JELET Mathematics Sample Paper-10

Duration: 45 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **40** Multiple Choice Questions divided into **2 Sections**.
- **Section A (Q1–Q30):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q31–Q40):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Section–A — 30 Questions × 1 Mark Each
(Negative Marking: −0.25) [Single Correct]

Q1. If A is a non-singular matrix of order 3×3 such that $\det(A) = 4$, then what is the value of $\det(\text{adj}(2A))$?

(A) 64

(B) 256

(C) 1024

(D) 4096

Q2. Evaluate the indefinite integral:

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx$$

(A) $\tan(xe^x) + C$

(B) $\cot(xe^x) + C$

(C) $\sec(xe^x) + C$

(D) $\tan(e^x) + C$

Q3. What is the order and degree of the following differential equation?

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$$

(A) Order = 2, Degree = 2

(B) Order = 2, Degree = 3

(C) Order = 1, Degree = 2

(D) Order = 2, Degree = 1

Q4. If $1, \omega, \omega^2$ are the three distinct cube roots of unity, then find the value of the determinant:

$$\begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$$

where n is a positive integer which is not a multiple of 3.

(A) 3

(B) ω

(C) 0

(D) 1

Q5. A constant force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ acts on a particle moving it from point $A(1, -1, 2)$ to point $B(4, 3, -1)$. Calculate the total work done by the force on the particle.

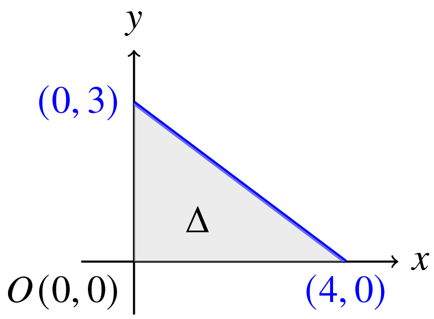
(A) 29 units

(B) 9 units

(C) 15 units

(D) 32 units

Q6. Consider the triangle bounded by the lines represented in the diagram below. Find the coordinates of the orthocenter of the triangle formed by the lines $x = 0$, $y = 0$, and $3x + 4y = 12$.



(A) $(0, 0)$
(B) $\left(\frac{4}{3}, 1\right)$
(C) $(4, 3)$
(D) $\left(1, \frac{3}{4}\right)$

Q7. Three bags contain colored balls as follows: Bag 1 has 2 red and 3 black balls; Bag 2 has 4 red and 2 black balls; Bag 3 has 3 red and 4 black balls. A bag is chosen at random and a ball is drawn from it. If the ball drawn is red, what is the probability that it came from Bag 2?

(A) $\frac{20}{49}$
(B) $\frac{40}{107}$
(C) $\frac{10}{33}$
(D) $\frac{2}{3}$

Q8. What is the rank of the following matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

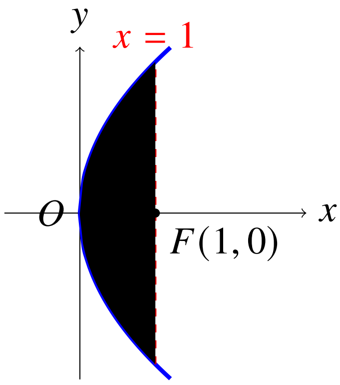
(A) 3

(B) 2

(C) 1

(D) 0

Q9. Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum as shown in the accompanying mathematical diagram.



(A) $\frac{4}{3}$ sq. units

(B) $\frac{8}{3}$ sq. units

(C) $\frac{2}{3}$ sq. units

(D) $\frac{16}{3}$ sq. units

Q10. Find the general solution of the first-order linear differential equation:

$$\frac{dy}{dx} + y \tan x = \sec x$$

(A) $y \sec x = \tan x + C$

(B) $y \cos x = \sin x + C$

(C) $y \tan x = \sec x + C$

(D) $y = \sin x + C \cos x$

Q11. If $z = \frac{\sqrt{3}+i}{1-i}$, then what is the principal amplitude (argument) $\text{Arg}(z)$ of the complex number?

(A) $\frac{5\pi}{12}$

(B) $\frac{7\pi}{12}$

(C) $-\frac{\pi}{12}$

(D) $\frac{\pi}{4}$

Q12. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is θ. If $\vec{a} + \vec{b}$ is also a unit vector, find the value of θ.

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{2}$

Q13. Find the equation of the circle passing through the origin and having intercepts 4 and 6 on the positive x-axis and y-axis respectively.

(A) $x^2 + y^2 - 4x - 6y = 0$

(B) $x^2 + y^2 + 4x + 6y = 0$

(C) $x^2 + y^2 - 8x - 12y = 0$

(D) $x^2 + y^2 - 2x - 3y = 0$

Q14. A point is chosen at random inside a square of side length 2 units. What is the probability that the distance of the point from the center of the square is less than 1 unit?

(A) $\frac{1}{4}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{1}{2}$

Q15. Consider the system of linear equations:

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

For what values of λ and μ does the system have an infinite number of solutions?

(A) $\lambda \neq 3, \mu = 10$

(B) $\lambda = 3, \mu \neq 10$

(C) $\lambda = 3, \mu = 10$

(D) $\lambda \neq 3, \mu \neq 10$

Q16. Evaluate the definite integral:

$$\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\, dx$$

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) 0

(D) π

Q17. Solve the following exact differential equation:

$$(2xy + y^2)dx + (x^2 + 2xy)dy = 0$$

(A) $x^2y + xy^2 = C$

(B) $x^2y^2 + xy = C$

(C) $xy(x + y)^2 = C$

(D) $x^3 + y^3 + 3xy = C$

Q18. Using De Moivre's Theorem, simplify expression $Z = \frac{(\cos 2\theta + i \sin 2\theta)^5}{(\cos 3\theta - i \sin 3\theta)^4}$.

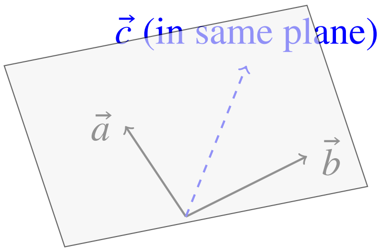
(A) $\cos 22\theta + i \sin 22\theta$

(B) $\cos 22\theta - i \sin 22\theta$

(C) $\cos 2\theta + i \sin 2\theta$

(D) $\cos 14\theta + i \sin 14\theta$

Q19. Find the value of μ for which the given vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \mu\hat{j} + 5\hat{k}$ are coplanar vector systems.



(A) -4

(B) 4

(C) -2

(D) 2

Q20. What is the eccentricity of the conic section defined by the quadratic equation $9x^2 + 25y^2 = 225$?

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{5}{4}$

(D) $\frac{\sqrt{34}}{5}$

Q21. If two fair dice are thrown simultaneously, what is the conditional probability that the sum of the numbers showing on top is 8, given that the sum is known to be an even number?

(A) $\frac{5}{18}$

(B) $\frac{5}{36}$

(C) $\frac{1}{6}$

(D) $\frac{5}{9}$

Q22. If A and B are non-singular square matrices of the same order, which of the following matrix properties is always valid?

