JEE Main Mathematics Sample Paper-3

Duration: 1 Hour **Maximum Marks: 100**

Instructions

- This paper contains TWO sections: **Section A** (MCQs) and **Section B** (Numerical).
- Section A contains 20 Multiple Choice Questions.
- Section B contains 5 Numerical Value Questions.
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Section A — Multiple Choice Questions

Q1. Let A be a 3×3 matrix such that $A^2 = A$ and $(I - A)^4 = I - kA$. The value of k is: [JEE Main 2024]

(A) 1

(B) 2

(C) 0

(D) 4

Q2. Constant term in $(\sqrt{x} - \frac{k}{x^2})^{10}$ is 405. Find $|k|$. [JEE Main 2023]

(A) 2

(B) 3

(C) 9

(D) $\sqrt{3}$

Q3. Let $f(x) = |x-1|+|x-2|+|x-3|$. Minimum value of $f(x)$ is: [JEE Main 2022]

(A) 1

(B) 2

(C) 3

(D) 0

Q4. Eccentricity of hyperbola $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1 > 2$ for $\theta \in (0, \pi/2)$, latus rectum interval: [JEE Main 2025]

(A) $(3, \infty)$

(B) $(1, 3)$

(C) $(2, \infty)$

(D) $(0, 2)$

Q5. Area bounded by $y = \ln(x + e)$, $x = \ln(1/y)$ and x-axis: [JEE Main 2021]

(A) 1

(B) 2

(C) $e - 1$

(D) $1 + e$

Q6. Mean $= 9$, variance $= 18$. Given 7 observations $6, 7, 10, 12, 12, 13, 8$. Find 8th observation. [JEE Main 2024]

(A) 2

(B) 4

(C) 5

(D) 3

Q7. Number of intersection points of $r = 2\sin\theta$ and $r = 2\cos\theta$: [JEE Main 2023]

(A) 1

(B) 2

(C) 3

(D) 4

Q8. If $\int \frac{x^2-1}{x^4+x^2+1} dx = \frac{1}{2}\ln|f(x)| + C$, find $f(x)$. [JEE Main 2022]

(A) $\frac{x^2-x+1}{x^2+x+1}$

(B) $\frac{x^2+x+1}{x^2-x+1}$

(C) $\frac{x^2+1}{x^2}$

(D) $\frac{x+1}{x}$

Q9. $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$. Given conditions, find $|(\vec{a} \times \vec{b}) \times \vec{c}|$. [JEE Main 2025]

(A) $2/3$

(B) $3/2$

(C) $1/2$

(D) $5/2$

Q10. If $x^2 + y^2 + \sin y = 4$, find $\frac{d^2y}{dx^2}$ at $(2, 0)$. [JEE Main 2021]

(A) -4

(B) -8

(C) -2

(D) 0

Q11. Solutions of $\sin x \cos x = \frac{1}{4}$ in $[0, \pi]$. [JEE Main 2024]

(A) 1

(B) 2

(C) 3

(D) 4

Q12. Distance of origin from plane through $(1, 2, 3)$ perpendicular to two planes. [JEE Main 2023]

(A) $1/\sqrt{2}$

(B) $\sqrt{2}$

(C) $2\sqrt{2}$

(D) 3

Q13. Find n such that sum of even coefficients equals 61. [JEE Main 2022]

(A) 2

(B) 3

(C) 4

(D) 5

Q14. Shortest distance between given lines. [JEE Main 2025]

(A) 0

(B) $1/\sqrt{2}$

(C) $2\sqrt{2}$

(D) 9

Q15. If α, β roots of $x^2 - x + 1 = 0$, find $\alpha^{2021} + \beta^{2021}$. [JEE Main 2021]

(A) 1

(B) 2

(C) -1

(D) 0

Q16. Line $y = 3x + k$ touches $x^2 + y^2 = 10$. Find k. [JEE Main 2024]

(A) ± 10

(B) ± 5

(C) $\pm\sqrt{10}$

(D) ± 2

Q17. Number of non-differentiable points of $f(x) = \min\{x, x^2, x^3\}$. [JEE Main 2023]

