KCET 2026 April 24 Mathematics

Question Paper with Solutions PDF

Conducted by KEA

General Instructions

- **Duration:** The total duration of the examination is 80 minutes.
- **Total Marks:** The complete paper carries a maximum of 60 marks.
- **Compulsory Questions:** All 60 questions are compulsory.
- Each question has four options. Only **one** option is correct.
- **Correct Answer:** +1 marks.
- **Incorrect Answer:** There is no Negative marking for incorrect answers.

1. $\tan^{-1}\left(\frac{1}{1+1\cdot 2}\right)+\tan^{-1}\left(\frac{1}{1+2\cdot 3}\right)+\cdots+\tan^{-1}\left(\frac{1}{1+n\cdot(n+1)}\right)=$

(A) $\tan^{-1}\left(\frac{n}{n+2}\right)$
(B) $\tan^{-1}\left(\frac{n+1}{n}\right)$
(C) $\tan^{-1}\left(\frac{n}{n+1}\right)$
(D) $\tan^{-1}\left(\frac{n+2}{n}\right)$

Correct Answer: (A) $\tan^{-1}\left(\frac{n}{n+2}\right)$

Solution:

Step 1: Understanding the Concept:
This problem requires evaluating the sum of a series involving inverse trigonometric functions. The general strategy is to express each term as a difference of two inverse tangents to create a telescoping series, where intermediate terms cancel out.

Step 2: Key Formula or Approach:

Use the inverse tangent difference formula:

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

We need to rewrite the k-th term, $T_k = \tan^{-1}\left(\frac{1}{1+k(k+1)}\right)$, in a form that matches the right side of this identity.

Step 3: Detailed Explanation:

Let's analyze the general k-th term of the series:

$$T_k = \tan^{-1}\left(\frac{1}{1+k(k+1)}\right)$$

We can cleverly rewrite the numerator 1 as $(k+1)-k$.

$$T_k = \tan^{-1}\left(\frac{(k+1)-k}{1+k\cdot(k+1)}\right)$$

Comparing this with the formula $\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1} x - \tan^{-1} y$, we can see that $x = k+1$ and $y = k$. Therefore, we can write T_k as a difference:

$$T_k = \tan^{-1}(k+1) - \tan^{-1}(k)$$

Now, we need to find the sum of the first n terms, $S_n = \sum_{k=1}^{n} T_k$:

$$S_n = \sum_{k=1}^{n}[\tan^{-1}(k+1) - \tan^{-1}(k)]$$

Writing out the terms to see the pattern: For $k=1$: $\tan^{-1}(2) - \tan^{-1}(1)$

For $k=2$: $\tan^{-1}(3) - \tan^{-1}(2)$

For $k=3$: $\tan^{-1}(4) - \tan^{-1}(3)$

...

For $k=n$: $\tan^{-1}(n+1) - \tan^{-1}(n)$

Adding all these terms together, we see that almost all terms cancel out (this is a telescoping sum):

$$S_n = [\tan^{-1}(2) - \tan^{-1}(1)] + [\tan^{-1}(3) - \tan^{-1}(2)] + \cdots + [\tan^{-1}(n+1) - \tan^{-1}(n)]$$

$$S_n = \tan^{-1}(n+1) - \tan^{-1}(1)$$

We know that $\tan^{-1}(1) = \frac{\pi}{4}$, but it's more useful to apply the difference formula again to combine these two terms into a single inverse tangent expression:

$$S_n = \tan^{-1}(n+1) - \tan^{-1}(1) = \tan^{-1}\left(\frac{(n+1)-1}{1+(n+1)\cdot 1}\right)$$

$$S_n = \tan^{-1}\left(\frac{n}{1+n+1}\right)$$

$$S_n = \tan^{-1}\left(\frac{n}{n+2}\right)$$

Step 4: Final Answer:

The sum of the series is $\tan^{-1}\left(\frac{n}{n+2}\right)$.

Quick Tip: For series summation involving $\tan^{-1}$, always try to express the argument in the form $\frac{x-y}{1+xy}$. This allows you to split the term into $\tan^{-1} x - \tan^{-1} y$, which almost always leads to a telescoping series where most terms neatly cancel out.

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2. The corner points of the feasible region determined by the system of linear constraints are (0, 10), (5, 5), (15, 15), (0, 20). Let $z = px + qy$, where $p, q > 0$. The relation between p and q, so that the maximum z occurs at both points (15, 15) and (0, 20) is

(A) $p = q$

(B) $p = 2q$

(C) $q = 2p$

(D) $q = 3p$

Correct Answer: (D) $q = 3p$

Solution:

Step 1: Understanding the Concept:

In linear programming, the fundamental theorem states that if an optimal value (maximum or minimum) exists, it must occur at one or more corner points of the feasible region. If the maximum value occurs at two different corner points, it means the value of the objective function evaluated at these two points must be exactly the same.

Step 2: Key Formula or Approach:

1. Evaluate the objective function $z = px + qy$ at the first given point $(15, 15)$.
2. Evaluate the objective function $z = px + qy$ at the second given point $(0, 20)$.
3. Set these two expressions equal to each other because they both represent the same maximum value.
4. Solve the resulting equation to find the relationship between p and q.

Step 3: Detailed Explanation:

The objective function is given as $z = px + qy$. Let's find the value of z at the corner point $(15, 15)$:

$$z_1 = p(15) + q(15) = 15p + 15q$$

Next, find the value of z at the corner point $(0, 20)$:

$$z_2 = p(0) + q(20) = 0 + 20q = 20q$$

The problem states that the maximum value of z occurs at both of these points. Therefore, the value of z must be equal at these two points:

$$z_1 = z_2$$

Substituting our expressions:

$$15p + 15q = 20q$$

Now, solve for the relation between p and q. Subtract $15q$ from both sides:

$$15p = 20q - 15q$$

$$15p = 5q$$

Divide both sides by 5:

$$3p = q$$

This can be written as $q = 3p$.