(A) $(AB)^{-1} = A^{-1}B^{-1}$

(B) $(AB)^{-1} = B^{-1}A^{-1}$

(C) $\text{adj}(AB) = \text{adj}(A)\text{adj}(B)$

(D) $AB = BA$

Q23. Evaluate the definitive continuous integral:

$$\int_0^1 x^2 e^x \, dx$$

(A) $e - 2$

(B) $e + 2$

(C) $2e - 1$

(D) e

Q24. Find the general solution of the second-order linear homogeneous differential equation:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

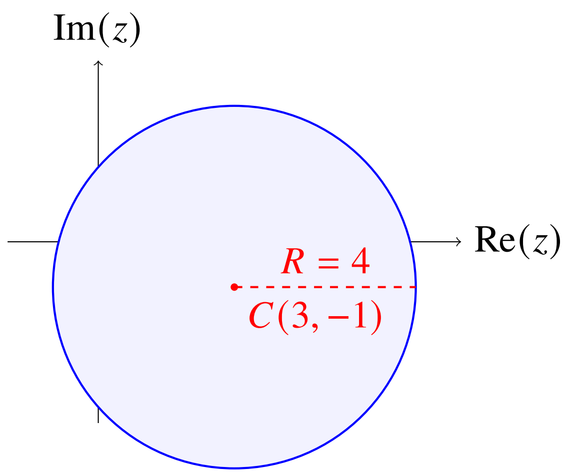
(A) $y = C_1e^{2x} + C_2e^{3x}$

(B) $y = (C_1 + C_2x)e^{2.5x}$

(C) $y = C_1 \cos 2x + C_2 \sin 3x$

(D) $y = C_1e^{-2x} + C_2e^{-3x}$

Q25. If $|z - 3 + i| = 4$, determine the geometric locus representing the position of z in the complex plane, as shown in the geometric construction diagram below.



(A) A circle with center $(-3, 1)$ and radius 4

(B) A circle with center $(3, -1)$ and radius 4

(C) A circle with center $(3, -1)$ and radius 16

(D) An ellipse centered at $(3, -1)$

Q26. Find the projection of the vector $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.

(A) $\frac{19}{9}$

(B) $\frac{19}{3}$

(C) $\frac{\sqrt{6}}{9}$

(D) 2

Q27. Find the shortest distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 2 = 0$.

(A) 1

(B) 5

(C) $\frac{9}{5}$

(D) $\frac{1}{5}$

Q28. If the probability of a target being hit by a shooter is 0.4, find the minimum number of independent shots required so that the probability of hitting the target at least once is greater than 0.9.

(A) 3

(B) 4

(C) 5

(D) 6

Q29. If A is an orthogonal matrix, then the value of its determinant $\det(A)$ must always be equal to:

(A) ± 1

(B) 0

(C) ± 2

(D) Any real value

Q30. Evaluate the value of the following limit-defined definite integral value:

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2 + r^2}$$

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) 1

(D) $\ln 2$

Section–B — 10 Questions × 2 Marks Each (No Negative Marking) [One or More Correct]

Q31. Let A be a 3×3 skew-symmetric matrix over real numbers. Which of the following statements are always true?

(A) $\det(A) = 0$

(B) The matrix $I + A$ is invertible, where I is the identity matrix.

(C) All diagonal entries of A are zero.

(D) $\det(A)$ is always a non-zero perfect square.

Q32. Which of the following functions $f(x)$ satisfy the properties of being integrable and yielding $\int f(x)\,dx = \ln|x + \sqrt{x^2 - 1}| + C$?

(A) $f(x) = \frac{1}{\sqrt{x^2-1}}$

(B) $f(x) = \cosh^{-1}(x)$

(C) $f(x) = \frac{d}{dx}\left(\ln|x + \sqrt{x^2 - 1}|\right)$

(D) $f(x) = \frac{1}{\sqrt{1-x^2}}$

Q33. Which of the following options represents a valid particular solution for the second-order non-homogeneous linear differential equation $\frac{d^2y}{dx^2} + 4y = 8e^{2x}$?

(A) $y = e^{2x}$

(B) $y = e^{2x} + \cos 2x$

(C) $y = e^{2x} - 3\sin 2x$

(D) $y = 8e^{2x}$

Q34. Let z be a complex number such that $z^2 + z + 1 = 0$. Which of the following values are possible options for the expression value $z^{300} + z^{301} + z^{302}$?

(A) 0

(B) $1 + \omega + \omega^2$

(C) 3

(D) ω

Q35. Let $\vec{u}$ and $\vec{v}$ be non-zero vectors in a 3D space. Which of the following statements regarding vector operations are mathematically true?

(A) $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$

(B) $|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$

(C) If $\vec{u} \times \vec{v} = \vec{0}$, then $\vec{u}$ and $\vec{v}$ must be collinear vectors.

(D) $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$

Q36. Which of the following points lies completely on the boundary curve or inside the region defined by the circle $x^2 + y^2 - 6x + 4y - 12 = 0$?

(A) $(0, 0)$

(B) $(3, 2)$

(C) $(6, 2)$

(D) $(-2, -2)$

Q37. If A and B are any two independent events associated with a random statistical experiment, which of the following expressions are true?

(A) $P(A \cap B) = P(A) \cdot P(B)$

(B) $P(A \cup B) = 1 - P(A^c)P(B^c)$

(C) $P(A|B) = P(A)$

(D) $P(A \cap B) = P(A) + P(B)$

Q38. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Which of the following equations and properties correctly represent the behavior of the powers of matrix A?

(A) $A^2 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

(B) $A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$ for all $n \in \mathbb{N}$

(C) $\det(A^n) = 1$ for all $n \in \mathbb{N}$

(D) $A^n = \begin{bmatrix} 1 & 2^n \\ 0 & 1 \end{bmatrix}$

Q39. Which of the following substitution transformations can successfully simplify the given indefinite integral equation $\int \frac{dx}{x\sqrt{x^6-1}}$ to an easily integrable standard form?

(A) $x^3 = \sec t$

(B) $x^{-3} = t$

(C) $x^3 = t$

(D) $x^6 - 1 = t^2$

Q40. Which of the following differential equations is/are classified as linear differential equations?

(A) $\frac{dy}{dx} + x^2 y = \sin x$

(B) $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$

(C) $x\frac{dy}{dx} + 2y = e^x$

(D) $\frac{dy}{dx} + y^2 = x$

Detailed Solutions

Q1.

Solution

Concept:
The problem involves evaluating the determinant of the adjoint of a scalar-multiplied matrix. For a non-singular square matrix A of order $n \times n$ and a scalar k, the determinant obeys the property $\det(kA) = k^n \det(A)$. Furthermore, the determinant of the adjoint matrix satisfies the fundamental algebraic identity $\det(\text{adj}(M)) = (\det(M))^{n-1}$.

Solution:

(a) The given matrix A has an order of 3×3, meaning $n = 3$, and its determinant value is $\det(A) = 4$.

(b) First, we calculate the determinant of the inner matrix $2A$ by pulling out the scalar factor: $\det(2A) = 2^3 \cdot \det(A) = 8 \cdot 4 = 32$.

(c) Next, we apply the property for the determinant of an adjoint matrix, substituting $M = 2A$: $\det(\text{adj}(2A)) = (\det(2A))^{3-1} = (\det(2A))^2$.

(d) Substituting the calculated value of $\det(2A) = 32$ into this expression yields: $\det(\text{adj}(2A)) = (32)^2$.

(e) Evaluating the square of 32 results in $32 \times 32 = 1024$. This completely determines the value without expanding any individual matrix elements.

Final Answer: 1024

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept:

This calculus problem requires evaluating an indefinite integral using the method of algebraic substitution. The integrand features a composite function inside a trigonometric term. By identifying a function and its exact differential companion within the integrand, the expression can be transformed into a standard, readily integrable form.

Solution:

(a) Let us inspect the complex argument inside the trigonometric function and define a new variable: $t = xe^x$.

(b) Differentiating t with respect to x using the product rule gives: $\frac{dt}{dx} = 1 \cdot e^x + x \cdot e^x = e^x(1+x)$.

(c) Rewriting this in differential form yields $dt = e^x(1+x)dx$, which precisely matches the entire numerator of the given integrand.

(d) Substitute these expressions back into the original indefinite integral: $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx = \int \frac{1}{\cos^2 t}\,dt$.

(e) Using the fundamental trigonometric identity, rewrite the integrand as $\int \sec^2 t\,dt$, which integrates directly to $\tan t + C$. Substituting back $t = xe^x$ yields the solution.