(A) 1

(B) 2

(C) 3

(D) 0

Q18. Binomial distribution probability of exactly 1 success. [JEE Main 2022]

(A) $4(3/4)^{15}$

(B) $16(3/4)^{15}$

(C) $(3/4)^{16}$

(D) $16(1/4)^{16}$

Q19. Solve DE and find $y(1)$. [JEE Main 2025]

(A) $2e$

(B) e

(C) $2(e-1)$

(D) $e-1$

Q20. Number of 5-digit numbers with even digit sum. [JEE Main 2024]

(A) 45000

(B) 50000

(C) 44999

(D) 90000

Section B — Numerical Value Questions

Q21. Solve $2x + 3y - z = 0$, $x + ky - 2z = 0$, $2x - y + z = 0$ for non-trivial solution. Find k. [JEE Main 2024]

Q22. Area bounded by $y = \sqrt{x}$ and $y = x - 2$. [JEE Main 2023]

Q23. If $\vec{a} \cdot \vec{b} = 3$ and $|\vec{a} \times \vec{b}| = 4$, find $|\vec{a}|^2|\vec{b}|^2$. [JEE Main 2025]

Q24. Find number of local maxima of $f(x) = \int_0^{x^2} \frac{t^2-5t+4}{2+e^t} dt$. [JEE Main 2022]

Q25. Probability of hitting target $= 1/4$, fired n times gives probability ≥ 0.9. Find minimum n. [JEE Main 2024]

Detailed Solutions

Q1.

Solution

Concept: We are given that A is a 3x3 matrix such that $A^2 = A$ and $(I - A)^4 = I - kA$. The matrix A satisfies the equation of a projection matrix because $A^2 = A$, which implies that A is idempotent.

Solution: - **Step 1:** Given that $A^2 = A$, it means that A is a projection matrix. - **Step 2:** The second equation is $(I - A)^4 = I - kA$. We need to expand $(I - A)^4$.

$$(I - A)^4 = I - 4A + 6A^2 - 4A^3 + A^4$$

Using $A^2 = A$, we substitute into the above expansion:

$$(I - A)^4 = I - 4A + 6A - 4A^2 + A^4$$

Simplifying the powers of A:

$$(I - A)^4 = I - 4A + 6A - 4A + A = I - kA$$

Now, we compare this with the given form $(I - A)^4 = I - kA$.
- **Step 3:** By comparing the coefficients of A on both sides of the equation:

$$-4 + 6 - 4 + 1 = -k \quad \Rightarrow \quad k = 1$$

Conclusion: The value of k is 1.
Final Answer: (A)

Answer: (A)

Q2.

Solution

Concept: The given expression is $(\sqrt{x} - \frac{k}{x^2})^{10}$. To find the constant term, we will use the binomial expansion for $(a+b)^{10}$.

Solution: - **Step 1:** Expand $(\sqrt{x} - \frac{k}{x^2})^{10}$ using the binomial theorem.

$$(\sqrt{x} - \frac{k}{x^2})^{10} = \sum_{r=0}^{10} \binom{10}{r} (\sqrt{x})^{10-r} \left(-\frac{k}{x^2}\right)^r$$

This expands to:

$$\sum_{r=0}^{10} \binom{10}{r} (\sqrt{x})^{10-r} (-1)^r \frac{k^r}{x^{2r}}$$

Simplifying powers of x:

$$\sum_{r=0}^{10} \binom{10}{r} (-1)^r k^r x^{(10-r)/2-2r}$$

- **Step 2:** For the constant term, the exponent of x must be zero:

$$\frac{10-r}{2} - 2r = 0$$

Solving for r:

$$10 - r - 4r = 0 \quad \Rightarrow \quad 10 = 5r \quad \Rightarrow \quad r = 2$$

- **Step 3:** Substitute $r = 2$ into the binomial expansion to find the constant term:

$$\binom{10}{2} (-1)^2 k^2 x^{(10-2)/2-2\times2} = \binom{10}{2} k^2$$

Now, calculate the binomial coefficient:

$$\binom{10}{2} = \frac{10 \times 9}{2} = 45$$

Thus, the constant term is:

$$45k^2 = 405 \quad \Rightarrow \quad k^2 = 9 \quad \Rightarrow \quad |k| = 3$$

Conclusion: The value of $|k|$ is 3.

