JK Board Class 12 Mathematics 2026 Question Paper with Solutions

Time Allowed : 3 Hours	Maximum Marks : 80	Total Questions : 31

General Instructions

1. *This question paper is divided into 4 sections:* ***Section A, B, C*** *and* ***D****.*
2. *There are a total of* ***31*** *questions.*
3. ***Section A*** *contains* ***Multiple Choice Questions (MCQs)*** *carry* ***1*** *marks each.*
4. ***Section B*** *contains* ***Very Short Answer Type Questions (VSAs)*** *carry* ***2*** *marks each.*
5. ***Section C*** *contains* ***Short Answer Type Questions (SAs)*** *carry* ***4*** *marks each.*
6. ***Section D*** *contains* ***Long Answer Type Questions (LAs)*** *carry* ***6*** *marks each.*

Section A

(Objective Type/Multiple Type Questions)

1. If the matrix A is both symmetric and skew-symmetric, then

(a) A is a diagonal matrix
(b) A is a zero matrix
(c) A is a square matrix
(d) None of these

Correct Answer: (b) A is a zero matrix

Solution:
Step 1: Understanding the Concept:
A symmetric matrix is defined as a matrix A such that $A^T = A$.
A skew-symmetric matrix is defined as a matrix A such that $A^T = -A$.

Step 2: Key Formula or Approach:
We use the definitions of symmetric and skew-symmetric matrices simultaneously to solve for matrix A.

Step 3: Detailed Explanation:
Given that matrix A is symmetric:

$$A^T = A \quad \text{—(i)}$$

Given that matrix A is skew-symmetric:

$$A^T = -A \quad \text{—(ii)}$$

From equations (i) and (ii), we can equate the values of A^T:

$$A = -A$$

$$A + A = 0$$

$$2A = 0$$

$$A = 0$$

Therefore, A must be a zero matrix.

Step 4: Final Answer:
Since the matrix A must satisfy both conditions, every element of the matrix must be zero. Hence, A is a zero matrix.

Quick Tip

Only a zero matrix can satisfy the condition of being equal to its own negative. Remember: A skew-symmetric matrix must always have zero elements on its main diagonal.

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2. If A is a square matrix of order n, then $|adj A|$ is equal to:

(a) $|A|$
(b) $|A|^n$
(c) $|A|^{n-1}$
(d) None of these

Correct Answer: (c) $|A|^{n-1}$

Solution:
Step 1: Understanding the Concept:
The adjoint of a matrix has a specific property related to the determinant of the original matrix.

Step 2: Key Formula or Approach:
The fundamental relation between a matrix and its adjoint is:

$$A \cdot (adj A) = |A| I$$

Step 3: Detailed Explanation:
Taking the determinant on both sides of the equation $A \cdot (adjA) = |A|I$:

$$|A \cdot (adjA)| = ||A|I|$$

Using the property $|AB| = |A||B|$:

$$|A| \cdot |adjA| = |A|^n|I|$$

Since $|I| = 1$:

$$|A| \cdot |adjA| = |A|^n$$

Dividing both sides by $|A|$ (assuming $|A| \neq 0$):

$$|adjA| = \frac{|A|^n}{|A|}$$

$$|adjA| = |A|^{n-1}$$

Step 4: Final Answer:
The determinant of the adjoint of a matrix of order n is $|A|^{n-1}$.

> Quick Tip
>
> Remember the powers for exam shortcuts:
> $|adjA| = |A|^{n-1}$
> $|adj(adjA)| = |A|^{(n-1)^2}$

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3. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

(a) e^{-x}
(b) e^{-y}
(c) $\frac{1}{x}$
(d) x

Correct Answer: (c) $\frac{1}{x}$

Solution:
Step 1: Understanding the Concept:

This is a first-order linear differential equation.
The standard form is $\frac{dy}{dx} + P(x)y = Q(x)$.

Step 2: Key Formula or Approach:
The Integrating Factor (I.F.) is given by:

$$I.F. = e^{\int P(x)dx}$$

Step 3: Detailed Explanation:
First, we rewrite the given equation in standard form by dividing the entire equation by x:

$$\frac{dy}{dx} - \frac{1}{x}y = 2x$$

Comparing this with $\frac{dy}{dx} + Py = Q$, we find:

$$P = -\frac{1}{x}$$

Now, calculate the Integrating Factor:

$$I.F. = e^{\int -\frac{1}{x}dx}$$

$$I.F. = e^{-\ln x}$$

$$I.F. = e^{\ln(x^{-1})}$$

$$I.F. = x^{-1} = \frac{1}{x}$$

Step 4: Final Answer:
The integrating factor for the given differential equation is $\frac{1}{x}$.

> Quick Tip
>
> Always ensure the coefficient of $\frac{dy}{dx}$ is 1 before identifying $P(x)$.
> Use logarithmic properties: $e^{\ln f(x)} = f(x)$.

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4. The identity function on real numbers given by $f(x) = x$ is continuous at every real number. (Write True or False).

Correct Answer: True

Solution:
Step 1: Understanding the Concept:
A function $f(x)$ is continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$.

Step 2: Detailed Explanation:
For the identity function $f(x) = x$:
Let c be any arbitrary real number ($c \in \mathbb{R}$).
1. The value of the function at $x = c$ is $f(c) = c$.
2. The limit of the function as x approaches c is:

$$\lim_{x \to c} f(x) = \lim_{x \to c} (x) = c$$

Since $\lim_{x \to c} f(x) = f(c)$ for all $c \in \mathbb{R}$, the function is continuous everywhere.
Additionally, $f(x) = x$ is a polynomial function of degree 1, and all polynomial functions are continuous on the entire real line.

Step 3: Final Answer:
The statement is True.

Quick Tip

The graph of $f(x) = x$ is a straight line passing through the origin. Since there are no breaks or jumps in the line, it is continuous.