Step 4: Final Answer:

The required relation between p and q is $q = 3p$.

Quick Tip: Whenever a problem states that an objective function attains its optimal value at multiple specific points, simply evaluate the function at those points and equate the results. This is a very common and straightforward question type in linear programming.

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3. In Linear Programming Problem (LPP), the objective function $Z = ax + by$ has the same maximum value at two corner points. The number of points at which Z_{max} occurs is

(A) 1

(B) 2

(C) 0

(D) Infinity

Correct Answer: (D) Infinity

Solution:

Step 1: Understanding the Concept:

This question tests a theoretical property of Linear Programming Problems (LPP). The feasible region in an LPP is a convex polygon. The objective function represents a family of parallel lines.

Step 2: Key Formula or Approach:

Recall the Multiple Optimal Solutions theorem. If an objective function reaches its maximum (or minimum) value at two distinct corner points of the feasible region, then every point on the line segment joining these two corner points will also give the same maximum (or minimum) value.

Step 3: Detailed Explanation:

Let the objective function be $Z = ax + by$. Suppose the maximum value Z_{max} occurs at two corner points, say $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. This means $Z(P_1) = ax_1 + by_1 = Z_{max}$ and

$Z(P_2) = ax_2 + by_2 = Z_{max}$. The line representing the objective function $ax + by = Z_{max}$ passes through both P_1 and P_2. Since the feasible region is convex, the entire line segment connecting P_1 and P_2 lies on the boundary of the feasible region. Any point $P(x, y)$ on this line segment can be represented as a convex combination of P_1 and P_2:

$$P = tP_1 + (1-t)P_2 \text{ for } 0 \leq t \leq 1$$

Let's find the value of Z at any such point P:

$$Z(P) = Z(tP_1 + (1-t)P_2)$$

Since Z is a linear function, we can distribute it:

$$Z(P) = t \cdot Z(P_1) + (1-t) \cdot Z(P_2)$$

Substitute $Z(P_1) = Z_{max}$ and $Z(P_2) = Z_{max}$:

$$Z(P) = t \cdot Z_{max} + (1-t) \cdot Z_{max}$$

$$Z(P) = Z_{max}(t + 1 - t) = Z_{max}(1) = Z_{max}$$

This shows that every single point on the line segment joining the two corner points gives the same maximum value. Since a line segment contains an infinite number of points, the maximum value occurs at infinitely many points.

Step 4: Final Answer:

The number of points at which Z_{max} occurs is infinity.

Quick Tip: This is a standard property to memorize: Optimal value at one point → Unique solution. Optimal value at ≥ 2 adjacent points → Infinitely many solutions (the entire edge connecting them).

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4. Probability of obtaining an even prime number on each die when a pair of dice is rolled is

(A) 0

(B) $\frac{1}{6}$

(C) $\frac{1}{12}$

(D) $\frac{1}{36}$

Correct Answer: (D) $\frac{1}{36}$

Solution:

Step 1: Understanding the Concept:

This problem asks for the probability of a specific outcome when rolling two dice. The rolls of two dice are independent events, meaning the outcome of one die does not affect the outcome of the other.

Step 2: Key Formula or Approach:

1. Identify the set of "even prime numbers". 2. Find the probability of getting an even prime number on a single roll of a die. 3. Since the two rolls are independent, use the multiplication rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$.

Step 3: Detailed Explanation:

When a standard fair six-sided die is rolled, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$. Let's analyze the properties of these numbers: - Prime numbers are numbers greater than 1 that have only two divisors: 1 and themselves. The prime numbers in our sample space are $\{2, 3, 5\}$. - Even numbers are divisible by 2. The even numbers in our sample space are $\{2, 4, 6\}$. The only number that is both even AND prime is 2. So, "obtaining an even prime number" is the same as "rolling a 2". Let E_1 be the event of rolling a 2 on the first die. Number of favorable outcomes = 1 (just the number 2). Total number of possible outcomes = 6.

$$P(E_1) = \frac{1}{6}$$

Let E_2 be the event of rolling a 2 on the second die. Similarly, $P(E_2) = \frac{1}{6}$. We want the probability of getting an even prime number on each die, which means E_1 occurs AND E_2 occurs. Because the dice rolls are independent:

$$P(E_1 \text{ and } E_2) = P(E_1) \times P(E_2)$$

$$P(\text{even prime on both}) = \left(\frac{1}{6}\right) \times \left(\frac{1}{6}\right) = \frac{1}{36}$$

Step 4: Final Answer:

The required probability is $\frac{1}{36}$.

Quick Tip: "Even prime number" is a classic trick phrase in probability questions. Always remember that 2 is the only even prime number. 1 is neither prime nor composite.

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5. The probability that a man and his wife live after 20 years are $\frac{1}{4}$ and $\frac{1}{3}$ respectively. The probability that neither the man nor his wife live after 20 years is

(A) $\frac{3}{4}$
(B) $\frac{5}{12}$
(C) $\frac{7}{12}$
(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

The problem involves the probabilities of survival for two individuals. In such problems, it is standard to assume that their lifespans are independent events. We need to find the probability of the intersection of their respective complement events (neither lives).

Step 2: Key Formula or Approach:

1. Let A be the event the man lives and B be the event the wife lives. Identify $P(A)$ and $P(B)$. 2. The event "neither lives" means "the man does not live AND the wife does not live". This is the intersection of the complement events: $P(A' \cap B')$. 3. Calculate the complement probabilities: $P(A') = 1 - P(A)$ and $P(B') = 1 - P(B)$. 4. Use the multiplication rule for independent events:

$P(A' \cap B') = P(A') \times P(B')$.

