break this into two inequalities:

$$\frac{4+3x}{2-x} \geq -1 \quad \text{and} \quad \frac{4+3x}{2-x} \leq 1$$

Solving the first:

$$\frac{4+3x}{2-x} + 1 \geq 0 \Rightarrow \frac{4+3x+2-x}{2-x} = \frac{6+2x}{2-x} \geq 0$$

Solving the second:

$$\frac{4+3x}{2-x} - 1 \leq 0 \Rightarrow \frac{4+3x-2+x}{2-x} = \frac{2+4x}{2-x} \leq 0$$

Combining both:

$$\left(\frac{6+2x}{2-x} \geq 0\right) \cap \left(\frac{2+4x}{2-x} \leq 0\right)$$

Multiply the two expressions:

$$\frac{(6+2x)(2+4x)}{(2-x)^2} \leq 0$$

Solve the inequality:

$$x \in \left[-3, -\frac{1}{2}\right]$$

Now take the intersection of both conditions:

$$x \in \left(-\infty, -\frac{5}{4}\right) \cup \left(\frac{3}{2}, \infty\right) \quad \cap \quad x \in \left[-3, -\frac{1}{2}\right] \Rightarrow x \in \left[-3, -\frac{5}{4}\right)$$

Thus, the domain of $f(x)$ is:

$$x \in \left[-3, -\frac{5}{4}\right)$$

Let $\alpha = -3$, $\beta = -\frac{5}{4}$

Then,

$$\alpha^2 + 4\beta = (-3)^2 + 4 \cdot \left(-\frac{5}{4}\right) = 9 - 5 = \boxed{4}$$

**Quick Tip**

For logarithmic functions, the argument must be strictly positive. For inverse sine functions, the argument must lie between -1 and 1, inclusive.

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**9. If $\sum_{r=1}^{9} \left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \alpha \left(\frac{3}{2}\right)^9 - \beta$, $\alpha, \beta \in N$, then $(\alpha + \beta)^2$ is equal to**

(1) 27
(2) 9
(3) 81
(4) 18

**Correct Answer:** (3) 81

**Solution: Given:**

$$\sum_{r=1}^{9} \left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \alpha \left(\frac{3}{2}\right)^9 - \beta, \quad \alpha, \beta \in N$$

We split the sum as:

$$\sum_{r=1}^{9} \left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \sum_{r=1}^{9} \left(\frac{r}{2^r} \cdot {}^9C_r\right) + \sum_{r=1}^{9} \left(\frac{3}{2^r} \cdot {}^9C_r\right)$$

Now, use the identity:

$$\frac{r}{2^r} \cdot {}^9C_r = \frac{9}{2^r} \cdot {}^8C_{r-1}$$

So,

$$\sum_{r=1}^{9} \frac{r}{2^r} \cdot {}^9C_r = 9 \sum_{r=1}^{9} \frac{1}{2^r} \cdot {}^8C_{r-1} = \frac{9}{2} \sum_{r=1}^{9} {}^8C_{r-1} \cdot \left(\frac{1}{2}\right)^{r-1}$$

Make the substitution $s = r - 1 \Rightarrow s = 0$ to 8:

$$= \frac{9}{2} \sum_{s=0}^{8} {}^8C_s \left(\frac{1}{2}\right)^s = \frac{9}{2} \cdot \left(1 + \frac{1}{2}\right)^8 = \frac{9}{2} \cdot \left(\frac{3}{2}\right)^8$$

Now for the second term:

$$\sum_{r=1}^{9} \frac{3}{2^r} \cdot {}^9C_r = 3 \left(\sum_{r=0}^{9} {}^9C_r \cdot \left(\frac{1}{2}\right)^r - {}^9C_0 \cdot 1\right) = 3 \left(\left(1 + \frac{1}{2}\right)^9 - 1\right) = 3 \left(\left(\frac{3}{2}\right)^9 - 1\right)$$

Adding both parts:

$$\frac{9}{2} \cdot \left(\frac{3}{2}\right)^8 + 3 \left(\left(\frac{3}{2}\right)^9 - 1\right) = \left(\frac{9}{2} \cdot \frac{2}{3} + 3\right) \cdot \left(\frac{3}{2}\right)^9 - 3 = (3 + 3) \cdot \left(\frac{3}{2}\right)^9 - 3 = 6 \left(\frac{3}{2}\right)^9 - 3$$

Thus, $\alpha = 6$, $\beta = 3$

$$(\alpha + \beta)^2 = (6 + 3)^2 = \boxed{81}$$

Quick Tip

Use the identity $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$ to simplify the summation. Also, remember the binomial expansion $(1 + x)^n = \sum_{r=0}^{n} {}^nC_r x^r$.

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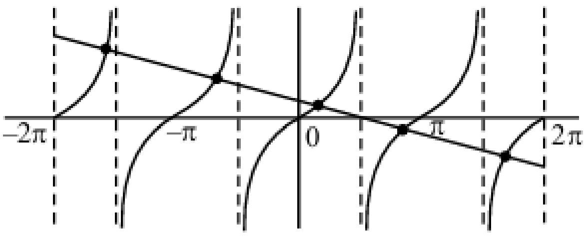
**10. The number of solutions of the equation $2x + 3\tan x = \pi$, $x \in [-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$ is**

(1) 6
(2) 5
(3) 4
(4) 3

**Correct Answer:** (2) 5

**Solution:** Given equation is $2x + 3\tan x = \pi$ Rearranging the terms, we get $\tan x = \frac{\pi - 2x}{3}$ Let $f(x) = \tan x$ and $g(x) = \frac{\pi - 2x}{3}$ We need to find the number of intersection points of these two functions in the given interval $[-2\pi, 2\pi]$.
$f(x) = \tan x$ has vertical asymptotes at $x = \pm\frac{\pi}{2}$ and $x = \pm\frac{3\pi}{2}$. $g(x) = \frac{\pi - 2x}{3}$ is a straight line with slope $-\frac{2}{3}$ and y-intercept $\frac{\pi}{3}$.
We can analyze the intersection points graphically or by analyzing intervals. In the interval $[-2\pi, 2\pi]$, we have the following intervals to consider: $[-2\pi, -\frac{3\pi}{2})$, $(-\frac{3\pi}{2}, -\frac{\pi}{2})$, $(-\frac{\pi}{2}, \frac{\pi}{2})$, $(\frac{\pi}{2}, \frac{3\pi}{2})$, $(\frac{3\pi}{2}, 2\pi]$.
By sketching the graphs or analyzing the behavior of the functions in each interval, we can observe that there are 5 intersection points.




Therefore, the number of solutions is 5.

Quick Tip

To find the number of solutions of an equation involving trigonometric and linear functions, sketch the graphs of both functions and count the number of intersection points.

**11. If** $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$**,** $x \in R$**, then** $\frac{d^2y}{dx^2} + y$ **is equal to**

(1) -1
(2) 28
(3) 27
(4) 1

**Correct Answer:** (1) -1

**Solution:** Given $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$
Applying $C_3 \to C_3 - C_1$, we get $y(x) = \begin{vmatrix} \sin x & \cos x & \cos x + 1 \\ 27 & 28 & 0 \\ 1 & 1 & 0 \end{vmatrix}$
Expanding the determinant, we have $y(x) = -(\cos x + 1) \begin{vmatrix} 27 & 28 \\ 1 & 1 \end{vmatrix}$
$y(x) = -(\cos x + 1)(27 - 28)$ $y(x) = \cos x + 1$
Differentiating with respect to x, we get $\frac{dy}{dx} = -\sin x$
Differentiating again with respect to x, we get $\frac{d^2y}{dx^2} = -\cos x$
Therefore, $\frac{d^2y}{dx^2} + y = -\cos x + \cos x + 1 = 1$

Quick Tip

Use determinant properties to simplify the expression before differentiation.

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**12. Let g be a differentiable function such that** $\int_0^x g(t)dt = x - \int_0^x tg(t)dt$**,** $x \geq 0$ **and let** $y = y(x)$ **satisfy the differential equation** $\frac{dy}{dx} - y \tan x = 2(x+1) \sec x g(x)$**,** $x \in \left[0, \frac{\pi}{2}\right)$**. If** $y(0) = 0$**, then** $y\left(\frac{\pi}{3}\right)$ **is equal to**

(1) $\frac{2\pi}{3\sqrt{3}}$
(2) $\frac{4\pi}{3}$
(3) $\frac{2\pi}{3}$
(4) $\frac{4\pi}{3\sqrt{3}}$

**Correct Answer:** (2) $\frac{4\pi}{3}$

**Solution: Given:**

$$\int_0^x g(t)\,dt = x - \int_0^x tg(t)\,dt$$

Differentiate both sides with respect to $x$:

$$g(x) = 1 - xg(x) \Rightarrow g(x)(1+x) = 1 \Rightarrow g(x) = \frac{1}{1+x}$$

Now consider the differential equation:

$$\frac{dy}{dx} - y\tan x = 2(x+1)\sec x \cdot g(x) = 2(x+1)\sec x \cdot \frac{1}{1+x} = 2\sec x$$

So the equation becomes:

$$\frac{dy}{dx} - y\tan x = 2\sec x$$

This is a linear differential equation. The integrating factor is:

$$\text{IF} = e^{\int -\tan x\, dx} = e^{\ln|\cos x|} = \cos x$$

Multiplying both sides by the integrating factor:

$$\cos x \cdot \frac{dy}{dx} - y\cos x\tan x = 2 \Rightarrow \frac{d}{dx}(y\cos x) = 2$$

Integrating both sides:

$$y\cos x = \int 2\, dx = 2x + C$$

Apply the initial condition $y(0) = 0$:

$$0 \cdot \cos 0 = 2 \cdot 0 + C \Rightarrow C = 0$$

Therefore, the solution is:

$$y\cos x = 2x \Rightarrow y = \frac{2x}{\cos x} = 2x\sec x$$

Now, compute $y\left(\frac{\pi}{3}\right)$:

$$y\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\pi}{3} \cdot \sec\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\pi}{3} \cdot 2 = \boxed{\frac{4\pi}{3}}$$

**Quick Tip**

Recognize and solve the linear differential equation using the integrating factor method.

---

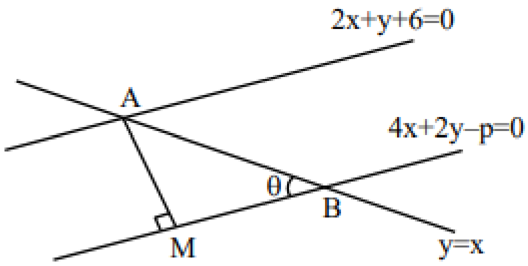
**13. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines $L_1 : 2x + y + 6 = 0$ and $L_2 : 4x + 2y - p = 0$, $p > 0$, at the points A and B, respectively. If $AB = \frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line $L_2$ is M, then $\frac{AM}{BM}$ is equal to**

(1) 5
(2) 4
(3) 2

(4) 3

**Correct Answer:** (4) 3

**Solution:** The line passing through the origin and making equal angles with the positive coordinate axes is $y = x$.




To find the coordinates of A, we solve $2x + y + 6 = 0$ and $y = x$: $2x + x + 6 = 0$ $3x = -6$ $x = -2$ $y = -2$ So, A is $(-2, -2)$.
To find the coordinates of B, we solve $4x + 2y - p = 0$ and $y = x$: $4x + 2x - p = 0$ $6x = p$ $x = \frac{p}{6}$ $y = \frac{p}{6}$ So, B is $\left(\frac{p}{6}, \frac{p}{6}\right)$.
Given $AB = \frac{9}{\sqrt{2}}$, we have: $\sqrt{\left(\frac{p}{6}+2\right)^2 + \left(\frac{p}{6}+2\right)^2} = \frac{9}{\sqrt{2}}$ $\sqrt{2\left(\frac{p}{6}+2\right)^2} = \frac{9}{\sqrt{2}}$ $\sqrt{2}\left|\frac{p}{6}+2\right| = \frac{9}{\sqrt{2}}$ $\left|\frac{p}{6}+2\right| = \frac{9}{2}$
Since $p > 0$, $\frac{p}{6} + 2 = \frac{9}{2}$ $\frac{p}{6} = \frac{9}{2} - 2 = \frac{5}{2}$ $p = 15$
Therefore, B is $\left(\frac{15}{6}, \frac{15}{6}\right) = \left(\frac{5}{2}, \frac{5}{2}\right)$.
The slope of $L_2$ is $m_2 = -2$. The slope of $y = x$ is $m_1 = 1$.
Let $\theta$ be the angle between the lines $y = x$ and $L_2$. $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{1-(-2)}{1+1(-2)}\right| = \left|\frac{3}{-1}\right| = 3$
From the geometry, $\tan\theta = \frac{AM}{BM}$. Therefore, $\frac{AM}{BM} = 3$.