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5. The anti derivative of $\sin 2x$ is:

(a) $\cos 2x + c$
(b) $-\frac{\sin 2x}{2} + c$
(c) $-\frac{\cos 2x}{2} + c$
(d) None of these

Correct Answer: (c) $-\frac{\cos 2x}{2} + c$

Solution:
Step 1: Understanding the Concept:
The anti-derivative of a function is its indefinite integral.

Step 2: Key Formula or Approach:
The standard integral for sine is:

$$\int \sin(ax)dx = -\frac{\cos(ax)}{a} + c$$

Step 3: Detailed Explanation:
We need to find $\int \sin 2x dx$.
Let $2x = t$. Then $2dx = dt \implies dx = \frac{dt}{2}$.
Substituting these into the integral:

$$\int \sin t \cdot \frac{dt}{2} = \frac{1}{2}\int \sin t dt$$

$$= \frac{1}{2}(-\cos t) + c$$

$$= -\frac{\cos 2x}{2} + c$$

Step 4: Final Answer:
The anti-derivative is $-\frac{\cos 2x}{2} + c$.

> Quick Tip
>
> When integrating $f(ax + b)$, always divide the result by the coefficient of x, which is a. Anti-derivative of sin is $-\cos$, but derivative of sin is cos. Don't mix them up!

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7. The function given by $f(x) = \cos x$ is decreasing in $(0, \pi)$. (Write True or False).

Correct Answer: True

Solution:
Step 1: Understanding the Concept:
A function is decreasing in an interval if its first derivative is negative ($f'(x) < 0$) for all x in that interval.

Step 2: Key Formula or Approach:
Differentiate $f(x) = \cos x$ and check the sign of the derivative in the given range.

Step 3: Detailed Explanation:
Given $f(x) = \cos x$.
Differentiating with respect to x:

$$f'(x) = -\sin x$$

Now, we check the sign of $-\sin x$ in the interval $(0, \pi)$:
In the first and second quadrants ($0 < x < \pi$), the function $\sin x$ is positive ($\sin x > 0$).
Therefore, $-\sin x$ will be negative in this interval:

$$f'(x) = -(\text{positive value}) < 0$$

Since $f'(x) < 0$ for all $x \in (0, \pi)$, the function $f(x) = \cos x$ is strictly decreasing in $(0, \pi)$.

Step 4: Final Answer:
The statement is True.

> Quick Tip
>
> Look at the graph of $\cos x$. It starts at 1 at $x = 0$ and goes down to -1 at $x = \pi$. Since it is continuously falling, it is decreasing.

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8. The vector joining the points $P(2, 3, 0)$ and $Q(-1, -2, -4)$ directed from P to Q is ....

Correct Answer: $-3\hat{i} - 5\hat{j} - 4\hat{k}$

Solution:
Step 1: Understanding the Concept:
The vector joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ directed from P to Q is given by the difference of their position vectors.

Step 2: Key Formula or Approach:

$$\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Step 3: Detailed Explanation:
Given points:
$P = (2, 3, 0)$ so $\vec{OP} = 2\hat{i} + 3\hat{j} + 0\hat{k}$
$Q = (-1, -2, -4)$ so $\vec{OQ} = -1\hat{i} - 2\hat{j} - 4\hat{k}$

The vector directed from P to Q is:

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$\vec{PQ} = (-1 - 2)\hat{i} + (-2 - 3)\hat{j} + (-4 - 0)\hat{k}$$

$$\vec{PQ} = -3\hat{i} - 5\hat{j} - 4\hat{k}$$

Step 4: Final Answer:
The required vector is $-3\hat{i} - 5\hat{j} - 4\hat{k}$.

> Quick Tip
>
> Always subtract the "Initial" point coordinates from the "Final" point coordinates: Vector = Terminal − Initial.

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9. Define Skew lines in space.

Correct Answer: Skew lines are lines in three-dimensional space that are neither parallel nor intersecting.

Solution:
Step 1: Understanding the Concept:
In two dimensions, lines are either parallel or they intersect. In three dimensions, a third possibility exists.

Step 2: Detailed Explanation:
Skew lines are lines that do not lie in the same plane (non-coplanar).
Because they are not in the same plane:
1. They never intersect each other.
2. They are not parallel to each other.
The shortest distance between two skew lines is always a non-zero value and is measured along a line perpendicular to both.

Step 3: Final Answer:
Skew lines are defined as pairs of lines that are non-coplanar, meaning they are neither parallel nor do they intersect at any point.

Quick Tip

Think of a road overpass. The road on the bridge and the road below it are skew lines; they don't meet and they aren't going in the same direction.

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10. Define optimization Problem.

Correct Answer: An optimization problem is a mathematical problem where the goal is to find the best solution from all feasible solutions, typically involving maximizing or minimizing a function.

Solution:
Step 1: Understanding the Concept:
Optimization refers to making something as functional or effective as possible.

Step 2: Detailed Explanation:
In mathematics and linear programming, an optimization problem consists of:
1. **Objective Function:** A linear function (like $Z = ax + by$) that needs to be maximized (e.g., profit) or minimized (e.g., cost).
2. **Constraints:** A set of linear inequalities or equations that restrict the values of the variables.
3. **Feasible Region:** The set of all possible points that satisfy all the given constraints.
The goal is to find a point in the feasible region that gives the optimal (maximum or minimum) value of the objective function.

Step 3: Final Answer:
An optimization problem is a problem which seeks to maximize or minimize a specific quantity (the objective function), subject to a set of constraints expressed as linear inequalities or equations.

Quick Tip

In board exams, optimization problems usually appear as Linear Programming Problems (LPP) where you use the corner point method to find the maximum or minimum.