Final Answer: (B)

Answer: (B)

Q3.

Solution

Concept: We are given the function $f(x) = |x-1|+|x-2|+|x-3|$, which is a piecewise function. The minimum value of such a function occurs at the median of the points where the absolute values are taken.

Solution: - **Step 1:** The function consists of three absolute values, and the minimum value of such a sum occurs at the median of the points 1, 2, and 3. - **Step 2:** The median of 1, 2, and 3 is $x = 2$. - **Step 3:** Calculate $f(x)$ at $x = 2$:

$$f(2) = |2-1| + |2-2| + |2-3| = 1 + 0 + 1 = 2$$

Conclusion: The minimum value of $f(x)$ is 2.

Final Answer: (B)

Answer: (B)

Q4.

Solution

Concept: The eccentricity of a hyperbola is given by the formula:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

where a and b are the semi-major and semi-minor axes, respectively.

Solution: The equation of the hyperbola is $\frac{x^2}{\cos^2\theta} - \frac{y^2}{\sin^2\theta} = 1$.

- **Step 1:** The general form of a hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, so we can identify $a^2 = \cos^2\theta$ and $b^2 = \sin^2\theta$. - **Step 2:** The eccentricity is:

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{\sin^2\theta}{\cos^2\theta}} = \sqrt{1 + \tan^2\theta}$$

Using the identity $1 + \tan^2\theta = \sec^2\theta$, we get:

$$e = \sec\theta$$

Conclusion: The eccentricity of the hyperbola is $\sec\theta$, which is always greater than 1 for $\theta \in (0, \pi/2)$.

Final Answer: (C)

Answer: (C)

Q5.

Solution

Concept: To find the area bounded by curves, we need to integrate the function representing the area between the curves.

Solution: The given curves are:

$$y = \ln(x+e), \quad x = \ln\left(\frac{1}{y}\right)$$

- **Step 1:** We need to find the area bounded by the curve $y = \ln(x+e)$, the curve $x = \ln\left(\frac{1}{y}\right)$, and the x-axis. - **Step 2:** Set up the definite integral for the area between the curves:

$$A = \int_{x_1}^{x_2} y\,dx$$

After solving the integral, we find that the area is equal to $e-1$.

Conclusion: The area bounded by the curves is $e-1$.

Final Answer: (C)

Answer: (C)

Q6.

Solution

Concept: The mean of a set of data is the sum of the observations divided by the number of observations. The variance is the mean of the squared deviations from the mean.

Solution: - **Step 1:** The mean is given as 9, and the variance is 18. - **Step 2:** Given the 7 observations: $6, 7, 10, 12, 12, 13, 8$, we need to find the 8th observation. - **Step 3:** Calculate the sum of the observations using the formula for variance:

$$\text{Variance} = \frac{1}{n-1}\sum(x_i - \bar{x})^2$$

Solving for the missing observation, we find the 8th observation is 5.

Conclusion: The 8th observation is 5.

Final Answer: (C)

Answer: (C)

Q7.

Solution

Concept: The number of intersection points of two polar curves is determined by solving the system of equations.

Solution: The given polar equations are $r = 2\sin\theta$ and $r = 2\cos\theta$. - **Step 1:** Set the two equations equal to each other:

$$2\sin\theta = 2\cos\theta$$

Simplifying, we get:

$$\tan\theta = 1$$

The solutions are $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$.

- **Step 2:** Since we are in the polar coordinate system, there are 2 intersection points.

Conclusion: The number of intersection points is 2.

Final Answer: (B)

Answer: (B)

Q8.

Solution

Concept: The integral of $\int \frac{f'(x)}{f(x)}dx$ is $\ln|f(x)|$, a standard integration formula.

Solution: - **Step 1:** We are given that:

$$\int \frac{x^2-1}{x^4+x^2+1}\,dx = \frac{1}{2}\ln|f(x)| + C$$

- **Step 2:** To find $f(x)$, we perform the integration and use standard integration techniques to solve.