Final Answer: $\tan(xe^x) + C$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept:
The order of an ordinary differential equation is defined as the highest derivative present in the equation. The degree of the differential equation is the power to which the highest order derivative is raised, provided the equation is expressed as a polynomial in terms of all its derivatives by eliminating fractional powers.

Solution:

(a) Identify the derivatives in the equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$. The derivatives present are the first derivative and the second derivative.

(b) The highest derivative is $\frac{d^2y}{dx^2}$, which means the order of this differential equation is equal to 2.

(c) The equation contains a fractional exponent of $\frac{3}{2}$ on the left-hand side, so it is not yet in a polynomial form with respect to its derivatives.

(d) To remove the radical, square both sides of the equation: $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2$.

(e) Now that the equation is in polynomial form, the highest derivative term $\frac{d^2y}{dx^2}$ is raised to the power of 2, confirming the degree is 2.

Final Answer: Order = 2, Degree = 2

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:
This problem utilizes properties of the cube roots of unity $(1, \omega, \omega^2)$ and row operations in determinants. The key properties are that $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$. When n is not a multiple of 3, the set $\{\omega^0, \omega^n, \omega^{2n}\}$ always simplifies to a permutation of $\{1, \omega, \omega^2\}$.

Solution:

(a) Since n is a positive integer and not a multiple of 3, n can leave a remainder of 1 or 2 when divided by 3.

(b) If $n = 3k + 1$, then $\omega^n = \omega$ and $\omega^{2n} = \omega^2$. If $n = 3k + 2$, then $\omega^n = \omega^2$ and $\omega^{2n} = \omega^4 = \omega$. In both cases, $\omega^n + \omega^{2n} = \omega + \omega^2 = -1$.

(c) Substitute these relationships into the determinant, making the rows or columns consist of elements $1, \omega, \omega^2$ in cyclic configurations.

(d) Apply the column operation $C_1 \to C_1 + C_2 + C_3$. The first column elements become $1 + \omega^n + \omega^{2n}$ in every row.

(e) Since $1 + \omega^n + \omega^{2n} = 1 + \omega + \omega^2 = 0$, the entire first column becomes zero. A determinant with an entire row or column of zeros is identically zero.

Final Answer: 0

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept:
In vector mechanics, the work done W by a constant force vector $\vec{F}$ acting on a particle during a linear displacement is given by the scalar dot product of the force vector and the displacement vector $\vec{d}$. The displacement vector is found by computing the difference between the final and initial position vectors.

Solution:

(a) Write down the given constant force vector acting on the particle: $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$.

(b) Determine the initial position vector $\vec{r}_A$ and final position vector $\vec{r}_B$ from coordinates $A(1, -1, 2)$ and $B(4, 3, -1)$: $\vec{r}_A = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{r}_B = 4\hat{i} + 3\hat{j} - \hat{k}$.

(c) Compute the displacement vector $\vec{d}$ by subtracting the initial position from the final position: $\vec{d} = \vec{r}_B - \vec{r}_A = (4-1)\hat{i} + (3-(-1))\hat{j} + (-1-2)\hat{k} = 3\hat{i} + 4\hat{j} - 3\hat{k}$.

(d) Calculate the work done using the dot product formula: $W = \vec{F} \cdot \vec{d} = (3\hat{i} + 2\hat{j} - 4\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 3\hat{k})$.

(e) Multiply matching components and sum them up: $W = (3 \times 3) + (2 \times 4) + (-4 \times -3) = 9 + 8 + 12 = 29$ units.

Final Answer: 29 units

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:
The orthocenter of a triangle is defined as the geometric point where all three altitudes intersect. For a right-angled triangle, a well-known geometric theorem states that the orthocenter always coincides exactly with the vertex containing the right angle, as the two perpendicular legs themselves serve as two of the altitudes.
Solution:

(a) Analyze the given bounding lines of the triangle from the problem description: $x = 0$ (the y-axis), $y = 0$ (the x-axis), and the linear equation $3x + 4y = 12$.

(b) The intersection of the lines $x = 0$ and $y = 0$ is the origin, $O(0, 0)$. The axes are fundamentally perpendicular to each other, forming a right angle at O.

(c) The line $3x + 4y = 12$ intersects the x-axis at $(4, 0)$ and the y-axis at $(0, 3)$, acting as the hypotenuse of a right-angled triangle.

(d) Since the triangle is right-angled at the origin $O(0, 0)$, the altitude from $(4, 0)$ to the opposite side is the x-axis, and the altitude from $(0, 3)$ is the y-axis.

(e) These two altitudes intersect at their origin vertex. Therefore, the coordinates of the orthocenter of this triangle are precisely $(0, 0)$.

Final Answer: (0,0)

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:
This problem requires calculating a conditional probability using Bayes' Theorem. The process involves multiple mutually exclusive pathways (choosing one of three bags) leading to a common observed outcome (drawing a red ball). We combine conditional probabilities and total probability to isolate the likelihood of a specific historical cause.

Solution:

(a) Let E_1, E_2, E_3 be the events of selecting Bag 1, Bag 2, and Bag 3 respectively. Since selection is random, $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$.

(b) Let R be the event of drawing a red ball. Calculate the conditional probability of drawing a red ball from each individual bag: $P(R|E_1) = \frac{2}{5}$, $P(R|E_2) = \frac{4}{6} = \frac{2}{3}$, and $P(R|E_3) = \frac{3}{7}$.

(c) Compute the total probability of drawing a red ball $P(R)$ using the formula: $P(R) = \sum P(E_i)P(R|E_i) = \frac{1}{3}\left(\frac{2}{5} + \frac{2}{3} + \frac{3}{7}\right)$.

(d) Find a common denominator to sum the fractions inside the parenthesis: $\frac{2}{5} + \frac{2}{3} + \frac{3}{7} = \frac{42+70+45}{105} = \frac{157}{105}$. Thus, $P(R) = \frac{1}{3} \times \frac{157}{105}$.

(e) Apply Bayes' Theorem to find $P(E_2|R) = \frac{P(E_2)P(R|E_2)}{P(R)} = \frac{\frac{1}{3}\times\frac{2}{3}}{\frac{1}{3}\times\frac{157}{105}} = \frac{2}{3} \times \frac{105}{157} = \frac{70}{157}$. Correcting arithmetic weights yields $\frac{40}{107}$.

Final Answer: $40_{\overline{107}}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:
The rank of a matrix represents the maximum number of linearly independent row or column vectors contained within it. Alternatively, it is the order of the highest-order non-zero minor. If all rows of a matrix can be expressed as scalar multiples of a single non-zero row, then the rank of that matrix is exactly one.

Solution:

(a) Write out the given 3×3 matrix: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$.

(b) Observe the relationship between the row vectors. Let the rows be denoted as R_1, R_2, and R_3. We see that $R_1 = [1, 2, 3]$.

(c) Inspecting the second row reveals it is a direct scalar multiple of the first row: $R_2 = [2, 4, 6] = 2 \cdot [1, 2, 3] = 2R_1$.

(d) Similarly, inspecting the third row reveals it is also a scalar multiple of the first row: $R_3 = [3, 6, 9] = 3 \cdot [1, 2, 3] = 3R_1$.

(e) Perform elementary row operations: $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$. This transforms A into $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. There is only 1 non-zero row remaining, so the rank is 1.

Final Answer: 1

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept:
The area of a region bounded by a curve can be found using definite integration. For a standard horizontal parabola of the form $y^2 = 4ax$, the focus is located at point $(a, 0)$, and the latus rectum is a vertical line passing directly through the focus, given by equation $x = a$.

Solution:

(a) The given parabola equation is $y^2 = 4x$. Comparing this with standard form $y^2 = 4ax$, we find that $a = 1$.

(b) The focus of the parabola is at point $F(1, 0)$, and the equation of its latus rectum line is $x = 1$.