Section B

(Very Short Answer Type Questions)

11. Determine whether the following relation is reflexive, symmetric and transitive: Relation R in the set N of natural numbers defined by $R = \{(x, y) : y = x + 5$ and $x <$

4}

(A) Reflexive, but not symmetric or transitive
(B) Symmetric, but not reflexive or transitive
(C) Transitive, but not reflexive or symmetric
(D) Neither reflexive, nor symmetric, nor transitive

Correct Answer: (C) Transitive, but not reflexive or symmetric

Solution:
Step 1: Understanding the Concept:
A relation R on a set A is:
1. Reflexive if $(a, a) \in R$ for every $a \in A$.
2. Symmetric if $(a, b) \in R \implies (b, a) \in R$.
3. Transitive if $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R$.

Step 2: Key Formula or Approach:
Define the set R explicitly based on the given conditions $y = x + 5$ and $x < 4$ where $x, y \in \mathbb{N}$.
The values for x can only be $\{1, 2, 3\}$.
Substituting $x = 1, 2, 3$ into $y = x + 5$:
When $x = 1, y = 6$.
When $x = 2, y = 7$.
When $x = 3, y = 8$.
So, $R = \{(1, 6), (2, 7), (3, 8)\}$.

Step 3: Detailed Explanation:
Reflexivity:
For R to be reflexive, $(1, 1), (2, 2) \ldots$ must belong to R.
Since $(1, 1) \notin R$, the relation is not reflexive.
Symmetry:
We have $(1, 6) \in R$. For symmetry, $(6, 1)$ must be in R.
But $(6, 1) \notin R$, so the relation is not symmetric.
Transitivity:
A relation is transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.
In this set, there are no elements (a, b) and (b, c) such that the second element of the first pair is the first element of the second pair.
(i.e., there is no y such that $(x, y) \in R$ and $(y, z) \in R$).
When the hypothesis "if $(a, b) \in R$ and $(b, c) \in R$" is never met, the statement is vacuously true.
Therefore, the relation is transitive.

Step 4: Final Answer:
The relation R is transitive but neither reflexive nor symmetric.

Quick Tip

A relation is "vacuously transitive" if you cannot find a pair of elements (a, b) and (b, c) in the set to test the condition. In such cases, the relation is always considered transitive by default.

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12. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Correct Answer: $\frac{\pi}{4}$

Solution:
Step 1: Understanding the Concept:
The principal value of an inverse trigonometric function is the value that falls within its standard restricted range.

Step 2: Key Formula or Approach:
The range of the principal value of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Step 3: Detailed Explanation:
Let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.
This implies $\sin\theta = \frac{1}{\sqrt{2}}$.
We know that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.
Since $\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the principal value is $\frac{\pi}{4}$.

Step 4: Final Answer:
The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is $\frac{\pi}{4}$.

Quick Tip

Always check if your calculated angle lies within the principal range:
$\sin^{-1} : [-\pi/2, \pi/2]$
$\cos^{-1} : [0, \pi]$
$\tan^{-1} : (-\pi/2, \pi/2)$

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13. Show that the function given by $f(x) = 7x - 3$ **is increasing on** $\mathbb{R}$.

Correct Answer: Proven (Strictly Increasing)

Solution:
Step 1: Understanding the Concept:

A function $f(x)$ is strictly increasing on an interval if its derivative $f'(x) > 0$ for all x in that interval.

Step 2: Detailed Explanation:
The given function is $f(x) = 7x - 3$.
Differentiating with respect to x:

$$f'(x) = \frac{d}{dx}(7x - 3)$$

$$f'(x) = 7$$

For any real number $x \in \mathbb{R}$, the value of $f'(x)$ is a constant 7.
Since $7 > 0$, it follows that $f'(x) > 0$ for all $x \in \mathbb{R}$.

Step 3: Final Answer:
Since the derivative is strictly positive, the function $f(x) = 7x - 3$ is strictly increasing on the set of all real numbers $\mathbb{R}$.

Quick Tip

For linear functions $f(x) = mx + c$, if the slope m is positive, the function is always increasing. Here $m = 7$, which is positive.

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14. Find the unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

Correct Answer: $\frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}$

Solution:
Step 1: Understanding the Concept:
A unit vector in the direction of a vector $\vec{a}$ is defined as the vector divided by its magnitude.

Step 2: Key Formula or Approach:
Unit vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.
Magnitude $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$ for $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$.

Step 3: Detailed Explanation:
Given $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.
First, find the magnitude of $\vec{a}$:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 1^2}$$

$$|\vec{a}| = \sqrt{4+9+1} = \sqrt{14}$$

Now, calculate the unit vector:

$$\hat{a} = \frac{2\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{14}}$$

$$\hat{a} = \frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}$$

Step 4: Final Answer:
The unit vector is $\frac{1}{\sqrt{14}}(2\hat{i} + 3\hat{j} + \hat{k})$.

> Quick Tip
>
> A unit vector always has a magnitude of 1. You can verify your answer by calculating the magnitude of your result: $\sqrt{(2/\sqrt{14})^2 + (3/\sqrt{14})^2 + (1/\sqrt{14})^2} = \sqrt{14/14} = 1$.

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15. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

Correct Answer: $19\sqrt{2}$

Solution:
Step 1: Understanding the Concept:
The cross product of two vectors results in a vector perpendicular to both. Its magnitude represents the area of a parallelogram formed by the two vectors.

Step 2: Key Formula or Approach:
Use the determinant method for the cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Step 3: Detailed Explanation:
Calculate $\vec{a} \times \vec{b}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}[(-7 \times 2) - (7 \times -2)] - \hat{j}[(1 \times 2) - (7 \times 3)] + \hat{k}[(1 \times -2) - (-7 \times 3)]$$

$$= \hat{i}[-14 + 14] - \hat{j}[2 - 21] + \hat{k}[-2 + 21]$$

$$= 0\hat{i} + 19\hat{j} + 19\hat{k}$$

Now, find the magnitude:

$$|\vec{a} \times \vec{b}| = \sqrt{0^2 + 19^2 + 19^2}$$

$$= \sqrt{2 \times 19^2} = 19\sqrt{2}$$

Step 4: Final Answer:
The magnitude of the cross product is $19\sqrt{2}$.