**Quick Tip**

Use the distance formula and the formula for the angle between two lines to solve the problem.

---

**14. Let $z \in C$ be such that $\frac{z+3i}{z-2+i} = 2 + 3i$. Then the sum of all possible values of $z$ is**

(1) $19 - 2i$
(2) $-19 - 2i$
(3) $19 + 2i$
(4) $-19 + 2i$

**Correct Answer:** (2) $-19 - 2i$

**Solution: Given:**

$$\frac{z + 3i}{z - 2 + i} = 2 + 3i$$

Multiply both sides by $z - 2 + i$:

$$z + 3i = (2 + 3i)(z - 2 + i)$$

Now expand the right-hand side:

$$\begin{aligned} z + 3i &= (2 + 3i)(z - 2 + i) \\ &= 2(z - 2 + i) + 3i(z - 2 + i) \\ &= 2z - 4 + 2i + 3iz - 6i + 3i^2 \\ &= 2z + 3iz - 4 - 4i - 3 \quad (\text{since } i^2 = -1) \\ &= 2z + 3iz - 7 - 4i \end{aligned}$$

Bring all terms to one side:

$$z + 3i - (2z + 3iz - 7 - 4i) = 0 \Rightarrow -z - 3iz + 10i + 7 = 0 \Rightarrow z(1 + 3i) = 7 + 7i$$

Now solve for $z$:

$$z = \frac{7 + 7i}{1 + 3i} = \frac{(7 + 7i)(1 - 3i)}{(1 + 3i)(1 - 3i)}$$

= 7(1 - 3i) + 7i(1 - 3i)$\overline{\phantom{x}}$ $1+9=\frac{7-21i+7i-21i^2}{10}=\frac{28-14i}{10}=\frac{14-7i}{5}$

So one value of $z$ is:

$$z = \boxed{\frac{14 - 7i}{5}}$$

Now observe that the original equation:

$$\frac{z + 3i}{z - 2 + i} = 2 + 3i \Rightarrow z + 3i = (2 + 3i)(z - 2 + i)$$

This can be written as a quadratic in $z$. Let's proceed:
Let's expand again:

$$z+3i = (2+3i)(z-2+i) = (2+3i)(z)+(2+3i)(-2+i) = 2z+3iz+(-4+2i-6i+3i^2) = 2z+3iz-4-4i-3 \quad (\text{si}$$

Now bring all terms to one side again:

$$z + 3i - 2z - 3iz + 7 + 4i = 0 \Rightarrow -z - 3iz + 7 + 7i = 0 \Rightarrow z(1 + 3i) = 7 + 7i$$

Multiply both sides by $1 + 3i$ to form a quadratic:

$$z(1 + 3i) = 7 + 7i \Rightarrow z^2 + 3iz = z(2 + 3i) - 7 - 4i \Rightarrow z^2 - (2 + 3i)z + 7 + 7i = 0$$

So the quadratic equation is:

$$z^2 - (2 + 3i)z + (7 + 7i) = 0$$

Let the roots be $z_1$ and $z_2$. Then:

$$z_1 + z_2 = 2 + 3i, \quad z_1 z_2 = 7 + 7i$$

Now compute:

$$\begin{aligned} z_1^2 + z_2^2 &= (z_1 + z_2)^2 - 2z_1 z_2 \\ &= (2 + 3i)^2 - 2(7 + 7i) \\ &= 4 + 12i - 9 - 14 - 14i \\ &= -19 - 2i \end{aligned}$$

$$\boxed{z_1^2 + z_2^2 = -19 - 2i}$$

**Quick Tip**

If the equation leads to a quadratic, the sum of roots can be found using the coefficients.

---

**15. Let $f(x) = \int x^3\sqrt{3 - x^2}dx$. If $5f(\sqrt{2}) = -4$, then $f(1)$ is equal to**

(1) $-\frac{2\sqrt{2}}{5}$
(2) $-\frac{8\sqrt{2}}{5}$
(3) $-\frac{4\sqrt{2}}{5}$
(4) $-\frac{6\sqrt{2}}{5}$

**Correct Answer:** (4) $-\frac{6\sqrt{2}}{5}$

**Solution:** Let $3 - x^2 = t^2$ $-2xdx = 2tdt$ $xdx = -tdt$
$f(x) = \int x^3\sqrt{3 - x^2}dx$ $f(x) = \int x^2\sqrt{3 - x^2}xdx$ $f(x) = \int(3 - t^2) \cdot t \cdot (-tdt) + c$
$f(x) = \int(t^4 - 3t^2)dt + c$ $f(x) = \frac{t^5}{5} - t^3 + c$ $f(x) = \frac{(3-x^2)^{5/2}}{5} - (3 - x^2)^{3/2} + c$
Given $5f(\sqrt{2}) = -4$, we have $5\left(\frac{(3-2)^{5/2}}{5} - (3 - 2)^{3/2} + c\right) = -4$ $5\left(\frac{1}{5} - 1 + c\right) = -4$
$1 - 5 + 5c = -4$ $-4 + 5c = -4$ $5c = 0$ $c = 0$
Therefore, $f(x) = \frac{(3-x^2)^{5/2}}{5} - (3 - x^2)^{3/2}$
Now, we need to find $f(1)$: $f(1) = \frac{(3-1)^{5/2}}{5} - (3 - 1)^{3/2}$ $f(1) = \frac{2^{5/2}}{5} - 2^{3/2}$ $f(1) = 2^{3/2}\left(\frac{2}{5} - 1\right)$
$f(1) = 2^{3/2}\left(-\frac{3}{5}\right)$ $f(1) = -\frac{3}{5} \cdot 2\sqrt{2}$ $f(1) = -\frac{6\sqrt{2}}{5}$

**Quick Tip**

Use substitution to simplify the integral and then use the given condition to find the constant of integration.

---

**16. Let $a_1, a_2, a_3, ...$ be a G.P. of increasing positive numbers. If $a_3a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to**

(1) 131
(2) 130
(3) 129
(4) 128

**Correct Answer:** (3) 129

**Solution:** Let the $1^{st}$ term of G.P. be a and common ratio be r. Given $a_3a_5 = 729$
$ar^2 \cdot ar^4 = 729$ $a^2r^6 = 729$ $ar^3 = 27$ ... (i)
Also, $a_2 + a_4 = \frac{111}{4}$ $ar + ar^3 = \frac{111}{4}$ $ar = \frac{111}{4} - 27 = \frac{111-108}{4} = \frac{3}{4}$ ... (ii)
Dividing (i) by (ii): $\frac{ar^3}{ar} = \frac{27}{3/4}$ $r^2 = 36$ $r = 6$ (since the G.P. is increasing, r ¿ 0)
From (ii): $a(6) = \frac{3}{4}$ $a = \frac{1}{8}$
Now, $24(a_1 + a_2 + a_3) = 24(a + ar + ar^2) = 24a(1 + r + r^2) = 24 \cdot \frac{1}{8}(1 + 6 + 36) = 3(43) = 129$

**Quick Tip**

Use the given conditions to form equations and solve for the first term and common ratio of the G.P.

---

**17. Let the domain of the function $f(x) = \log_2 \log_4 \log_6(3 + 4x - x^2)$ be (a, b). If $\int_0^{a+b}[x^2]dx = p - q\sqrt{r}$, $p, q, r \in N$, gcd(p, q, r) = 1, where [.] is the greatest integer function, then p + q + r is equal to**

(1) 10
(2) 8
(3) 11
(4) 9

**Correct Answer:** (1) 10

**Solution:** $\log_6(3 + 4x - x^2) > 1$ $3 + 4x - x^2 > 6$ $x^2 - 4x + 3 < 0$ $(x - 1)(x - 3) < 0$ $x \in (1, 3)$
So $a = 1$ and $b = 3$
$\Rightarrow \int_0^2 [x^2]dx = ?$
$I = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{2} [x^2]dx$
$= 0 + |x|_1^{\sqrt{2}} + 2|x|_{\sqrt{2}}^{\sqrt{3}} + 3|x|_{\sqrt{3}}^{2}$
$= (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$
$= 5 - \sqrt{2} - \sqrt{3}$
$\Rightarrow p + q + r = 10$

**Quick Tip**

Find the domain of the function by solving the inequalities and then evaluate the integral by splitting it into intervals where $[x^2]$ is constant.

---

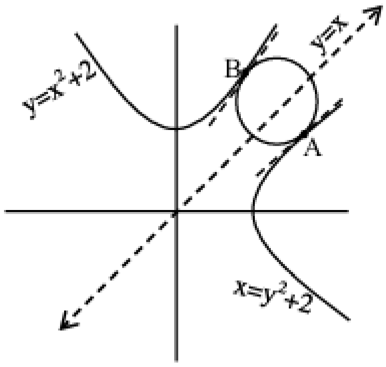
**18. The radius of the smallest circle which touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is**

(1) $\frac{7\sqrt{2}}{2}$

(2) $\frac{7\sqrt{2}}{16}$
(3) $\frac{7\sqrt{2}}{4}$
(4) $\frac{7\sqrt{2}}{8}$

**Correct Answer:** (4) $\frac{7\sqrt{2}}{8}$

**Solution:** The given parabolas are symmetric about the line $y = x$.




Tangents at A and B must be parallel to $y = x$ line, so slope of the tangents $= 1$.
$\left(\frac{dy}{dx}\right)_{\min A} = 1 = \left(\frac{dy}{dx}\right)_{\min B}$
For $y = x^2 + 2$, $\frac{dy}{dx} = 2x$ $2x = 1$ $x = \frac{1}{2}$ $y = \left(\frac{1}{2}\right)^2 + 2 = \frac{1}{4} + 2 = \frac{9}{4}$ So, point A is $\left(\frac{1}{2}, \frac{9}{4}\right)$.
For $x = y^2 + 2$, $1 = 2y\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{1}{2y}$ $\frac{1}{2y} = 1$ $y = \frac{1}{2}$ $x = \left(\frac{1}{2}\right)^2 + 2 = \frac{1}{4} + 2 = \frac{9}{4}$ So, point B is $\left(\frac{9}{4}, \frac{1}{2}\right)$.
Distance between A and B: $AB = \sqrt{\left(\frac{9}{4} - \frac{1}{2}\right)^2 + \left(\frac{1}{2} - \frac{9}{4}\right)^2}$ $AB = \sqrt{2\left(\frac{7}{4}\right)^2}$ $AB = \frac{7\sqrt{2}}{4}$
The radius of the smallest circle is half of the distance AB. Radius $= \frac{AB}{2} = \frac{7\sqrt{2}}{8}$

**Quick Tip**

Use the symmetry of the parabolas about the line $y = x$ to find the points of tangency and then calculate the distance between them.

---

**19. Let** $f(x) = \begin{cases} (1+ax)^{1/x} & , x < 0 \\ 1 + b & , x = 0 \\ \frac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2} & , x > 0 \end{cases}$ **be continuous at x = 0. Then** $e^a bc$ **is equal to**

(1) 64
(2) 72
(3) 48
(4) 36

**Correct Answer:** (3) 48

**Solution:** For continuity at x = 0, we need $f(0^-) = f(0) = f(0^+)$.

$f(0^-) = \lim_{x \to 0^-} (1 + ax)^{1/x} = e^{\lim_{x \to 0^-} \frac{1}{x} \ln(1+ax)} = e^{\lim_{x \to 0^-} \frac{ax}{x}} = e^a$
$f(0) = 1 + b$
$f(0^+) = \lim_{x \to 0^+} \frac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2}$ Using L'Hopital's rule: $f(0^+) = \lim_{x \to 0^+} \frac{\frac{1}{2}(x+4)^{-1/2}}{\frac{1}{3}(x+c)^{-2/3}}$
$f(0^+) = \frac{\frac{1}{2}(4)^{-1/2}}{\frac{1}{3}(c)^{-2/3}} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{3}c^{-2/3}} = \frac{\frac{1}{4}}{\frac{1}{3}c^{-2/3}} = \frac{3}{4}c^{2/3}$
From $f(0^-) = f(0)$, we have $e^a = 1 + b$. From $f(0) = f(0^+)$, we have $1 + b = \frac{3}{4}c^{2/3}$.
Also, we know that if $(x + c)^{1/3} - 2$ is in the denominator, then $(x + c)^{1/3} - 2 = 0$ at x = 0.
$c^{1/3} - 2 = 0$ $c^{1/3} = 2$ $c = 8$
Now, $1 + b = \frac{3}{4}(8)^{2/3} = \frac{3}{4}(2^3)^{2/3} = \frac{3}{4} \cdot 4 = 3$ $b = 2$
Also, $e^a = 1 + b = 3$ $a = \ln 3$
Therefore, $e^a bc = 3 \cdot 2 \cdot 8 = 48$

**Quick Tip**

For continuity, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$. Use L'Hopital's rule when dealing with indeterminate forms.