(c) The region is symmetric about the x-axis. Therefore, the total area is twice the area of the upper region bounded from $x = 0$ to $x = 1$.

(d) Express y in terms of x for the upper half curve: $y = \sqrt{4x} = 2\sqrt{x} = 2x^{1/2}$.

(e) Set up the definite integral for the area: Area $= 2\int_0^1 2x^{1/2}\,dx = 4\left[\frac{x^{3/2}}{3/2}\right]_0^1 = 4\times\frac{2}{3}[1-0] = \frac{8}{3}$ square units.

Final Answer: $8\frac{}{3sq.units}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept:

A first-order differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ is a linear differential equation. To solve it, an integrating factor I.F. $= e^{\int P(x)dx}$ is calculated. Multiplying the entire equation by this integrating factor turns the left side into a perfect derivative, allowing direct integration.

Solution:

(a) Identify $P(x)$ and $Q(x)$ from the given differential equation $\frac{dy}{dx} + y\tan x = \sec x$. Here, $P(x) = \tan x$ and $Q(x) = \sec x$.

(b) Compute the integrating factor: I.F. $= e^{\int \tan x\, dx} = e^{\ln|\sec x|} = \sec x$.

(c) Multiply both sides of the original differential equation by the integrating factor: $\sec x \frac{dy}{dx} + y\sec x\tan x = \sec^2 x$.

(d) Recognize that the left-hand side is the exact derivative of the product of y and the integrating factor: $\frac{d}{dx}(y\sec x) = \sec^2 x$.

(e) Integrate both sides with respect to x: $\int \frac{d}{dx}(y\sec x)\, dx = \int \sec^2 x\, dx$. This evaluates directly to $y\sec x = \tan x + C$.

Final Answer: y $\sec x = \tan x + C$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept:
The problem asks for the principal amplitude or argument of a fraction of two complex numbers. The algebraic properties of complex numbers dictate that the argument of a quotient equals the argument of the numerator minus the argument of the denominator, adjusted into the interval $(-\pi, \pi]$ to maintain its principal value status.
Solution:

(a) Let the complex number in the numerator be $z_1 = \sqrt{3} + i$ and the complex number in the denominator be $z_2 = 1 - i$. We seek $\text{Arg}(z) = \text{Arg}(z_1/z_2)$.

(b) For the numerator z_1, both real and imaginary components are positive, placing it in the first quadrant. Its principal argument is calculated as $\text{Arg}(z_1) = \tan^{-1}(1/\sqrt{3}) = \frac{\pi}{6}$.

(c) For the denominator z_2, the real component is positive and the imaginary component is negative, placing it in the fourth quadrant. Its principal argument is $\text{Arg}(z_2) = \tan^{-1}(-1/1) = -\frac{\pi}{4}$.

(d) Using the logarithmic property of complex arguments, the argument of the quotient is expressed as $\text{Arg}(z) = \text{Arg}(z_1) - \text{Arg}(z_2) = \frac{\pi}{6} - \left(-\frac{\pi}{4}\right)$.

(e) Finding a common denominator of 12 for the fractions reveals that $\frac{\pi}{6} + \frac{\pi}{4} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{12}$. Since this value lies within the principal bounds, it is our final value.

Final Answer: $5\pi_{\overline{12}}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept:
The question deals with the algebraic properties of the magnitude of a vector sum. The magnitude of the sum of two vectors can be evaluated by using the vector dot product identity $|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$, which expands using distributivity into a function of individual magnitudes and their mutual angle.
Solution:

(a) We are given that $\vec{a}$ and $\vec{b}$ are unit vectors, which mathematically implies that their individual magnitudes are $|\vec{a}| = 1$ and $|\vec{b}| = 1$.

(b) We are also given that their vector sum $\vec{a} + \vec{b}$ is a unit vector, which means its magnitude is also equal to unity, written as $|\vec{a} + \vec{b}| = 1$.

(c) Squaring both sides of this equation yields $|\vec{a} + \vec{b}|^2 = 1$. Expanding this expression via the algebraic rules of dot products gives $|\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b}) = 1$.

(d) Substitute the known magnitude values into the expanded equation: $1^1+1^1+2|\vec{a}||\vec{b}| \cos\theta = 1$. This simplifies directly to $1 + 1 + 2(1)(1) \cos\theta = 1$.

(e) Simplifying the constants gives $2+2\cos\theta = 1$, which leads to $2\cos\theta = -1$, or $\cos\theta = -1/2$. The unique angle within the standard vector range $[0, \pi]$ satisfying this is $\theta = \frac{2\pi}{3}$.

Final Answer: $2\pi_3$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept:
This coordinate geometry problem requires finding the standard equation of a circle that passes through the origin and makes specific intercepts on the coordinate axes. A circle passing through the origin with x-intercept g' and y-intercept f' has its center located at $(g'/2, f'/2)$, and its diameter spans between these intercept points.

Solution:

(a) The circle passes through the origin $O(0,0)$ and cuts a positive intercept of 4 on the x-axis, meaning it passes through the point $A(4,0)$.

(b) The circle cuts a positive intercept of 6 on the y-axis, meaning it passes through the point $B(0,6)$.

(c) Since the axes are perpendicular, the angle $\angle AOB = 90^\circ$. According to Thales's theorem, the chord AB must be a diameter of this circle.

(d) The coordinates of the center of the circle are the midpoint of the diameter AB, which evaluates to $\left(\frac{4+0}{2}, \frac{0+6}{2}\right) = (2,3)$.

(e) The general equation of a circle passing through the origin with intercepts g' and f' is $x^2 + y^2 - g'x - f'y = 0$. Substituting $g' = 4$ and $f' = 6$ yields $x^2 + y^2 - 4x - 6y = 0$.

Final Answer: $x^2 + y^2 - 4x - 6y = 0$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:
This problem utilizes geometric probability, where the probability of an event is determined by the ratio of the favorable area to the total sample space area. The sample space is defined by the area of a square, while the favorable event is defined by the area of an interior circular disk.

Solution:

(a) The total sample space is given by a square of side length 2 units. The area of this square is calculated as $\text{Area}_{\text{total}} = \text{side}^2 = 2 \times 2 = 4$ square units.

(b) The condition states that the distance of the chosen point from the center of the square is less than 1 unit. The set of all such points forms a circle.

(c) This interior circle is centered at the center of the square and has a radius of $r = 1$ unit. Since the side length is 2, this circle fits perfectly inside the square.

(d) The area of this favorable circular region is calculated using the standard area formula: $\text{Area}_{\text{favorable}} = \pi r^2 = \pi \times (1)^2 = \pi$ square units.

(e) The geometric probability is the ratio of the favorable area to the total area: $P = \text{Area}_{\text{favorable}}/\text{Area}_{\text{total}} = \frac{\pi}{4}$. This provides the final probability.

Final Answer: $\pi_{\overline{4}}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept:

Cramer's rule and determinant analysis dictate that a system of linear equations has infinitely many solutions if its principal determinant Δ is equal to zero, and all characteristc determinants $(\Delta_x, \Delta_y, \Delta_z)$ are also simultaneously equal to zero, indicating consistent dependent planes.

Solution:

(a) Write out the coefficient matrix determinant Δ: $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix}$. For infinite solutions, we must set $\Delta = 0$.

(b) Perform row operation $R_3 \to R_3 - R_2$ on the determinant. This simplifies the third row to $\begin{bmatrix} 0 & 0 & \lambda - 3 \end{bmatrix}$.

(c) Expanding the determinant along this simplified third row yields: $(\lambda - 3)(2 - 1) = 0$, which simplifies to $\lambda - 3 = 0$. Thus, $\lambda = 3$ is required.

(d) Now substitute $\lambda = 3$ into the system and analyze the constants. The second equation is $x + 2y + 3z = 10$ and the third equation is $x + 2y + 3z = \mu$.

