JEE Main Mathematics Sample Paper-5

Duration: 1 Hour **Maximum Marks: 100**

Instructions

- This paper contains TWO sections: **Section A** (MCQs) and **Section B** (Numerical).
- Section A contains 20 Multiple Choice Questions.
- Section B contains 5 Numerical Value Questions.
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Section A — Multiple Choice Questions

Q1. Let $f(x) = \min\{[x], |x|, x^2\}$ for $x \in [-2, 2]$, where $[x]$ denotes the greatest integer function. The number of points where $f(x)$ is non-differentiable is: [JEE Main 2023]

(A) 3

(B) 4

(C) 5

(D) 6

Q2. If $\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then the value of $a + b + c$ is: [JEE Main 2021]

(A) 0

(B) 2

(C) 4

(D) 6

Q3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x}{1+e^{1/x}}$ for $x \neq 0$ and $f(0) = 0$. Then at $x = 0$, f is: [JEE Main 2024]

(A) Continuous and differentiable

(B) Continuous but not differentiable

(C) Discontinuous

(D) Differentiable but not continuous

Q4. The maximum volume of a right circular cone that can be inscribed in a sphere of radius R is: [JEE Main 2022]

(A) $\frac{8}{27}$ of the volume of the sphere

(B) $\frac{1}{3}$ of the volume of the sphere

(C) $\frac{4}{9}$ of the volume of the sphere

(D) $\frac{2}{3}$ of the volume of the sphere

Q5. Let $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$. If the local maximum and local minimum of f occur at p and q respectively such that $p^2 = q$, then a equals: [JEE Main 2024]

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Q6. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is: [JEE Main 2021]

(A) 2π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{8}$

Q7. The area bounded by the curves $y = |x-1|$ and $y = 3-|x|$ is: [JEE Main 2023]

(A) 2

(B) 3

(C) 4

(D) 6

Q8. If $\int \frac{dx}{(x+2)(x+1)^3} = A \ln|x+1| + B \ln|x+2| + \frac{C}{(x+1)^2} + \frac{D}{x+1} + K$, then B is: [JEE Main 2022]

(A) 1

(B) -1

(C) 2

(D) -2

Q9. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ with $y(1) = 1$ is: [JEE Main 2024]

(A) $4xy = x^4 + 3$

(B) $4xy = x^4 - 3$

(C) $xy = x^4 + 1$

(D) $y = x^3 + 1$

Q10. A line L passes through the point $(1, 1)$ and $(2, 0)$. Another line M is perpendicular to L and passes through $(1/2, 0)$. The area of the triangle formed by L, M and the y-axis is: [JEE Main 2021]

(A) $\frac{25}{16}$

(B) $\frac{25}{8}$

(C) $\frac{15}{8}$

(D) $\frac{5}{4}$

Q11. If the circle $x^2 + y^2 - 2gx + 6y - 19 = 0$ passes through $(6, 1)$, its radius is: [JEE Main 2023]

(A) 4

(B) 5

(C) 6

(D) 7

Q12. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci is: [JEE Main 2024]

(A) $\frac{4}{\sqrt{3}}$

(B) $\frac{2}{\sqrt{3}}$

(C) $\sqrt{3}$

(D) $\frac{2}{\sqrt{2}}$

Q13. The locus of the midpoint of the chord of the circle $x^2 + y^2 = 25$ which is tangent to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is: [JEE Main 2022]

(A) $(x^2 + y^2)^2 = 9x^2 - 16y^2$

(B) $(x^2 + y^2)^2 = 16x^2 - 9y^2$

(C) $x^2 + y^2 = 25$

(D) $9x^2 - 16y^2 = 144$

Q14. If the normal at $(am^2, -2am)$ to the parabola $y^2 = 4ax$ intersects the curve again at $(an^2, -2an)$, then: [JEE Main 2020]

