JKBOSE Class 10 Mathematics Question Paper with Solutions(Memory Based)

Time Allowed :3 Hours	Maximum Marks :100	Total questions :35

General Instructions

Read the following instructions carefully and strictly adhere to them:

1. All questions are compulsory and must be answered in complete sentences; one-word or fragmented answers will not be awarded full marks.
2. Graphic organizers such as web diagrams, flow charts, and tables must be drawn neatly and exactly as presented in the question paper with the correct information filled in.
3. In reference to point 2, answers provided without the required diagrams or tables will not be considered for credit.
4. Use only a blue or black ballpoint/gel pen for writing and drawing; the use of pencils for diagrams is permitted, but colour pencils or sketch pens are strictly prohibited.
5. If multiple responses are provided for a single-answer activity, the entire attempt will be treated as invalid and no marks will be assigned.
6. Maintain the chronological sequence of Sections, Question Numbers, and Sub-activities as per the question paper to ensure systematic evaluation.

1. Prove that $\sqrt{3}$ or $\sqrt{5}$ is an irrational number.

Solution:

Concept: A number is irrational if it cannot be expressed as a ratio of two integers. A common method to prove irrationality is **proof by contradiction**:

- Assume the number is rational.

- Express it in lowest terms $\frac{p}{q}$.
- Derive a contradiction (both numerator and denominator becoming divisible by the same number).

Step 1: Prove that $\sqrt{3}$ is irrational.

Assume $\sqrt{3}$ is rational. Then it can be written as:

$$\sqrt{3} = \frac{p}{q}, \quad \text{where } p, q \in \mathbb{Z},\ q \neq 0,\ \text{and } \gcd(p, q) = 1.$$

Squaring both sides:

$$3 = \frac{p^2}{q^2} \quad \Rightarrow \quad p^2 = 3q^2.$$

Thus, p^2 is divisible by 3, so p must also be divisible by 3. Let $p = 3k$.

Substitute back:

$$(3k)^2 = 3q^2 \Rightarrow 9k^2 = 3q^2 \Rightarrow q^2 = 3k^2.$$

So q^2 is divisible by 3, hence q is divisible by 3.

This means both p and q are divisible by 3, contradicting the assumption that $\frac{p}{q}$ is in lowest terms.

$$\therefore \sqrt{3} \text{ is irrational.}$$

Step 2: Prove that $\sqrt{5}$ is irrational.

Assume $\sqrt{5} = \frac{p}{q}$ in lowest terms.

Squaring:

$$5 = \frac{p^2}{q^2} \Rightarrow p^2 = 5q^2.$$

Thus, p is divisible by 5. Let $p = 5k$.

Substitute:

$$25k^2 = 5q^2 \Rightarrow q^2 = 5k^2.$$

Hence q is also divisible by 5, contradicting lowest terms assumption.

$$\therefore \sqrt{5} \text{ is irrational.}$$

Conclusion: Both $\sqrt{3}$ and $\sqrt{5}$ are irrational numbers.

Quick Tip

To prove square roots of non-perfect squares are irrational, assume the number is rational and use contradiction by showing numerator and denominator share a common factor.

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2. Find HCF and LCM using the prime factorisation method for: (i) 56 and 72 (ii) 6, 72, and 120

Solution:

Concept: Using the **prime factorisation method**:

- HCF (Highest Common Factor): Product of the lowest powers of common prime factors.
- LCM (Least Common Multiple): Product of the highest powers of all prime factors.

(i) For 56 and 72

Step 1: Prime factorisation

$$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

Step 2: HCF

Common prime factors: only 2 Lowest power of 2 is 2^3

$$\text{HCF} = 2^3 = 8$$

Step 3: LCM

Take highest powers of all primes:

$$2^3, \quad 3^2, \quad 7$$

$$\text{LCM} = 2^3 \times 3^2 \times 7 = 8 \times 9 \times 7 = 504$$

$$\therefore \text{HCF} = 8, \quad \text{LCM} = 504$$

(ii) For 6, 72, and 120

Step 1: Prime factorisation

$$6 = 2 \times 3 = 2^1 \times 3^1$$

$$72 = 2^3 \times 3^2$$

$$120 = 2^3 \times 3 \times 5 = 2^3 \times 3^1 \times 5^1$$

Step 2: HCF

Common prime factors in all numbers: 2 and 3

Lowest powers:

$$2^1, \quad 3^1$$

$$\text{HCF} = 2 \times 3 = 6$$

Step 3: LCM

Take highest powers of all primes:

$$2^3, \quad 3^2, \quad 5^1$$

$$\text{LCM} = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$$

$$\therefore \text{HCF} = 6, \quad \text{LCM} = 360$$

Quick Tip

For HCF, take the smallest powers of common primes. For LCM, take the largest powers of all primes present in any number.