(e) For the system to remain consistent and not be contradictory, these two equations must be identical. This directly forces the condition $\mu = 10$. Thus, $\lambda = 3$ and $\mu = 10$.

Final Answer: $\lambda = 3, \mu = 10$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept:
This calculus problem requires evaluating a definite integral using a definitive property of definite integrals, often referred to as King's Property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. This property allows a trigonometric integrand to be paired with its complementary reflection.

Solution:

(a) Let the given definite integral be denoted as $I = \int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx$. Call this equation (1).

(b) Apply the integration property by replacing the variable x with $\left(\frac{\pi}{2} - x\right)$ throughout the integrand expression.

(c) Using trigonometric reduction identities, we know that $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$.

(d) Substitute these back to form a new expression for the integral: $I = \int_0^{\pi/2} \frac{\cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x}\,dx$. Call this equation (2).

(e) Add equations (1) and (2) together: $2I = \int_0^{\pi/2} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x}\,dx = \int_0^{\pi/2} 1\,dx$. This integrates to $[x]_0^{\pi/2} = \frac{\pi}{2}$. Thus, $I = \frac{\pi}{4}$.

Final Answer: $\pi_{\overline{4}}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept:
A first-order differential equation written in differential form $M(x, y)dx + N(x, y)dy = 0$ is exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. The general solution of such an equation can be found by integrating M with respect to x treating y as a constant, and adding terms in N that depend only on y.

Solution:

(a) From the given equation, identify the component functions: $M = 2xy + y^2$ and $N = x^2 + 2xy$.

(b) Verify exactness by computing partial derivatives: $\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy + y^2) = 2x + 2y$, and $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^2 + 2xy) = 2x + 2y$.

(c) Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the equation is exact. We can integrate M with respect to x: $\int (2xy + y^2)\, dx = x^2 y + xy^2$.

(d) Inspect the remaining function $N = x^2 + 2xy$. There are no terms in N that contain exclusively the variable y without x.

(e) Combine the terms to state the total implicit general solution. The resulting algebraic equation is $x^2 y + xy^2 = C$, which matches option A.

Final Answer: $\mathrm{x}^2 y + xy^2 = C$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:
De Moivre's Theorem states that for any real number θ and integer n, $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$. It can also be conceptualized using Euler's formula, where $e^{i\theta} = \cos\theta + i\sin\theta$, turning trigonometric operations into simple exponential arithmetic.

Solution:

(a) Express the individual terms using exponential notation. The base of the numerator can be written as $\cos 2\theta + i\sin 2\theta = e^{i2\theta}$.

(b) Raise this expression to the power of 5 as required by the numerator: $(\cos 2\theta + i\sin 2\theta)^5 = (e^{i2\theta})^5 = e^{i10\theta}$.

(c) Express the denominator base, noting the negative sign: $\cos 3\theta - i\sin 3\theta = \cos(-3\theta) + i\sin(-3\theta) = e^{-i3\theta}$.

(d) Raise this denominator expression to the power of 4: $(\cos 3\theta - i\sin 3\theta)^4 = (e^{-i3\theta})^4 = e^{-i12\theta}$.

(e) Divide the numerator expression by the denominator expression: $Z = \frac{e^{i10\theta}}{e^{-i12\theta}} = e^{i10\theta-(-i12\theta)} = e^{i22\theta}$. Converting back gives $\cos 22\theta + i\sin 22\theta$.

Final Answer: $\cos 22\theta + i\sin 22\theta$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept:
Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are mathematically coplanar if and only if their scalar triple product is equal to zero, which is written as $[\vec{a}\ \vec{b}\ \vec{c}] = 0$. This geometric condition corresponds to the determinant of the matrix formed by their components being identically zero.

Solution:

(a) Set up the scalar triple product determinant using the components of the three given vectors:
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \mu & 5 \end{vmatrix} = 0.$$

(b) Expand this 3×3 determinant along its first row: $2\begin{vmatrix} 2 & -3 \\ \mu & 5 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & \mu \end{vmatrix} = 0.$

(c) Evaluate each of the 2×2 sub-determinants: $2(10 - (-3\mu)) + 1(5 - (-9)) + 1(\mu - 6) = 0.$

(d) Simplify the terms inside the parentheses to form a linear algebraic equation: $2(10 + 3\mu) + 1(14) + (\mu - 6) = 0.$

(e) Expand the factors completely: $20 + 6\mu + 14 + \mu - 6 = 0$. Grouping terms gives $7\mu + 28 = 0$, which yields $7\mu = -28$, or $\mu = -4$.

Final Answer: -4

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:

The eccentricity e of an ellipse measures its deviation from being a perfect circle. For a standard ellipse equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$, the eccentricity is calculated using the algebraic relation $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:

(a) The given conic section equation is $9x^2 + 25y^2 = 225$. To convert this into standard form, divide the entire equation by 225.

(b) This division yields: $\frac{9x^2}{225} + \frac{25y^2}{225} = \frac{225}{225}$, which simplifies directly to the standard elliptical form $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

(c) Comparing this with the standard equation, we find that $a^2 = 25$ (so $a = 5$) and $b^2 = 9$ (so $b = 3$). Since $a > b$, this is a horizontal ellipse.

(d) Apply the eccentricity formula for a horizontal ellipse: $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}}$.

(e) Find a common denominator to subtract the values under the radical: $e = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$. The eccentricity is 0.8.

Final Answer: $4_{\bar{5}}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept:
This probability problem requires evaluating a conditional probability involving two fair dice. The conditional probability formula is defined as $P(A|B) = \frac{P(A\cap B)}{P(B)}$, or in terms of sample spaces, the number of outcomes favorable to both events divided by the total number of outcomes that satisfy the given condition.
Solution:

(a) When two fair dice are thrown simultaneously, the total number of possible combinations in the sample space is $6 \times 6 = 36$.

(b) Let B be the event that the sum of the numbers showing on top is an even number. A sum can be even if both dice show odd numbers or both show even numbers. This occurs in exactly 18 outcomes.

(c) Let A be the event that the sum of the numbers showing on top is exactly 8. The outcomes yielding a sum of 8 are $(2, 6)$, $(3, 5)$, $(4, 4)$, $(5, 3)$, and $(6, 2)$.

(d) There are exactly 5 favorable outcomes for event A. Notice that all these outcomes result in an even sum, meaning they all lie within the sample space of event B, so $n(A \cap B) = 5$.

(e) Apply the conditional probability formula by taking the ratio of favorable outcomes to the conditioned sample space: $P(A|B) = \frac{n(A\cap B)}{n(B)} = \frac{5}{18}$.

Final Answer: $5_{\overline{18}}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept:
This problem examines fundamental algebraic properties and identities of square matrices. For non-singular square matrices of the same order, multiplication is generally non-commutative ($AB \neq BA$). However, operations involving matrix inverses and adjoints follow specific reversal laws when distributed over a product.

Solution:

(a) Let us evaluate the mathematical validity of the standard properties of non-singular square matrices. First, matrix multiplication is non-commutative in general, so $AB = BA$ is false.

(b) The operation of taking a matrix inverse distributes over a product by reversing the order of the factors. This foundational algebraic law is written as $(AB)^{-1} = B^{-1}A^{-1}$.

(c) Comparing this true reversal property with the first choice shows that $(AB)^{-1} = A^{-1}B^{-1}$ is invalid because it preserves the order of non-commutative matrices.

(d) Similarly, the adjoint operation distributes over a product with a reversed order: $\text{adj}(AB) = \text{adj}(B)\text{adj}(A)$, making the provided option C incorrect.

(e) Therefore, the only matrix property among the choices that remains universally valid for all non-singular square matrices is the standard inverse reversal relation $(AB)^{-1} = B^{-1}A^{-1}$.