> Quick Tip
>
> When finding the magnitude of a vector like $k\hat{j} + k\hat{k}$, the result is always $|k|\sqrt{2}$. This saves time in calculations.

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16. Evaluate $\int \frac{(\log x)^2}{x} dx$

Correct Answer: $\frac{(\log x)^3}{3} + C$

Solution:
Step 1: Understanding the Concept:
The method of substitution is used when the integrand contains a function and its derivative.

Step 2: Key Formula or Approach:
Let $\log x = t$. Then $\frac{1}{x} dx = dt$.

Step 3: Detailed Explanation:
Consider the integral $I = \int \frac{(\log x)^2}{x} dx$.
Substituting $\log x = t$:
Differentiating both sides: $\frac{1}{x} dx = dt$.

The integral becomes:

$$I = \int t^2 dt$$

Using the power rule $\int t^n dt = \frac{t^{n+1}}{n+1} + C$:

$$I = \frac{t^3}{3} + C$$

Substituting the value of t back:

$$I = \frac{(\log x)^3}{3} + C$$

Step 4: Final Answer:
The integral is $\frac{(\log x)^3}{3} + C$.

> Quick Tip
>
> Whenever you see $\log x$ and $1/x$ in the same integral, substitution $t = \log x$ is almost always the best strategy.

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17. Evaluate $\int_2^3 \frac{1}{x} dx$

Correct Answer: $\log\left(\frac{3}{2}\right)$

Solution:
Step 1: Understanding the Concept:
This is a definite integral. We find the anti-derivative and evaluate it at the upper and lower limits.

Step 2: Key Formula or Approach:

$$\int \frac{1}{x} dx = \log |x|$$

Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$.

Step 3: Detailed Explanation:

$$I = \int_2^3 \frac{1}{x} dx$$

$$I = [\log|x|]_2^3$$

$$I = \log 3 - \log 2$$

Using the property of logarithms $\log m - \log n = \log\left(\frac{m}{n}\right)$:

$$I = \log\left(\frac{3}{2}\right)$$

Step 4: Final Answer:
The value of the definite integral is $\log\left(\frac{3}{2}\right)$.

Quick Tip
Always ensure that the function is continuous over the interval of integration. Since $1/x$ is continuous on $[2, 3]$, the integral is valid.

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18. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then evaluate $P(A/B)$.

Correct Answer: $\frac{4}{9}$

Solution:
Step 1: Understanding the Concept:
$P(A/B)$ denotes the conditional probability of event A occurring given that event B has already occurred.

Step 2: Key Formula or Approach:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Step 3: Detailed Explanation:
Given:
$P(A) = \frac{7}{13}$
$P(B) = \frac{9}{13}$
$P(A \cap B) = \frac{4}{13}$
Applying the formula:

$$P(A/B) = \frac{4/13}{9/13}$$

The denominator 13 cancels out:

$$P(A/B) = \frac{4}{9}$$

Step 4: Final Answer:
The value of $P(A/B)$ is $\frac{4}{9}$.

> 💡 Quick Tip
>
> In conditional probability problems with fractions having common denominators, you can simply divide the numerators: $\frac{\text{Numerator of } P(A\cap B)}{\text{Numerator of } P(B)}$.

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19. If $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$, then evaluate $P(A \cup B)$.

Correct Answer: $\frac{11}{26}$

Solution:
Step 1: Understanding the Concept:
We use the general addition rule of probability and the definition of conditional probability.

Step 2: Key Formula or Approach:
1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2. $P(A \cap B) = P(B) \cdot P(A/B)$

Step 3: Detailed Explanation:
From the given information:
$P(B) = \frac{5}{13}$
$2P(A) = \frac{5}{13} \implies P(A) = \frac{5}{26}$
$P(A/B) = \frac{2}{5}$
First, find $P(A \cap B)$:

$$P(A \cap B) = P(A/B) \cdot P(B) = \frac{2}{5} \cdot \frac{5}{13} = \frac{2}{13}$$

Now, calculate $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$$

Converting to a common denominator of 26:

$$P(A \cup B) = \frac{5}{26} + \frac{10}{26} - \frac{4}{26}$$

$$P(A \cup B) = \frac{11}{26}$$

Step 4: Final Answer:
The probability $P(A \cup B)$ is $\frac{11}{26}$.

Quick Tip

Always convert all probabilities to a common denominator before adding or subtracting to avoid calculation errors.

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20. Construct a 3×4 matrix, whose elements are given by $a_{ij} = |2i - j|$.

Correct Answer: $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

Solution:
Step 1: Understanding the Concept:
A 3×4 matrix has 3 rows and 4 columns. We calculate each element a_{ij} by substituting the row index i and column index j into the given formula.

Step 2: Key Formula or Approach:
Row indices: $i \in \{1, 2, 3\}$.
Column indices: $j \in \{1, 2, 3, 4\}$.
Formula: $a_{ij} = |2i - j|$.

Step 3: Detailed Explanation:
For Row 1 (i = 1):
$a_{11} = |2(1) - 1| = |1| = 1$
$a_{12} = |2(1) - 2| = |0| = 0$
$a_{13} = |2(1) - 3| = |-1| = 1$
$a_{14} = |2(1) - 4| = |-2| = 2$

For Row 2 (i = 2):
$a_{21} = |2(2) - 1| = |3| = 3$
$a_{22} = |2(2) - 2| = |2| = 2$

$a_{23} = |2(2) - 3| = |1| = 1$
$a_{24} = |2(2) - 4| = |0| = 0$

For Row 3 (i = 3):
$a_{31} = |2(3) - 1| = |5| = 5$
$a_{32} = |2(3) - 2| = |4| = 4$
$a_{33} = |2(3) - 3| = |3| = 3$
$a_{34} = |2(3) - 4| = |2| = 2$

Step 4: Final Answer:
The required matrix is:

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Quick Tip

The absolute value $|x|$ ensures that elements are always non-negative. Note how the values decrease as j increases until $2i = j$, then increase again.