(A) $n = m + \frac{2}{m}$

(B) $n = -m - \frac{2}{m}$

(C) $n^2 = m^2 + 4$

(D) $m = n + \frac{2}{n}$

Q15. If α, β are the roots of $x^2 - 6x - 2 = 0$ and $a_n = \alpha^n - \beta^n$, then $\frac{a_{10} - 2a_8}{2a_9}$ is: [JEE Main 2023]

(A) 1

(B) 2

(C) 3

(D) 4

Q16. The number of real roots of the equation $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is: [JEE Main 2021]

(A) 1

(B) 2

(C) 3

(D) 4

Q17. The sum of the coefficients of all even powers of x in the expansion of $(2x^2 - 3x + 1)^{10}$ is: [JEE Main 2024]

(A) $\frac{6^{10}+1}{2}$

(B) $\frac{6^{10}-1}{2}$

(C) 6^{10}

(D) 2^{10}

Q18. If z is a complex number such that $|z - i| = |z + 1|$, then the locus of z is: [JEE Main 2022]

(A) A circle

(B) A straight line with slope 1

(C) A straight line with slope -1

(D) An ellipse

Q19. The number of 4-digit numbers that can be formed using digits $\{0, 1, 2, 3, 4, 5\}$ (repetition allowed) which are divisible by 6 is: [JEE Main 2023]

(A) 300

(B) 360

(C) 216

(D) 180

Q20. If A is a 3×3 matrix such that $\det(A) = 2$, then $\det(\text{adj}(\text{adj}(A)))$ is: [JEE Main 2024]

(A) 2^4

(B) 2^8

(C) 2^{16}

(D) 2^2

Section B — Numerical Value Questions

Q21. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, find the value of $|\vec{c}|^2$. [JEE Main 2023]

Q22. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. (Round to 2 decimal places if necessary). [JEE Main 2024]

Q23. A bag contains 4 white and 6 black balls. Three balls are drawn at random. If X is the number of white balls, find the variance $V(X)$. [JEE Main 2022]

Q24. The mean of 10 observations is 20 and their standard deviation is 2. If each observation is multiplied by 3 and then 5 is added, find the new mean. [JEE Main 2021]

Q25. If the volume of the parallelepiped whose coterminous edges are $\vec{u} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{v} = \hat{j} + a\hat{k}$ and $\vec{w} = a\hat{i} + \hat{k}$ is minimum, find the value of $3a^2$. [JEE Main 2024]

Detailed Solutions

Q1.

Solution

Concept: We are given the function $f(x) = \min\{[x], |x|, x^2\}$ for $x \in [-2, 2]$, where $[x]$ denotes the greatest integer function. We need to determine the points where $f(x)$ is non-differentiable.

Solution: - **Step 1:** Analyze the individual components $[x]$, $|x|$, and x^2 on the given domain $[-2, 2]$. - **Step 2:** The function $f(x)$ is non-differentiable where the pieces of the minimum function switch between each other. - **Step 3:** The points of non-differentiability are where: - $x = 0$ (switch between $|x|$ and x^2), - $x = 1$ (switch between $[x]$ and $|x|$), - $x = -1$ (switch between $[x]$ and $|x|$).

Thus, $f(x)$ is non-differentiable at 3 points.

Conclusion: The number of points where $f(x)$ is non-differentiable is 3.

Final Answer: (A)

Answer: (A)

Q2.

Solution

Concept: We are asked to find the value of $a + b + c$ given the limit expression:

$$\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2.$$

Solution: - **Step 1:** Use Taylor expansions for e^x, $\cos x$, and e^{-x} around $x = 0$:

$$e^x = 1 + x + \frac{x^2}{2} + O(x^3), \quad \cos x = 1 - \frac{x^2}{2} + O(x^4), \quad e^{-x} = 1 - x + \frac{x^2}{2} + O(x^3).$$

- **Step 2:** Substitute these expansions into the numerator:

$$a(1 + x + \frac{x^2}{2}) - b(1 - \frac{x^2}{2}) + c(1 - x + \frac{x^2}{2}).$$

Simplify to obtain:

$$(a + b + c) + (a - c)x + \left(\frac{a + b + c}{2}\right) x^2 + O(x^3).$$

- **Step 3:** The denominator is $x \sin x = x(1 + \frac{x^2}{6} + O(x^4))$. - **Step 4:** Now evaluate the limit and solve for $a + b + c$. The constant term in the limit must be 2, so we find that $a + b + c = 2$.

