Jee Main 2026 B. Planning Memory Based Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :300	Total questions :75

Important Instructions

1. **The test is of 3 hours duration.**
2. **This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.**
3. **This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry, and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.**
4. **Section-A: Attempt all questions.**
5. **Section-B: Attempt all questions.**
6. **Section-A (01 – 20):** Contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for the correct answer and –1 mark for the wrong answer.
7. **Section-B (21 – 25):** Contains 5 Numerical value-based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for the correct answer and –1 mark for the wrong answer.

1. If both the roots of the equation

$$x^2 - 2ax + a^2 - 1 = 0 \quad (a \in \mathbb{R})$$

lie in the interval $(-2, 2)$**, then the equation**

$$x^2 - (a^2 + 1)x - (a^2 + 2) = 0$$

has:
(1) both roots in $(-3, 0)$
(2) one root in $(0, 2)$ and another root in $(-2, 0)$
(3) one root in $(2, 3)$ and another root in $(-2, 0)$
(4) one root in $(-3, -2)$ and another root in $(0, 2)$

Correct Answer: (3)

Solution:
Step 1: Analyze the first equation

$$x^2 - 2ax + a^2 - 1 = 0$$

Rewrite:

$$(x - a)^2 = 1$$

So the roots are:

$$x = a \pm 1$$

Given that both roots lie in $(-2, 2)$:

$$-2 < a - 1 < 2 \quad \text{and} \quad -2 < a + 1 < 2$$

From these:

$$-1 < a < 3 \quad \text{and} \quad -3 < a < 1$$

Combining:

$$-1 < a < 1$$

Step 2: Analyze the second equation

$$x^2 - (a^2 + 1)x - (a^2 + 2) = 0$$

Sum of roots:

$$\alpha + \beta = a^2 + 1 > 0$$

Product of roots:

$$\alpha\beta = -(a^2 + 2) < 0$$

Hence, one root is positive and the other is negative.

Step 3: Locate the roots

Evaluate the polynomial at key points (using $|a| < 1$):

$$f(0) = -(a^2 + 2) < 0$$

$$f(2) = 4 - 2(a^2 + 1) - (a^2 + 2) = -3a^2 < 0$$

$$f(3) = 9 - 3(a^2 + 1) - (a^2 + 2) = 4 - 4a^2 > 0$$

Thus, the positive root lies in $(2, 3)$.

Now check negative side:

$$f(-2) = 4 + 2(a^2 + 1) - (a^2 + 2) = 4 + a^2 > 0$$

Since $f(-2) > 0$ and $f(0) < 0$, the negative root lies in $(-2, 0)$.

Final Conclusion:

One root lies in $(2, 3)$ and the other lies in $(-2, 0)$.

$$\boxed{\text{Option (3)}}$$

Quick Tip

When roots are restricted to an interval, first determine the parameter range, then use sign analysis of the polynomial to locate roots in specific subintervals.

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2. If the system of equations

$$\begin{cases} 2x + y + pz = -1 \\ 3x - 2y + z = q \\ 5x - 8y + 9z = 5 \end{cases}$$

has more than one solution, then $q - p$ is equal to:
(1) 2
(2) -2
(3) 4
(4) -4

Correct Answer: (3)

Solution:
For a system of three linear equations to have **more than one solution**, the determinant of the coefficient matrix must be zero and the system must be consistent.
Step 1: Determinant of coefficient matrix

$$\begin{vmatrix} 2 & 1 & p \\ 3 & -2 & 1 \\ 5 & -8 & 9 \end{vmatrix} = 2((-2)\cdot 9 - 1\cdot(-8)) - 1(3\cdot 9 - 1\cdot 5) + p(3\cdot(-8) - (-2)\cdot 5)$$

$$= 2(-18 + 8) - 1(27 - 5) + p(-24 + 10) = -20 - 22 - 14p$$

$$\Rightarrow -42 - 14p = 0 \Rightarrow p = -3$$

Step 2: Consistency condition
With $p = -3$, observe that the rows satisfy:

$$-2R_1 + 3R_2 = R_3$$

For consistency, the constants must satisfy the same relation:

$$-2(-1) + 3q = 5 \Rightarrow 2 + 3q = 5 \Rightarrow q = 1$$

Step 3: Compute $q - p$

$$q - p = 1 - (-3) = 4$$

$$\boxed{4}$$

Quick Tip

For infinitely many solutions, ensure both:

- determinant of coefficient matrix is zero,
- constants follow the same linear dependence as the equations.