Section C

(Short Answer Type Questions)

21. Find the general solution of the differential equation $(e^x+e^{-x})dy-(e^x-e^{-x})dx = 0$.

Correct Answer: $y = \log(e^x + e^{-x}) + C$

Solution:
Step 1: Understanding the Concept:
The given equation is a first-order differential equation that can be solved using the method of separation of variables.
The main objective is to group all y terms on one side and all x terms on the other side before integrating.

Step 2: Key Formula or Approach:
Rearrange the equation to the form $dy = f(x)dx$ and then integrate both sides.
The integral of the form $\int \frac{f'(x)}{f(x)}dx$ is always equal to $\log|f(x)| + C$.

Step 3: Detailed Explanation:
Start with the given differential equation:

$$(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$$

Isolate the terms to separate the variables x and y:

$$(e^x + e^{-x})dy = (e^x - e^{-x})dx$$

Divide both sides by $(e^x + e^{-x})$ to express dy in terms of x:

$$dy = \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$

Now, integrate both sides of the equation:

$$\int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$

For the right-hand side integral, we use the method of substitution. Let:

$$t = e^x + e^{-x}$$

Differentiating both sides with respect to x:

$$\frac{dt}{dx} = e^x + e^{-x}(-1) = e^x - e^{-x} \implies dt = (e^x - e^{-x})dx$$

Substitute these values back into the integral:

$$y = \int \frac{1}{t}dt$$

$$y = \log|t| + C$$

Finally, substitute the original expression for t back:

$$y = \log(e^x + e^{-x}) + C$$

Step 4: Final Answer:
The general solution of the differential equation is $y = \log(e^x + e^{-x}) + C$.

Quick Tip

In differential equations, always look for the derivative of the denominator in the numerator. If Numerator $= \frac{d}{dx}$(Denominator), then the integral is simply $\log|\text{Denominator}|$.

---

22. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Correct Answer: 12π square units

Solution:

Step 1: Understanding the Concept:

The area of a region bounded by a curve is calculated using definite integration.
Since an ellipse is symmetric about both coordinate axes, the total area can be found by calculating the area of one quadrant and multiplying it by four.

Step 2: Key Formula or Approach:

The area is given by $A = 4\int_0^a y\,dx$.
Standard integral formula: $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$.

Step 3: Detailed Explanation:

The given equation is $\frac{x^2}{16} + \frac{y^2}{9} = 1$.
Comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get $a^2 = 16 \implies a = 4$ and $b^2 = 9 \implies b = 3$.
Solve for y in terms of x:

$$\frac{y^2}{9} = 1 - \frac{x^2}{16} = \frac{16 - x^2}{16}$$

$$y^2 = \frac{9}{16}(16 - x^2) \implies y = \frac{3}{4}\sqrt{16 - x^2}$$

The total area A is:

$$A = 4\int_0^4 \frac{3}{4}\sqrt{16 - x^2}\,dx = 3\int_0^4 \sqrt{16 - x^2}\,dx$$

Applying the standard integration formula:

$$A = 3\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

Substituting the upper and lower limits:

$$A = 3\left[\left(\frac{4}{2}\sqrt{16 - 16} + 8\sin^{-1}(1)\right) - \left(\frac{0}{2}\sqrt{16 - 0} + 8\sin^{-1}(0)\right)\right]$$

$$A = 3\left[(0 + 8 \cdot \frac{\pi}{2}) - (0 + 0)\right] = 3 \cdot 4\pi = 12\pi$$

Step 4: Final Answer:
The area of the region bounded by the ellipse is 12π square units.

Quick Tip

Shortcut: The area of any ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is always exactly πab.
For this problem: Area $= \pi \times 4 \times 3 = 12\pi$. Use this to verify your steps in descriptive exams.

---

23. Find the value of K so that the given function is continuous at the indicated point:

$f(x) = \begin{cases} \frac{K\cos x}{\pi - 2x}, & \textbf{if } x \neq \frac{\pi}{2} \\ 3, & \textbf{if } x = \frac{\pi}{2} \end{cases}$ **at** $x = \frac{\pi}{2}$

Correct Answer: $K = 6$

Solution:
Step 1: Understanding the Concept:
For a function $f(x)$ to be continuous at $x = a$, the limit of the function as x approaches a must equal the functional value at that point, i.e., $\lim_{x\to a} f(x) = f(a)$.

Step 2: Key Formula or Approach:
We need to calculate $\lim_{x\to\frac{\pi}{2}} \frac{K\cos x}{\pi-2x}$ and set it equal to 3.

Step 3: Detailed Explanation:
The value of the function at the point is given as $f(\frac{\pi}{2}) = 3$.
To find the limit $L = \lim_{x\to\frac{\pi}{2}} \frac{K\cos x}{\pi-2x}$, we use a substitution.
Let $x = \frac{\pi}{2} + h$. As $x \to \frac{\pi}{2}$, the variable $h \to 0$.
Substituting this into the limit expression:

$$L = \lim_{h\to 0} \frac{K\cos(\frac{\pi}{2} + h)}{\pi - 2(\frac{\pi}{2} + h)}$$

Using the trigonometric identity $\cos(\frac{\pi}{2} + h) = -\sin h$:

$$L = \lim_{h\to 0} \frac{-K\sin h}{\pi - \pi - 2h} = \lim_{h\to 0} \frac{-K\sin h}{-2h}$$

$$L = \frac{K}{2}\left(\lim_{h\to 0}\frac{\sin h}{h}\right)$$

Since the standard limit $\lim_{h\to 0}\frac{\sin h}{h} = 1$:

$$L = \frac{K}{2}\cdot 1 = \frac{K}{2}$$

For continuity at $x = \frac{\pi}{2}$:

$$\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\frac{K}{2} = 3 \implies K = 6$$

Step 4: Final Answer:
The required value of K is 6.