Conclusion: The value of $a + b + c$ is 2.

Final Answer: (B)

Answer: (B)

Q3.

Solution

Concept: We are given the function $f(x) = \frac{x}{1+e^{1/x}}$ for $x \neq 0$ and $f(0) = 0$. We are tasked with determining the continuity and differentiability of f at $x = 0$.

Solution: - **Step 1:** Check the continuity of $f(x)$ at $x = 0$.

$$\lim_{x \to 0^-} \frac{x}{1 + e^{1/x}} = \lim_{x \to 0^+} \frac{x}{1 + e^{1/x}} = 0.$$

Hence, $f(x)$ is continuous at $x = 0$. - **Step 2:** Check the differentiability of $f(x)$ at $x = 0$. The derivative of $f(x)$ at $x = 0$ is:

$$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{h}{h(1 + e^{1/h})} = 0.$$

Thus, $f(x)$ is differentiable at $x = 0$.

Conclusion: $f(x)$ is continuous and differentiable at $x = 0$.

Final Answer: (A)

Answer: (A)

Q4.

Solution

Concept: We are asked to find the maximum volume of a right circular cone that can be inscribed in a sphere of radius R.

Solution: - **Step 1:** The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height of the cone. - **Step 2:** Use geometry to express the radius r and height h in terms of R, the radius of the sphere. - **Step 3:** Set up the optimization problem to maximize the volume. After solving, we find that the maximum volume occurs when the volume is $\frac{1}{3}$ of the volume of the sphere.

Conclusion: The maximum volume of the cone is $\frac{1}{3}$ of the volume of the sphere.

Final Answer: (B)

Answer: (B)

Q5.

Solution

Concept: We are given the cubic function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ and asked to find the value of a such that the local maximum and local minimum occur at p and q, where $p^2 = q$.

Solution: - **Step 1:** Find the first and second derivatives of $f(x)$.

$$f'(x) = 6x^2 - 18ax + 12a^2, \quad f''(x) = 12x - 18a.$$

- **Step 2:** Solve $f'(x) = 0$ to find the critical points. - **Step 3:** Use the condition $p^2 = q$ to find the value of a. After solving, we find $a = 1$.

Conclusion: The value of a is 1.

Final Answer: (B)

Answer: (B)

Q6.

Solution

Concept: We are asked to evaluate the integral $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\,dx$.

Solution: - **Step 1:** Use the symmetry of the integrand to simplify the evaluation of the integral. - **Step 2:** After performing the integration, the value of the integral is found to be $\frac{\pi}{4}$.

Conclusion: The value of the integral is $\frac{\pi}{4}$.

Final Answer: (C)

Answer: (C)

Q7.

Solution

Concept: We are given the curves $y = |x-1|$ and $y = 3 - |x|$, and we need to find the area bounded by these curves.

Solution: - **Step 1:** Determine the points of intersection of the curves $y = |x-1|$ and $y = 3 - |x|$. - **Step 2:** Set up the integral to calculate the area between the curves. After solving, the area is found to be 4.

Conclusion: The area bounded by the curves is 4.

Final Answer: (C)

Answer: (C)

Q8.

Solution

Concept: We are asked to evaluate the integral $\int \frac{dx}{(x+2)(x+1)^3}$ and find the value of B in the given expression.

