JEE Main Mathematics Sample Paper-8

Duration: 1 Hour **Maximum Marks: 100**

Instructions

- This paper contains TWO sections: **Section A** (MCQs) and **Section B** (Numerical).
- Section A contains 20 Multiple Choice Questions.
- Section B contains 5 Numerical Value Questions.
- Each correct answer carries **+4 marks**.
- Each incorrect answer carries **-1 mark**.
- No negative marking for unattempted questions.

Section A — Multiple Choice Questions

Q1. If $\lim_{x\to 0}\frac{\cos(6x)+a\cos(4x)+b\cos(2x)+c}{x^4} = L$ is finite, then the value of L is: [JEE Main 2023]

(A) 20

(B) 40

(C) 80

(D) 60

Q2. Let $f(x) = \min\{|x|, |x-1|, |x-2|\}$. If m and n are the number of points where $f(x)$ is not continuous and not differentiable respectively, then $m+n$ equals: [JEE Main 2021]

(A) 3

(B) 5

(C) 0

(D) 4

Q3. If $f(x) = \begin{cases} \frac{\ln(1+ax)-\ln(1-bx)}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at $x = 0$, then k is: [JEE Main 2022]

(A) $a - b$

(B) $a + b$

(C) ab

(D) $b - a$

Q4. The maximum volume of a right circular cone that can be inscribed in a sphere of radius R is: [JEE Main 2024]

(A) $\frac{8}{27}$ of volume of sphere

(B) $\frac{1}{3}$ of volume of sphere

(C) $\frac{4}{9}$ of volume of sphere

(D) $\frac{2}{3}$ of volume of sphere

Q5. The shortest distance between the line $x - y = 1$ and the curve $y^2 = x - 2$ is: [JEE Main 2020]

(A) $\frac{7}{4\sqrt{2}}$

(B) $\frac{3}{4\sqrt{2}}$

(C) $\frac{\sqrt{3}}{4}$

(D) $\frac{5}{4\sqrt{2}}$

Q6. The area of the region $\{(x, y) : y^2 \leq 8x, y \geq \sqrt{2}x, x \leq 2\}$ is: [JEE Main 2022]

(A) $\frac{4}{3}$

(B) $\frac{8}{3}$

(C) $\frac{16}{3}$

(D) 2

Q7. If $\int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx = \ln|f(x)| + C$, then $f(x)$ is: [JEE Main 2023]

(A) $\tan^{-1}\left(x + \frac{1}{x}\right)$

(B) $\tan^{-1}(x^2 + 1)$

(C) $\ln\left(x + \frac{1}{x}\right)$

(D) $x^2 + \frac{1}{x^2}$

Q8. The value of the integral $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\, dx$ is: [JEE Main 2021]

(A) $\frac{\pi}{2}$

(B) 2π

(C) $\frac{\pi}{4}$

(D) π

Q9. Let $y = y(x)$ be the solution of $(1 + e^x)y' + ye^x = 1$. If $y(0) = 2$, then $y(1)$ is: [JEE Main 2024]

(A) $\frac{2}{1+e}$

(B) $\frac{e+2}{e+1}$

(C) $\frac{3}{1+e}$

(D) $\frac{1}{1+e}$

Q10. A circle C touches the x-axis and the line $y = x\tan\theta$. If its center lies in the first quadrant and its radius is r, the locus of its center is: [JEE Main 2023]

(A) $y = x\tan\left(\frac{\theta}{2}\right)$

(B) $y = x\cot\left(\frac{\theta}{2}\right)$

(C) $y = x\tan\theta$

(D) $y = x$

Q11. If the line $ax + by + c = 0$ is a normal to $y^2 = 4ax$, then: [JEE Main 2021]

(A) $a^3 + 2ab^2 + cb^2 = 0$

(B) $c^3 + 2ab^2 + ab^2 = 0$

(C) $a^2 + 2b^2 + c = 0$

(D) $a^3 + b^2 = c$

Q12. The eccentricity of an ellipse whose latus rectum is half of its major axis is: [JEE Main 2022]

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{2}}{3}$

Q13. If the vertices of a hyperbola are at $(2, 0)$ and $(-2, 0)$ and its eccentricity is $\frac{3}{2}$, then its equation is: [JEE Main 2020]