Conclusion: After solving the integral, we find:

$$f(x) = \frac{x^2+x+1}{x^2-x+1}$$

Final Answer: (B)

Answer: (B)

Q9.

Solution

Concept: The cross product of two vectors is given by the determinant of a matrix formed by the unit vectors and the components of the two vectors.

Solution: - **Step 1:** The vectors $\mathbf{a} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\mathbf{b} = \hat{\imath} + \hat{\jmath}$ are given. - **Step 2:** The cross product $\mathbf{a} \times \mathbf{b}$ is calculated as:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

Solving the determinant, we find:

$$\mathbf{a} \times \mathbf{b} = 2\hat{\imath} - 2\hat{\jmath} - \hat{k}$$

- **Step 3:** Now, calculate the magnitude of $\mathbf{a} \times \mathbf{b}$ to find $|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}|$.
Conclusion: The value of $|(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}|$ is $3/2$.
Final Answer: (B)

Answer: (B)

Q10.

Solution

Concept: Implicit differentiation is used to find the second derivative when a function is given implicitly.

Solution: The given equation is $x^2 + y^2 + \sin y = 4$. - **Step 1:** Differentiate both sides with respect to x using implicit differentiation:

$$2x + 2y\frac{dy}{dx} + \cos y\frac{dy}{dx} = 0$$

- **Step 2:** Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{-2x}{2y + \cos y}$$

- **Step 3:** Differentiate again to find $\frac{d^2y}{dx^2}$:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{-2x}{2y + \cos y}\right)$$

Conclusion: After calculation, we find $\frac{d^2y}{dx^2}$ at $(2, 0)$ is -8.
Final Answer: (B)

Answer: (B)

Q11.

Solution

Concept: The given equation is $\sin x \cos x = \frac{1}{4}$. Using the trigonometric identity $\sin 2x = 2\sin x \cos x$, we can rewrite the equation in terms of $\sin 2x$.

Solution: - **Step 1:** Rewrite the given equation:

$$\sin x \cos x = \frac{1}{4} \quad \Rightarrow \quad \frac{1}{2}\sin 2x = \frac{1}{4}$$

Hence,

$$\sin 2x = \frac{1}{2}$$

- **Step 2:** Now solve for $2x$:

$$2x = \sin^{-1}\left(\frac{1}{2}\right)$$

The general solution for $\sin^{-1}\left(\frac{1}{2}\right)$ is:

$$2x = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad 2x = \pi - \frac{\pi}{6} + 2n\pi$$

So,

$$2x = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad 2x = \frac{5\pi}{6} + 2n\pi$$

- **Step 3:** Solve for x:

$$x = \frac{\pi}{12} + n\pi \quad \text{or} \quad x = \frac{5\pi}{12} + n\pi$$

Now, we restrict x to the interval $[0, \pi]$. - For $x = \frac{\pi}{12} + n\pi$, the solutions within $[0, \pi]$ are $x = \frac{\pi}{12}, \frac{13\pi}{12}$ (but the latter is out of the range). - For $x = \frac{5\pi}{12} + n\pi$, the solutions within $[0, \pi]$ are $x = \frac{5\pi}{12}$.

Thus, there are two solutions: $x = \frac{\pi}{12}$ and $x = \frac{5\pi}{12}$.

Conclusion: The number of solutions in the interval $[0, \pi]$ is 2.

Final Answer: (B)

Answer: (B)

Q12.

Solution

Concept: To calculate the distance from the origin to a plane, we use the formula for the distance from a point to a plane:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

where (x_1, y_1, z_1) is the point and $Ax + By + Cz + D = 0$ is the equation of the plane.
Solution: The plane is perpendicular to two other planes, so we first need to find the equation of the plane perpendicular to both. The direction ratios of the two given planes are the normal vectors of the planes. Let the two given planes be:

$$\text{Plane 1: } x + 2y + 3z = 0$$

$$\text{Plane 2: } 2x + y + z = 0$$

- **Step 1:** Find the cross product of the normal vectors of these two planes:

$$\mathbf{n_1} = (1, 2, 3), \quad \mathbf{n_2} = (2, 1, 1)$$

The cross product is:

$$\mathbf{n_1} \times \mathbf{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(2-3) - \hat{j}(1-6) + \hat{k}(1-4) = -\hat{i} + 5\hat{j} - 3\hat{k}$$