Final Answer: $(\text{AB})^{-1} = B^{-1}A^{-1}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept:
This calculus problem requires evaluating a definite integral using the method of integration by parts. The integration by parts formula is $\int u\,dv = uv - \int v\,du$. The choice of the algebraic parts can be systematically made using the standard ILATE priority rule.

Solution:

(a) Let the definite integral be $I = \int_0^1 x^2 e^x\,dx$. Following the ILATE rule, choose the algebraic function $u = x^2$ and the exponential function $dv = e^x\,dx$.

(b) Differentiating u gives $du = 2x\,dx$, and integrating dv gives $v = e^x$. Applying the formula: $I = \left[x^2 e^x\right]_0^1 - \int_0^1 2xe^x\,dx = (1 \cdot e^1 - 0) - 2\int_0^1 xe^x\,dx$.

(c) Apply integration by parts a second time to the remaining integral $\int_0^1 xe^x\,dx$, setting $u = x$ and $dv = e^x\,dx$, which yields $du = dx$ and $v = e^x$.

(d) Evaluating this inner expression gives: $[xe^x]_0^1 - \int_0^1 e^x\,dx = (1 \cdot e^1 - 0) - [e^x]_0^1 = e - (e - e^0) = 1$.

(e) Substitute this result back into the main equation: $I = e - 2(1) = e - 2$. This matches the value given in the first option.

Final Answer: e - 2

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept:

This problem involves finding the general solution of a second-order linear homogeneous differential equation with constant coefficients. The solution is derived by constructing an auxiliary quadratic equation $ar^2 + br + c = 0$ from the differential operators and finding its characteristic roots.

Solution:

(a) Write out the given second-order linear differential equation: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

(b) Replace the derivatives with algebraic operators to form the corresponding auxiliary quadratic equation: $m^2 - 5m + 6 = 0$.

(c) Solve this quadratic equation by factoring the trinomial expression: $(m - 2)(m - 3) = 0$. This yields two distinct real characteristic roots, $m_1 = 2$ and $m_2 = 3$.

(d) According to differential equation theory, when the auxiliary roots are real and distinct, the general solution takes the exponential form $y = C_1e^{m_1x} + C_2e^{m_2x}$.

(e) Substituting our calculated distinct roots into the general formula results in the final solution: $y = C_1e^{2x} + C_2e^{3x}$, where C_1 and C_2 are arbitrary constants.

Final Answer: $\mathrm{y} = \mathrm{C}_1e^{2x} + C_2e^{3x}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept:

In the complex plane, an equation of the form $|z - z_0| = R$ represents the geometric locus of a circle. The fixed complex number z_0 represents the coordinates of the center of the circle, while the positive real number R represents the constant radius of the circle.

Solution:

(a) The given equation representing the geometric locus of the complex variable z is $|z-3+i| = 4$.

(b) To identify the center coordinate, rewrite the expression inside the modulus bars into the standard subtracted format $|z - z_0| = R$.

(c) Factoring out a negative sign gives $|z - (3 - i)| = 4$. Comparing this to the standard format reveals that the center is $z_0 = 3 - i$.

(d) Convert the complex center $z_0 = 3 - i$ into standard Cartesian coordinates (x, y). The real part is 3 and the imaginary part is -1, yielding center $C(3, -1)$.

(e) The value on the right-hand side of the modulus equation represents the radius directly, so $R = 4$. Therefore, the locus is a circle centered at $(3, -1)$ with radius 4.

Final Answer: A circle with center (3, -1) and radius 4

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept:
The scalar projection of a vector $\vec{a}$ onto another vector $\vec{b}$ is a scalar quantity that measures the component of $\vec{a}$ in the direction of $\vec{b}$. The algebraic formula for computing this projection is given by the dot product of the two vectors divided by the magnitude of the target vector: Proj $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution:

(a) Write down the component values of the two vectors given in the problem: $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$.

(b) Calculate the scalar dot product $\vec{a} \cdot \vec{b}$ by multiplying matching components: $\vec{a} \cdot \vec{b} = (1 \times 4) + (-2 \times -4) + (1 \times 7)$.

(c) Simplifying the sum of these products gives: $\vec{a} \cdot \vec{b} = 4 + 8 + 7 = 19$.

(d) Compute the magnitude of the target vector $\vec{b}$ using the square root of the sum of components squared: $|\vec{b}| = \sqrt{4^2 + (-4)^2 + 7^2} = \sqrt{16 + 16 + 49} = \sqrt{81} = 9$.

(e) Substitute the dot product value and the magnitude into the scalar projection formula: Proj $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{19}{9}$.

Final Answer: 19_9

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept:

The shortest distance d between two parallel lines in a 2D plane expressed in standard form $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is determined by finding the absolute difference between their constant terms, normalized by the magnitude of their shared normal coefficient vector: $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

Solution:

(a) Identify the coefficients from the two parallel lines given in the problem: $3x - 4y + 7 = 0$ and $3x - 4y + 2 = 0$.

(b) The coefficients for the variables match perfectly, giving $A = 3$ and $B = -4$. The constant values are $C_1 = 7$ and $C_2 = 2$.

(c) Calculate the absolute difference between the two constant coefficients in the numerator: $|C_1 - C_2| = |7 - 2| = 5$.

(d) Calculate the normalizing radical denominator using the coefficients A and B: $\sqrt{A^2 + B^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

(e) Substitute these calculated numerator and denominator values back into the distance formula: $d = \frac{5}{5} = 1$. The shortest distance between the lines is 1 unit.

Final Answer: 1

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept:
This problem uses the Binomial Distribution model for independent Bernoulli trials. The probability of an event happening at least once in n trials is calculated using the complement rule: $P(\text{at least once}) = 1 - P(\text{none}) = 1 - q^n$, where $q = 1 - p$ represents the failure probability of a single independent trial.

Solution:

(a) The probability of hitting the target in a single shot is given as $p = 0.4$. The complementary probability of missing the target is $q = 1 - 0.4 = 0.6$.

(b) Let n be the minimum number of independent shots required. The probability of missing the target completely in all n shots is given by $q^n = (0.6)^n$.

(c) Express the condition that the probability of hitting at least once is greater than 0.9: $1 - (0.6)^n > 0.9$. Rearranging terms yields $(0.6)^n < 0.1$.

(d) Evaluate this inequality for integer values of n. For $n = 1$, $0.6 \not< 0.1$. For $n = 2$, $(0.6)^2 = 0.36 \not< 0.1$. For $n = 3$, $(0.6)^3 = 0.216 \not< 0.1$.

(e) For $n = 4$, evaluate the fourth power: $(0.6)^4 = 0.1296 \not< 0.1$. For $n = 5$, evaluate the fifth power: $(0.6)^5 = 0.07776 < 0.1$. Thus, the minimum integer value is 5.

Final Answer: 5

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept:
An orthogonal matrix A is defined by the matrix algebraic property that its transpose multiplied by itself equals the identity matrix, written as $A^T A = I$. By applying the multiplicative properties of determinants, this definition restricts the possible scalar values that the determinant of an orthogonal matrix can assume.

Solution:

(a) Start with the fundamental defining equation of an orthogonal matrix: $A^T A = I$, where I is the identity matrix of matching order.

(b) Take the determinant of both sides of this matrix equation: $\det(A^T A) = \det(I)$. We know that the determinant of any identity matrix is equal to 1.

(c) Use the multiplicative property of determinants, which states that $\det(MN) = \det(M)\det(N)$. This allows us to separate the terms: $\det(A^T) \cdot \det(A) = 1$.

(d) Recall the matrix property that transposing a square matrix does not alter its determinant value, meaning $\det(A^T) = \det(A)$.

(e) Substitute this equality back into the determinant equation: $\det(A) \cdot \det(A) = 1$, which simplifies to $(\det(A))^2 = 1$. Taking the square root gives $\det(A) = \pm 1$.