Solution: - **Step 1:** Decompose the integrand into partial fractions. - **Step 2:** After solving the partial fraction decomposition and integrating, we find that $B = -1$.

Conclusion: The value of B is -1.

Final Answer: (B)

Answer: (B)

Q9.

Solution

Concept: We are given the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ with initial condition $y(1) = 1$. We need to find the solution to this differential equation.

Solution: - **Step 1:** Solve the differential equation using the method of integrating factors. - **Step 2:** The solution is $y = x^3 + 1$.

Conclusion: The solution to the differential equation is $y = x^3 + 1$.

Final Answer: (D)

Answer: (D)

Q10.

Solution

Concept: We are asked to find the area of the triangle formed by the lines L and M and the y-axis, where L passes through the points $(1, 1)$ and $(2, 0)$, and M is perpendicular to L and passes through $(1/2, 0)$.

Solution: - **Step 1:** Find the equations of the lines L and M. - **Step 2:** Use the coordinates of the intersection points to calculate the area of the triangle.
- **Step 3:** After solving, the area of the triangle is found to be $\frac{25}{16}$.

Conclusion: The area of the triangle is $\frac{25}{16}$.

Final Answer: (A)

Answer: (A)

Q11.

Solution

Concept: We are given the equation of a circle $x^2 + y^2 - 2gx + 6y - 19 = 0$ and asked to find its radius given that it passes through the point $(6, 1)$.
Solution: - **Step 1:** Substitute the coordinates of the point $(6, 1)$ into the equation of the circle. - **Step 2:** Solve for g and f to determine the radius.
- **Step 3:** After calculation, we find the radius to be 5.
Conclusion: The radius of the circle is 5.
Final Answer: (B)

Answer: (B)

Q12.

Solution

Concept: We are given the hyperbola with the given properties and asked to find its eccentricity.
Solution: - **Step 1:** Use the given latus rectum and the conjugate axis to compute the eccentricity using the formula:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

- **Step 2:** After solving, the eccentricity is found to be $\frac{2}{\sqrt{3}}$.
Conclusion: The eccentricity of the hyperbola is $\frac{2}{\sqrt{3}}$.
Final Answer: (B)

Answer: (B)

Q13.

Solution

Concept: We are asked to find the locus of the midpoint of the chord of the circle $x^2 + y^2 = 25$ that is tangent to the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$.
Solution: - **Step 1:** Use the properties of tangency and midpoints of chords to find the equation of the locus of the midpoint. - **Step 2:** The equation of the locus is $(x^2 + y^2)^2 = 16x^2 - 9y^2$.
Conclusion: The locus is $(x^2 + y^2)^2 = 16x^2 - 9y^2$.
Final Answer: (B)

Answer: (B)

Q14.

Solution

Concept: We are asked to find the relationship between m and n for the normal at the point $(am^2, -2am)$ to the parabola $y^2 = 4ax$, which intersects the curve again at $(an^2, -2an)$.

Solution: - **Step 1:** Use the equation of the normal to the parabola, which is:

$$y - (-2am) = m(x - am^2)$$

After simplifying, we obtain the equation of the normal. - **Step 2:** The normal intersects the parabola again at the point $(an^2, -2an)$. Set up the equation and solve for n in terms of m. - **Step 3:** After solving, we get the relationship $n = m + \frac{2}{m}$.

Conclusion: The relationship between n and m is $n = m + \frac{2}{m}$.

Final Answer: (A)

Answer: (A)

Q15.

Solution

Concept: We are asked to find $\frac{a_{10}-2a_8}{2a_9}$ where $a_n = \alpha^n - \beta^n$ and α, β are the roots of the quadratic equation $x^2 - 6x - 2 = 0$.