---

**20. Line $L_1$ passes through the point (1, 2, 3) and is parallel to z-axis. Line $L_2$ passes through the point $(\lambda, 5, 6)$ and is parallel to y-axis. Let for $\lambda = \lambda_1, \lambda_2, \lambda_2 < \lambda_1$, the shortest distance between the two lines be 3. Then the square of the distance of the point $(\lambda_1, \lambda_2, 7)$ from the line $L_1$ is**

(1) 40
(2) 32
(3) 25
(4) 37

**Correct Answer:** (3) 25

**Solution:** The equation of line $L_1$ is: $\frac{x-1}{0} = \frac{y-2}{0} = \frac{z-3}{1}$
The equation of line $L_2$ is: $\frac{x-\lambda}{0} = \frac{y-5}{1} = \frac{z-6}{0}$
The shortest distance (SD) between the lines is given by: $SD = \frac{\begin{vmatrix} \lambda - 1 & 5 - 2 & 6 - 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}{\sqrt{0^2+1^2+0^2}}$
$SD = \begin{vmatrix} \lambda - 1 & 3 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$ $SD = |\lambda - 1|$
Given SD = 3, we have: $|\lambda - 1| = 3$ $\lambda - 1 = \pm 3$ $\lambda = 4, -2$
Since $\lambda_2 < \lambda_1$, we have: $\lambda_1 = 4$ $\lambda_2 = -2$
The point is $P(4, -2, 7)$.
Let Q be the foot of the perpendicular from P to $L_1$. Q is of the form $(1, 2, 3 + t)$.
The direction vector of PQ is $(1 - 4, 2 - (-2), 3 + t - 7) = (-3, 4, t - 4)$. The direction vector of $L_1$ is $(0, 0, 1)$.

Since PQ is perpendicular to $L_1$, their dot product is 0. $(-3, 4, t-4)\cdot(0,0,1) = 0$ $t - 4 = 0$ $t = 4$
So, Q is $(1, 2, 7)$.
Now, $PQ^2 = (4-1)^2 + (-2-2)^2 + (7-7)^2$ $PQ^2 = 3^2 + (-4)^2 + 0^2$ $PQ^2 = 9 + 16 = 25$

**Quick Tip**

Use the formula for the shortest distance between two skew lines. To find the foot of the perpendicular, use the dot product of the direction vectors.

---

**21. All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number $n$ be denoted by $W_n$. Let the probability $P(W_n)$ of choosing the word $W_n$ satisfy $P(W_n) = 2P(W_{n-1})$, $n > 1$. If $P(CDBEA) = \frac{2^\alpha}{2^\beta - 1}$, $\alpha, \beta \in N$, then $\alpha + \beta$ is equal to :**

**Correct Answer:** (183)

**Solution:** Let $P(W_1) = x$. Given $P(W_n) = 2P(W_{n-1})$. Then, $P(W_2) = 2x$, $P(W_3) = 2^2x$, ..., $P(W_n) = 2^{n-1}x$.
Since $\sum_{i=1}^{120} P(W_i) = 1$, we have

$$x + 2x + 2^2x + \cdots + 2^{119}x = 1$$

$$x(1 + 2 + 2^2 + \cdots + 2^{119}) = 1$$

$$x \cdot \frac{2^{120} - 1}{2 - 1} = 1$$

$$x(2^{120} - 1) = 1$$

$$x = \frac{1}{2^{120} - 1} \tag{1}$$

Now, let's find the rank of CDBEA. A_ _ _ _ $= 4! = 24$ B_ _ _ _ $= 4! = 24$ CA_ _ _ $= 3! = 6$ CB_ _ _ $= 3! = 6$ CDA_ _ $= 2! = 2$ CDBAE $= 1$ CDBEA $= 1$
So, the rank of CDBEA is

$$24 + 24 + 6 + 6 + 2 + 1 = 63$$

Thus, CDBEA is $W_{64}$.
Therefore,

$$P(CDBEA) = P(W_{64}) = 2^{63}\cdot P(W_1) = 2^{63}\cdot\frac{1}{2^{120} - 1}$$

$$P(CDBEA) = \frac{2^{63}}{2^{120} - 1}$$

Comparing with $P(CDBEA) = \frac{2^\alpha}{2^\beta - 1}$, we have:

$$\alpha = 63, \quad \beta = 120$$

$$\alpha + \beta = 63 + 120 = \boxed{183}$$

**Quick Tip**

Use the given recursive relation to find the probability of each word. Calculate the rank of the given word to find its serial number.

---

**22. Let the product of the focal distances of the point P(4, $2\sqrt{3}$) on the hyperbola H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be 32. Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then $p^2 + q^2$ is equal to .....**

**Correct Answer:** (120)

**Solution:** Given hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (1)

Point $P(4, 2\sqrt{3})$ lies on $H$. Given $PS_1 \cdot PS_2 = 32$ Also, $|PS_1 - PS_2| = 2a$

Since $P(4, 2\sqrt{3})$ lies on $H$:

$$\frac{16}{a^2} - \frac{12}{b^2} = 1$$

$$16b^2 - 12a^2 = a^2b^2 \quad (2)$$

$$|PS_1 - PS_2|^2 = 4a^2$$

$$PS_1^2 + PS_2^2 - 2 \cdot PS_1 \cdot PS_2 = 4a^2$$

$$(ae - 4)^2 + 12 + (ae + 4)^2 + 12 - 2(32) = 4a^2$$

$$2a^2e^2 - 8 = 4a^2$$

$$a^2e^2 - 4 = 2a^2$$

$$b^2 = a^2(e^2 - 1) = 2a^2 \Rightarrow b^2 - a^2 = 4 \quad (3)$$

From (2) and (3):

$$16(a^2 + 4) - 12a^2 = a^2(a^2 + 4)$$

$$16a^2 + 64 - 12a^2 = a^4 + 4a^2$$

$$a^4 = 64 \Rightarrow a^2 = 8$$

$$b^2 = 12$$

$$p = 2b \quad \text{(length of conjugate axis)}$$

$$q = \frac{2b^2}{a} \quad \text{(length of latus rectum)}$$

$$p^2 + q^2 = 4b^2 + \frac{4b^4}{a^2}$$

$$p^2 + q^2 = 4(12) + \frac{4(12^2)}{8}$$

$$p^2 + q^2 = 48 + 72 = \boxed{120}$$

Quick Tip

Use the properties of hyperbola, including the relationship between focal distances and the equation of the hyperbola, to solve for the required values.

---

**23. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$. If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d} + \mu\vec{c}|^2$ is equal to ____.**

**Correct Answer:** (5)

**Solution:** Given $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$

$$\vec{a} \times \hat{d} - \vec{b} \times \hat{d} = 0$$

$$(\vec{a} - \vec{b}) \times \hat{d} = 0$$

This means $\vec{a} - \vec{b}$ is parallel to $\hat{d}$.

$$\vec{a} - \vec{b} = (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} + 2\hat{j} - \hat{k}) = -2\hat{i} - \hat{j} + 2\hat{k}$$

Let

$$\hat{d} = \frac{\vec{a} - \vec{b}}{|\vec{a} - \vec{b}|} = \frac{-2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{(-2)^2 + (-1)^2 + 2^2}} = \frac{-2\hat{i} - \hat{j} + 2\hat{k}}{3}$$

Given $\vec{c} \cdot \hat{d} = 1$:

$$(\lambda\hat{j} + \mu\hat{k}) \cdot \frac{-2\hat{i} - \hat{j} + 2\hat{k}}{3} = 1$$

$$\frac{-\lambda + 2\mu}{3} = 1$$

$$-\lambda + 2\mu = 3 \qquad (1)$$

Given $\vec{c}$ is perpendicular to $\vec{a}$, so $\vec{c} \cdot \vec{a} = 0$:

$$(\lambda\hat{j} + \mu\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\lambda + \mu = 0 \Rightarrow \mu = -\lambda \qquad (2)$$

Substituting (2) in (1), we get:

$$-\lambda - 2\lambda = 3$$

$$-3\lambda = 3 \Rightarrow \lambda = -1, \quad \mu = 1$$

So,

$$\vec{c} = -\hat{j} + \hat{k}$$

Now,

$$3\lambda\hat{d} + \mu\vec{c} = 3(-1) \cdot \frac{-2\hat{i} - \hat{j} + 2\hat{k}}{3} + (-\hat{j} + \hat{k})$$

$$= 2\hat{i} + \hat{j} - 2\hat{k} - \hat{j} + \hat{k} = 2\hat{i} - \hat{k}$$

$$|3\lambda\hat{d} + \mu\vec{c}|^2 = |2\hat{i} - \hat{k}|^2 = 2^2 + (-1)^2 = 4 + 1 = \boxed{5}$$

Quick Tip

Use the given vector relations to find the vectors $\hat{d}$ and $\vec{c}$. Remember that if $\vec{a} \times \vec{b} = 0$, then $\vec{a}$ and $\vec{b}$ are parallel.

---

**24. If the number of seven-digit numbers, such that the sum of their digits is even, is $m \cdot n \cdot 10^a$; $m, n \in \{1, 2, 3, ..., 9\}$, then $m + n$ is equal to**

**Correct Answer:** (14)

**Solution:** Total 7 digit numbers = 9000000
7 digit numbers having sum of digits even = 4500000 = $9.5 \cdot 10^5$
$m = 9$, $n = 5$
$m + n = 14$

Quick Tip

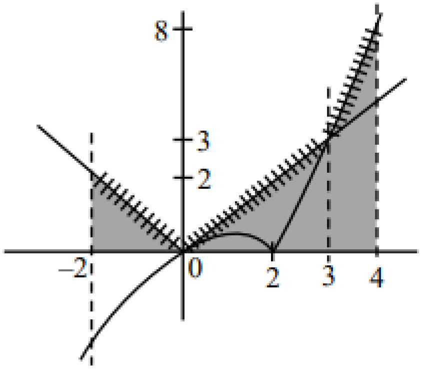
Half of the total 7 digit numbers will have an even sum of digits.

---

**25. The area of the region bounded by the curve $y = \max\{|x|, |x - 2|\}$, then x-axis and the lines x = -2 and x = 4 is equal to ____.**

**Correct Answer:** (12)

**Solution:** As given in the picture, the area is calculated as:




Required Area $= \frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 1 \times 11$
Required Area $= 2 + \frac{9}{2} + \frac{11}{2}$
Required Area $= 2 + \frac{20}{2}$
Required Area $= 2 + 10$
Required Area $= 12$
Thus, following the given solution, the area is 12.

Quick Tip

The solution provided in the picture calculates the area as a sum of triangular areas.

---

**26. During the melting of a slab of ice at 273 K at atmospheric pressure:**

(1) Internal energy of ice-water system remains unchanged.
(2) Positive work is done by the ice-water system on the atmosphere.
(3) Internal energy of the ice-water system decreases.
(4) Positive work is done on the ice-water system by the atmosphere.

**Correct Answer:** (4)

**Solution:** Volume decreases during melting of ice so positive work is done on ice water system by atmosphere. Heat absorbed by ice water so $\Delta Q$ is positive, work done by ice water system is negative. Hence by first law of thermodynamics $\Delta U = \Delta Q + \Delta W =$ Positive So internal energy increases.

Quick Tip

Apply the first law of thermodynamics and consider the sign conventions for work and heat. Melting ice results in a decrease in volume, indicating work done on the system.

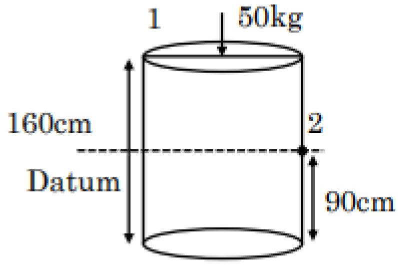
---

**27. Consider a completely full cylindrical water tank of height 1.6 m and cross-sectional area 0.5 $m^2$. It has a small hole in its side at a height 90 cm from the bottom. Assume, the cross-sectional area of the hole to be negligibly small as compared to that of the water tank. If a load 50 kg is applied at the top surface of the water in the tank then the velocity of the water coming out at the instant when the hole is opened is : (g = 10 $m/s^2$)**

(1) 3 m/s
(2) 5 m/s
(3) 2 m/s
(4) 4 m/s

**Correct Answer:** (4) 4 m/s

**Solution:**




Apply Bernoulli equation between points 1 and 2.
$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h = P_2 + \frac{1}{2}\rho v_2^2 + 0$
$P_0 + \frac{mg}{A} + \rho g \frac{70}{100} = P_0 + \frac{1}{2}\rho v_2^2$
$\frac{5000}{0.5} + 10^3 \times 10 \times \frac{70}{100} = \frac{1}{2} \times 10^3 v_2^2$
$10^4 + 10^3 \times 7 = \frac{10^3}{2} v_2^2$
$v_2^2 = 16$
$v_2 = 4m/s$
As the tank area is large $v_1$ is negligible compared to $v_2$.