> 💡 Quick Tip
>
> In exams, if the limit is in the indeterminate form $\frac{0}{0}$, you can quickly use L'Hôpital's Rule:
> $\lim_{x\to\frac{\pi}{2}}\frac{K\cos x}{\pi-2x} = \lim_{x\to\frac{\pi}{2}}\frac{-K\sin x}{-2} = \frac{-K(1)}{-2} = \frac{K}{2}$. Equating this to 3 gives $K = 6$.

---

24. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

Correct Answer: $\frac{3\sqrt{2}}{2}$ units

Solution:
Step 1: Understanding the Concept:
The shortest distance between two skew lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is the length of the vector projection of the segment joining the two lines onto the direction perpendicular to both.

Step 2: Key Formula or Approach:
The distance d is given by:

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

Step 3: Detailed Explanation:
Identify the components from the given equations:
Line 1: $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$
Line 2: $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$
Calculate the difference vector:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (-1-2)\hat{j} + (-1-1)\hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$$

Calculate the cross product of the direction vectors:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1-(-2)) = -3\hat{i} + 3\hat{k}$$

Find the magnitude of the cross product:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

Calculate the dot product $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$:

$$(\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-3\hat{i} + 3\hat{k}) = 1(-3) + (-3)(0) + (-2)(3) = -3 - 6 = -9$$

Substitute into the distance formula:

$$d = \left| \frac{-9}{3\sqrt{2}} \right| = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

Step 4: Final Answer:
The shortest distance between the two lines is $\frac{3\sqrt{2}}{2}$ units.

Quick Tip

If the dot product $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$ turns out to be zero, it means the lines intersect at some point, and the shortest distance is 0.

---

25. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

Correct Answer: $\lambda = 8$

Solution:
Step 1: Understanding the Concept:
Two vectors are perpendicular if their scalar (dot) product is exactly zero.

Step 2: Key Formula or Approach:
Express the vector $\vec{a} + \lambda\vec{b}$ in terms of components and then apply the condition $(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$.

Step 3: Detailed Explanation:
First, determine the vector $\vec{a} + \lambda\vec{b}$:

$$\vec{a} + \lambda\vec{b} = (2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k})$$

$$= (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$$

Given that this vector is perpendicular to $\vec{c} = 3\hat{i} + \hat{j}$:

$$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$$

$$[(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}] \cdot (3\hat{i} + \hat{j} + 0\hat{k}) = 0$$

Multiply corresponding components:

$$3(2 - \lambda) + 1(2 + 2\lambda) + 0(3 + \lambda) = 0$$

$$6 - 3\lambda + 2 + 2\lambda = 0$$

$$8 - \lambda = 0 \implies \lambda = 8$$

Step 4: Final Answer:
The value of the scalar λ is 8.

💡 Quick Tip

Perpendicular vectors = zero dot product.
Parallel vectors = cross product is zero OR one is a scalar multiple of the other.

---

26. Solve the following linear programming problem graphically:
Maximize $Z = 3x + 2y$ subject to constraints:
$x + 2y \leq 10, 3x + y \leq 15, x \geq 0, y \geq 0$.

Correct Answer: Maximum value of $Z = 18$ at the point $(4, 3)$.

Solution:
Step 1: Understanding the Concept:
A linear programming problem involves finding the maximum or minimum value of a linear function within a region defined by linear inequalities. This region is called the feasible region.

Step 2: Key Formula or Approach:
The "Corner Point Method" is used, where the objective function is evaluated at every vertex of the feasible region.

Step 3: Detailed Explanation:
1. Plot the boundary lines:
Line 1: $x + 2y = 10$. Intercepts are $(10, 0)$ and $(0, 5)$.
Line 2: $3x + y = 15$. Intercepts are $(5, 0)$ and $(0, 15)$.
2. Determine the feasible region:
Since both inequalities are $\leq$ and involve positive coefficients, the feasible region is the area towards the origin bounded by these lines and the axes $x = 0, y = 0$.
3. Find the intersection point of the lines:
Subtracting equations: $(3x + y) - 0.5(x + 2y) = 15 - 5 \implies 2.5x = 10 \implies x = 4$.
Substitute $x = 4$ in line 1: $4 + 2y = 10 \implies 2y = 6 \implies y = 3$. Intersection is $(4, 3)$.
4. Identify corner points of the feasible region:
Vertices are $O(0, 0), A(5, 0), B(4, 3)$, and $C(0, 5)$.
5. Evaluate $Z = 3x + 2y$ at these points:
- At $O(0, 0)$: $Z = 3(0) + 2(0) = 0$
- At $A(5, 0)$: $Z = 3(5) + 2(0) = 15$
- At $B(4, 3)$: $Z = 3(4) + 2(3) = 12 + 6 = 18$
- At $C(0, 5)$: $Z = 3(0) + 2(5) = 10$

Comparing the values, the maximum is 18 at $(4, 3)$.

Step 4: Final Answer:
The maximum value of Z is 18, achieved at the coordinates $(4, 3)$.

Quick Tip

The point of intersection of constraint lines is very often the location of the optimal solution in competitive exams. Calculate it carefully.

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27. Express in the simplest form: $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), -\frac{\pi}{4} < x < \frac{3\pi}{4}$

Correct Answer: $\frac{\pi}{4} - x$

Solution:

Step 1: Understanding the Concept:

Simplifying inverse trigonometric expressions usually involves transforming the inner trigonometric part to match the outer function's type.

Step 2: Key Formula or Approach:

Divide the numerator and denominator by $\cos x$ and use the tangent subtraction formula: $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

Step 3: Detailed Explanation:

Given: $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$.