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3. Find the zeroes of a quadratic polynomial (e.g., $x^2 - 3x - 10$ or $4u^2 + 8u$) and verify the relationship between zeroes and coefficients.

Solution:

Concept: For a quadratic polynomial $ax^2 + bx + c$ with zeroes α and β:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

We find the zeroes and verify these relations.

Example 1: $x^2 - 3x - 10$

Step 1: Factorisation

$$x^2 - 3x - 10 = (x - 5)(x + 2)$$

Step 2: Zeroes

$$x - 5 = 0 \Rightarrow x = 5$$

$$x + 2 = 0 \Rightarrow x = -2$$

So, zeroes are 5 and -2.

Step 3: Verify relations

Here $a = 1, b = -3, c = -10$

$$\alpha + \beta = 5 + (-2) = 3$$
$$-\frac{b}{a} = -\frac{-3}{1} = 3 \quad \checkmark$$

$$\alpha\beta = 5 \times (-2) = -10$$
$$\frac{c}{a} = \frac{-10}{1} = -10 \quad \checkmark$$

Relations verified.

Example 2: $4u^2 + 8u$

Step 1: Factorisation

$$4u^2 + 8u = 4u(u + 2)$$

Step 2: Zeroes

$$4u = 0 \Rightarrow u = 0$$

$$u + 2 = 0 \Rightarrow u = -2$$

So, zeroes are 0 and -2.

Step 3: Verify relations

Here $a = 4, b = 8, c = 0$

$$\alpha + \beta = 0 + (-2) = -2$$
$$-\frac{b}{a} = -\frac{8}{4} = -2 \quad \checkmark$$

$$\alpha\beta = 0 \times (-2) = 0$$
$$\frac{c}{a} = \frac{0}{4} = 0 \quad \checkmark$$

Relations verified.

Conclusion: For both polynomials, the sum and product of zeroes satisfy:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Quick Tip

For any quadratic $ax^2 + bx + c$: Sum of zeroes = $-\frac{b}{a}$, Product of zeroes = $\frac{c}{a}$. Always verify after finding roots.

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4. Find the HCF of 18 and 24.

Solution:

Concept: The HCF (Highest Common Factor) is the greatest number that divides both numbers exactly. We use the **prime factorisation method**:

- Write each number as a product of prime factors.
- Take the common primes with the smallest powers.

Step 1: Prime factorisation

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

Step 2: Identify common prime factors

Common primes: 2 and 3

Lowest powers:

$$2^1, \quad 3^1$$

Step 3: Find HCF

$$\text{HCF} = 2 \times 3 = 6$$

$$\therefore \text{HCF of 18 and 24 is } 6.$$

Quick Tip

HCF = product of common primes with smallest powers. LCM uses highest powers instead.

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5. State whether the following is True or False: $\sqrt{7}$ is a rational number.

Solution:

Concept: A rational number can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. The square root of a number is rational only if the number is a perfect square.

Step 1: Examine the number

Since 7 is not a perfect square, its square root cannot be written as a ratio of two integers.

Step 2: Conclusion

$$\sqrt{7} \text{ is an irrational number.}$$

Therefore, the given statement is **False**.

> **Quick Tip**
> The square root of any non-perfect square number is always irrational.

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6. What is the probability of an impossible event?

Solution:

Concept: Probability measures the chance of an event occurring and lies between 0 and 1.

- Probability = 0 $\Rightarrow$ Impossible event
- Probability = 1 $\Rightarrow$ Certain event

Step 1: Definition of an impossible event

An impossible event is one that can never happen. Example: Getting 7 on a standard die.

Step 2: Probability value

Since the event cannot occur at all, the number of favourable outcomes is 0.

$$\text{Probability} = \frac{0}{\text{Total outcomes}} = 0$$

Final Answer:

$$\boxed{0}$$

> **Quick Tip**
> Probability always lies between 0 and 1. Impossible event $\to$ 0, Certain event $\to$ 1.

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7. Write the n^{th} term of an Arithmetic Progression (AP).

Solution:

Concept: An Arithmetic Progression (AP) is a sequence of numbers in which the difference between consecutive terms is constant. This constant difference is called the **common difference** d.

Let:

- First term = a
- Common difference = d

Step 1: General form of an AP

$$a,\ a+d,\ a+2d,\ a+3d,\ \ldots$$

Step 2: Pattern of terms

$$\text{1st term} = a$$
$$\text{2nd term} = a+d$$
$$\text{3rd term} = a+2d$$
$$\text{4th term} = a+3d$$

We observe that the coefficient of d is always one less than the term number.

Step 3: Formula for the n^{th} term

$$a_n = a + (n-1)d$$

Final Answer:

$$\boxed{a_n = a + (n-1)d}$$

Quick Tip

To find any term in an AP, use: Term $=$ First term $+ (n-1) \times$ Common difference.