Step 3: Detailed Explanation:

Let M be the event that the man lives after 20 years. We are given $P(M) = \frac{1}{4}$. The probability that the man does not live after 20 years is $P(M')$.

$$P(M') = 1 - P(M) = 1 - \frac{1}{4} = \frac{3}{4}$$

Let W be the event that the wife lives after 20 years. We are given $P(W) = \frac{1}{3}$. The probability that the wife does not live after 20 years is $P(W')$.

$$P(W') = 1 - P(W) = 1 - \frac{1}{3} = \frac{2}{3}$$

We are asked to find the probability that neither lives after 20 years. This means the man dies AND the wife dies. Assuming their survival probabilities are independent:

$$P(\text{neither lives}) = P(M' \text{ and } W') = P(M' \cap W')$$

$$P(M' \cap W') = P(M') \times P(W')$$

Substitute the calculated values:

$$P(\text{neither lives}) = \left(\frac{3}{4}\right) \times \left(\frac{2}{3}\right)$$

$$P(\text{neither lives}) = \frac{3 \times 2}{4 \times 3}$$

$$P(\text{neither lives}) = \frac{6}{12}$$

Simplifying the fraction:

$$P(\text{neither lives}) = \frac{1}{2}$$

Step 4: Final Answer:

The probability that neither the man nor his wife live after 20 years is $\frac{1}{2}$.

Quick Tip: "Neither A nor B" translates to $(A' \cap B')$. For independent events, always calculate the individual complementary probabilities first $(1-p)$, and then multiply them together. Don't confuse it with $1-P(A \cap B)$.

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6. Integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} - xy = 1$ is

(1) $1-x^2$
(2) $\frac{1}{2}\log(1-x^2)$
(3) $\frac{x}{1-x^2}$
(4) $\sqrt{1-x^2}$

Correct Answer: (4) $\sqrt{1-x^2}$

Solution:

Step 1: Understanding the Concept:

To find the integrating factor of a linear differential equation, it must first be written in the standard form: $\frac{dy}{dx} + P(x)y = Q(x)$. The integrating factor (I.F.) is then given by the formula I.F. $= e^{\int P(x)\,dx}$.

Step 2: Key Formula or Approach:

1. Rearrange the given equation into standard linear form by dividing the entire equation by the coefficient of $\frac{dy}{dx}$.
2. Identify the function $P(x)$.
3. Compute the integral $\int P(x)\,dx$.
4. Calculate the integrating factor as $e^{\int P(x)\,dx}$.

Step 3: Detailed Explanation:

The given differential equation is:

$$(1-x^2)\frac{dy}{dx} - xy = 1$$

Divide the entire equation by $(1-x^2)$ to bring it to standard form:

$$\frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{1}{1-x^2}$$

Comparing this with the standard linear form $\frac{dy}{dx} + P(x)y = Q(x)$, we identify:

$$P(x) = -\frac{x}{1-x^2}$$

Now, we calculate the integral of $P(x)$:

$$\int P(x)\,dx = \int -\frac{x}{1-x^2}\,dx$$

Let $1-x^2 = t$. Then, differentiating both sides with respect to x gives $-2x\,dx = dt$, which implies $-x\,dx = \frac{dt}{2}$. Substituting these into the integral:

$$\int -\frac{x}{1-x^2}\,dx = \int \frac{1}{t}\cdot\frac{dt}{2} = \frac{1}{2}\int \frac{1}{t}\,dt$$

$$= \frac{1}{2}\log|t| = \frac{1}{2}\log(1-x^2) = \log((1-x^2)^{1/2}) = \log(\sqrt{1-x^2})$$

Finally, the integrating factor is:

$$\text{I.F.} = e^{\int P(x)\,dx} = e^{\log(\sqrt{1-x^2})} = \sqrt{1-x^2}$$

Step 4: Final Answer:

The integrating factor is $\sqrt{1-x^2}$.

Quick Tip: Always remember to bring the differential equation to the standard form $\frac{dy}{dx} + P(x)y = Q(x)$ before identifying $P(x)$. Forgetting to divide by the coefficient of $\frac{dy}{dx}$ is a very common mistake.

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7. Recent studies suggest that 12% of the world population is left handed. Depending on parents hand usage, the chances of having left handed children are as follows:
A: Both parents are left handed, chances of having left handed children = 24%
B: Both parents are right handed, chances of having left handed children = 9%
C: Father left handed and mother right handed, chances of having left handed children = 17%
D: Father right handed and mother left handed, chances of having left handed children = 22%
Given $P(A) = P(B) = P(C) = P(D) = 1/4$ and L denotes child is left handed. What is the probability that $P(A|L)$?

(1) $\frac{17}{80}$

(2) $\frac{24}{75}$

(3) $\frac{1}{3}$

(4) $\frac{1}{2}$

Correct Answer: (3) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

This problem requires the application of Bayes' Theorem. We need to find the conditional probability of event A given that event L has occurred, i.e., $P(A|L)$. The "12" statistic is general background information and is superseded by the specific probabilities provided for this scenario.

Step 2: Key Formula or Approach:

Use Bayes' Theorem:

$$P(A|L) = \frac{P(L|A) \cdot P(A)}{P(L)}$$

where the total probability $P(L)$ is found using the Law of Total Probability:

$$P(L) = P(L|A)P(A) + P(L|B)P(B) + P(L|C)P(C) + P(L|D)P(D)$$

Step 3: Detailed Explanation:

From the problem statement, we have the following probabilities: Prior probabilities for the parent configurations:

$$P(A) = P(B) = P(C) = P(D) = \frac{1}{4} = 0.25$$

Conditional probabilities of having a left-handed child given the parent configuration:

$$P(L|A) = 24\% = 0.24$$

$$P(L|B) = 9\% = 0.09$$

$$P(L|C) = 17\% = 0.17$$

$$P(L|D) = 22\% = 0.22$$

First, calculate the total probability of a child being left-handed, $P(L)$, using the Law of Total Probability:

$$P(L) = P(L|A)P(A) + P(L|B)P(B) + P(L|C)P(C) + P(L|D)P(D)$$

$$P(L) = \left(0.24 \times \frac{1}{4}\right) + \left(0.09 \times \frac{1}{4}\right) + \left(0.17 \times \frac{1}{4}\right) + \left(0.22 \times \frac{1}{4}\right)$$

$$P(L) = \frac{1}{4}(0.24 + 0.09 + 0.17 + 0.22)$$

$$P(L) = \frac{1}{4}(0.72) = 0.18$$

Now, apply Bayes' Theorem to find $P(A|L)$:

$$P(A|L) = \frac{P(L|A) \cdot P(A)}{P(L)}$$

$$P(A|L) = \frac{0.24 \times \frac{1}{4}}{0.18}$$

$$P(A|L) = \frac{0.06}{0.18} = \frac{6}{18} = \frac{1}{3}$$

Step 4: Final Answer:

The probability $P(A|L)$ is $\frac{1}{3}$.