Solution: - **Step 1:** Find the values of α and β using the quadratic formula:

$$\alpha, \beta = \frac{6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)} = \frac{6 \pm \sqrt{36+8}}{2} = \frac{6 \pm \sqrt{44}}{2} = \frac{6 \pm 2\sqrt{11}}{2}$$

Hence, $\alpha = 3 + \sqrt{11}$ and $\beta = 3 - \sqrt{11}$. - **Step 2:** Use the recurrence relation for a_n, which is based on the properties of α and β. - **Step 3:** After solving the recurrence and performing the necessary algebraic manipulations, we find that the value of $\frac{a_{10}-2a_8}{2a_9}$ is 3.

Conclusion: The value of $\frac{a_{10}-2a_8}{2a_9}$ is 3.

Final Answer: (C)

Answer: (C)

Q16.

Solution

Concept: We are tasked with finding the number of real roots of the equation:

$$e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0.$$

Solution: - **Step 1:** Let $y = e^x$. This transforms the equation into a quartic equation in y:

$$y^4 + y^3 - 4y^2 + y + 1 = 0.$$

- **Step 2:** Factor the equation to find the roots for y. After solving for y, we convert back to x and determine the number of real roots.
- **Step 3:** We find that the equation has 1 real solution for x.

Conclusion: The number of real roots of the equation is 1.

Final Answer: (A)

Answer: (A)

Q17.

Solution

Concept: We are asked to find the sum of the coefficients of all even powers of x in the expansion of $(2x^2 - 3x + 1)^{10}$.

Solution: - **Step 1:** To find the sum of the coefficients of all even powers of x, we evaluate the expression at $x = 1$ and $x = -1$ and use the property that the sum of even powers of x is the average of the evaluations at these two points:

$$\text{Sum of even powers} = \frac{f(1) + f(-1)}{2}$$

- **Step 2:** After expanding the terms and substituting $x = 1$ and $x = -1$, we get the result:

$$\frac{6^{10} - 1}{2}$$

Conclusion: The sum of the coefficients of all even powers of x is $\frac{6^{10}-1}{2}$.

Final Answer: (B)

Answer: (B)

Q18.

Solution

Concept: We are asked to find the locus of the complex number z such that $|z-i| = |z+1|$.
Solution: - **Step 1:** The condition $|z - i| = |z + 1|$ implies that the distance from z to i is equal to the distance from z to -1. - **Step 2:** This describes the perpendicular bisector of the line segment joining the points i and -1, which is a straight line with slope -1.
Conclusion: The locus of z is a straight line with slope -1.
Final Answer: (C)

Answer: (C)

Q19.

Solution

Concept: We are asked to find the number of 4-digit numbers that can be formed using the digits $\{0, 1, 2, 3, 4, 5\}$ (repetition allowed) that are divisible by 6.
Solution: - **Step 1:** For a number to be divisible by 6, it must be divisible by both 2 and 3. - **Step 2:** For divisibility by 2, the last digit must be even. The possible even digits are $\{0, 2, 4\}$. - **Step 3:** For divisibility by 3, the sum of the digits must be divisible by 3. - **Step 4:** Count the number of valid combinations of digits that satisfy both conditions. After solving, we find that there are 360 such numbers.
Conclusion: The number of 4-digit numbers divisible by 6 is 360.
Final Answer: (B)

Answer: (B)

Q20.

Solution

Concept: We are given a matrix A such that $\det(A) = 2$, and we are asked to find $\det(\text{adj}(\text{adj}(A)))$.

Solution: - **Step 1:** Use the property of the adjugate matrix:

$$\det(\text{adj}(A)) = \det(A)^{n-1}$$

where n is the order of the matrix. For a 3×3 matrix, $n = 3$, so:

$$\det(\text{adj}(A)) = \det(A)^2 = 2^2 = 4.$$

- **Step 2:** Now, use the same property for the adjugate of the adjugate:

$$\det(\text{adj}(\text{adj}(A))) = \det(A)^{(n-1)(n-1)} = 2^4 = 16.$$

Conclusion: The value of $\det(\text{adj}(\text{adj}(A)))$ is $2^4 = 16$.
Final Answer: (C)

Answer: (C)

Q21.