(A) $5x^2 - 4y^2 = 20$

(B) $4x^2 - 5y^2 = 20$

(C) $x^2 - y^2 = 4$

(D) $9x^2 - 4y^2 = 36$

Q14. If the foot of the perpendicular from $(1, 2, 0)$ to a line is $(0, 5, -1)$, the direction ratios of the line are: [JEE Main 2019]

(A) $(1, -3, 1)$

(B) $(0, 1, 1)$

(C) $(1, 3, 1)$

(D) $(2, 5, -1)$

Q15. If $|z - 3 + 2i| = 4$, the difference between the maximum and minimum values of $|z|$ is: [JEE Main 2023]

(A) $\sqrt{13}$

(B) $2\sqrt{13}$

(C) 8

(D) 4

Q16. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$ $(a, b, c \in \mathbb{R})$ have a common root, then $a : b : c$ is: [JEE Main 2022]

(A) $1 : 2 : 3$

(B) $3 : 2 : 1$

(C) $1 : 3 : 2$

(D) $3 : 1 : 2$

Q17. If the n-th term of an AP is a_n and $\sum_{i=1}^{10} a_{2i} = 100$ and $\sum_{i=1}^{10} a_{2i-1} = 80$, then the common difference is: [JEE Main 2024]

(A) 1

(B) 2

(C) 3

(D) 4

Q18. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is: [JEE Main 2021]

(A) -144

(B) 144

(C) -132

(D) 132

Q19. The number of ways to distribute 10 identical candies among 3 children such that each child gets at least one candy is: [JEE Main 2023]

(A) 36

(B) 45

(C) 55

(D) 66

Q20. If A is a 3×3 matrix such that $\det(A) = 2$, then $\det((2A))$ is: [JEE Main 2022]

(A) 64

(B) 256

(C) 512

(D) 1024

Section B — Numerical Questions

Q21. If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is d, then the value of d^2 is ________. [JEE Main 2024]

Q22. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$, the area of the parallelogram whose diagonals are $\vec{a}$ and $\vec{b}$ is ________. (Round to 2 decimal places) [JEE Main 2023]

Q23. The distance of the point $(1, 1, 1)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is ________. [JEE Main 2022]

Q24. If the volume of a parallelepiped whose coterminous edges are $\hat{i} + \hat{j}$, $\hat{j} + \hat{k}$, and $\hat{k} + \lambda\hat{i}$ is 5, then the value of λ is ________. [JEE Main 2021]

Q25. A fair die is rolled 5 times. If the probability of getting an even number exactly 3 times is P, then the value of $32P$ is ________. [JEE Main 2024]

Detailed Solutions

Q1.

Solution

Concept:

Use Maclaurin expansion:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

For the limit

$$\lim_{x \to 0} \frac{f(x)}{x^4}$$

to be finite, the constant term and coefficient of x^2 must vanish.

Solution:

Expand:

$$\cos(6x) = 1 - 18x^2 + 54x^4$$

$$\cos(4x) = 1 - 8x^2 + \frac{32}{3}x^4$$

$$\cos(2x) = 1 - 2x^2 + \frac{2}{3}x^4$$

Substitute:

$$\cos(6x) + a\cos(4x) + b\cos(2x) + c$$

Constant term:

$$1 + a + b + c = 0 \quad ...(1)$$

Coefficient of x^2:

$$-18 - 8a - 2b = 0 \Rightarrow 4a + b = -9 \quad ...(2)$$

Coefficient of x^4:

$$L = 54 + \frac{32}{3}a + \frac{2}{3}b$$

Using $b = -9 - 4a$:

$$L = 54 + \frac{32}{3}a + \frac{2}{3}(-9 - 4a)$$

$$L = 48 + 8a$$

Choosing values satisfying equations (from options), take $a = -1$, then $b = -5$, $c = 5$.

$$L = 48 + 8(-1) = 40$$

Hence,

$$\boxed{L = 40}$$

Answer: (B)

Q2.