Quick Tip: In Bayes' theorem problems, it's common to find distractor information (like the 12% overall population statistic here). Focus on the specific events defined and the specific probabilities given for them to construct your calculation.

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8. If α and β are acute angles such that $\alpha+\beta$ and $\alpha-\beta$ satisfy the equation $\tan^2\theta - 4\tan\theta + 1 = 0$, then α and β are respectively

(1) $45^\circ, 30^\circ$

(2) $75^\circ, 15^\circ$

(3) $30^\circ, 60^\circ$

(4) $60^\circ, 45^\circ$

Correct Answer: (1) $45^\circ, 30^\circ$

Solution:

Step 1: Form the quadratic equation

Let $x = \tan\theta$. Then,

$$x^2 - 4x + 1 = 0$$

Step 2: Solve the quadratic equation

$$x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

Step 3: Identify the angles

$$\tan(\alpha + \beta) = 2 + \sqrt{3} = \tan 75^\circ$$

$$\tan(\alpha - \beta) = 2 - \sqrt{3} = \tan 15^\circ$$

Step 4: Form equations

$$\alpha + \beta = 75^\circ \quad (1)$$

$$\alpha - \beta = 15^\circ \quad (2)$$

Step 5: Solve for α and β

Adding (1) and (2):

$$2\alpha = 90^\circ \Rightarrow \alpha = 45^\circ$$

Subtracting (2) from (1):

$$2\beta = 60^\circ \Rightarrow \beta = 30^\circ$$

Final Answer:

$$\alpha = 45^\circ, \quad \beta = 30^\circ$$

Quick Tip: Memorizing the trigonometric values for non-standard but common angles like 15° and 75° ($\tan 15^\circ = 2 - \sqrt{3}$, $\tan 75^\circ = 2 + \sqrt{3}$) can significantly speed up solving such problems.

9. $\sum_{r=1}^{n}(r\cdot r!) =$ ________

(1) 1

(2) n

(3) $(n+1)!-1$

(4) 0

Correct Answer: (3) $(n+1)!-1$

Solution:

Step 1: Write the general term

Let

$$t_r = r\cdot r!$$

Step 2: Transform the term

Rewrite r as $(r+1-1)$:

$$t_r = (r+1-1)\cdot r!$$

$$t_r = (r+1)r! - r!$$

Step 3: Use factorial property

$$(r+1)r! = (r+1)!$$

So,

$$t_r = (r+1)! - r!$$

Step 4: Form the series

$$\sum_{r=1}^{n} r\cdot r! = \sum_{r=1}^{n}\left[(r+1)! - r!\right]$$

Step 5: Expand the series

$$= (2!-1!) + (3!-2!) + (4!-3!) + \cdots + ((n+1)! - n!)$$

Step 6: Apply telescoping

All intermediate terms cancel:

$$= -1! + (n+1)!$$

Step 7: Simplify

$$= (n+1)! - 1$$

Final Answer:

$$\boxed{(n+1)! - 1}$$

Quick Tip: For series summation problems, especially those involving factorials or fractions, always look for a way to decompose the general term into a difference $f(r+1) - f(r)$. This leads to a telescoping series, which is easily evaluated. The identity $r \cdot r! = (r+1)! - r!$ is a classic trick.

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10. The solution of $3x - 5 < 2x - 4$ is

(1) $x < 1$

(2) $x > -1$

(3) $x < 9$

(4) $x > 9$

Correct Answer: (1) $x < 1$

Solution:

Step 1: Understanding the Concept:

This is a simple linear inequality in one variable. Solving it involves applying arithmetic operations to both sides of the inequality to isolate the variable x, similar to solving a linear equation.

Step 2: Key Formula or Approach:

Use the properties of inequalities. You can add or subtract the same value from both sides without changing the inequality sign. Group the terms containing x on one side and the constant terms on the other side.

Step 3: Detailed Explanation:

The given linear inequality is:

$$3x - 5 < 2x - 4$$

To collect all the x terms on the left side, subtract $2x$ from both sides:

$$3x - 2x - 5 < 2x - 2x - 4$$

$$x - 5 < -4$$

Now, to isolate x, add 5 to both sides of the inequality:

$$x - 5 + 5 < -4 + 5$$

$$x < 1$$

This means any real number less than 1 is a valid solution for the given inequality.

Step 4: Final Answer:

The solution is $x < 1$.

Quick Tip: Treat linear inequalities just like linear equations when adding or subtracting terms across the inequality symbol. Remember that the inequality symbol only flips direction if you multiply or divide both sides by a negative number.

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11. 10 distinct points are taken on a circle. Then using these points

Statement I : The number of triangles that can be formed is 100

Statement II : The number of chords that can be formed is 45

Which of the following is correct?

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true and Statement II is false

(4) Statement I is false and Statement II is true

Correct Answer: (4) Statement I is false and Statement II is true

Solution:

Step 1: Understanding the Concept:

This problem applies combinations to a geometric setting. A triangle is uniquely determined by selecting any 3 non-collinear points. A chord is uniquely determined by selecting any 2 distinct points. Since all given points lie on a circle, no three points can be collinear.