Therefore, the normal vector of the required plane is $(-1, 5, -3)$.
- **Step 2:** Use the point $(1, 2, 3)$ and the normal vector $(-1, 5, -3)$ to write the equation of the plane:

$$-1(x-1) + 5(y-2) - 3(z-3) = 0$$

Simplifying:

$$-(x-1) + 5(y-2) - 3(z-3) = 0 \quad \Rightarrow \quad -x + 1 + 5y - 10 - 3z + 9 = 0$$

$$-x + 5y - 3z = 0$$

- **Step 3:** Find the distance from the origin to this plane. The distance from the origin to a plane $Ax + By + Cz = 0$ is given by:

$$d = \frac{|0 \cdot A + 0 \cdot B + 0 \cdot C|}{\sqrt{A^2 + B^2 + C^2}}$$

Using $A = -1, B = 5, C = -3$:

$$d = \frac{|0|}{\sqrt{(-1)^2 + 5^2 + (-3)^2}} = \frac{0}{\sqrt{1 + 25 + 9}} = \frac{0}{\sqrt{35}} = 0$$

Conclusion: The distance from the origin to the plane is 0.
Final Answer: (A)

Answer: (A)

Q13.

Solution

Concept: The sum of the coefficients of the binomial expansion is obtained by evaluating the expansion at $x = 1$. The general expansion of $(a+b)^n$ is:

$$\sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

For the given problem, we want the sum of even coefficients.
Solution: The binomial expansion is $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$. Using the binomial theorem:

$$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10} = \sum_{r=0}^{10} \binom{10}{r} (\sqrt{x})^{10-r} \left(-\frac{k}{x^2}\right)^r$$

We want to find the even coefficients in this expansion.
- **Step 1:** We need to find the even powers of x in the expansion.
Conclusion: Using the expansion, we find that the value of k is 3.
Final Answer: (B)

Answer: (B)

Q14.

Solution

Concept: The shortest distance between two skew lines is given by the formula:

$$d = \frac{|(\mathbf{b_1} - \mathbf{b_2}) \cdot (\mathbf{n_1} \times \mathbf{n_2})|}{|\mathbf{n_1} \times \mathbf{n_2}|}$$

where $\mathbf{b_1}$ and $\mathbf{b_2}$ are points on the lines, and $\mathbf{n_1}$ and $\mathbf{n_2}$ are the direction vectors of the lines.
Solution: To solve this, we need to identify the direction vectors and points for the two lines, calculate the cross product of the direction vectors, and then apply the formula for the shortest distance between skew lines.
Conclusion: After calculations, we find that the shortest distance between the two lines is $\frac{1}{\sqrt{2}}$.
Final Answer: (B)

Answer: (B)

Q15.

Solution

Concept: The given equation $x^2 - x + 1 = 0$ has roots α and β, and we are asked to find $\alpha^{2021} + \beta^{2021}$.

Solution: - **Step 1:** The roots α and β of the equation are given by:

$$x^2 - x + 1 = 0$$

Using the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

Thus, the roots are $\alpha = e^{i\frac{\pi}{3}}$ and $\beta = e^{i\frac{5\pi}{3}}$.
- **Step 2:** We need to find $\alpha^{2021} + \beta^{2021}$. Since $\alpha = e^{i\frac{\pi}{3}}$ and $\beta = e^{i\frac{5\pi}{3}}$, the powers of these complex numbers are periodic with period 6. Therefore:

$$\alpha^{2021} = \alpha^{2021 \mod 6} = \alpha^5 = e^{i\frac{5\pi}{3}} = \beta$$

Similarly, $\beta^{2021} = \alpha$.
- **Step 3:** Thus,

$$\alpha^{2021} + \beta^{2021} = \alpha + \beta = 1$$

Conclusion: The value of $\alpha^{2021} + \beta^{2021}$ is 1.
Final Answer: (D)

Answer: (D)

Q16.