**Quick Tip**

Use Bernoulli's equation to relate the pressure, velocity, and height at two points in a fluid flow. Consider the pressure due to the applied load and the hydrostatic pressure.

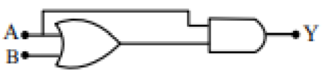
---

**28. Choose the correct logic circuit for the given truth table having inputs A and B.**

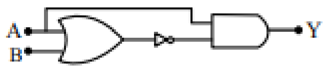
| Inputs | Output | |
|---|---|---|
| A | B | Y |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(1)

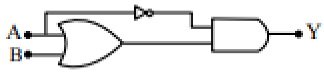

(2)




(3)




(4)




**Correct Answer:** (2)

**Solution:** Only option (2) matches with the truth table.

> **Quick Tip**
>
> Analyze each logic gate combination and compare its output with the given truth table to find the correct match.

---

**29. The radiation pressure exerted by a 450 W light source on a perfectly reflecting surface placed at 2m away from it, is :**

(1) 1.5 $\times 10^{-4}$ Pascals
(2) 0
(3) 6 $\times 10^{-5}$ Pascals
(4) 3 $\times 10^{-5}$ Pascals

**Correct Answer:** (3) 6 $\times 10^{-5}$ Pascals

**Solution:** $P_{rad} = \frac{2I}{C}$ Where I = intensity at surface C = Speed of light
$Power = \frac{450}{Area} = \frac{450}{4\pi r^2}$ $I = \frac{450}{4\pi \times 4} = \frac{450}{16\pi}$
$P_{rad} = \frac{2\times 450}{16\pi \times 3\times 10^8} = \frac{150}{8\pi \times 10^8}$
$= 5.97 \times 10^{-8} \approx 6 \times 10^{-8}$ Pascals

> **Quick Tip**
>
> Use the formula for radiation pressure on a perfectly reflecting surface. Remember to calculate the intensity of the light at the given distance.

---

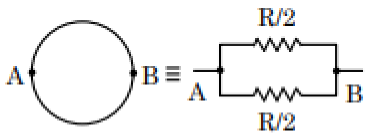
**30. A wire of length $25\,\text{m}$ and cross-sectional area $5\,\text{mm}^2$ having resistivity $2 \times 10^{-6}\,\Omega\cdot\text{m}$ is bent into a complete circle. The resistance between diametrically opposite points will be:**

(1) 12.5 $\Omega$

(2) 50 Ω
(3) 100 Ω
(4) 25 Ω

**Correct Answer:** (Bonus) NTA Ans. (4)

**Solution:** The wire is bent into a circle. The resistance between diametrically opposite points will be the resistance of two semicircles in parallel.



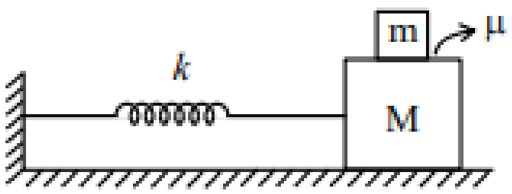

$L = 25$ m, $A = 5mm^2 = 5 \times 10^{-6}m^2$ $\rho = 2 \times 10^{-6}\Omega m$
$R_{wire} = \frac{\rho L}{A} = \frac{2\times10^{-6}\times25}{5\times10^{-6}} = 10\Omega$
The resistance of each semicircle is $\frac{R_{wire}}{2} = \frac{10}{2} = 5\Omega$
The equivalent resistance of two semicircles in parallel is: $R_{eq} = \frac{R/2}{2} = \frac{10}{4} = 2.5\Omega$
Answer does not match with NTA option.

**Quick Tip**

When a wire is bent into a circle, the resistance between diametrically opposite points is the parallel combination of the resistances of the two semicircles.

---

**31. Two blocks of masses m and M, (M ¿ m), are placed on a frictionless table as shown in figure. A massless spring with spring constant k is attached with the lower block. If the system is slightly displaced and released then ($\mu$ = coefficient of friction between the two blocks)**

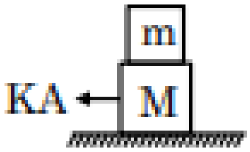


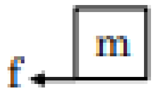

(A) The time period of small oscillation of the two blocks is T $= 2\pi\sqrt{\frac{M+m}{k}}$
(B) The acceleration of the blocks is a $= \frac{kx}{M+m}$ (x = displacement of the blocks from the mean position)
(C) The magnitude of the frictional force on the upper block is $\frac{m\mu x}{M+m}$
(D) The maximum amplitude of the upper block, if it does not slip, is $\frac{\mu(M+m)g}{k}$
(E) Maximum frictional force can be $\mu(M+m)g$.
Choose the **correct** answer from the options given below :

(1) A, B, D Only
(2) B, C, D Only
(3) C, D, E Only
(4) A, B, C Only

**Correct Answer:** (1) A, B, D Only

**Solution:** (A) As both blocks moving together so time period = $2\pi\sqrt{\frac{m}{k}}$; where m = M + m
T = $2\pi\sqrt{\frac{M+m}{k}}$
(B) Let block is displaced by x in (+ve) direction so force on block will be in (-ve) direction F = Kx (M + m)a = -Kx a = $\frac{-Kx}{M+m}$
(C) As upper block is moving due to friction thus f = ma = $\frac{mKx}{M+m}$
(D) This option is like two block problem in friction for maximum amplitude, force on block is also maximum, for which both blocks are moving together.



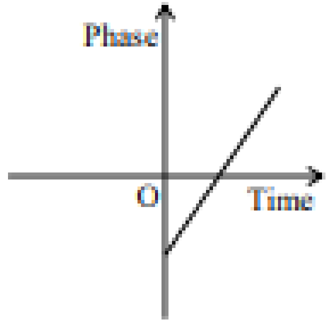

KA = (M + m)a a = $\frac{KA}{M+m}$ f = ma = $\frac{mKA}{M+m}$ $f_{max} = f_L = \mu mg$ f = $\mu mg$ $\frac{mKA}{M+m} = \mu mg$ A = $\frac{\mu(M+m)g}{K}$
(E) Maximum friction can be $\mu mg$ as force is acting between blocks and normal force here is mg.

**Quick Tip**

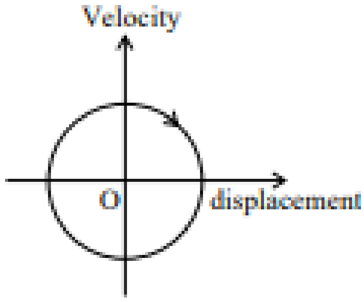
Analyze the forces acting on the blocks and apply Newton's second law to determine the acceleration and frictional force. Consider the conditions for maximum amplitude and maximum frictional force.

---

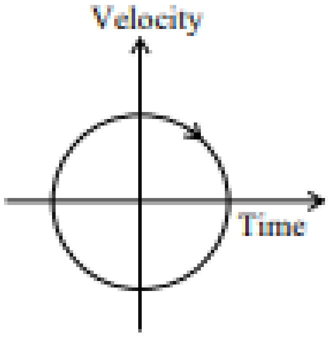
**32. Which of the following curves possibly represent one-dimensional motion of a particle?**



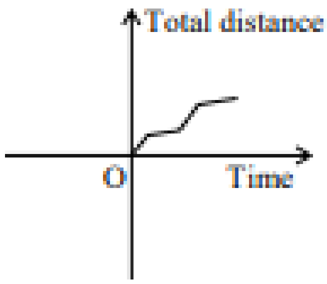

(A)




(B)




(C)




(D)

Choose the **correct** answer from the options given below :

(1) A, B and D only
(2) A, B and C only
(3) A and B only
(4) A, C and D only

**Correct Answer:** (1) A, B and D only

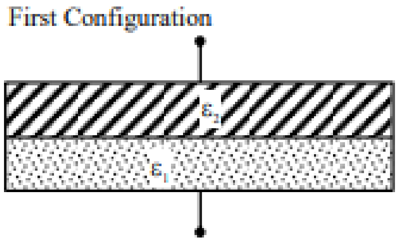
**Solution:** For option (A) $\phi = kt + C$ it can be 1D motion eg -¿ $x = A \sin \phi$ (SHM)
For option (B) $v^2 + x^2 =$ constant yes 1D
For option (C) time can't be negative Not possible
For option (D) Possible
A, B and D only

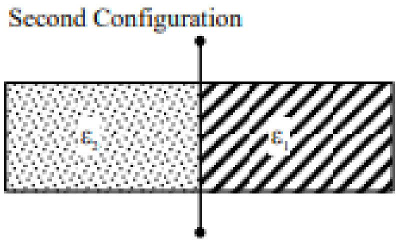
Quick Tip

Analyze each graph to determine if it can represent one-dimensional motion. Consider the physical quantities involved and their relationships.

---

**33. A parallel plate capacitor is filled equally (half) with two dielectrics of dielectric constant $\epsilon_1$ and $\epsilon_2$, as shown in figures. The distance between the plates is d and area of each plate is A. If capacitance in first configuration and second configuration are $C_1$ and $C_2$ respectively, then $\frac{C_1}{C_2}$ is:**




(1) $\frac{\epsilon_1\epsilon_2}{(\epsilon_1+\epsilon_2)^2}$
(2) $\frac{4\epsilon_1\epsilon_2}{(\epsilon_1+\epsilon_2)^2}$
(3) $\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}$
(4) $\frac{\epsilon_0(\epsilon_1+\epsilon_2)}{2}$

**Correct Answer:** (2) $\frac{4\epsilon_1\epsilon_2}{(\epsilon_1+\epsilon_2)^2}$

**Solution:** Let $C_0 = \frac{\epsilon_0 A}{d}$
First Configuration: Area of plate is A. Then $C = \frac{\epsilon_2\epsilon_0 A}{d/2} = \frac{2\epsilon_2\epsilon_0 A}{d} = 2\epsilon_2 C_0$
$C' = \frac{\epsilon_1\epsilon_0 A}{d/2} = \frac{2\epsilon_1\epsilon_0 A}{d} = 2\epsilon_1 C_0$
C and C' are in series.
$C_1 = \frac{CC'}{C+C'} = \frac{4\epsilon_1\epsilon_2 C_0^2}{2C_0(\epsilon_2+\epsilon_1)}$ $C_1 = \frac{2\epsilon_2\epsilon_1 C_0}{(\epsilon_2+\epsilon_1)}$
Second Configuration:
Here $C = \frac{\epsilon_1\epsilon_0 A}{2d} = \frac{\epsilon_1 C_0}{2}$ $C' = \frac{\epsilon_2 C_0}{2}$ C and C' are in parallel.
$C_2 = C' + C = (\epsilon_1 + \epsilon_2)\frac{C_0}{2}$
Thus $\frac{C_1}{C_2} = \frac{2\epsilon_1\epsilon_2 C_0}{(\epsilon_2+\epsilon_1)} \times \frac{2}{(\epsilon_1+\epsilon_2)C_0}$ $\frac{C_1}{C_2} = \frac{4\epsilon_1\epsilon_2}{(\epsilon_2+\epsilon_1)^2}$

Quick Tip

Use the formula for capacitance in series and parallel configurations. Remember to define a common term $C_0$ to simplify the calculations.