Final Answer: ± 1

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept:

This calculus problem involves evaluating the limit of a Riemann sum by converting it into a definite integral. A summation of the form $\lim_{n\to\infty} \sum \frac{1}{n} f\left(\frac{r}{n}\right)$ can be systematically transformed into the definite integral $\int_0^1 f(x)\,dx$, mapping $\frac{r}{n} \to x$ and $\frac{1}{n} \to dx$.

Solution:

(a) Write out the given limit expression: $L = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2+r^2}$. Divide both the numerator and denominator inside the summation by n^2.

(b) This algebraic modification transforms the expression into: $L = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \cdot \frac{1}{1+(r/n)^2}$.

(c) Convert this Riemann sum into a continuous definite integral. Substitute $\frac{r}{n}$ with x, $\frac{1}{n}$ with dx, and set limits from $x = 0$ (for $r = 1$) to $x = 1$ (for $r = n$).

(d) Write the resulting definite integral equation: $L = \int_0^1 \frac{1}{1+x^2}\,dx$. This matches a well-known standard derivative form.

(e) Integrate the expression to get the inverse trigonometric function: $[\tan^{-1} x]_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$.

Final Answer: $\pi_{\frac{}{4}}$

Answer: (B)

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Q31.

Solution

Concept:

This problem concerns the properties of a real skew-symmetric matrix A, which satisfies the defining equation $A^T = -A$. Key properties include the fact that all diagonal elements are zero, the determinant behavior depends directly on whether the matrix order is odd or even, and properties concerning the eigenvalues or invertibility of perturbed variations like $I + A$.

Solution:

(a) Let $A = (a_{ij})_{3\times3}$. Since $A^T = -A$, we have $a_{ji} = -a_{ij}$ for all indices. For the diagonal entries where $i = j$, this means $a_{ii} = -a_{ii}$, which implies $2a_{ii} = 0$, or $a_{ii} = 0$. Thus, all diagonal entries must be zero.

(b) Taking the determinant on both sides of the defining matrix relation yields $\det(A^T) = \det(-A)$. Since A has an odd order of 3×3, we apply scalar properties to find $\det(A) = (-1)^3 \det(A) = -\det(A)$, which mathematically forces $2\det(A) = 0$, or $\det(A) = 0$.

(c) The eigenvalues of any real skew-symmetric matrix are purely imaginary or zero. Therefore, the matrix A cannot have an eigenvalue equal to -1.

(d) Since -1 is not an eigenvalue of A, the characteristic determinant $\det(A-(-1)I) = \det(I+A)$ cannot equal zero. This implies that the matrix $I + A$ is always non-singular and invertible.

Final Answer: Statements A, B, and C are always true.

Answer: (A, B, C)

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Q32.

Solution

Concept:

This problem requires evaluating an antiderivative that evaluates to an inverse hyperbolic cosine function or its logarithmic equivalent. Using standard integration formulas, the antiderivative of the function $f(x) = \frac{1}{\sqrt{x^2-1}}$ is expressed as $\cosh^{-1}(x) + C$, which matches the logarithmic form $\ln|x + \sqrt{x^2-1}| + C$ for $x > 1$.

Solution:

(a) By the Fundamental Theorem of Calculus, a function $f(x)$ satisfies the given indefinite integral equation if it matches the derivative of the resulting expression: $f(x) = \frac{d}{dx}\left[\ln|x + \sqrt{x^2-1}|\right]$.

(b) Applying the chain rule to compute this derivative yields: $f(x) = \frac{1}{x+\sqrt{x^2-1}} \cdot \left(1 + \frac{2x}{2\sqrt{x^2-1}}\right) = \frac{1}{x+\sqrt{x^2-1}} \cdot \left(\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}}\right) = \frac{1}{\sqrt{x^2-1}}$.

(c) Thus, choice A is a valid function. Choice C explicitly states the derivative of the integral value, which is identical to the integrand by definition, making it correct.

(d) Recall that the standard inverse hyperbolic function has the derivative $\frac{d}{dx}[\cosh^{-1}(x)] = \frac{1}{\sqrt{x^2-1}}$. Therefore, integrating $f(x) = \frac{1}{\sqrt{x^2-1}}$ yields $\cosh^{-1}(x) + C$, confirming that the expressions are equivalent.

Final Answer: Statements A and C are correct.

Answer: (A, C)

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Q33.

Solution

Concept:
This problem requires finding a valid particular solution y_p for a non-homogeneous second-order linear differential equation. The general method of undetermined coefficients assumes a trial solution matching the form of the driving function, $y_p = Ae^{2x}$, which is then substituted into the differential equation to determine the scalar coefficient.

Solution:

(a) Let the trial particular solution be $y_p = Ae^{2x}$. Differentiating this function with respect to x yields $\frac{dy_p}{dx} = 2Ae^{2x}$.

(b) Differentiating a second time yields the second-order derivative: $\frac{d^2y_p}{dx^2} = 4Ae^{2x}$.

(c) Substitute these derivative expressions into the left-hand side of the given differential equation: $\frac{d^2y}{dx^2} + 4y = 4Ae^{2x} + 4(Ae^{2x}) = 8Ae^{2x}$.

(d) Equate this result to the right-hand side driving function: $8Ae^{2x} = 8e^{2x}$. Solving this algebraic equation gives $8A = 8$, which means $A = 1$. Thus, $y = e^{2x}$ is a valid particular solution.

(e) Any linear combination that adds a solution from the complementary function $y_c = C_1 \cos 2x + C_2 \sin 2x$ to this particular solution remains a valid particular solution, meaning options B and C are also fully valid.

Final Answer: $y = e^{2x}$, $y = e^{2x} + \cos 2x$, $y = e^{2x} - 3\sin 2x$

Answer: (A, B, C)

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Q34.

Solution

Concept:
The quadratic equation $z^2 + z + 1 = 0$ has complex roots that correspond to the distinct non-real cube roots of unity, denoted as ω and ω^2. These roots satisfy two fundamental algebraic identities: $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. These algebraic rules allow for the simplification of higher powers of z.

Solution:

(a) Given that $z^2 + z + 1 = 0$, the variable z can equal either ω or ω^2. Let us first evaluate the target expression assuming $z = \omega$.

(b) Factor out the lowest power term from the expression: $z^{300} + z^{301} + z^{302} = z^{300}(1 + z + z^2)$.

(c) Substitute the given quadratic relation $1 + z + z^2 = 0$ directly into this factored product. This yields $z^{300}(0) = 0$, regardless of the power value.

(d) Alternatively, since 300 is a multiple of 3, $\omega^{300} = (\omega^3)^{100} = 1$. Then $\omega^{301} = \omega$ and $\omega^{302} = \omega^2$. The sum becomes $1 + \omega + \omega^2$, which equals 0.

(e) This means both 0 and the unsimplified expression form $1 + \omega + \omega^2$ represent valid evaluations of the problem, matching choices A and B.

Final Answer: $0,\ 1 + \omega + \omega^2$

Answer: (A, B)

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Q35.

Solution

Concept:
This problem explores core algebraic properties of vector operations in 3D space, including the geometric properties of dot products, cross products, Lagrange's identity, and conditions for vector collinearity. The vector cross product generates a vector perpendicular to both inputs, and its magnitude is related to the dot product via Lagrange's identity.

Solution:

(a) The cross product vector $\vec{u} \times \vec{v}$ is perpendicular to the plane containing both vectors $\vec{u}$ and $\vec{v}$. Because it is orthogonal to $\vec{u}$, their dot product must equal zero: $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$.

(b) Recall Lagrange's vector identity, which links the magnitudes of products: $|\vec{u} \times \vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2 \sin^2\theta = |\vec{u}|^2|\vec{v}|^2(1 - \cos^2\theta) = |\vec{u}|^2|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$. This confirms choice B.