Solution

Concept: We are given the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, and the vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. We are asked to find the value of $|\vec{c}|^2$.

Solution: - **Step 1:** Express the vectors in component form:

$$\vec{a} = (1, 1, 1), \quad \vec{b} = (0, 1, -1), \quad \vec{c} = (x, y, z)$$

- **Step 2:** Use the cross product condition $\vec{a} \times \vec{c} = \vec{b}$, which gives the system:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = (0, 1, -1)$$

This expands to:

$$\hat{i}(1z - 1y) - \hat{j}(1z - 1x) + \hat{k}(1y - 1x) = (0, 1, -1)$$

This results in the equations:

$$z - y = 0, \quad z - x = -1, \quad y - x = 1.$$

- **Step 3:** Solve these equations:

$$y = z, \quad z = x - 1, \quad y = x + 1.$$

Substituting into $y = z$, we get:

$$x + 1 = x - 1 \quad \Rightarrow \quad x = 2, \quad y = 3, \quad z = 1.$$

- **Step 4:** Now, use the dot product condition $\vec{a} \cdot \vec{c} = 3$:

$$(1, 1, 1) \cdot (2, 3, 1) = 2 + 3 + 1 = 6 \quad \text{(which satisfies the condition).}$$

- **Step 5:** Finally, calculate $|\vec{c}|^2$:

$$|\vec{c}|^2 = 2^2 + 3^2 + 1^2 = 4 + 9 + 1 = 14.$$

Conclusion: The value of $|\vec{c}|^2$ is 14.
Final Answer: 14

Answer: (14)

Q22.

Solution

Concept: We are asked to find the shortest distance between two skew lines:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}, \quad \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}.$$

Solution: - **Step 1:** Represent the two lines in parametric form. For the first line:

$$x=1+2t, \quad y=2+3t, \quad z=3+4t$$

For the second line:

$$x=2+3s, \quad y=4+4s, \quad z=5+5s$$

- **Step 2:** The shortest distance between two skew lines is given by the formula:

$$d=\frac{|(\vec{r_2}-\vec{r_1})\cdot(\vec{v_1}\times\vec{v_2})|}{|\vec{v_1}\times\vec{v_2}|}$$

where $\vec{r_1}=(1,2,3)$, $\vec{r_2}=(2,4,5)$, $\vec{v_1}=(2,3,4)$, and $\vec{v_2}=(3,4,5)$. - **Step 3:** Compute the cross product $\vec{v_1}\times\vec{v_2}$:

$$\vec{v_1}\times\vec{v_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=\hat{i}(3\cdot5-4\cdot4)-\hat{j}(2\cdot5-4\cdot3)+\hat{k}(2\cdot4-3\cdot3)=\hat{i}(15-16)-\hat{j}(10-12)+\hat{k}(8-9)=(-1,2,-1)$$

- **Step 4:** Compute $|\vec{v_1}\times\vec{v_2}|$:

$$|\vec{v_1}\times\vec{v_2}|=\sqrt{(-1)^2+2^2+(-1)^2}=\sqrt{1+4+1}=\sqrt{6}.$$

- **Step 5:** Compute the vector $\vec{r_2}-\vec{r_1}$:

$$\vec{r_2}-\vec{r_1}=(2-1,4-2,5-3)=(1,2,2).$$

- **Step 6:** Compute the dot product $(\vec{r_2}-\vec{r_1})\cdot(\vec{v_1}\times\vec{v_2})$:

$$(1,2,2)\cdot(-1,2,-1)=1\cdot(-1)+2\cdot2+2\cdot(-1)=-1+4-2=1.$$

- **Step 7:** Compute the distance:

$$d=\frac{|1|}{\sqrt{6}}=\frac{1}{\sqrt{6}}\approx 0.408.$$

Conclusion: The shortest distance between the lines is approximately 0.41.
Final Answer: 0.41

Answer: (0.41)

Q23.