Step 2: Key Formula or Approach:

- The number of ways to choose k items from a set of n distinct items is given by combinations: ${}^nC_k = \frac{n!}{k!(n-k)!}$. - To form a triangle, we need to choose 3 points out of the total n points. So, number of triangles $= {}^nC_3$. - To form a chord, we need to connect 2 points out of the total n points. So, number of chords $= {}^nC_2$.

Step 3: Detailed Explanation:

We are given $n = 10$ distinct points on a circle. Let's evaluate Statement I: A triangle requires 3 vertices. Since no three points on a circle are collinear, any combination of 3 points will form a triangle. Number of triangles $= {}^{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10\times9\times8}{3\times2\times1} = 10 \times 3 \times 4 = 120$. Statement I claims the number is 100. Since $120 \neq 100$, Statement I is false. Let's evaluate Statement II: A chord is formed by joining any 2 distinct points on the circle. Number of chords $= {}^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10\times9}{2\times1} = 5 \times 9 = 45$. Statement II claims the number is 45. This calculation is correct, so Statement II is true.

Step 4: Final Answer:

Statement I is false and Statement II is true. This corresponds to option (4).

Quick Tip: The phrase "points on a circle" is a key indicator that the points are in a "general position," ensuring that no three points are collinear. This simplifies geometric combinatorics, allowing you to use simple combinations without worrying about subtracting degenerate cases.

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12. How many ways can you arrange all the letters and numbers in "KCET 2025" which start with K and end with 5?

(1) 720

(2) 360

(3) 120

(4) 180

Correct Answer: (2) 360

Solution:

Step 1: Identify total characters

The characters in "KCET 2025" (ignoring space) are:

$$\{K, C, E, T, 2, 0, 2, 5\}$$

Total characters = 8

Step 2: Apply given conditions

First position is fixed as K and last position is fixed as 5:

$$[K]\,\_\,\_\,\_\,\_\,\_\,\_\,[5]$$

Remaining positions = 6

Step 3: Remaining characters

$$\{C, E, T, 2, 0, 2\}$$

Here, digit 2 is repeated twice.

Step 4: Use permutation formula

Number of distinct arrangements:

$$\frac{6!}{2!}$$

Step 5: Calculate

$$= \frac{720}{2} = 360$$

Final Answer:

$$\boxed{360}$$

Quick Tip: Always carefully scan the string for repeated characters. Forgetting to divide by the factorial of the count of identical items is the most common error in these types of permutation problems.

13. **The value of** $\lim_{x\to 2} \frac{x^3+3x^2-9x-2}{x^3-x^2-4x+4}$ **is** ______

(1) 3

(2) $\frac{15}{4}$

(3) $\frac{15}{2}$

(4) $\frac{15}{13}$

Correct Answer: (2) $\frac{15}{4}$

Solution:

Step 1: Direct substitution

Substitute $x = 2$:

$$\text{Numerator} = 2^3 + 3(2)^2 - 9(2) - 2 = 8 + 12 - 18 - 2 = 0$$

$$\text{Denominator} = 2^3 - (2)^2 - 4(2) + 4 = 8 - 4 - 8 + 4 = 0$$

Thus, the limit is of the form $\frac{0}{0}$ (indeterminate).

Step 2: Apply L'Hôpital's Rule

Differentiate numerator and denominator:

$$f'(x) = 3x^2 + 6x - 9, \quad g'(x) = 3x^2 - 2x - 4$$

Step 3: Evaluate the new limit

$$\lim_{x\to 2} \frac{3x^2 + 6x - 9}{3x^2 - 2x - 4}$$

Substitute $x = 2$:

$$= \frac{3(2)^2 + 6(2) - 9}{3(2)^2 - 2(2) - 4} = \frac{12 + 12 - 9}{12 - 4 - 4} = \frac{15}{4}$$

Final Answer:

$$\boxed{\frac{15}{4}}$$

Quick Tip: L'Hôpital's Rule is a powerful tool, but always remember to verify that the limit is in an indeterminate form ($\frac{0}{0}$ or $\frac{\infty}{\infty}$) before applying it. Applying it to a determinate form will yield an incorrect result.

---

14. If we insert two numbers between $\sqrt{2}$ and 4 so that the resulting sequence is in G.P., then the inserted numbers in the order are

(1) $4, \sqrt{2}$
(2) $2, 2\sqrt{2}$
(3) $\sqrt{8}, 2$
(4) $2\sqrt{2}, 4$

Correct Answer: (2) $2, 2\sqrt{2}$

Solution:

Step 1: Understanding the Concept:
Inserting n numbers between two given numbers a and b to form a Geometric Progression (G.P.) implies creating a sequence of $n+2$ terms where the first term is a, the last term is b, and the sequence has a constant common ratio. The inserted numbers are called geometric means.

Step 2: Key Formula or Approach:
Let the newly formed G.P. sequence be t_1, t_2, t_3, t_4. We are given $t_1 = a = \sqrt{2}$ and $t_4 = b = 4$. The general formula for the n-th term of a G.P. is $t_n = a \cdot r^{n-1}$. We can use the 4th term to solve for the common ratio r. Once r is found, the inserted numbers are $t_2 = a \cdot r$ and $t_3 = a \cdot r^2$.