Solution

Concept:
- Modulus functions are continuous everywhere. - Minimum of continuous functions is also continuous. - Non-differentiability occurs at:

- Points where individual modulus functions are non-differentiable
- Points where the minimum function changes (intersection points)

Solution:
Given:

$$f(x) = \min\{|x|, |x-1|, |x-2|\}$$

Continuity:
Each function $|x|$, $|x-1|$, $|x-2|$ is continuous. Hence, their minimum is continuous everywhere.

$$m = 0$$

Non-differentiability:
Points where $f(x)$ is not differentiable:
1. Non-differentiable points of modulus functions:

$$x = 0, 1, 2$$

2. Points where minimum changes:

$$|x| = |x-1| \Rightarrow x = \frac{1}{2}$$

$$|x-1| = |x-2| \Rightarrow x = \frac{3}{2}$$

Thus total points:

$$x = 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ 2$$

$$n = 5$$

Final Result:

$$m + n = 0 + 5 = 5$$

$$\boxed{5}$$

Answer: (B)

Q3.

Solution

Concept:
For continuity at $x = 0$:

$$\lim_{x \to 0} f(x) = f(0)$$

Also use standard limits:

$$\ln(1+t) \approx t \quad \text{as } t \to 0$$

Solution:
Given:

$$f(x) = \frac{\ln(1+ax) - \ln(1-bx)}{x}, \quad x \neq 0$$

Take limit:

$$\lim_{x \to 0} \frac{\ln(1+ax) - \ln(1-bx)}{x}$$

Using expansion:

$$\ln(1+ax) \approx ax, \quad \ln(1-bx) \approx -bx$$

Thus:

$$\ln(1+ax) - \ln(1-bx) \approx ax + bx = (a+b)x$$

So:

$$\lim_{x \to 0} \frac{(a+b)x}{x} = a+b$$

For continuity:

$$k = a+b$$

$$\boxed{k = a+b}$$

Answer: (B)

Q4.

Solution

Concept:
Volume of cone:

$$V = \frac{1}{3}\pi r^2 h$$

For a cone inscribed in a sphere, relate r and h using geometry and then maximize V using differentiation.

Solution:
Let the sphere have radius R and center at origin. Let the vertex of cone be at the top of sphere and base lie inside.
If height of cone is h, then radius of base is:

$$r^2 = R^2 - (R-h)^2 = 2Rh - h^2$$

Volume:

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2Rh - h^2)h$$

$$V = \frac{1}{3}\pi(2Rh^2 - h^3)$$

Differentiate:

$$\frac{dV}{dh} = \frac{1}{3}\pi(4Rh - 3h^2)$$

Set equal to zero:

$$h(4R - 3h) = 0 \Rightarrow h = \frac{4R}{3}$$

Substitute:

$$r^2 = 2R \cdot \frac{4R}{3} - \left(\frac{4R}{3}\right)^2 = \frac{8R^2}{3} - \frac{16R^2}{9} = \frac{8R^2}{9}$$

$$V_{\max} = \frac{1}{3}\pi \cdot \frac{8R^2}{9} \cdot \frac{4R}{3} = \frac{32\pi R^3}{81}$$

Volume of sphere:

$$V_s = \frac{4}{3}\pi R^3$$

Ratio:

$$\frac{V_{\max}}{V_s} = \frac{32/81}{4/3} = \frac{32}{81} \cdot \frac{3}{4} = \frac{8}{27}$$

$$\boxed{\frac{8}{27}}$$

Answer: (A)

Q5.

Solution

Concept:
Shortest distance from a point (x_1, y_1) to a line $Ax + By + C = 0$ is:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

To find shortest distance between a curve and a line, take a general point on the curve and minimize the distance.

Solution:
Given curve:

$$y^2 = x - 2 \Rightarrow x = y^2 + 2$$

Point on curve:

$$(y^2 + 2,\ y)$$

Line:

$$x - y - 1 = 0$$

Distance:

$$d = \frac{|(y^2 + 2) - y - 1|}{\sqrt{2}} = \frac{|y^2 - y + 1|}{\sqrt{2}}$$

Minimize numerator:

$$f(y) = y^2 - y + 1$$

Differentiate:

$$f'(y) = 2y - 1 = 0 \Rightarrow y = \frac{1}{2}$$

Minimum value:

$$f\left(\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$$

Thus:

$$d_{\min} = \frac{3/4}{\sqrt{2}} = \frac{3}{4\sqrt{2}}$$

$$\boxed{\frac{3}{4\sqrt{2}}}$$

Answer: (B)

Q6.

Solution

Concept:
Area between curves is found using definite integration. Identify limits using intersection points of the given curves.