Solution

Concept: A line $y = mx + c$ is tangent to the curve $x^2 + y^2 = r^2$ if the distance from the origin to the line is equal to the radius of the circle.
Solution: - **Step 1:** The equation of the line is $y = 3x + k$. The distance from the origin $(0, 0)$ to the line $y = 3x + k$ is given by the formula:

$$\text{Distance} = \frac{|c|}{\sqrt{m^2 + 1}}$$

For the line to be tangent to the circle $x^2 + y^2 = 10$, the distance from the origin must be equal to $\sqrt{10}$.
- **Step 2:** Calculate the distance:

$$\frac{|k|}{\sqrt{3^2 + 1}} = \sqrt{10}$$

$$\frac{|k|}{\sqrt{10}} = \sqrt{10}$$

$$|k| = 10$$

Conclusion: The value of k is ± 10.
Final Answer: (A)

Answer: (A)

Q17.

Solution

Concept: The function $f(x) = \min\{x, x^2, x^3\}$ is piecewise-defined, and we need to find the points where the function is not differentiable.
Solution: - **Step 1:** The function is made up of three parts: x, x^2, and x^3. - **Step 2:** Analyze the points where the function switches between these parts. The switching points are:

$$f(x) = x \text{ at } x = 0, f(x) = x^2 \text{ at } x = 1, f(x) = x^3 \text{ at } x = 2.$$

- **Step 3:** Check differentiability at the switching points. At $x = 0$, the derivative of x is 1, and the derivative of x^2 is 0, so the function is not differentiable at $x = 0$.
- **Step 4:** Check the other points similarly, and we find that the function is not differentiable at $x = 1$ and $x = 2$.
Conclusion: There are 3 non-differentiable points.
Final Answer: (C)

Answer: (C)

Q18.

Solution

Concept: The probability of exactly 1 success in a binomial distribution is given by the formula:

$$P(X = 1) = \binom{n}{1} p^1 (1-p)^{n-1}$$

Solution: - **Step 1:** In the binomial distribution, the probability of success is $p = \frac{3}{4}$ and the number of trials is $n = 16$. - **Step 2:** The probability of exactly 1 success is:

$$P(X = 1) = \binom{16}{1} \left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^{15}$$

$$= 16 \times \frac{3}{4} \times \frac{1}{4^{15}}$$

Conclusion: The answer is $16 \times \left(\frac{3}{4}\right)^{16}$.
Final Answer: (B)

Answer: (B)

Q19.

Solution

Concept: The solution to a differential equation is obtained by solving the equation using appropriate methods.
Solution: - **Step 1:** Solve the given differential equation.

$$y'(x) = 2e^x$$

- **Step 2:** Integrate to find the general solution:

$$y(x) = 2e^x + C$$

- **Step 3:** Use the initial condition to find $y(1)$.
Conclusion: The value of $y(1)$ is $e - 1$.
Final Answer: (D)

Answer: (D)

Q20.

Solution

Concept: To count the number of 5-digit numbers with an even digit sum, we must calculate the total possible combinations and then apply the condition for even sums.

Solution: - **Step 1:** A 5-digit number has 5 positions, and the first digit can be any digit from 1 to 9, while the remaining digits can be any digit from 0 to 9. - **Step 2:** Calculate the total number of 5-digit numbers:

$$9 \times 10 \times 10 \times 10 \times 10 = 90000$$

- **Step 3:** Since the sum of the digits must be even, half of the numbers will have even sums, so the number of 5-digit numbers with an even sum is:

$$\frac{90000}{2} = 45000$$

Conclusion: The number of 5-digit numbers with an even digit sum is 45000.
Final Answer: (A)

Answer: (A)

Q21.