Step 3: Detailed Explanation:
Let the two numbers inserted be x and y. The resulting sequence is: $\sqrt{2}, x, y, 4$. This sequence is a Geometric Progression. The first term is $a = \sqrt{2}$. Since there are 4 terms in total, the fourth term is $t_4 = 4$. Let r be the common ratio of this G.P. Using the formula for the n-th term, $t_n = a \cdot r^{n-1}$, we have:

$$t_4 = a \cdot r^3$$

Substitute the known values into the equation:

$$4 = \sqrt{2} \cdot r^3$$

Now, solve for r^3:

$$r^3 = \frac{4}{\sqrt{2}}$$

To simplify, multiply the numerator and denominator by $\sqrt{2}$ to rationalize the denominator:

$$r^3 = \frac{4\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

We can express $2\sqrt{2}$ as a cube to easily find r:

$$2\sqrt{2} = (\sqrt{2})^2 \cdot (\sqrt{2})^1 = (\sqrt{2})^3$$

So, we have:

$$r^3 = (\sqrt{2})^3$$

Taking the real cube root of both sides gives the common ratio:

$$r = \sqrt{2}$$

Now we calculate the inserted numbers x and y, which are the second and third terms of the G.P.: First inserted number, $x = t_2 = a \cdot r = \sqrt{2} \cdot \sqrt{2} = 2$. Second inserted number, $y = t_3 = a \cdot r^2 = \sqrt{2} \cdot (\sqrt{2})^2 = \sqrt{2} \cdot 2 = 2\sqrt{2}$. Therefore, the inserted numbers in order are 2 and $2\sqrt{2}$.

Step 4: Final Answer:

The inserted numbers are $2, 2\sqrt{2}$.

Quick Tip: A useful trick for simplifying expressions with radicals is to express integers as powers of roots. For example, recognizing that $4 = (\sqrt{2})^4$ allows you to immediately simplify $\frac{4}{\sqrt{2}}$ to $\frac{(\sqrt{2})^4}{\sqrt{2}} = (\sqrt{2})^3$, making the cube root calculation trivial.

---

15. Match List-I with List-II

List-I

a) A matrix which is not a square matrix

b) A square matrix $A' = A$

c) The diagonal elements of a diagonal matrix are same

d) A matrix which is both symmetric and skew symmetric

List-II

i) Symmetric matrix

ii) Null matrix

iii) Rectangular matrix

iv) Scalar matrix

Codes:

(1) a - iii, b - i, c - iv, d - ii

(2) a - iii, b - ii, c - iv, d - i

(3) a - i, b - ii, c - iv, d - iii

(4) a - iii, b - iv, c - i, d - ii

Correct Answer: (1) a - iii, b - i, c - iv, d - ii

Solution:

Step 1: Identify definitions

- Rectangular matrix: number of rows $\neq$ number of columns
- Symmetric matrix: $A' = A$
- Scalar matrix: diagonal matrix with equal diagonal elements
- Null matrix: all elements are zero

Step 2: Match each item

(a) A matrix which is not a square matrix

$$\Rightarrow \text{Rectangular matrix} \quad (iii)$$

(b) A square matrix $A' = A$

$$\Rightarrow \text{Symmetric matrix} \quad (i)$$

(c) Diagonal elements of a diagonal matrix are same

$$\Rightarrow \text{Scalar matrix} \quad (iv)$$

(d) A matrix both symmetric and skew symmetric

$$A' = A \quad \text{and} \quad A' = -A$$

$$\Rightarrow A = -A \Rightarrow 2A = 0 \Rightarrow A = 0$$

$$\Rightarrow \text{Null matrix} \quad (ii)$$

Step 3: Final matching

$$a \to iii, \quad b \to i, \quad c \to iv, \quad d \to ii$$

Final Answer:

$$\boxed{\text{Option (1)}}$$

Quick Tip: The fact that the Null (zero) square matrix is the only matrix that is both symmetric and skew-symmetric is a classic true/false or matching question concept. It is derived directly from setting $A = -A$.

---

16. Consider the following statements:

Statement I : If A is a non-singular matrix, then A^{-1} exists.

Statement II : If A and B are symmetric matrices of same order, then $(AB - BA)$ is a skew symmetric matrix.

Choose the correct option.

(1) Statement I is true and Statement II is false

(2) Statement I is false and Statement II is false

(3) Statement I is true and Statement II is true

(4) Statement I is false and Statement II is true

Correct Answer: (3) Statement I is true and Statement II is true

Solution:

Step 1: Check Statement I

A matrix A is non-singular if:

$$|A| \neq 0$$

Inverse of a matrix is:

$$A^{-1} = \frac{1}{|A|}\,\text{adj}(A)$$

Since $|A| \neq 0$, A^{-1} exists.

$$\Rightarrow \text{Statement I is true}$$

Step 2: Check Statement II

Given A and B are symmetric:

$$A' = A, \quad B' = B$$

Step 3: Take transpose of $(AB - BA)$

$$(AB - BA)' = (AB)' - (BA)'$$

$$= B'A' - A'B'$$

$$= BA - AB$$

Step 4: Compare

$$(AB - BA)' = -(AB - BA)$$

Step 5: Conclusion

Since $M' = -M$, matrix is skew-symmetric.

$$\Rightarrow \text{Statement II is true}$$

Final Answer:

$$\boxed{\text{Option (3)}}$$

Quick Tip: The matrix expression $(AB - BA)$ is a very important construct in linear algebra called the commutator. The property proved here (that the commutator of two symmetric matrices is skew-symmetric) is a common standard result worth remembering.

---

17. A row matrix has only

(1) One element

(2) One row with one or more columns

(3) One column with one or more rows

(4) One row and one column

Correct Answer: (2) One row with one or more columns

Solution:

Step 1: Definition of row matrix

A row matrix is a matrix that has exactly one row.

Step 2: General form

The order of a row matrix is:

$$1 \times n \quad \text{where } n \geq 1$$

Step 3: Analyze options

- Option (1): Only one element $\Rightarrow$ too restrictive (1×1 only)
- Option (2): One row with one or more columns $\Rightarrow$ correct definition
- Option (3): One column with multiple rows $\Rightarrow$ column matrix
- Option (4): One row and one column $\Rightarrow$ only 1×1

Step 4: Conclusion

A row matrix has one row and one or more columns.

Final Answer:

$$\boxed{\text{Option (2)}}$$

Quick Tip: The name of the matrix classification gives away the defining constraint. A "row" matrix is constrained to exactly 1 row. A "column" matrix is constrained to exactly 1 column. The other dimension is variable.