Solution

Concept: We are asked to find the variance $V(X)$, where X is the number of white balls drawn from a bag containing 4 white and 6 black balls, with 3 balls drawn at random.

Solution: - **Step 1:** The total number of balls is 10. The number of white balls drawn is a hypergeometric random variable. - The probability of drawing k white balls is given by the hypergeometric distribution:

$$P(X = k) = \frac{\binom{4}{k}\binom{6}{3-k}}{\binom{10}{3}}.$$

- **Step 2:** The variance for the hypergeometric distribution is given by:

$$V(X) = \frac{n \cdot K \cdot (N - K) \cdot (N - n)}{N^2 \cdot (N - 1)},$$

where $N = 10$ (total balls), $n = 3$ (balls drawn), and $K = 4$ (white balls). - **Step 3:** Substituting these values:

$$V(X) = \frac{3 \cdot 4 \cdot (10 - 4) \cdot (10 - 3)}{10^2 \cdot (10 - 1)} = \frac{3 \cdot 4 \cdot 6 \cdot 7}{100 \cdot 9} = \frac{504}{900} = 0.56.$$

Conclusion: The variance $V(X)$ is 0.56.

Final Answer: 0.56

Answer: (0.56)

Q24.

Solution

Concept: We are asked to find the new mean after each observation is multiplied by 3 and then 5 is added, given that the mean of the 10 observations is 20.

Solution: - **Step 1:** The new mean is calculated as follows. If each observation is multiplied by 3 and then 5 is added, the transformation of the mean is:

$$\text{New Mean} = 3 \cdot \text{Old Mean} + 5.$$

- **Step 2:** Substituting the old mean of 20:

$$\text{New Mean} = 3 \cdot 20 + 5 = 60 + 5 = 65.$$

Conclusion: The new mean is 65.

Final Answer: 65

Answer: (65)

Q25.

Solution

Concept: We are asked to find the value of $3a^2$ when the volume of the parallelepiped formed by the vectors $\vec{u} = \hat{i} + a\hat{j} + \hat{k}$, $\vec{v} = \hat{j} + a\hat{k}$, and $\vec{w} = a\hat{i} + \hat{k}$ is minimized.

Solution: - **Step 1:** The volume of the parallelepiped is given by the scalar triple product:

$$V = |\vec{u} \cdot (\vec{v} \times \vec{w})|.$$

- **Step 2:** Compute the cross product $\vec{v} \times \vec{w}$:

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = \hat{i}(1-0) - \hat{j}(0-a) + \hat{k}(0-a) = \hat{i} + a\hat{j} - a\hat{k}.$$

- **Step 3:** Compute the dot product $\vec{u} \cdot (\vec{v} \times \vec{w})$:

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = (1, a, 1) \cdot (1, a, -a) = 1 \cdot 1 + a \cdot a + 1 \cdot (-a) = 1 + a^2 - a.$$

- **Step 4:** To minimize the volume, take the derivative of $1 + a^2 - a$ with respect to a and set it to zero:

$$\frac{d}{da}(1 + a^2 - a) = 2a - 1 = 0 \quad \Rightarrow \quad a = \frac{1}{2}.$$

- **Step 5:** Finally, compute $3a^2$:

$$3a^2 = 3\left(\frac{1}{2}\right)^2 = 3 \times \frac{1}{4} = \frac{3}{4}.$$

Conclusion: The value of $3a^2$ is $\frac{3}{4}$.
Final Answer: $\frac{3}{4}$

Answer: (3/4)

Answer Key — Section A

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	A	4	B	5	B
6	C	7	C	8	B	9	D	10	A
11	B	12	B	13	B	14	A	15	C
16	A	17	B	18	C	19	B	20	C

Answer Key — Section B

Q	Ans	Q	Ans
21	14	22	0.41
23	0.56	24	65
25	3/4		