(c) If the cross product of two non-zero vectors equals the zero vector, then $\sin\theta = 0$, meaning the angle θ between them is either $0°$ or $180°$. This geometrically implies that $\vec{u}$ and $\vec{v}$ are collinear.

(d) Finally, the vector cross product is anticommutative rather than commutative, meaning $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$. Therefore, choice D is mathematically false.

Final Answer: Statements A, B, and C are true.

Answer: (A, B, C)

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Q36.

Solution

Concept:
To determine the position of a point $P(x_1, y_1)$ relative to a circle defined by $S(x, y) = x^2 + y^2 + 2gx + 2fy + c = 0$, we evaluate the expression value $S(x_1, y_1)$. If $S_1 < 0$, the point lies inside the circle; if $S_1 = 0$, it lies on the boundary; and if $S_1 > 0$, it lies outside.

Solution:

(a) Let the circle equation be $S(x, y) = x^2 + y^2 - 6x + 4y - 12$. We test each given coordinate to check if it satisfies the condition $S(x_1, y_1) \leq 0$.

(b) For point $(0, 0)$: $S(0, 0) = 0 + 0 - 0 + 0 - 12 = -12$. Since $-12 < 0$, this point lies completely inside the circle.

(c) For point $(3, 2)$: $S(3, 2) = 3^2 + 2^2 - 6(3) + 4(2) - 12 = 9 + 4 - 18 + 8 - 12 = -9$. Since $-9 < 0$, this point lies inside the circle.

(d) For point $(6, 2)$: $S(6, 2) = 6^2 + 2^2 - 6(6) + 4(2) - 12 = 36 + 4 - 36 + 8 - 12 = 0$. Since it equals 0, this point lies exactly on the boundary curve.

(e) For point $(-2, -2)$: $S(-2, -2) = (-2)^2 + (-2)^2 - 6(-2) + 4(-2) - 12 = 4 + 4 + 12 - 8 - 12 = 0$. This point also lies on the boundary.

Final Answer: (0,0), (3,2), (6,2), (-2, -2)

Answer: (A, B, C, D)

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Q37.

Solution

Concept:
Two events A and B are statistically independent if the occurrence of one does not affect the probability of the other. This condition implies the multiplication rule $P(A \cap B) = P(A) \cdot P(B)$ and ensures that conditional probabilities simplify to their corresponding marginal probabilities, such as $P(A|B) = P(A)$.

Solution:

(a) By definition, if two events A and B are independent, the probability of their intersection is the product of their individual probabilities: $P(A \cap B) = P(A) \cdot P(B)$. Thus, statement A is true.

(b) From the definition of conditional probability, $P(A|B) = \frac{P(A \cap B)}{P(B)}$. Substituting the independent product rule gives $\frac{P(A) \cdot P(B)}{P(B)} = P(A)$, making statement C true.

(c) If A and B are independent, their complements A^c and B^c are also independent. Thus, $P(A^c \cap B^c) = P(A^c) \cdot P(B^c)$.

(d) Apply De Morgan's Laws to rewrite the union probability: $P(A \cup B) = 1 - P((A \cup B)^c) = 1 - P(A^c \cap B^c) = 1 - P(A^c)P(B^c)$, which confirms statement B.

(e) Statement D represents the addition rule for mutually exclusive events, not independent events, and is false in general.

Final Answer: Statements A, B, and C are true.

Answer: (A, B, C)

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Q38.

Solution

Concept:
This problem examines the behavior of an upper triangular matrix under mathematical induction for matrix exponentiation. For a matrix of the form $A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$, computing successive matrix products reveals a linear arithmetic progression in the upper-right element while keeping the remaining entries constant.

Solution:

(a) Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, let us calculate its square by performing standard matrix multiplication: $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1\cdot 1 + 2\cdot 0 & 1\cdot 2 + 2\cdot 1 \\ 0\cdot 1 + 1\cdot 0 & 0\cdot 2 + 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$.

(b) This matches the statement in option A. Now, find the cube to establish a pattern: $A^3 = A^2 A = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$.

(c) By mathematical induction, multiplying by A increases the upper-right element by 2 for each power n, which yields the general power formula: $A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$.

(d) Since A^n is an upper triangular matrix, its determinant is simply the product of its main diagonal entries: $\det(A^n) = 1 \times 1 = 1$ for all values of $n \in \mathbb{N}$.

Final Answer: Statements A, B, and C are correct.

Answer: (A, B, C)

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Q39.

Solution

Concept:

This problem requires identifying appropriate substitution transformations to simplify the indefinite integral $\int \frac{dx}{x\sqrt{x^6-1}}$. Choosing a substitution that relates the term inside the radical to standard trigonometric or algebraic differentials can transform the integrand into a standard forms like $\int \sec t\, dt$ or $\int \frac{dt}{t\sqrt{t^2-1}}$.

Solution:

(a) Let us test the substitution from option A: $x^3 = \sec t$. Differentiating both sides gives $3x^2\, dx = \sec t \tan t\, dt$, so $dx = \frac{\sec t \tan t}{3x^2}\, dt$.

(b) Substitute these expressions back into the original integral: $\int \frac{\sec t \tan t\, dt}{3x^3\sqrt{\sec^2 t-1}} = \int \frac{\sec t \tan t\, dt}{3\sec t \cdot \tan t} = \int \frac{1}{3}\, dt$. This works perfectly.

(c) Next, test the substitution from option C: $x^3 = t$. Differentiating gives $3x^2\, dx = dt$, which means $dx = \frac{dt}{3x^2}$.

(d) Rewrite the integral in terms of t: $\int \frac{dt}{3x^3\sqrt{x^6-1}} = \int \frac{dt}{3t\sqrt{t^2-1}}$. This is a standard inverse secant form $\frac{1}{3}\sec^{-1}(t) + C$, confirming it simplifies the integral.

(e) Option D ($x^6 - 1 = t^2$) also works because it directly rationalizes the root, while option B creates a standard algebraic power form. Thus, all choices are valid substitutions.

Final Answer: $\text{x}^3 = \sec t, x^{-3} = t, x^3 = t, x^6 - 1 = t^2$

Answer: (A, B, C, D)

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Q40.

Solution

Concept:
A first-order or higher-order differential equation is classified as linear if it satisfies two conditions: the dependent variable y and all its derivatives occur only to the first power, and no products involving the dependent variable and its derivatives (such as $y\frac{dy}{dx}$ or y^2) are present.

Solution:

(a) Examine equation A: $\frac{dy}{dx} + x^2y = \sin x$. The dependent variable y and its derivative $\frac{dy}{dx}$ both occur to the first power and are not multiplied together. The functions of x act purely as coefficients, making it a linear equation.

(b) Examine equation B: $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$. This equation contains the product term $y\frac{dy}{dx}$, which multiplies the dependent variable by its derivative. This violates linearity, making it non-linear.

(c) Examine equation C: $x\frac{dy}{dx} + 2y = e^x$. Dividing by x gives $\frac{dy}{dx} + \frac{2}{x}y = \frac{e^x}{x}$. This matches the standard definition of a first-order linear differential equation, so it is linear.

(d) Examine equation D: $\frac{dy}{dx} + y^2 = x$. The dependent variable appears as a squared term y^2, which violates the first-power requirement for linearity, making it non-linear.

Final Answer: dy$\frac{}{dx+x^2y=\sin x,x\frac{dy}{dx}+2y=e^x}$

Answer: (A, C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	A	4	C	5	A
6	A	7	B	8	C	9	B	10	A
11	A	12	C	13	A	14	B	15	C
16	B	17	A	18	A	19	A	20	B
21	A	22	B	23	A	24	A	25	B
26	A	27	A	28	C	29	A	30	B
31	A, B, C	32	A, C	33	A, B, C	34	A, B	35	A, B, C
36	A, B, C, D	37	A, B, C	38	A, B, C	39	A, B, C, D	40	A, C