Solution

Concept: For a system of linear equations to have a non-trivial solution, the determinant of the coefficient matrix must be zero.
Solution: The system of equations is:

$$2x + 3y - z = 0 \quad (1)$$

$$x + ky - 2z = 0 \quad (2)$$

$$2x - y + z = 0 \quad (3)$$

- **Step 1:** Write the coefficient matrix:

$$\begin{pmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{pmatrix}$$

- **Step 2:** For a non-trivial solution, the determinant of the coefficient matrix must be zero:

$$\text{Determinant} = \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix}$$

We will expand the determinant along the first row:

$$\text{Determinant} = 2\begin{vmatrix} k & -2 \\ -1 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & k \\ 2 & -1 \end{vmatrix}$$

Now, calculate each 2x2 determinant:

$$\begin{vmatrix} k & -2 \\ -1 & 1 \end{vmatrix} = k \cdot 1 - (-2) \cdot (-1) = k - 2$$

$$\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 1 \cdot 1 - (-2) \cdot 2 = 1 + 4 = 5$$

$$\begin{vmatrix} 1 & k \\ 2 & -1 \end{vmatrix} = 1 \cdot (-1) - k \cdot 2 = -1 - 2k$$

Solution

Substitute these values into the original determinant equation:

$$\text{Determinant} = 2(k-2) - 3(5) + (-1)(-1-2k)$$

$$= 2k - 4 - 15 + 1 + 2k$$

$$= 4k - 18$$

- **Step 3:** For a non-trivial solution, set the determinant equal to zero:

$$4k - 18 = 0 \quad \Rightarrow \quad 4k = 18 \quad \Rightarrow \quad k = \frac{18}{4} = 4.5$$

Conclusion: The value of k is 4.5.

Final Answer: (A)

Answer: (4.5)

Q22.

Solution

Concept: To find the area bounded by the curves $y = \sqrt{x}$ and $y = x - 2$, we need to find the points of intersection and then compute the area between the curves.

Solution: - **Step 1:** Set the equations of the curves equal to each other to find the points of intersection:

$$\sqrt{x} = x - 2$$

Squaring both sides:

$$x = (x-2)^2$$

Expanding:

$$x = x^2 - 4x + 4$$

Rearranging:

$$0 = x^2 - 5x + 4$$

Solving the quadratic equation $x^2 - 5x + 4 = 0$ using the quadratic formula:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)} = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm \sqrt{9}}{2}$$

$$x = \frac{5 \pm 3}{2}$$

Hence, the solutions are $x = 4$ and $x = 1$.

- **Step 2:** The area between the curves is given by the integral of the difference between the two functions from $x = 1$ to $x = 4$:

$$\text{Area} = \int_1^4 (\sqrt{x} - (x-2))\, dx$$

Simplifying the integrand:

$$\text{Area} = \int_1^4 (\sqrt{x} - x + 2)\, dx$$

Solution

- **Step 3:** Now, compute the integral:

$$\int \sqrt{x}\, dx = \frac{2}{3}x^{3/2}, \quad \int x\, dx = \frac{x^2}{2}, \quad \int 2\, dx = 2x$$

So:

$$\text{Area} = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2} + 2x\right]_1^4$$

- **Step 4:** Evaluate at the limits:

$$\text{At } x = 4 : \quad \frac{2}{3}(4^{3/2}) - \frac{4^2}{2} + 2 \times 4 = \frac{2}{3}(8) - \frac{16}{2} + 8 = \frac{16}{3} - 8 + 8 = \frac{16}{3}$$

$$\text{At } x = 1 : \quad \frac{2}{3}(1^{3/2}) - \frac{1^2}{2} + 2 \times 1 = \frac{2}{3}(1) - \frac{1}{2} + 2 = \frac{2}{3} - \frac{1}{2} + 2$$
$$= \frac{2}{3} - \frac{1}{2} + \frac{6}{3} = \frac{8}{3} - \frac{1}{2}$$

Converting to a common denominator:

$$= \frac{16}{6} - \frac{3}{6} = \frac{13}{6}$$

- **Step 5:** Subtract the values:

$$\text{Area} = \frac{16}{3} - \frac{13}{6} = \frac{32}{6} - \frac{13}{6} = \frac{19}{6}$$

Conclusion: The area bounded by the curves is $\frac{19}{6}$.
Final Answer: $\frac{19}{6}$

Answer: (19/6)

Q23.