3. All the words (with or without meaning) formed using all the five letters of the word GOING are arranged as in a dictionary. Then the word OGGIN occurs at the place which is:
(1) 48^{th}
(2) 49^{th}
(3) 50^{th}
(4) 51^{th}

Correct Answer: (2)

Solution:
Step 1: Arrange letters in alphabetical order
Letters of the word GOING are:

$$G,\ O,\ I,\ N,\ G$$

Alphabetical order:

$$G,\ G,\ I,\ N,\ O$$

Step 2: Find rank of the word OGGIN
We calculate the number of words that come before **OGGIN**.
First letter: O
Letters smaller than O are G, G, I, N.
Number of permutations using remaining 4 letters (with two G's):

$$\frac{4!}{2!} = 12$$

Number of such letters before O: 4

$$\Rightarrow 4 \times 12 = 48$$

Step 3: Remaining letters
After fixing O, the remaining word is **GGIN**. This is already the smallest possible arrangement of these letters.
So, no additional words are added.
Step 4: Final rank

$$\text{Rank} = 48 + 1 = 49$$

$$\boxed{49^{\text{th}}}$$

Quick Tip

For rank problems with repeated letters: always divide by factorial of repetitions and count only letters **strictly smaller** at each position.

4. Let f be a differentiable function satisfying

$$f(x+y) = f(x) + f(y) - xy \quad \textbf{for all } x, y \in \mathbb{R}.$$

If

$$\lim_{h \to 0} \frac{f(h)}{h} = 3,$$

then the value of

$$\sum_{n=1}^{10} f(n)$$

is equal to:

(1) $-\frac{55}{2}$
(2) $\frac{275}{2}$
(3) $-\frac{55}{4}$
(4) $\frac{225}{4}$

Correct Answer: (1)

Solution:
Step 1: Find the general form of $f(x)$
Given:

$$f(x+y) = f(x) + f(y) - xy$$

Assume $f(x)$ is a polynomial of degree ≤ 2:

$$f(x) = ax^2 + bx + c$$

Substitute into the functional equation:

$$a(x+y)^2 + b(x+y) + c = ax^2 + bx + c + ay^2 + by + c - xy$$

Comparing coefficients:

$$2a = -1 \Rightarrow a = -\frac{1}{2}$$

Constant term:

$$c = 0$$

So,

$$f(x) = -\frac{x^2}{2} + bx$$

Step 2: Use the given limit

$$\lim_{h \to 0} \frac{f(h)}{h} = \lim_{h \to 0} \left(-\frac{h}{2} + b \right) = b$$

Given the limit is 3:

$$b = 3$$

Hence,

$$f(x) = 3x - \frac{x^2}{2}$$

Step 3: Compute the required sum

$$\sum_{n=1}^{10} f(n) = \sum_{n=1}^{10} \left(3n - \frac{n^2}{2}\right)$$

$$= 3\sum_{n=1}^{10} n - \frac{1}{2}\sum_{n=1}^{10} n^2$$

$$= 3 \cdot \frac{10 \cdot 11}{2} - \frac{1}{2} \cdot \frac{10 \cdot 11 \cdot 21}{6}$$

$$= 165 - \frac{385}{2} = \frac{330 - 385}{2} = -\frac{55}{2}$$

$$\boxed{-\frac{55}{2}}$$

Quick Tip

Functional equations involving $f(x + y)$ often lead to polynomial solutions. Always use the given limit to fix remaining constants.

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5. The function

$$f(x) = \sin 2x + 2\cos x, \qquad x \in \left(-\frac{3\pi}{4}, \frac{3\pi}{4}\right)$$

has:

(1) no critical point
(2) a point of local maxima and a point of local minima
(3) a point of local maxima and a point of inflection
(4) a point of local minima and a point of inflection

Correct Answer: (3)

Solution:
Step 1: Find the first derivative

$$f(x) = \sin 2x + 2\cos x$$

$$f'(x) = 2\cos 2x - 2\sin x$$

Set $f'(x) = 0$:

$$2(\cos 2x - \sin x) = 0 \Rightarrow \cos 2x = \sin x$$

Using $\cos 2x = 1 - 2\sin^2 x$:

$$1 - 2\sin^2 x = \sin x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -1$$

In the interval $\left(-\frac{3\pi}{4}, \frac{3\pi}{4}\right)$,

$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$$

($\sin x = -1$ gives $x = -\frac{\pi}{2}$, which is a boundary point for critical behaviour.)

Step 2: Second derivative test

$$f''(x) = -4\sin 2x - 2\cos x$$

At $x = \frac{\pi}{6}$:

$$f''\left(\frac{\pi}{6}\right) = -4\sin\frac{\pi}{3} - 2\cos\frac{\pi}{6} = -4 \cdot \frac{\sqrt{3}}{2} - 2 \cdot \frac{\sqrt{3}}{2} < 0$$

Hence, $x = \frac{\pi}{6}$ is a point of **local maxima**.

Step 3: Check for inflection point

At $x = -\frac{\pi}{2}$,

$$f'(x) = 0$$

but $f''(x)$ changes sign across this point, hence it is a **point of inflection**.

Final Conclusion:

The function has a point of local maxima and a point of inflection.

$$\boxed{\text{Option (3)}}$$

Quick Tip

A point where $f'(x) = 0$ but no sign change in monotonicity occurs is not an extremum — always check the second derivative or sign of $f'(x)$.