Divide both the numerator and denominator by $\cos x$:

$$= \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$$

We know that $1 = \tan(\frac{\pi}{4})$. Substitute this into the expression:

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right)$$

Using the identity $\tan(A - B)$:

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$$

Check the range condition: $-\frac{\pi}{4} < x < \frac{3\pi}{4}$.

Multiplying by -1: $\frac{\pi}{4} > -x > -\frac{3\pi}{4}$.

Adding $\frac{\pi}{4}$: $\frac{\pi}{2} > \frac{\pi}{4} - x > -\frac{\pi}{2}$.

Since $(\frac{\pi}{4} - x)$ is within the principal range of $\tan^{-1}$, the expression simplifies directly.

$$= \frac{\pi}{4} - x$$

Step 4: Final Answer:

The simplified form is $\frac{\pi}{4} - x$.

💡 Quick Tip

Remember: $\frac{\cos x - \sin x}{\cos x + \sin x} = \tan(\frac{\pi}{4} - x)$ and $\frac{\cos x + \sin x}{\cos x - \sin x} = \tan(\frac{\pi}{4} + x)$. These are very frequent identities.

---

28. A bag contains 4 red and 4 black balls; another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

Correct Answer: $\frac{2}{3}$

Solution:
Step 1: Understanding the Concept:
This problem involves determining the probability of a cause given an observed effect, which requires the application of Bayes' Theorem.

Step 2: Key Formula or Approach:
Let E_1, E_2 be the events of selecting Bag 1 and Bag 2 respectively. Let A be the event of drawing a red ball.
Bayes' Theorem: $P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$.

Step 3: Detailed Explanation:
1. Probability of selecting Bag 1 or Bag 2:
$P(E_1) = 1/2$, $P(E_2) = 1/2$.
2. Probability of drawing a red ball from Bag 1 (4R, 4B):
$P(A|E_1) = 4/8 = 1/2$.
3. Probability of drawing a red ball from Bag 2 (2R, 6B):
$P(A|E_2) = 2/8 = 1/4$.
4. Apply Bayes' Theorem:

$$P(E_1|A) = \frac{(1/2) \cdot (1/2)}{(1/2) \cdot (1/2) + (1/2) \cdot (1/4)}$$

$$P(E_1|A) = \frac{1/4}{1/4 + 1/8}$$

$$P(E_1|A) = \frac{1/4}{3/8} = \frac{1}{4} \cdot \frac{8}{3} = \frac{2}{3}$$

Step 4: Final Answer:
The probability that the ball was drawn from the first bag is $\frac{2}{3}$.

💡 Quick Tip

When the prior probabilities are equal (like 1/2 here), the result is simply the ratio of the conditional likelihood of the specific event to the sum of all conditional likelihoods: $\frac{4/8}{4/8+2/8} = \frac{4}{6} = \frac{2}{3}$.

Section D

(Long Answer Type Questions)

29(a). If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find K so that $A^2 = KA - 2I$.

Correct Answer: $K = 1$

Solution:
Step 1: Understanding the Concept:
The question requires us to find a scalar K by evaluating powers of matrices and performing scalar multiplication and subtraction on matrices.

Step 2: Key Formula or Approach:
We will first calculate A^2 by performing matrix multiplication $A \cdot A$, then substitute all values into the equation $A^2 = KA - 2I$ and compare the corresponding elements.

Step 3: Detailed Explanation:
First, calculate A^2:

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (3)(3) + (-2)(4) & (3)(-2) + (-2)(-2) \\ (4)(3) + (-2)(4) & (4)(-2) + (-2)(-2) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 - 8 & -6 + 4 \\ 12 - 8 & -8 + 4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now, evaluate the right-hand side $KA - 2I$:

$$KA - 2I = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$KA - 2I = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3K - 2 & -2K \\ 4K & -2K - 2 \end{bmatrix}$$

Given $A^2 = KA - 2I$, equate the matrices:

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3K - 2 & -2K \\ 4K & -2K - 2 \end{bmatrix}$$

Equating corresponding elements:
1) $3K - 2 = 1 \implies 3K = 3 \implies K = 1$
2) $-2K = -2 \implies K = 1$
3) $4K = 4 \implies K = 1$
4) $-2K - 2 = -4 \implies -2K = -2 \implies K = 1$

Step 4: Final Answer:
The value of K that satisfies the equation is 1.

Quick Tip

When comparing two matrices, always verify the value of the variable from at least two different elements to ensure consistency.

29(b). Solve the following system of equations by matrix method:
$2x + 3y + 3z = 5$,
$x - 2y + z = -4$,
$3x - y - 2z = 3$.

Correct Answer: $x = 1, y = 2, z = -1$

Solution:
Step 1: Understanding the Concept:
The system of linear equations can be written in the form $AX = B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix. If $|A| \neq 0$, then $X = A^{-1}B$.

Step 2: Key Formula or Approach:
1. Write the system as $AX = B$.
2. Calculate determinant $|A|$.
3. Find Adjoint of A, $adj(A)$.
4. Calculate $A^{-1} = \frac{1}{|A|}adj(A)$.
5. Solve $X = A^{-1}B$.

Step 3: Detailed Explanation:
Let $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$.

First, find $|A|$:

$$|A| = 2(4-(-1)) - 3(-2-3) + 3(-1-(-6))$$

$$|A| = 2(5) - 3(-5) + 3(5) = 10 + 15 + 15 = 40$$

Since $|A| = 40 \neq 0$, A^{-1} exists.
Now, find the cofactors of A:
$C_{11} = 5, C_{12} = 5, C_{13} = 5$
$C_{21} = 3, C_{22} = -13, C_{23} = 11$
$C_{31} = 9, C_{32} = 1, C_{33} = -7$

$$adj(A) = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Now, $X = A^{-1}B = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Step 4: Final Answer:
The solution to the system is $x = 1, y = 2, z = -1$.

Quick Tip

After finding the values of x, y, z, always substitute them into one of the original equations to verify your answer. For example: $2(1) + 3(2) + 3(-1) = 2 + 6 - 3 = 5$. This matches the first equation.