---

**34. Match the LIST-I with LIST-II**

| LIST-I | LIST-II |
|---|---|
| A. Gravitational constant | I. $[LT^{-2}]$ |
| B. Gravitational potential energy | II. $[L^2T^{-2}]$ |
| C. Gravitational potential | III. $[ML^2T^{-2}]$ |
| D. Acceleration due to gravity | IV. $[M^{-1}L^3T^{-2}]$ |

Choose the **correct** answer from the options given below :

(1) A-IV, B-III, C-II, D-I
(2) A-III, B-II, C-I, D-IV
(3) A-II, B-IV, C-III, D-I
(4) A-I, B-III, C-IV, D-II

**Correct Answer:** (1) A-IV, B-III, C-II, D-I

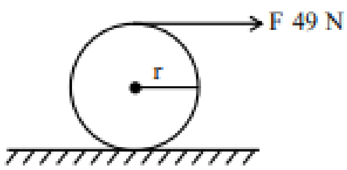
**Solution:** (A) $G = \frac{Fr^2}{m^2}$ $[G] = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$ (IV)
(B) P.E. = mgh = $[MLT^{-2}L] = [ML^2T^{-2}]$ (III)
(C) Gravitational Potential = $\frac{GM}{r}$ $[M^{-1}L^3T^{-2}]\frac{[M]}{[L]} = [M^0L^2T^{-2}] = [L^2T^{-2}]$ (II)
(D) Acceleration due to gravity = $\frac{GM}{r^2}$ $[M^{-1}L^3T^{-2}]\frac{[M]}{[L^2]} = [M^0LT^{-2}] = [LT^{-2}]$ (I)

Quick Tip

Use the formulas for gravitational constant, gravitational potential energy, gravitational potential, and acceleration due to gravity to derive their dimensional formulas.

---

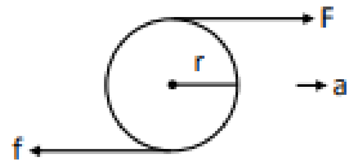
**35. A force of 49 N acts tangentially at the highest point of a sphere (solid) of mass 20 kg, kept on a rough horizontal plane. If the sphere rolls without slipping, then the acceleration of the center of the sphere is**



(1) 3.5 $m/s^2$
(2) 0.35 $m/s^2$
(3) 2.5 $m/s^2$
(4) 0.25 $m/s^2$

**Correct Answer:** (1) 3.5 $m/s^2$

**Solution:**



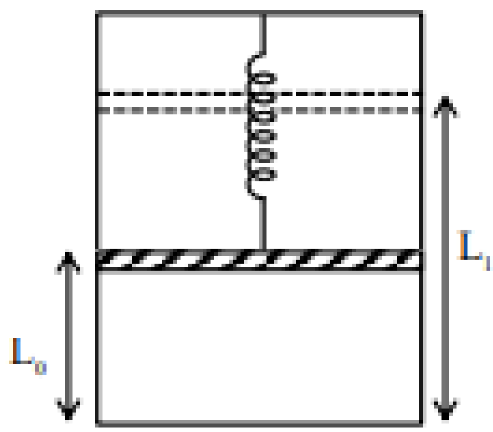

Torque about bottom point: $F \times 2r = I\alpha$ $49 \times 2r = \frac{7}{5}mr^2\alpha$ $14 - 4r\alpha$ As sphere rolls without slipping $a = r\alpha$ $a = \frac{14}{4} = \frac{7}{2} = 3.5m/s^2$

Quick Tip

Apply the torque equation about the point of contact to find the angular acceleration. Use the relation between linear acceleration and angular acceleration for rolling without slipping.

---

**36. A piston of mass M is hung from a massless spring whose restoring force law goes as F = -kx, where k is the spring constant of appropriate dimension. The piston separates the vertical chamber into two parts, where the bottom part is filled with 'n' moles of an ideal gas. An external work is done on the gas isothermally (at a constant temperature T) with the help of a heating filament (with negligible volume) mounted in lower part of the chamber, so that the piston goes up from a height $L_0$ to $L_1$, the total energy delivered by the filament is (Assume spring to be in its natural length before heating)**



(1) $3nRT\ln\left(\frac{L_1}{L_0}\right) + 2Mg(L_1 - L_0) + \frac{k}{3}(L_1^3 - L_0^3)$
(2) $nRT\ln\left(\frac{L_1}{L_0}\right) + \frac{Mg}{2}(L_1 - L_0) + \frac{k}{4}(L_1^4 - L_0^4)$
(3) $nRT\ln\left(\frac{L_1}{L_0}\right) + Mg(L_1 - L_0) + \frac{k}{4}(L_1^4 - L_0^4)$
(4) $nRT\ln\left(\frac{L_1}{L_0}\right) + Mg(L_1 - L_0) + \frac{3k}{4}(L_1^4 - L_0^4)$

**Correct Answer:** (3) $nRT\ln\left(\frac{L_1}{L_0}\right) + Mg(L_1 - L_0) + \frac{k}{4}(L_1^4 - L_0^4)$

**Solution:** Using WET: Total energy supplied = gravitational potential energy + spring potential energy + work done by gas
$Mg(L_1 - L_0) + \int_0^{L_1 - L_0} kxdx + nRT\ln\left(\frac{L_1}{L_0}\right) + W_{ext} = 0$
$\frac{k}{4}[x^4]_0^{L_1 - L_0} + Mg(L_1 - L_0) + \int_0^{L_1 - L_0} kxdx + nRT\ln\left(\frac{L_1}{L_0}\right) + W_{ext} = 0$
$\frac{k}{4}(L_1^4 - L_0^4) + Mg(L_1 - L_0) + nRT\ln\left(\frac{L_1}{L_0}\right) + W_{ext} = 0$
$W_{ext} = \frac{k}{4}(L_1^4 - L_0^4) + Mg(L_1 - L_0) + nRT\ln\left(\frac{L_1}{L_0}\right)$

Quick Tip

Apply the work-energy theorem (WET) to equate the total energy supplied to the change in potential energy and work done by the gas. Remember to consider the work done against gravity and the spring force.

---

**37. A gas is kept in a container having walls which are thermally non-conducting. Initially the gas has a volume of 800 $cm^3$ and temperature 27°C. The change in temperature when the gas is adiabatically compressed to 200 $cm^3$ is: (Take $\gamma = 1.5$ : $\gamma$ is the ratio of specific heats at constant pressure and at constant volume)**

(1) 327 K
(2) 600 K
(3) 522 K
(4) 300 K

**Correct Answer:** (4) 300 K

**Solution:** $V_1 = 800cm^3$ $V_2 = 200cm^3$ $T_1 = 300$ K
For adiabatic: $TV^{\gamma-1} =$ constant.
$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$
$(300)(800)^{1.5-1} = T_2(200)^{1.5-1}$
$T_2 = 300\left(\frac{800}{200}\right)^{0.5}$
$T_2 = 300(4)^{1/2}$
$T_2 = 600$ K
$\Delta T = 600 - 300 = 300$ K

Quick Tip

Use the adiabatic process equation to relate the initial and final temperatures and volumes. Remember to use consistent units for volume.

---

**38. Match the LIST-I with LIST-II**

| LIST-I | LIST-II |
|---|---|
| A. $^{236}_{92}U \rightarrow ^{94}_{38} Sr + ^{140}_{54} Xe + 2n$ | I. Chemical Reaction |
| B. $2H_2 + O_2 \rightarrow 2H_2O$ | II. Fusion with +ve Q value |
| C. $^{3}_{1}H + ^{2}_{1} H \rightarrow ^{4}_{2} He + n$ | III. Fission |
| D. $^{1}_{1}H + ^{3}_{1} H \rightarrow ^{4}_{2} H + \gamma$ | IV. Fusion with -ve Q value |

Choose the **correct** answer from the options given below :

(1) A-II, B-I, C-III, D-IV
(2) A-III, B-I, C-II, D-IV
(3) A-II, B-I, C-IV, D-III
(4) A-III, B-I, C-IV, D-II

**Correct Answer:** (2) A-III, B-I, C-II, D-IV

**Solution:** Conceptual

Quick Tip

Identify the type of reaction based on the reactants and products. Fission involves breaking a large nucleus into smaller ones, fusion involves combining smaller nuclei into a larger one, and chemical reactions involve changes in electron configuration.

---

**39. The electrostatic potential on the surface of uniformly charged spherical shell of radius R = 10 cm is 120 V. The potential at the centre of shell, at a distance r = 5 cm from centre, and at a distance r = 15 cm from the centre of the shell respectively, are:**

(1) 120V, 120V, 80V
(2) 40V, 40V, 80V
(3) 0V, 0V, 80V
(4) 0V, 120V, 40V

**Correct Answer:** (1) 120V, 120V, 80V

**Solution:** Potential inside shell is equal to potential on surface.
$V_{in} = V_{surface} = \frac{kQ}{R} = 120V$

at r = 15 cm $V = \frac{kQ}{r} = \frac{120 \times 10}{15} = 80V$

**Quick Tip**

Remember that the electrostatic potential inside a uniformly charged spherical shell is constant and equal to the potential on the surface. Outside the shell, the potential varies inversely with the distance from the center.

---

**40. The work function of a metal is 3 eV. The color of the visible light that is required to cause emission of photoelectrons is**

(1) Green
(2) Blue
(3) Red
(4) Yellow

**Correct Answer:** (2) Blue

**Solution:** $(KE)_{max} = \frac{hc}{\lambda} - \phi$
$\frac{hc}{\lambda} > \phi$ [for emission]
$\lambda < \frac{hc}{\phi}$
$\lambda < \frac{1242}{3} nm$
So blue light option (B)

**Quick Tip**

Use the photoelectric equation to relate the energy of incident light to the work function and kinetic energy of emitted electrons. Remember that shorter wavelengths correspond to higher energy photons, which are required to overcome the work function.

---

**41. A particle is released from height S above the surface of the earth. At certain height its kinetic energy is three times its potential energy. The height from the surface of the earth and the speed of the particle at that instant are respectively.**

(1) $\frac{S}{2}, \sqrt{\frac{3gS}{2}}$
(2) $\frac{S}{2}, \frac{3gS}{2}$
(3) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$
(4) $\frac{S}{4}, \frac{3gS}{2}$

**Correct Answer:** (3) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

**Solution:** $V^2 = 0 + 2g(S - x)$ $V^2 = 2g(S - x)$
At B, Potential energy = mgx Kinetic energy = $\frac{1}{2}mv^2$ $\frac{1}{2}mv^2 = 3mgx$ $gx = \frac{1}{6}v^2 = \frac{1}{6}2g(S - x)$
$4x = S$ $x = \frac{S}{4}$
$V = \sqrt{2g \times \frac{3S}{4}} = \sqrt{\frac{3gS}{2}}$

**Quick Tip**

Use the conservation of energy principle to relate the potential and kinetic energies at different heights. Remember that the potential energy is proportional to the height above the surface.

---

**42. A person measures mass of 3 different particles as 435.42 g, 226.3 g and 0.125 g. According to the rules for arithmetic operations with significant figures, the additions of the masses of 3 particles will be.**

(1) 661.845 g
(2) 662 g
(3) 661.8 g
(4) 661.84 g

**Correct Answer:** (3) 661.8 g

**Solution:** $m_1 + m_2 + m_3 = 435.42 + 226.3 + 0.125 = 661.845$ g
According to least significant digits m = 661.8 g

**Quick Tip**

When adding or subtracting measurements, the result should have the same number of decimal places as the measurement with the fewest decimal places.

---

**43. The radii of curvature for a thin convex lens are 10 cm and 15 cm respectively. The focal length of the lens is 12 cm. The refractive index of the lens material is**

(1) 1.2
(2) 1.4
(3) 1.5
(4) 1.8

**Correct Answer:** (3) 1.5

**Solution:** $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
$\frac{1}{12} = (\mu - 1)\left(\frac{1}{10} - \frac{1}{-15}\right)$

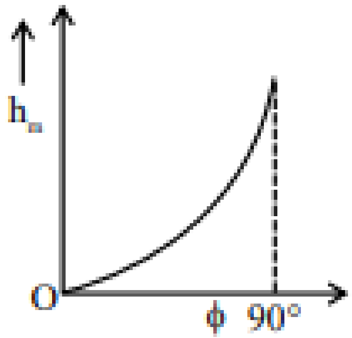
$\frac{1}{12} = (\mu - 1)\left(\frac{3+2}{30}\right)$
$\mu = \frac{3}{2}$

**Quick Tip**

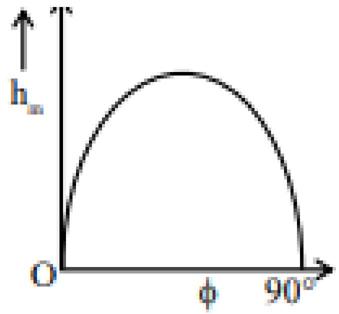
Use the lensmaker's formula to relate the focal length, radii of curvature, and refractive index of the lens material.