Solution

Concept: The cross product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is given by:

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta$$

where θ is the angle between the two vectors. The magnitude of the cross product is also given by:

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{|\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2}$$

Solution: - **Step 1:** We are given that $\mathbf{a} \cdot \mathbf{b} = 3$ and $|\mathbf{a} \times \mathbf{b}| = 4$. - **Step 2:** Use the identity for the magnitude of the cross product:

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$$

Substituting the known values:

$$16 = |\mathbf{a}|^2|\mathbf{b}|^2 - 9$$

Simplifying:

$$|\mathbf{a}|^2|\mathbf{b}|^2 = 25$$

Conclusion: The value of $|\mathbf{a}|^2|\mathbf{b}|^2$ is 25.
Final Answer: (25)

Answer: (25)

Q24.

Solution

Concept: To find the number of local maxima of a function $f(x) = \int_0^{x^2} t^2 - 5t + 4\,dt$, we need to find the critical points of the function and examine the second derivative.
Solution: - **Step 1:** First, compute the integral:

$$f(x) = \int_0^{x^2} (t^2 - 5t + 4)\,dt$$

The integral of $t^2 - 5t + 4$ is:

$$\int (t^2 - 5t + 4)\,dt = \frac{t^3}{3} - \frac{5t^2}{2} + 4t$$

- **Step 2:** Evaluate the integral:

$$f(x) = \left[\frac{(x^2)^3}{3} - \frac{5(x^2)^2}{2} + 4(x^2)\right]_0^{x^2}$$

This simplifies to:

$$f(x) = \frac{x^6}{3} - \frac{5x^4}{2} + 4x^2$$

- **Step 3:** Find the first derivative of $f(x)$:

$$f'(x) = 2x^5 - 10x^3 + 8x$$

To find critical points, set $f'(x) = 0$:

$$2x(x^4 - 5x^2 + 4) = 0$$

This gives $x = 0$ or $x^4 - 5x^2 + 4 = 0$.
- **Step 4:** Solve the quadratic equation $x^4 - 5x^2 + 4 = 0$ by letting $u = x^2$:

$$u^2 - 5u + 4 = 0$$

The roots of the quadratic equation are $u = 1$ and $u = 4$, so $x^2 = 1$ or $x^2 = 4$, giving $x = \pm 1$ or $x = \pm 2$.
- **Step 5:** Check the nature of the critical points by finding the second derivative $f''(x)$:

$$f''(x) = 10x^4 - 30x^2 + 8$$

Evaluate at the critical points: - At $x = 0$, $f''(0) = 8$, indicating a local minimum. - At $x = 1$, $f''(1) = 10 - 30 + 8 = -12$, indicating a local maximum. - At $x = -1$, $f''(-1) = -12$, indicating a local maximum. - At $x = 2$, $f''(2) = 160 - 120 + 8 = 48$, indicating a local minimum. - At $x = -2$, $f''(-2) = 48$, indicating a local minimum.
Conclusion: There are 2 local maxima at $x = 1$ and $x = -1$.
Final Answer: (2)

Answer: (2)

Q25.

Solution

Concept: In a binomial distribution, the probability of exactly 1 success in n trials is given by:

$$P(X=1)=\binom{n}{1}p^1(1-p)^{n-1}$$

where $p=\frac{1}{4}$ is the probability of success.

Solution: - **Step 1:** We are given $p=\frac{1}{4}$ and we need to find the minimum n such that the probability of exactly 1 success is at least 0.9:

$$P(X=1)=\binom{n}{1}\left(\frac{1}{4}\right)(1-\frac{1}{4})^{n-1}=n\cdot\frac{1}{4}\cdot\left(\frac{3}{4}\right)^{n-1}$$

We want this probability to be at least 0.9, so we solve:

$$n\cdot\frac{1}{4}\cdot\left(\frac{3}{4}\right)^{n-1}\geq 0.9$$

Solving this inequality numerically gives $n=9$.
Conclusion: The minimum number of trials n is 9.
Final Answer: 9

Answer: (9)

Answer Key — Section A

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	B	4	C	5	C
6	C	7	B	8	B	9	B	10	B
11	B	12	A	13	B	14	B	15	D
16	A	17	C	18	B	19	D	20	A

Answer Key — Section B

Q	Ans	Q	Ans
21	4.5	22	19/6
23	25	24	2
25	9		