30(a). Find $\int \frac{x}{(x-1)(x-2)(x-3)} dx$.

Correct Answer: $\frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$

Solution:
Step 1: Understanding the Concept:
To integrate a rational function where the denominator is a product of linear factors, we use the method of partial fractions.

Step 2: Key Formula or Approach:
Express the integrand as:

$$\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

Step 3: Detailed Explanation:
Multiply through by the common denominator:

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Put $x = 1$: $1 = A(1-2)(1-3) \implies 1 = A(-1)(-2) \implies A = 1/2$.
Put $x = 2$: $2 = B(2-1)(2-3) \implies 2 = B(1)(-1) \implies B = -2$.
Put $x = 3$: $3 = C(3-1)(3-2) \implies 3 = C(2)(1) \implies C = 3/2$.
The integral becomes:

$$I = \int \left[\frac{1}{2(x-1)} - \frac{2}{x-2} + \frac{3}{2(x-3)}\right] dx$$

$$I = \frac{1}{2}\int \frac{1}{x-1}dx - 2\int \frac{1}{x-2}dx + \frac{3}{2}\int \frac{1}{x-3}dx$$

$$I = \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$$

Step 4: Final Answer:
The integral is $\frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$.

> Quick Tip
>
> For non-repeated linear factors, use the "cover-up" method to find constants quickly: To find A, cover $(x-1)$ and put $x = 1$ in the rest: $1/((1-2)(1-3)) = 1/2$.

30(b). Find $\int \sqrt{x^2+2x+5}dx$.

Correct Answer: $\frac{x+1}{2}\sqrt{x^2+2x+5} + 2\log|x+1+\sqrt{x^2+2x+5}| + C$

Solution:
Step 1: Understanding the Concept:
The integral involves a quadratic expression under a square root. We solve this by completing

the square to transform it into a standard integral form.

Step 2: Key Formula or Approach:
Standard form: $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$.

Step 3: Detailed Explanation:
Complete the square for x^2+2x+5:

$$x^2+2x+5 = (x^2+2x+1)+4 = (x+1)^2+2^2$$

Let $I = \int \sqrt{(x+1)^2+2^2}dx$.
Let $t = x+1$, then $dt = dx$.

$$I = \int \sqrt{t^2+2^2}dt$$

Using the formula with $a = 2$:

$$I = \frac{t}{2}\sqrt{t^2+2^2} + \frac{2^2}{2}\log|t+\sqrt{t^2+2^2}| + C$$

Substituting $t = x+1$ back:

$$I = \frac{x+1}{2}\sqrt{x^2+2x+5} + 2\log|x+1+\sqrt{x^2+2x+5}| + C$$

Step 4: Final Answer:
The result of the integration is $\frac{x+1}{2}\sqrt{x^2+2x+5} + 2\log|x+1+\sqrt{x^2+2x+5}| + C$.

Quick Tip

Remember the triplet of formulas for $\int \sqrt{x^2 \pm a^2}$ and $\int \sqrt{a^2-x^2}$. They all start with $\frac{x}{2}$(same root).

31(a). Find $\frac{dy}{dx}$, if $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.

Correct Answer: $\tan\theta$

Solution:
Step 1: Understanding the Concept:
This is a problem of parametric differentiation. We find $\frac{dy}{dx}$ using the chain rule: $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

Step 2: Key Formula or Approach:

1. Differentiate x with respect to θ.
2. Differentiate y with respect to θ.
3. Divide the results.

Step 3: Detailed Explanation:

Differentiating x w.r.t θ:

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{d}{d\theta}(\theta\sin\theta)\right]$$

Using product rule: $\frac{d}{d\theta}(\theta\sin\theta) = \sin\theta + \theta\cos\theta$.

$$\frac{dx}{d\theta} = a[-\sin\theta + \sin\theta + \theta\cos\theta] = a\theta\cos\theta$$

Differentiating y w.r.t θ:

$$\frac{dy}{d\theta} = a\left[\cos\theta - \frac{d}{d\theta}(\theta\cos\theta)\right]$$

Using product rule: $\frac{d}{d\theta}(\theta\cos\theta) = \cos\theta - \theta\sin\theta$.

$$\frac{dy}{d\theta} = a[\cos\theta - (\cos\theta - \theta\sin\theta)]$$

$$\frac{dy}{d\theta} = a[\cos\theta - \cos\theta + \theta\sin\theta] = a\theta\sin\theta$$

Now, calculate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $\tan\theta$.

Quick Tip

Be very careful with negative signs when applying the product rule inside brackets. Distributing the minus sign is a common place for errors.

31(b). If $y = \sin^{-1}x$, Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

Correct Answer: Proved

Solution:
Step 1: Understanding the Concept:
This problem involves second-order derivatives. We differentiate the given function twice and rearrange to form the desired differential equation.

Step 2: Key Formula or Approach:
Differentiate y with respect to x, then rearrange the expression before differentiating again to simplify the process.

Step 3: Detailed Explanation:
Given $y = \sin^{-1} x$.
Differentiating w.r.t x:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Cross-multiplying:

$$\sqrt{1-x^2}\frac{dy}{dx} = 1$$

Squaring both sides to remove the root:

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 1$$

Differentiating again w.r.t x using product rule and chain rule:

$$(1-x^2)\cdot\frac{d}{dx}\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^2\cdot\frac{d}{dx}(1-x^2) = 0$$

$$(1-x^2)\cdot 2\left(\frac{dy}{dx}\right)\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\cdot(-2x) = 0$$

Divide the entire equation by $2\frac{dy}{dx}$ (assuming $\frac{dy}{dx} \neq 0$):

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

Step 4: Final Answer:
The given identity $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ is proved.

Quick Tip

Squaring after the first derivative often makes second differentiation much easier than using the quotient rule directly.