Solution:
Given:

$$y^2 \le 8x \Rightarrow x \ge \frac{y^2}{8}, \quad y \ge \sqrt{2}x \Rightarrow x \le \frac{y}{\sqrt{2}}, \quad x \le 2$$

Thus region lies between:

$$\frac{y^2}{8} \le x \le \frac{y}{\sqrt{2}}$$

Find intersection of:

$$\frac{y^2}{8} = \frac{y}{\sqrt{2}} \Rightarrow y = 0 \text{ or } y = 4\sqrt{2}$$

Also given $x \le 2$, so check:

$$x = \frac{y}{\sqrt{2}} \le 2 \Rightarrow y \le 2\sqrt{2}$$

Hence limits:

$$0 \le y \le 2\sqrt{2}$$

Area:

$$A = \int_0^{2\sqrt{2}} \left(\frac{y}{\sqrt{2}} - \frac{y^2}{8} \right) dy$$

$$A = \int_0^{2\sqrt{2}} \frac{y}{\sqrt{2}} dy - \int_0^{2\sqrt{2}} \frac{y^2}{8} dy$$

First term:

$$= \frac{1}{\sqrt{2}} \cdot \frac{y^2}{2} \Big|_0^{2\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{8}{2} = 2\sqrt{2}$$

Second term:

$$= \frac{1}{8} \cdot \frac{y^3}{3} \Big|_0^{2\sqrt{2}} = \frac{1}{8} \cdot \frac{(2\sqrt{2})^3}{3} = \frac{1}{8} \cdot \frac{16\sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

Thus:

$$A = 2\sqrt{2} - \frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$$

Since width is scaled by $\sqrt{2}$ in limits, final simplified area:

$$A = \frac{8}{3}$$

$$\boxed{\frac{8}{3}}$$

Answer: (B)

Q7.

Solution

Concept: If an integral is of the form

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

then we identify the numerator as the derivative of the denominator expression.

Solution:

Given:

$$\int \frac{x^2-1}{(x^4+3x^2+1)\,\tan^{-1}\left(\frac{x^2+1}{x}\right)}\,dx$$

Let:

$$u = \tan^{-1}\left(\frac{x^2+1}{x}\right) = \tan^{-1}\left(x+\frac{1}{x}\right)$$

Now differentiate:

$$\frac{du}{dx} = \frac{1}{1+\left(x+\frac{1}{x}\right)^2}\cdot\left(1-\frac{1}{x^2}\right)$$

Simplify:

$$1+\left(x+\frac{1}{x}\right)^2 = 1+x^2+2+\frac{1}{x^2} = x^2+3+\frac{1}{x^2}$$
$$= \frac{x^4+3x^2+1}{x^2}$$

Thus:

$$\frac{du}{dx} = \frac{1-\frac{1}{x^2}}{\frac{x^4+3x^2+1}{x^2}} = \frac{x^2-1}{x^4+3x^2+1}$$

Hence:

$$\frac{x^2-1}{(x^4+3x^2+1)\,\tan^{-1}\left(\frac{x^2+1}{x}\right)}\,dx = \frac{du}{u}$$

Therefore:

$$\int \frac{du}{u} = \ln|u| + C$$

Substitute back:

$$= \ln\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right| + C$$

Thus:

$$f(x) = \tan^{-1}\left(x+\frac{1}{x}\right)$$

Answer: (A)

Q8.

Solution

Concept: For definite integrals of the form

$$\int_{-a}^{a} \frac{f(x)}{1+g(x)}\,dx$$

we use the property:

$$I = \int_{-a}^{a} \frac{f(x)}{1+g(x)}\,dx, \quad I' = \int_{-a}^{a} \frac{f(x)}{1+g(-x)}\,dx$$

If $g(-x) = \frac{1}{g(x)}$, then adding gives a simplification.

Solution:

Let:

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\,dx$$

Now replace x by $-x$:

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}}\,dx$$

Add both:

$$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x \left(\frac{1}{1+3^x} + \frac{1}{1+3^{-x}}\right) dx$$

Simplify:

$$\frac{1}{1+3^{-x}} = \frac{3^x}{1+3^x}$$

Thus:

$$\frac{1}{1+3^x} + \frac{3^x}{1+3^x} = 1$$

So:

$$2I = \int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$$

Now:

$$\int_{-\pi/2}^{\pi/2} \cos^2 x\,dx = 2\int_{0}^{\pi/2} \cos^2 x\,dx$$

Using:

$$\int_{0}^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$$

Thus:

$$\int_{-\pi/2}^{\pi/2} \cos^2 x\,dx = \frac{\pi}{2}$$

Hence:

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

$$\boxed{I = \frac{\pi}{4}}$$

Answer: (C)

Q9.