---

**44. The angle of projection of a particle is measured from the vertical axis as $\phi$ and the maximum height reached by the particle is $h_m$. Here $h_m$ as function of $\phi$ can be presented as**



(1)



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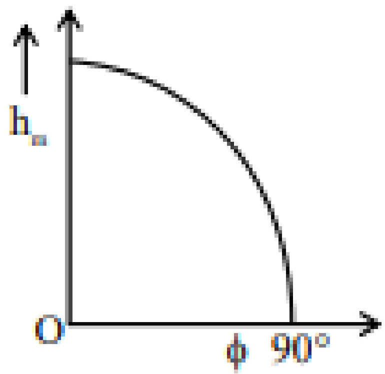

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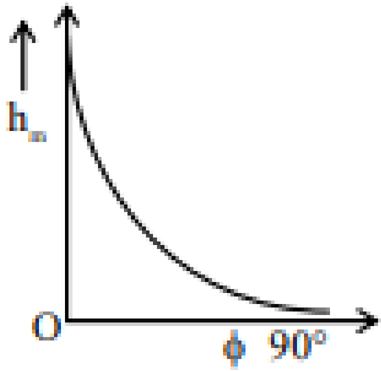
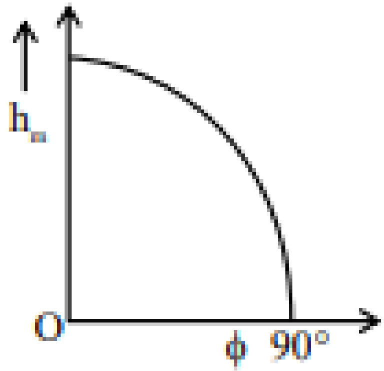
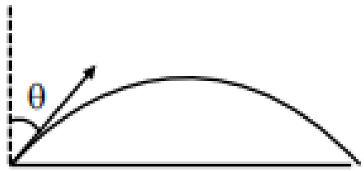
(4) 


**Correct Answer:** (3)

**Solution:**




$H_{max} = \frac{u^2 \cos^2 \phi}{2g}$

**Quick Tip**

Use the formula for maximum height in projectile motion. Since the angle is measured from the vertical, use $\cos\phi$ instead of $\sin\theta$.

---

**45. Consider following statements for refraction of light through prism, when angle of deviation is minimum.**

(A) The refracted ray inside prism becomes parallel to the base.
(B) Larger angle prisms provide smaller angle of minimum deviation.
(C) Angle of incidence and angle of emergence becomes equal.
(D) There are always two sets of angle of incidence for which deviation will be same except at minimum deviation setting.
(E) Angle of refraction becomes double of prism angle.
Choose the **correct** answer from the options given below:

(1) A, C and D Only
(2) B, C and D Only
(3) A, B and E Only
(4) B, D and E Only

**Correct Answer:** (1) A, C and D Only

**Solution:** $\delta = I + e - A$
For $\delta_{min} \Rightarrow I = e$ and refracted ray is parallel to base
A, C, D are correct

**Quick Tip**

Remember the conditions for minimum deviation in a prism. The refracted ray becomes parallel to the base, and the angle of incidence equals the angle of emergence.

---

**46. Three identical spheres of mass m, are placed at the vertices of an equilateral triangle of length a. When released, they interact only through gravitational force and collide after a time T = 4 seconds. If the sides of the triangle are increased to length 2a and also the masses of the spheres are made 2m, then they will collide after ______ seconds.**

**Correct Answer:** (8)

**Solution:**

$$T \propto m^x G^y a^z$$

$$T \propto M^x \left[M^{-1}L^3T^{-2}\right]^y [L]^z$$

$$T \propto M^{x-y} L^{3y+z} T^{-2y}$$

$$x - y = 0 \Rightarrow x = y$$

$$-2y = 1 \Rightarrow y = -\frac{1}{2}$$

$$3y + z = 0 \Rightarrow z = -3y = \frac{3}{2}$$

$$\Rightarrow T \propto m^{-\frac{1}{2}} G^{-\frac{1}{2}} a^{\frac{3}{2}}$$

$$T \propto \frac{a^{3/2}}{\sqrt{m}}$$

$$T = 4\left(\frac{2a}{a}\right)^{3/2} = 8s$$

**Quick Tip**

Use dimensional analysis to find the relationship between the collision time and the given parameters.

---

**47. A 4.0 cm long straight wire carrying a current of 8A is placed perpendicular to an uniform magnetic field of strength 0.15 T. The magnetic force on the wire is ______ mN.**

**Correct Answer:** (48)

**Solution:**

$$F = IlB$$
$$F = 8 \times \frac{4}{100} \times 0.15$$
$$F = \frac{48 \times 100}{10000} N$$
$$F = 48 \times 10^{-3} N$$
$$F = 48 \text{ mN}$$

**Quick Tip**

Use the formula for the magnetic force on a current-carrying wire in a magnetic field. Remember to convert the length to meters.

---

**48. Two coherent monochromatic light beams of intensities 4I and 9I are superimposed. The difference between the maximum and minimum intensities in the resulting interference pattern is xI. The value of x is ______.**

**Correct Answer:** (24)

**Solution:**

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$
$$I_{max} = (\sqrt{4I} + \sqrt{9I})^2$$
$$I_{max} = (2\sqrt{I} + 3\sqrt{I})^2$$
$$I_{max} = (5\sqrt{I})^2 = 25I$$

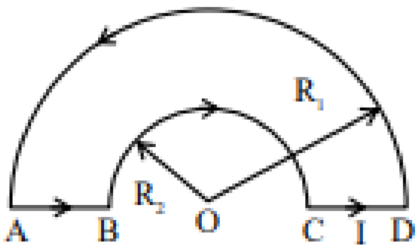
$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$
$$I_{min} = (\sqrt{4I} - \sqrt{9I})^2$$
$$I_{min} = (2\sqrt{I} - 3\sqrt{I})^2$$
$$I_{min} = (-\sqrt{I})^2 = I$$
$$I_{max} - I_{min} = 24I$$
$$x = 24$$

**Quick Tip**

Use the formulas for maximum and minimum intensities in interference. Remember that the intensities are proportional to the square of the amplitudes.

---

**49. A loop ABCD, carrying current $I = 12\,$A, is placed in a plane, consists of two semi-circular segments of radius $R_1 = 6\pi\,$m and $R_2 = 4\pi\,$m. The magnitude of the resultant magnetic field at center O is $k \times 10^{-7}\,$T. The value of $k$ is ______ (Given $\mu_0 = 4\pi \times 10^{-7}$ T m A$^{-1}$)**




**Correct Answer:** (1)

**Solution:**

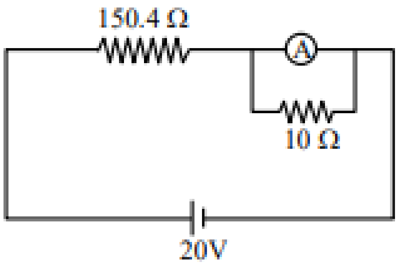
$$B_0 = |B_{R_1} - B_{R_2}|$$
$$B_0 = \left|\frac{\mu_0 I}{4R_2} - \frac{\mu_0 I}{4R_1}\right|$$
$$B_0 = \frac{4\pi \times 10^{-7} \times 12}{4}\left|\frac{1}{4\pi} - \frac{1}{6\pi}\right|$$
$$B_0 = 12\pi \times 10^{-7}\left|\frac{1}{12\pi}\right|$$
$$B_0 = 1 \times 10^{-7}T$$
$$k = 1$$

Quick Tip

Use the formula for the magnetic field due to a circular arc at its center. Remember to consider the direction of the magnetic field due to each segment and subtract them.

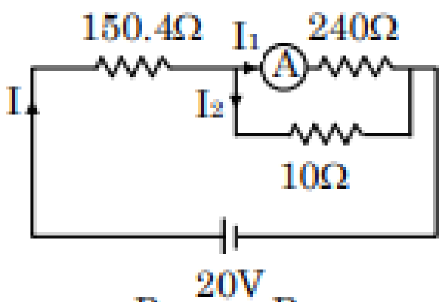
---

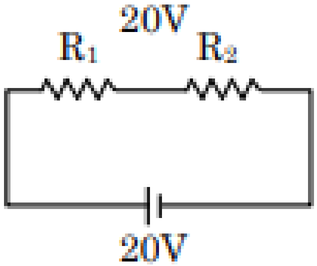
**50. In the figure shown below, a resistance of 150.4 Ω is connected in series to an ammeter A of resistance 240 Ω. A shunt resistance of 10 Ω is connected in parallel with the ammeter. The reading of the ammeter is ______ mA.**




**Correct Answer:** (5) NTA Ans. (125)

**Solution:**




$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 150.4 + \frac{240 \times 10}{250}$$

$$R_{eq} = 150.4 + 9.6 = 160\Omega$$

$$I_1 = \frac{IR_2}{240}$$

$$I_1 = \frac{I \times 9.6}{240}$$

$$I = \frac{20}{160}$$

$$I_1 = \frac{20}{160} \times \frac{9.6}{240} = \frac{1}{200} = 5 \times 10^{-3} A = 5mA$$

**Quick Tip**

Calculate the equivalent resistance of the circuit. Then use Ohm's law and current division to find the current through the ammeter.

---

**51. Which of the following postulate of Bohr's model of hydrogen atom in not in agreement with quantum mechanical model of an atom ?**

(1) An atom in a stationary state does not emit electromagnetic radiation as long as it stays in the same state
(2) An atom can take only certain distinct energies $E_1$, $E_2$, $E_3$, etc. These allowed states of constant energy are called the stationary states of atom
(3) When an electron makes a transition from a higher energy stationary state to a lower energy stationary state, then it emits a photon of light
(4) The electron in a H atom's stationary state moves in a circle around the nucleus

**Correct Answer:** (4)

**Solution:** The electron in a H-atom's stationary state moves in a spherical path.

**Quick Tip**

Bohr's model assumes electrons move in circular orbits, while quantum mechanics describes electrons in terms of probability distributions (orbitals).

---

**52. Given below are two statements: Statement I : The N-N single bond is weaker and longer than that of P-P single bond Statement II : Compounds of group 15 elements in +3 oxidation states readily undergo disproportionation reactions. In the light of above statements, choose the correct answer from the options given below**

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true

**Correct Answer:** (2)

**Solution:** N-N single bond weaker than P-P due to more lp-lp repulsion. Bond length $d_{N-N} > d_{P-P}$ (size↑, B.L.↑) In group 15 elements only N and P show disproportionation in +3 oxidation state. As, Sb and Bi have almost inert for disproportionation in +3 oxidation state. So both statements are false.

> **Quick Tip**
>
> Consider the bond strengths and lengths in nitrogen and phosphorus. Recall the trend of disproportionation reactions in group 15 elements.

---

**53. Given below are two statements:**
**Statement I:** A catalyst cannot alter the equilibrium constant ($K_c$) of the reaction, temperature remaining constant.
**Statement II:** A homogeneous catalyst can change the equilibrium composition of a system, temperature remaining constant.
**In the light of the above statements, choose the correct answer from the options given below.**

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is true but Statement II is false

**Correct Answer:** (2)

**Solution:** A catalyst can change equilibrium composition if it is added at constant pressure, but it can not change equilibrium constant.

> **Quick Tip**
>
> A catalyst affects the rate of a reaction, but not the equilibrium constant. It can alter the equilibrium composition by changing the forward and reverse reaction rates equally.

---

**54. The metal ions that have the calculated spin only magnetic moment value of 4.9 B.M. are A.** $Cr^{2+}$ **B.** $Fe^{2+}$ **C.** $Fe^{3+}$ **D.** $Co^{2+}$ **E.** $Mn^{2+}$ **Choose the correct answer from the options given below**

(1) A, C and E only
(2) B and E only
(3) B and E only
(4) A, B and E only

**Correct Answer:** (4)

**Solution: Given magnetic moment = 4.9 B.M.**
We know, M.M = $\sqrt{n(n+2)}$ B.M.
Where, $n$ = Number of unpaired electrons ($e^-$)
$4.9 = \sqrt{n(n+2)}$
We get $n = 4$
(A) $Cr^{2+}$ = [Ar] $3d^4$ (4 unpaired $e^-$)
(B) $Fe^{2+}$ = [Ar] $3d^6$ (4 unpaired $e^-$)
(C) $Fe^{3+}$ = [Ar] $3d^5$ (5 unpaired $e^-$)
(D) $Co^{2+}$ = [Ar] $3d^7$ (3 unpaired $e^-$)
(E) $Mn^{2+}$ = [Ar] $3d^5$ (5 unpaired $e^-$)

**Quick Tip**

Use the spin-only magnetic moment formula to calculate the number of unpaired electrons. Then, determine the electronic configurations of the given metal ions and identify those with 4 unpaired electrons.