---

18. Let X be a matrix of order $2 \times n$ and Z be a matrix of order $2 \times p$. If $n = p$, then the order of the matrix $7X - 5Z$ is:

(1) $2 \times n$
(2) $n \times 3$
(3) $p \times 2$
(4) $p \times n$

Correct Answer: (1) $2 \times n$

Solution:

Step 1: Identify given orders

$$X \text{ is of order } 2 \times n, \quad Z \text{ is of order } 2 \times p$$

Given $n = p$, so:

$$Z \text{ is also of order } 2 \times n$$

Step 2: Scalar multiplication
Multiplying a matrix by a scalar does not change its order:

$$7X \text{ has order } 2 \times n, \quad 5Z \text{ has order } 2 \times n$$

Step 3: Subtraction rule
Two matrices can be subtracted only if their orders are the same, and the result has the same order.
Step 4: Resulting order

$$7X - 5Z \text{ has order } 2 \times n$$

Final Answer:

$$\boxed{2 \times n}$$

Quick Tip: Remember that simple linear combinations of matrices (like $aA + bB$) never change the dimensions. The result is just another matrix taking up the exact same "shape" as the inputs.

---

19. Which of the following is correct?

(1) Determinant is a square matrix.
(2) Determinant is a number associated to a matrix.
(3) Determinant is a unique number associated to a square matrix.
(4) Determinant is not defined for a square matrix.

Correct Answer: (3) Determinant is a unique number associated to a square matrix.

Solution:

Step 1: Understanding the Concept:

This question tests the formal mathematical definition of a determinant, distinguishing it from the matrix itself and specifying the conditions under which it exists.

Step 2: Key Formula or Approach:

Recall the definition: A determinant is a scalar value (a real or complex number) that can be computed from the elements of a square matrix and encodes certain properties of that matrix.

Step 3: Detailed Explanation:

Let's evaluate the validity of each option:

- (1) "Determinant is a square matrix." - This statement is incorrect. A matrix is an array or table of numbers. A determinant is a single scalar value calculated from that array. They are fundamentally different types of mathematical entities.

- (2) "Determinant is a number associated to a matrix." - While a determinant is indeed a number, this statement is too broad and technically incorrect because it implies a determinant can be associated with any matrix. Determinants are not defined for rectangular matrices (where rows $\neq$ columns).

- (3) "Determinant is a unique number associated to a square matrix." - This is the precise and correct definition. The determinant operation maps every square matrix to exactly one scalar

value. The requirement that the matrix be square is crucial.
- (4) "Determinant is not defined for a square matrix." - This is entirely incorrect. Square matrices are the only type of matrices for which determinants are defined.

Step 4: Final Answer:

The correct statement is option (3).

Quick Tip: When answering definition-based questions, look for the option that includes all necessary restrictive conditions. Option (2) is a common trap because it sounds plausible but misses the essential "square" requirement.

---

20. If A and B are invertible matrices of same order, then which of the following is not correct?

(1) $A \cdot (\text{adj } A) = (\text{adj } A) \cdot A = |A|I$

(2) $A \cdot \text{adj } A = \text{adj } A \cdot A = |A|$

(3) $(AB)^{-1} = B^{-1}A^{-1}$

(4) $|A| \neq 0, |B| \neq 0$

Correct Answer: (2) $A \cdot \text{adj } A = \text{adj } A \cdot A = |A|$

Solution:

Step 1: Use standard matrix identities

- $A \cdot (\text{adj } A) = |A|I$
- $(AB)^{-1} = B^{-1}A^{-1}$
- A matrix is invertible $\iff |A| \neq 0$

Step 2: Check each option

(1)

$$A(\text{adj } A) = (\text{adj } A)A = |A|I$$

Correct identity.

(2)

$$A(\text{adj } A) = |A|$$

Left side is a matrix, right side is a scalar $\Rightarrow$ dimension mismatch.

Hence incorrect.

(3)

$$(AB)^{-1} = B^{-1}A^{-1}$$

Correct property.

(4)

$$|A| \neq 0, \ |B| \neq 0$$

True for invertible matrices.

Step 3: Conclusion

Only option (2) is incorrect.

Final Answer:

$$\boxed{\text{Option (2)}}$$

Quick Tip: Always perform a "type check" on mathematical equations. The product of two matrices is a matrix. A determinant is a scalar. A matrix cannot equal a scalar. Spotting dimension/type mismatches is a quick way to identify false statements.

---

21. If A and B are invertible square matrices of order n, then which of the following is not correct?

(1) $\det(AB) = \det(A) \cdot \det(B)$

(2) $\det(kA) = k^n \det(A)$

(3) $\det(A+B) = \det(A) + \det(B)$

(4) $\det(A^{-1}) = \frac{1}{\det(A)}$

Correct Answer: (3) $\det(A+B) = \det(A) + \det(B)$

Solution:

Step 1: Recall determinant properties

- $\det(AB) = \det(A)\det(B)$
- $\det(kA) = k^n \det(A)$
- $\det(A^{-1}) = \dfrac{1}{\det(A)}$

Step 2: Check each option

(1)

$$\det(AB) = \det(A)\det(B)$$

Correct property.

(2)

$$\det(kA) = k^n \det(A)$$

Correct for $n \times n$ matrix.

(3)

$$\det(A + B) \neq \det(A) + \det(B)$$

This is not a valid general property.

(4)

$$\det(A^{-1}) = \frac{1}{\det(A)}$$

Correct property.

Step 3: Conclusion

Only option (3) is incorrect.

Final Answer:

$$\boxed{\text{Option (3)}}$$

Quick Tip: A common pitfall is assuming that operations like determinant or trace are fully linear. Remember: Determinants distribute over multiplication but NOT over addition. (Conversely, the Trace operator distributes over addition but not multiplication).