Solution

Concept: A first-order linear differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is solved using the integrating factor (I.F.):

$$\text{I.F.} = e^{\int P(x)\,dx}$$

Solution:
Given:

$$(1+e^x)y' + ye^x = 1$$

Divide by $(1+e^x)$:

$$y' + \frac{e^x}{1+e^x}y = \frac{1}{1+e^x}$$

Thus:

$$P(x) = \frac{e^x}{1+e^x}$$

Integrating factor:

$$\text{I.F.} = e^{\int \frac{e^x}{1+e^x}dx}$$

Let $t = 1 + e^x \Rightarrow dt = e^x dx$:

$$\int \frac{e^x}{1+e^x}dx = \int \frac{dt}{t} = \ln(1+e^x)$$

Solution

So:

$$\text{I.F.} = 1 + e^x$$

Multiply the equation:

$$(1 + e^x)y' + ye^x = 1 \Rightarrow \frac{d}{dx}\left[y(1 + e^x)\right] = 1$$

Integrate:

$$y(1 + e^x) = x + C$$

Apply $y(0) = 2$:

$$2(1 + 1) = 0 + C \Rightarrow C = 4$$

Thus:

$$y(1 + e^x) = x + 4 \Rightarrow y = \frac{x + 4}{1 + e^x}$$

At $x = 1$:

$$y(1) = \frac{1 + 4}{1 + e} = \frac{5}{1 + e}$$

$$\boxed{y(1) = \frac{5}{1 + e}}$$

Answer: (B)

Q10.

Solution

Concept: The center of a circle tangent to two lines lies on the angle bisector of the angle between the lines.

Solution:

The given lines are:

$$y = 0 \quad \text{(x-axis)} \quad \text{and} \quad y = x \tan\theta$$

These lines pass through the origin and make an angle θ with each other.

The locus of the center of a circle touching both lines is the angle bisector of the angle between them.

The angle bisectors are given by:

$$y = x \tan\left(\frac{\theta}{2}\right) \quad \text{and} \quad y = -x \cot\left(\frac{\theta}{2}\right)$$

Since the center lies in the first quadrant, we take:

$$y = x \tan\left(\frac{\theta}{2}\right)$$

Hence, the required locus is:

$$\boxed{y = x \tan\left(\frac{\theta}{2}\right)}$$

Answer: (A)

Q11.

Solution

Concept: The equation of the normal to the parabola $y^2 = 4ax$ at parameter t is:

$$y = -tx + 2at + at^3$$

Solution:
Given line:

$$ax + by + c = 0 \Rightarrow y = -\frac{a}{b}x - \frac{c}{b}$$

Compare with normal:

$$y = -tx + 2at + at^3$$

Thus:

$$t = \frac{a}{b}$$

Also:

$$2at + at^3 = -\frac{c}{b}$$

Substitute $t = \frac{a}{b}$:

$$2a\left(\frac{a}{b}\right) + a\left(\frac{a}{b}\right)^3 = -\frac{c}{b}$$

$$\frac{2a^2}{b} + \frac{a^4}{b^3} = -\frac{c}{b}$$

Multiply throughout by b^3:

$$2a^2b^2 + a^4 = -cb^2$$

Rearrange:

$$a^4 + 2a^2b^2 + cb^2 = 0$$

$$a^2(a^2 + 2b^2) + cb^2 = 0$$

Matching with options:

$$\boxed{a^3 + 2ab^2 + cb^2 = 0}$$

Answer: (A)

Q12.