---

**55. In a reaction A + B → C, initial concentrations of A and B are related as $[\mathbf{A}]_0 = 8[\mathbf{B}]_0$. The half lives of A and B are 10 min and 40 min, respectively. If they start to disappear at the same time, both following first order kinetics, after how much time will the concentration of both the reactants be same?**

(1) 60 min
(2) 80 min
(3) 20 min
(4) 40 min

**Correct Answer:** (4)

**Solution:**

$$\text{Given: } [A]_0 = 8[B]_0$$

$$t_{1/2(A)} = 10 \text{ min}$$

$$t_{1/2(B)} = 40 \text{ min}$$

$$1^{st} \text{ order kinetics}$$

$$t = ?$$

$$[A] = [B]$$

$$-k_A \times t = \ln \frac{[A]}{[A]_0}$$

$$[A] = [A]_0 e^{-k_A t}$$

$$[B] = [B]_0 e^{-k_B t}$$

$$[A] = [B]$$

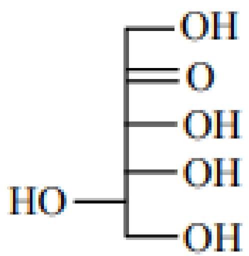
$$[A]_0 e^{-k_A t} = [B]_0 e^{-k_B t}$$
$$8[B]_0 e^{-k_A t} = [B]_0 e^{-k_B t}$$
$$8 = e^{(k_A - k_B)t}$$
$$\ln 8 = (k_A - k_B)t$$
$$t = \frac{\ln 8}{k_A - k_B}$$
$$t = \frac{\ln 8}{\frac{\ln 2}{10} - \frac{\ln 2}{40}}$$
$$t = \frac{\ln 2^3}{\frac{\ln 2}{10} - \frac{\ln 2}{40}}$$
$$t = \frac{3 \ln 2}{\ln 2 \left(\frac{1}{10} - \frac{1}{40}\right)}$$
$$t = \frac{3}{\frac{4-1}{40}} = \frac{3}{\frac{3}{40}}$$
$$t = 40 \text{ min}$$

**Quick Tip**

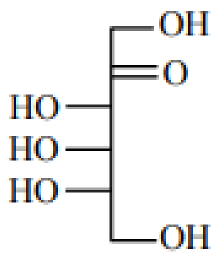
Use the first-order rate law and the given half-lives to find the time when the concentrations of A and B are equal.

---

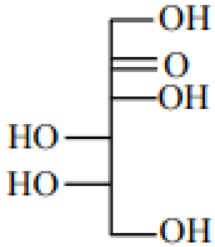
**56. Which of the following is the correct structure of L-fructose?**

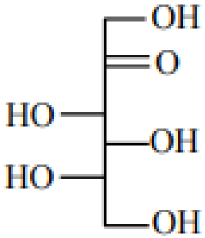



(1)



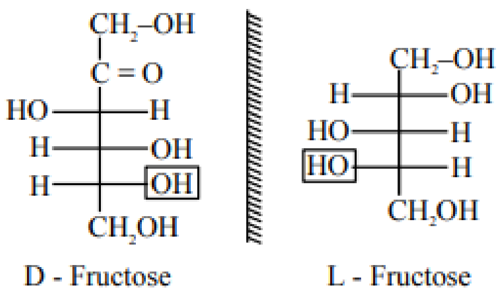

(2)

(3) 

(4) 

**Correct Answer:** (3)

**Solution:**

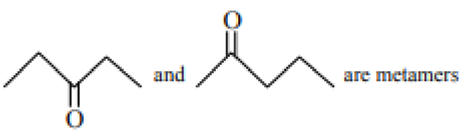



Quick Tip

Remember the structure of L-fructose, including the position of the hydroxyl groups and the orientation of the chiral centers.

---

**57. Identify the correct statements from the following**

A. 


B. 


C. 

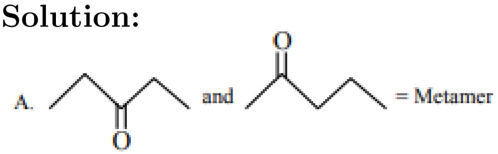

D. 


Choose the **correct** answer from the options given below

(1) C and D only
(2) B and C only
(3) A and B only
(4) A, B and C only

**Correct Answer:** (3)

**Solution:**





In option C are momologues to each - other and option D are only organic molecule not isomers.

**Quick Tip**

Remember the definitions of metamers, functional isomers, position isomers, and homologous series. Identify the relationships between the given pairs of compounds based on their structures and functional groups.

---

**58. Among $10^{-10}$ g (each) of the following elements, which one will have the highest number of atoms?**
Element : Pb, Po, Pr and Pt

(1) Po
(2) Pr
(3) Pb
(4) Pt

**Correct Answer:** (2)

**Solution:**

$$\text{No. of atoms} = \frac{\text{Mass in g}}{\text{Molar Mass (g/mol)}} \times N_A$$

Therefore for the same Mass element having the least Molar mass will have the higher no. of atoms.

$$M_{Pb} = 209$$

$$M_{Pr} = 141$$

$$M_{Po} = 207$$

$$M_{Pt} = 195$$

Quick Tip

The number of atoms in a given mass is inversely proportional to the molar mass of the element.

---

**59. Which of the following statements are correct? A. The process of the addition an electron to a neutral gaseous atom is always exothermic B. The process of removing an electron from an isolated gaseous atom is always endothermic C. The 1st ionization energy of the boron is less than that of the beryllium D. The electronegativity of C is 2.5 in $CH_4$ and $CCl_4$ E. Li is the most electropositive among elements of group 1 Choose the correct answer from the options given below**

(1) B and C only
(2) A, C and D only
(3) B and D only
(4) B, C and E only

**Correct Answer:** (1)

**Solution:**

(A) The process of adding an $e^-$ to a neutral gaseous atom is not always exothermic; it may be exothermic or endothermic.

(B) Be: $1s^2 2s^2$
B: $1s^2 2s^2 2p^1$
In Be, the 2s subshell is fully filled.
So, high energy is needed to remove an $e^-$ as compared to B.

(D) In $CCl_4$, due to partially positive charge, $Z_{eff} \uparrow$
So, EN of C: $CCl_4 > CH_4$

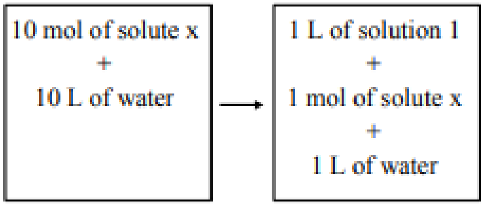
(E) Cs is most electropositive

Quick Tip

Review the definitions of electron affinity, ionization energy, electronegativity, and electropositivity. Consider the electronic configurations and trends in the periodic table.

---

**60. Which of the following properties will change when system containing solution 1 will become solution 2 ?**




(Solution 1) (Solution 2)

(1) Molar heat capacity
(2) Density
(3) Concentration
(4) Gibbs free energy

**Correct Answer:** (4)

**Solution:** Sol. Both solutions are having same composition, which is 1 mole of 'x' in 1L $H_2O$, so all the intensive properties will remain same, but as total amount is greater in solution '1' compared to solution '2'. So extensive properties will be different hence Gibbs free energy will be different.

**Quick Tip**

Distinguish between intensive and extensive properties. Intensive properties do not depend on the amount of substance, while extensive properties do.

---

**61. Number of molecules from below which cannot give iodoform reaction is:** Ethanol, Isopropyl alcohol, Bromoacetone, 2-Butanol, 2-Butanone, Butanal, 2-Pentanone, 3-Pentanone, Pentanal and 3-Pentanol

(1) 2
(2) 4
(3) 3
(4) 2

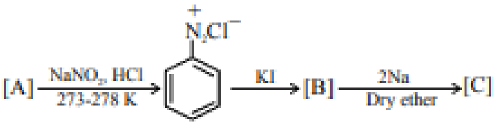
**Correct Answer:** (2)

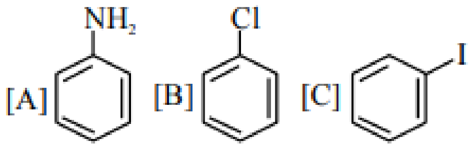
**Solution:** Following will not give iodoform reaction/test. (1) Butanal
(2) 2-Pentanone
(3) Pentanal
(4) 3-Pentanol

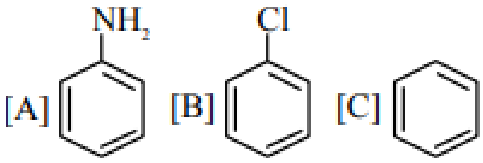
**Quick Tip**

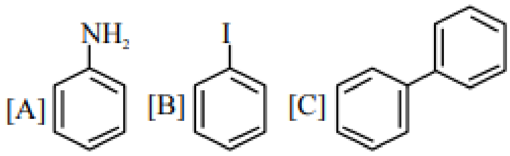
Remember the structural requirements for the iodoform test. The compound must have a $CH_3CO-$ or $CH_3CH(OH)-$ group.

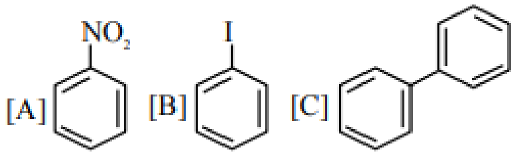
**62. Identify [A], [B], and [C], respectively in the following reaction sequence :**




(1) 

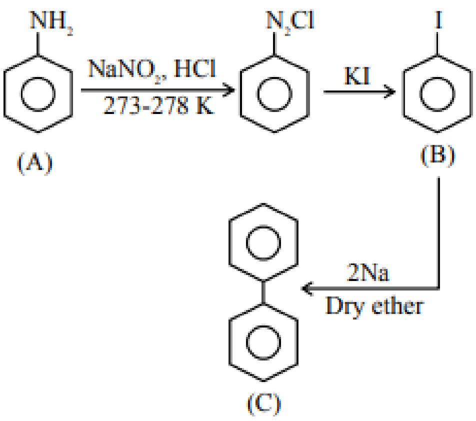

(2) 


(3) 


(4) 

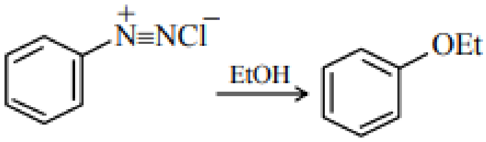

**Correct Answer:** (3)

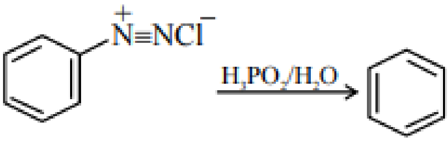
**Solution:**

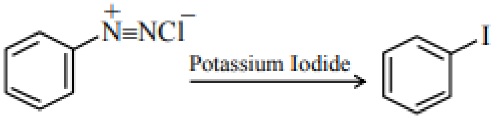


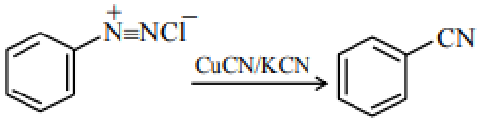

> **Quick Tip**
>
> Recognize the reactions involved: diazotization, Sandmeyer reaction, and coupling reaction.

**63. In the following reactions, which one is NOT correct?**

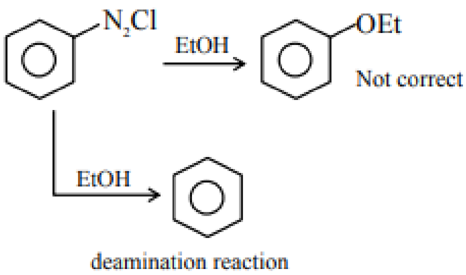
(1) 


(2) 


(3) 


(4) 


**Correct Answer:** (1)

**Solution:** 


Not correct

**Quick Tip**

Recognize the reactions involved: Sandmeyer reaction, hydrolysis, and nucleophilic substitution.

---

**64. The correct order of the complexes**

$[Co(NH_3)_5(H_2O)]^{3+}$ (A),
$[Co(NH_3)_6]^{3+}$ (B),
$[Co(CN)_6]^{3-}$ (C),
$[CoCl(NH_3)_5]^{2+}$ (D)

**in terms of wavelength of light absorbed is:**

(1) D > A > B > C

(2) C > B > D > A
(3) D > C > B > A
(4) C > B > A > D

**Correct Answer:** (1)

**Solution:**

$$\text{We know } E = h\nu = \frac{hC}{\lambda}$$

$$E \propto \frac{1}{\lambda}$$

Here all Co in +3 oxidation state. So, as the ligand field strength ↑, CFSE ↑ Order of field strength of ligand :

$$CN^- > NH_3 > H_2O > Cl^-$$

CFSE order : C > B > A > D Wavelength order : D > A > B > C

**Quick Tip**

Remember the spectrochemical series and how it relates to the strength of ligands and the energy of absorbed light.

---

**65. In the following system, $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ at equilibrium, upon addition of xenon gas at constant T and p, the concentration of**

(1) $PCl_5$ will increase
(2) $Cl_2$ will decrease
(3) $PCl_5$, $PCl_3$ and $Cl_2$ remain constant
(4) $PCl_3$ will increase

**Correct Answer:** (4)

**Solution:** On addition of inert gas at constant P and T, reaction moves in the direction of greater no. of moles so it will shift in forward direction, so $[PCl_5]$ decrease and $[PCl_3]$ and $[Cl_2]$ will increase.

**Quick Tip**

Adding an inert gas at constant pressure and temperature increases the volume of the container. This causes the equilibrium to shift towards the side with more moles of gas.

---

**66. 2 moles each of ethylene glycol and glucose are dissolved in 500 g of water. The boiling point of the resulting solution is:** (Given: Ebullioscopic constant of water $= 0.52\,\text{K kg mol}^{-1}$)

(1) 379.2 K
(2) 377.3 K
(3) 375.3 K
(4) 277.3 K

**Correct Answer:** (2)

**Solution:**

$$\Delta T_b = i_1 m_1 k_b + i_2 m_2 k_b$$
$$\Delta T_b = 1 \times \frac{2}{0.5} \times 0.52 + 1 \times \frac{2}{0.5} \times 0.52 = 4.16$$
$$(T_b)_{\text{solution}} = 373.16 + 4.16 = 377.3 \text{ K}$$

Quick Tip

Use the formula for boiling point elevation. Remember to account for the van't Hoff factor (i) for each solute and the total molality of the solution.

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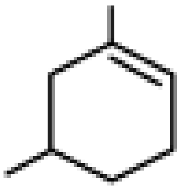
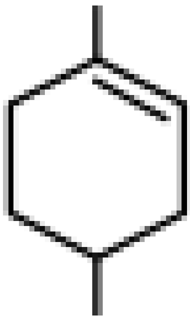
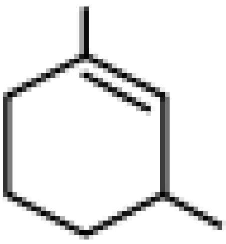
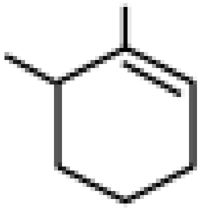
**67. Which compound would give 3-methyl-6-oxoheptanal upon ozonolysis ?**

(1) 

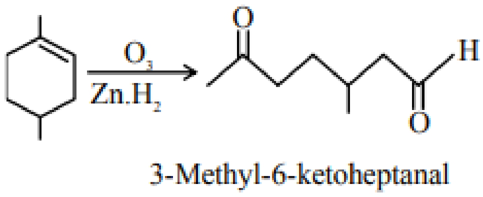
(2) 

(3) 



(4)

**Correct Answer:** (2)

**Solution:** 


3-Methyl-6-ketoheptanal

Quick Tip

Remember that ozonolysis of alkenes cleaves the double bond and forms carbonyl compounds.

---

**68. Match the LIST-I with LIST-II**

| LIST-I | LIST-II |
|---|---|
| A. $PF_5$ | I. $dsp^2$ |
| B. $SF_6$ | II. $sp^3d$ |
| C. $Ni(CO)_4$ | III. $sp^3d^2$ |
| D. $[PtCl_4]^{2-}$ | IV. $sp^3$ |

Choose the **correct** answer from the options given below :

(1) A-II, B-III, C-IV, D-I
(2) A-IV, B-I, C-II, D-III
(3) A-I, B-II, C-III, D-IV
(4) A-III, B-I, C-IV, D-II

**Correct Answer:** (1)

**Solution:**

$PF_5$**:** $5\sigma + 0$ lone pair $\Rightarrow sp^3d$ hybridisation

$SF_6$**:** $6\sigma + 0$ lone pair $\Rightarrow sp^3d^2$ hybridisation

$Ni(CO)_4$**:** Ni oxidation state $= 0$

In presence of ligand field:

Ni(0): [Ar] ↑↓ ↑↓ ↑↓ ↑↓ ↑↓ ----

Orbitals: $3d$ $4s$ $4p$

$\Rightarrow sp^3$ hybridisation

$[PtCl_4]^{2-}$**:** Pt oxidation state $= +2$

In presence of ligand field:

$Pt^{2+}$: [Kr] ↑↓ ↑↓ ↑↓ ↑↓ ---

Orbitals: $5d$ $6s$ $6p$

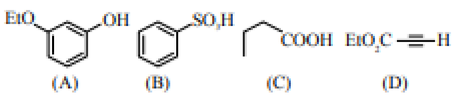
$\Rightarrow dsp^2$ hybridisation

**Quick Tip**

Determine the hybridization of the central atom in each molecule/ion by counting the number of sigma bonds and lone pairs.

---

**69. The least acidic compound, among the following is**




(1) D
(2) A
(3) B
(4) C

**Correct Answer:** (1)


**Solution:**
C.B. of terminal alkyne will be sp hybridisation and localised. In other C.B. will be resonance stabilised.

**Quick Tip**

Acidity depends on the stability of the conjugate base. Consider the hybridization of the carbon atom from which the proton is removed and the possibility of resonance stabilization.

**70. Correct order of limiting molar conductivity for cations in water at 298 K is :**

(1) $H^+ > K^+ > Ca^{2+} > Mg^{2+}$
(2) $H^+ > Ca^{2+} > Mg^{2+} > K^+$
(3) $Mg^{2+} > H^+ > Ca^{2+} > K^+$
(4) $H^+ > Na^+ > Ca^{2+} > Mg^{2+} > K^+$

**Correct Answer:** (2)

**Solution:** Limiting Molar Conductivities of ions :

$$\lambda^0_{H^+} : 349.8 \text{ Sem}^2\text{mol}^{-1}$$

$$\lambda^0_{Na^+} : 50.11 \text{ Sem}^2\text{mol}^{-1}$$

$$\lambda^0_{K^+} : 73.52 \text{ Sem}^2\text{mol}^{-1}$$

$$\lambda^0_{Ca^{2+}} : 119 \text{ Sem}^2\text{mol}^{-1}$$

$$\lambda^0_{Mg^{2+}} : 106.12 \text{ Sem}^2\text{mol}^{-1}$$

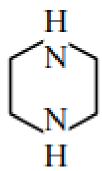
Therefore correct order of limiting molar conductivity of cations will be -

$$H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$$

**Quick Tip**

The limiting molar conductivity depends on the size and charge of the ion. Smaller and highly charged ions are more hydrated, leading to lower mobility and thus lower conductivity. However, $H^+$ has exceptionally high conductivity due to its movement through a proton hopping mechanism.

**71. During estimation of Nitrogen by Dumas' method of compound X (0.42 g) :**

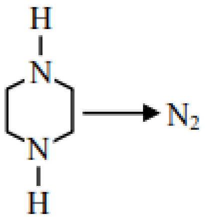



X

**mL of $N_2$ gas will be liberated at STP. (nearest integer)**
(Given molar mass in g $mol^{-1}$ : C : 12, H : 1, N : 14)

**Correct Answer:** (111)

**Solution:** M.wt of given compound = 86



Applying POAC on 'N' $n_N \times 2 = n_{N_2} \times 2$ $n_N = n_{N_2}$ Moles of N in 0.42 g compound $= \frac{0.42}{86} \times \frac{14 \times 1}{14} = \frac{0.42}{86}$ Moles of $N_2$ formed $= \frac{0.42}{86}$ Volume $(N_2)$ at STP $= \frac{0.42}{86} \times 22.4L$ Volume $(N_2)$ at STP $= 0.1108L = 110.8mL \approx 111mL$

**Quick Tip**

In Dumas' method, all the nitrogen in the organic compound is converted to $N_2$ gas. Use the molar ratio of nitrogen in the compound to the moles of $N_2$ produced and then calculate the volume at STP.

---

**72. 0.5 g of an organic compound on combustion gave 1.46 g of $CO_2$ and 0.9 g of $H_2O$. The percentage of carbon in the compound is ______ (Nearest integer)**
(Given : Molar mass (in g mol$^{-1}$) C : 12, H : 1, O : 16)

**Correct Answer:** (80)

**Solution:** Organic Compound → $CO_2$
Applying POAC on 'C'
(mole) of 'C' in compound $= n_{CO_2} \times 1$

So mass of 'C' in compound

$$= \frac{1.46}{44} \times 12$$

So, % of 'C' in compound =

$$\frac{1.46}{44} \times \frac{12}{0.5} \times 100$$
$$= 79.63$$

**Quick Tip**

During combustion of an organic compound, all the carbon present in the compound is converted to $CO_2$. Use the molar mass ratio to find the mass of carbon in the $CO_2$ produced and then calculate the percentage of carbon in the organic compound.

---

**73. The number of optical isomers exhibited by the iron complex (A) obtained from the following reaction is ______** $FeCl_3 + KOH + H_2C_2O_4 \rightarrow A$

**Correct Answer:** (2)

**Solution:**

$$FeCl_3 + KOH + H_2C_2O_4 \rightarrow K_3[Fe(C_2O_4)_3]$$

(A)

$$[Fe(C_2O_4)_3]^{3-} \text{ is } [M(AA)_3] \text{ type complex.}$$

So total optical isomers = 2

**Quick Tip**

For $[M(AA)_3]$ type complexes, where AA is a symmetrical bidentate ligand, the complex is chiral and exists as two optical isomers (d and l forms).

---

**74. Given:** $\Delta H_f^0[C(graphite)] = 710$ **kJ mol¹** $\Delta_c H^0 = 414$ **kJ mol¹** $\Delta_{H-H}^0 = 436$ **kJ mol¹** $\Delta_{C-H}^0 = 611$ **kJ mol¹ The**
$\Delta H_{C=C}^0$ **for** $CH_2 = CH_2$ **is ______ kJ mol**$^{-1}$ **(nearest integer value)**

**Correct Answer:** (25)

**Solution:**

$$[\Delta H_f^0]_{C_2H_4(g)} = (2 \times 710) + (2 \times 436) - 611 - 4 \times 414$$

$$[\Delta H_f^0]_{C_2H_4(g)} = 1420 + 872 - 611 - 1656$$

$$[\Delta H_f^0]_{C_2H_4(g)} = 2292 - 2267 = 25 \text{ kJ mol}^{-1}$$

Final Answer: The final answer is $\boxed{25}$

**Quick Tip**

The enthalpy of formation of a compound can be related to the enthalpies of formation of its constituent atoms and the bond enthalpies of the bonds within the molecule. Set up an equation relating the enthalpy of formation of ethylene to the sublimation enthalpy of carbon, the bond enthalpy of hydrogen, and the bond enthalpies of C=C and C-H bonds, then solve for the C=C bond enthalpy.

---

**75. Consider the following reactions** $A + HCl + H_2SO_4 \rightarrow CrO_2Cl_2 + SideProducts$ **Little amount** $CrO_2Cl_2(vapour) + NaOH \rightarrow B + NaCl + H_2O$ $B + H^+ \rightarrow C + H_2O$ **The number of terminal 'O' present in the compound 'C' is ______**

**Correct Answer:** (6)

**Solution:**

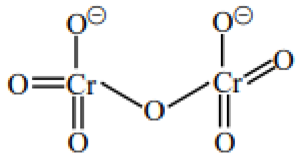
$$Cr_2O_7^{2-} + HCl + H_2SO_4 \rightarrow CrO_2Cl_2$$

$$CrO_2Cl_2(vapour) + NaOH \rightarrow Na_2CrO_4 + NaCl + H_2O$$

$$Na_2CrO_4 + H^+ \rightarrow Na_2Cr_2O_7 + H_2O$$

$$2Na_2CrO_4 + 2H^+ \rightarrow Na_2Cr_2O_7 + 2Na^+ + H_2O$$

$$CrO_4^{2-} \xrightarrow{H^+} Cr_2O_7^{2-}$$



No of terminal "O" = 6

**Quick Tip**

Identify the chromium-containing species formed in each reaction. Chromyl chloride ($CrO_2Cl_2$) reacts with NaOH to form chromate ions ($CrO_4^{2-}$), which in acidic solution convert to dichromate ions ($Cr_2O_7^{2-}$). Draw the structure of the dichromate ion to count the terminal oxygen atoms.

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