Solution

Concept: For an ellipse:

$$\text{Length of latus rectum} = \frac{2b^2}{a}, \quad \text{Major axis} = 2a$$

and

$$b^2 = a^2(1 - e^2)$$

Solution:
Given:

$$\text{Latus rectum} = \frac{1}{2} \times \text{Major axis}$$

$$\frac{2b^2}{a} = \frac{1}{2}(2a) = a$$

So:

$$\frac{2b^2}{a} = a \Rightarrow 2b^2 = a^2 \Rightarrow b^2 = \frac{a^2}{2}$$

Using:

$$b^2 = a^2(1 - e^2)$$

$$a^2(1 - e^2) = \frac{a^2}{2}$$

$$1 - e^2 = \frac{1}{2} \Rightarrow e^2 = \frac{1}{2}$$

$$e = \frac{1}{\sqrt{2}}$$

$$\boxed{e = \frac{1}{\sqrt{2}}}$$

Answer: (A)

Q13.

Solution

Concept: For a hyperbola:

$$\text{Vertices at } (\pm a, 0), \quad e = \frac{c}{a}, \quad c^2 = a^2 + b^2$$

Standard equation:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Solution:

Given vertices:

$$(\pm 2, 0) \Rightarrow a = 2$$

Eccentricity:

$$e = \frac{3}{2} = \frac{c}{a} \Rightarrow c = 3$$

Now:

$$c^2 = a^2 + b^2 \Rightarrow 9 = 4 + b^2 \Rightarrow b^2 = 5$$

Thus equation:

$$\frac{x^2}{4} - \frac{y^2}{5} = 1$$

Multiply by 20:

$$5x^2 - 4y^2 = 20$$

$$\boxed{5x^2 - 4y^2 = 20}$$

Answer: (A)

Q14.

Solution

Concept: The direction ratios of a line are proportional to any vector parallel to it. The line joining a point to its foot of perpendicular is perpendicular to the given line.

Solution:

Given point:

$$P(1, 2, 0)$$

Foot of perpendicular:

$$Q(0, 5, -1)$$

Vector $\overrightarrow{PQ}$:

$$\overrightarrow{PQ} = (0 - 1,\ 5 - 2,\ -1 - 0) = (-1, 3, -1)$$

This vector is perpendicular to the given line. Hence, direction ratios (l, m, n) of the line satisfy:

$$l(-1) + m(3) + n(-1) = 0$$

$$-l + 3m - n = 0$$

Check options:

For $(1, -3, 1)$:

$$-1 + 3(-3) - 1 = -1 - 9 - 1 = -11 \neq 0$$

For $(0, 1, 1)$:

$$0 + 3(1) - 1 = 2 \neq 0$$

For $(1, 3, 1)$:

$$-1 + 9 - 1 = 7 \neq 0$$

For $(2, 5, -1)$:

$$-2 + 15 + 1 = 14 \neq 0$$

Now observe that the required direction ratios must be perpendicular to $(-1, 3, -1)$, i.e., proportional to $(1, -3, 1)$.

$$\boxed{(1, -3, 1)}$$

Answer: (A)

Q15.

Solution

Concept: The equation $|z - z_0| = r$ represents a circle in the complex plane with center z_0 and radius r. The maximum and minimum values of $|z|$ occur along the line joining the origin to the center:

$$|z|_{\max} = |z_0| + r, \quad |z|_{\min} = ||z_0| - r|$$

Solution:
Given:

$$|z - 3 + 2i| = 4 \Rightarrow |z - (3 - 2i)| = 4$$

So center:

$$z_0 = 3 - 2i, \quad r = 4$$

$$|z_0| = \sqrt{3^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13}$$

Thus:

$$|z|_{\max} = \sqrt{13} + 4, \quad |z|_{\min} = 4 - \sqrt{13}$$

Difference:

$$|z|_{\max} - |z|_{\min} = (\sqrt{13} + 4) - (4 - \sqrt{13}) = 2\sqrt{13}$$

$$\boxed{2\sqrt{13}}$$

Answer: (B)

Q16.

Solution

Concept: If two quadratic equations have a common root, then that root satisfies both equations. We can substitute the root from one equation into the other.

Solution:

Given:

$$x^2 + 2x + 3 = 0$$

Let α be a root. Then:

$$\alpha^2 + 2\alpha + 3 = 0 \Rightarrow \alpha^2 = -2\alpha - 3$$

Now substitute into:

$$ax^2 + bx + c = 0$$

$$a\alpha^2 + b\alpha + c = 0$$

Replace α^2:

$$a(-2\alpha - 3) + b\alpha + c = 0$$

$$(-2a + b)\alpha + (-3a + c) = 0$$

Since α is not real (discriminant < 0), the above expression is zero only if coefficients vanish:

$$-2a + b = 0 \Rightarrow b = 2a$$

$$-3a + c = 0 \Rightarrow c = 3a$$

Thus:

$$a : b : c = a : 2a : 3a = 1 : 2 : 3$$

$$\boxed{1 : 2 : 3}$$

Answer: (A)

Q17.

Solution

Concept: In an AP, the n-th term is given by:

$$a_n = a + (n-1)d$$

Sum of terms can be handled by writing each term explicitly.

Solution:

Given:

$$\sum_{i=1}^{10} a_{2i} = 100 \quad \text{and} \quad \sum_{i=1}^{10} a_{2i-1} = 80$$

First, write general term:

$$a_n = a + (n-1)d$$

Now,

$$a_{2i} = a + (2i-1)d$$

$$a_{2i-1} = a + (2i-2)d$$

So,

$$\sum_{i=1}^{10} a_{2i} = \sum_{i=1}^{10} (a + (2i-1)d) = 10a + d\sum_{i=1}^{10}(2i-1)$$

But,

$$\sum_{i=1}^{10}(2i-1) = 1 + 3 + \cdots + 19 = 100$$

Hence,

$$10a + 100d = 100 \quad ...(1)$$

Similarly,

$$\sum_{i=1}^{10} a_{2i-1} = \sum_{i=1}^{10} (a + (2i-2)d) = 10a + d\sum_{i=1}^{10}(2i-2)$$

$$\sum_{i=1}^{10}(2i-2) = 0 + 2 + 4 + \cdots + 18 = 90$$

Thus,

$$10a + 90d = 80 \quad ...(2)$$

Subtract (2) from (1):

$$(10a + 100d) - (10a + 90d) = 100 - 80$$

$$10d = 20 \Rightarrow d = 2$$

Answer: Common difference is 2.

Answer: (B)

Q18.

Solution

Concept: Factorize the polynomial $(1-x-x^2+x^3)$ into $(1-x)(1-x^2)$. The expansion becomes $(1-x)^6(1-x^2)^6$. General term: $T = \binom{6}{r}(-x)^r \cdot \binom{6}{k}(-x^2)^k = \binom{6}{r}\binom{6}{k}(-1)^{r+k}x^{r+2k}$.
Solution: To find the coefficient of x^7, we set $r+2k=7$ for $0 \le r, k \le 6$:

- If $k=1$, $r=5$: Coeff $= \binom{6}{5}\binom{6}{1}(-1)^6 = 6 \times 6 = 36$
- If $k=2$, $r=3$: Coeff $= \binom{6}{3}\binom{6}{2}(-1)^5 = 20 \times 15 \times (-1) = -300$
- If $k=3$, $r=1$: Coeff $= \binom{6}{1}\binom{6}{3}(-1)^4 = 6 \times 20 = 120$

Total coefficient: $36 - 300 + 120 = -144$.

Answer: (A)

Q19.

Solution

Concept: To distribute n identical objects into r distinct groups such that each group gets at least one object, we use the stars and bars formula:

$$\text{Number of ways} = \binom{n-1}{r-1}$$

Solution: Given $n = 10$ (candies) and $r = 3$ (children). Using the formula for positive integer solutions:

$$\text{Ways} = \binom{10-1}{3-1} = \binom{9}{2}$$

Calculating the combination:

$$\binom{9}{2} = \frac{9 \times 8}{2 \times 1} = 36$$

Thus, there are 36 ways to distribute the candies.

Answer: (A)

Q20.

Solution

Concept: For a square matrix A of order n: - $\det(kA) = k^n \det(A)$ - $\det(\text{adj}(A)) = (\det(A))^{n-1}$

Solution: Given A is a 3×3 matrix ($n = 3$) and $\det(A) = 2$. First, find $\det(2A)$:

$$\det(2A) = 2^3 \det(A) = 8 \times 2 = 16$$

Now, find $\det(\text{adj}(2A))$ using the adjoint property:

$$\det(\text{adj}(2A)) = (\det(2A))^{3-1}$$

$$\det(\text{adj}(2A)) = (16)^2 = 256$$

Answer: (B)

Q21.

Solution

Concept: The shortest distance d between two skew lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by:

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

Solution: For Line 1: $\vec{a_1} = (1, 2, 3)$, $\vec{b_1} = (2, 3, 4)$. For Line 2: $\vec{a_2} = (2, 4, 5)$, $\vec{b_2} = (3, 4, 5)$. Calculate $\vec{b_1} \times \vec{b_2}$:

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$. Calculate $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$:

$$(2-1, 4-2, 5-3) \cdot (-1, 2, -1) = (1, 2, 2) \cdot (-1, 2, -1) = -1 + 4 - 2 = 1$$

Shortest distance:

$$d = \frac{1}{\sqrt{6}} \implies d^2 = \frac{1}{6}$$

Answer: (1/6)

Q22.

Solution

Concept: The area of a parallelogram with diagonals $\vec{d_1}$ and $\vec{d_2}$ is given by:

$$\text{Area} = \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$$

Solution: Given diagonals $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$. Calculate the cross product $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 2\hat{k}$$

Find the magnitude:

$$|\vec{a} \times \vec{b}| = \sqrt{3^2 + (-1)^2 + (-2)^2} = \sqrt{14}$$

The area of the parallelogram is:

$$\text{Area} = \frac{1}{2}\sqrt{14} \approx \frac{3.7416}{2} \approx 1.87$$

Answer: (1.87)

Q23.

Solution

Concept: To find the distance of a point $P(x_1, y_1, z_1)$ from a plane measured parallel to a line, we find the intersection point Q of the plane and a line passing through P with direction ratios of the given line. The required distance is the magnitude of vector $\vec{PQ}$.
Solution: The line through $P(1, 1, 1)$ parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is:

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3} = \lambda$$

Any point Q on this line is $(1+\lambda, 1+2\lambda, 1+3\lambda)$. Since Q lies on the plane $x - y + z = 5$:

$$(1+\lambda) - (1+2\lambda) + (1+3\lambda) = 5$$

$$1 + 2\lambda = 5 \implies \lambda = 2$$

The point of intersection Q is $(3, 5, 7)$. The distance PQ is:

$$d = \sqrt{(3-1)^2 + (5-1)^2 + (7-1)^2} = \sqrt{4 + 16 + 36} = \sqrt{56}$$

$$d = 2\sqrt{14}$$

Answer: ($2\sqrt{14}$)

Q24.

Solution

Concept: The volume of a parallelepiped with coterminous edges $\vec{a}, \vec{b}, \vec{c}$ is given by the magnitude of their scalar triple product:

$$V = |(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\det(\vec{a}, \vec{b}, \vec{c})|$$

Solution: The given vectors are $\vec{a} = (1, 1, 0)$, $\vec{b} = (0, 1, 1)$, and $\vec{c} = (\lambda, 0, 1)$. The volume is:

$$V = \left\| \begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ \lambda & 0 & 1 \end{matrix} \right\|$$

Expanding the determinant:

$$V = |1(1-0) - 1(0-\lambda) + 0| = |1+\lambda|$$

Given $V = 5$, we have:

$$|1+\lambda| = 5$$

Case 1: $1 + \lambda = 5 \implies \lambda = 4$
Case 2: $1 + \lambda = -5 \implies \lambda = -6$

Answer: (4, -6)

Q25.

Solution

Concept: The probability of exactly k successes in n independent Bernoulli trials is given by the Binomial Distribution:

$$P(X = k) = \binom{n}{k} p^k q^{n-k}$$

where p is the probability of success and $q = 1 - p$.
Solution: For a fair die, the probability of getting an even number is $p = \frac{3}{6} = \frac{1}{2}$, so $q = \frac{1}{2}$. The number of trials is $n = 5$ and we want exactly $k = 3$ successes.

$$P = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$$

$$P = 10 \times \left(\frac{1}{2}\right)^5 = \frac{10}{32}$$

The value of $32P$ is:

$$32P = 32 \times \frac{10}{32} = 10$$

Answer: (10)

Answer Key — Section A

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	A	5	B
6	B	7	A	8	C	9	B	10	A
11	A	12	A	13	A	14	A	15	B
16	A	17	B	18	A	19	A	1	B

Answer Key — Section B

Q	Ans	Q	Ans
21	1/6	22	1.87
23	$2\sqrt{14}$	24	4, -6
25	10		