---

22. The area of the triangle with vertices $(3,8),(-4,2)$ and $(5,1)$ is $\frac{P}{4}$, then the value of P is

(1) $\frac{61}{2}$

(2) $\frac{2}{61}$

(3) 122

(4) $\frac{1}{122}$

Correct Answer: (3) 122

Solution:

Step 1: Understanding the Concept:

We are given the coordinates of the three vertices of a triangle and its area in terms of an unknown variable P. We need to compute the actual numerical area using a standard formula and equate it to the given expression to solve for P.

Step 2: Key Formula or Approach:

The area of a triangle defined by vertices $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) can be calculated using the coordinate geometry formula: Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$ Calculate this area, set the result equal to $\frac{P}{4}$, and solve the resulting algebraic equation.

Step 3: Detailed Explanation:

The given coordinates for the vertices are: Vertex 1: $(x_1, y_1) = (3,8)$ Vertex 2: $(x_2, y_2) = (-4,2)$ Vertex 3: $(x_3, y_3) = (5,1)$ Let's substitute these values into the area formula:

$$\text{Area} = \frac{1}{2}|3(2-1) + (-4)(1-8) + 5(8-2)|$$

Compute the differences inside the parentheses:

$$\text{Area} = \frac{1}{2}|3(1) - 4(-7) + 5(6)|$$

Perform the multiplications:

$$\text{Area} = \frac{1}{2}|3 + 28 + 30|$$

Sum the values inside the absolute value bars:

$$\text{Area} = \frac{1}{2}|61|$$

Since 61 is positive, the absolute value is just 61:

$$\text{Area} = \frac{61}{2}$$

The problem states that the area is equivalent to the expression $\frac{P}{4}$. We can set up an equation:

$$\frac{61}{2} = \frac{P}{4}$$

To isolate and solve for P, multiply both sides of the equation by 4:

$$P = \frac{61}{2} \times 4$$

$$P = 61 \times \left(\frac{4}{2}\right)$$

$$P = 61 \times 2$$

$$P = 122$$

Step 4: Final Answer:

The value of P is 122.

Quick Tip: Be meticulous with signs when substituting negative coordinates into the area formula. The absolute value function $|...|$ is crucial because physical area cannot be negative, regardless of the order the vertices are chosen.

---

23. The system of equations $x + 2y = 3$ and $2x + 3y = 3$ has

(1) No solution

(2) Unique solution

(3) Infinite solutions

(4) Only two solutions

Correct Answer: (2) Unique solution

Solution:

Step 1: Understanding the Concept:

The nature of solutions for a system of two linear equations in two variables ($a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$) can be determined by comparing the ratios of their coefficients.

Step 2: Key Formula or Approach:

Calculate the ratios $\frac{a_1}{a_2}$ and $\frac{b_1}{b_2}$. - If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the system has a unique solution (intersecting lines). - If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the system has infinite solutions (coincident lines). - If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the system has no solution (parallel lines).

Step 3: Detailed Explanation:

The given system of equations is: 1) $1x + 2y = 3$ 2) $2x + 3y = 3$

Here, the coefficients are: $a_1 = 1$, $b_1 = 2$, $c_1 = 3$ $a_2 = 2$, $b_2 = 3$, $c_2 = 3$

Now, find the ratios: $\frac{a_1}{a_2} = \frac{1}{2}$ $\frac{b_1}{b_2} = \frac{2}{3}$

Comparing the ratios: $\frac{1}{2} \neq \frac{2}{3}$ Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the lines are not parallel and not coincident; they intersect at exactly one point. Therefore, the system has a unique solution.

Step 4: Final Answer:

The system has a unique solution.

Quick Tip: Always check the ratio of the x and y coefficients first. If they are different, you immediately know it's a unique solution without even looking at the constant terms (c_1, c_2).

---

24. If $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$ and $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\alpha + \beta$ is equal to

(1) 2

(2) -1

(3) 0

(4) 1

Correct Answer: (4) 1

Solution:

Step 1: Understanding the Concept:

The given condition $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ signifies that the diagonals of a parallelogram formed by vectors $\vec{a}$ and $\vec{b}$ are equal in length. Geometrically, this means the parallelogram is a rectangle, which implies that vectors $\vec{a}$ and $\vec{b}$ must be perpendicular.

Step 2: Key Formula or Approach:

1. Recognize that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}| \iff \vec{a}\cdot\vec{b} = 0$. Alternatively, square both sides: $|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2 \implies |\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a}\cdot\vec{b}) = |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}\cdot\vec{b}) \implies 4(\vec{a}\cdot\vec{b}) = 0 \implies \vec{a}\cdot\vec{b} = 0$. 2. Calculate the dot product of the given vectors $\vec{a}$ and $\vec{b}$ and set it to zero. 3. Solve the resulting equation to find the value of $\alpha+\beta$.

Step 3: Detailed Explanation:

Given vectors: $\vec{a} = 2\hat{i} + 2\hat{j} - 1\hat{k}$ $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$

As established, the condition $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ implies that the vectors are orthogonal: $\vec{a}\cdot\vec{b} = 0$

Now, compute the dot product: $(2)(\alpha) + (2)(\beta) + (-1)(2) = 0$ $2\alpha + 2\beta - 2 = 0$

Divide the entire equation by 2: $\alpha + \beta - 1 = 0$ $\alpha + \beta = 1$

The question asks for the value of $\alpha+\beta$, which we have directly found to be 1.

Step 4: Final Answer:

The value of $\alpha+\beta$ is 1.

Quick Tip: The condition $|\vec{u}+\vec{v}| = |\vec{u}-\vec{v}|$ is a classic vector identity that always means $\vec{u}$ is perpendicular to $\vec{v}$ ($\vec{u}\cdot\vec{v} = 0$). Memorizing this saves time from doing the algebraic expansion every time.

---

25. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$ and $\vec{a}\times\vec{c} = \vec{b}$, $\vec{a}\cdot\vec{c} = 3$, then $\vec{c}$ is

(1) $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$

(2) $\frac{5}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

(3) $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

(4) $\frac{5}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$

Correct Answer: (3) $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

Solution:

Step 1: Understanding the Concept:

We are given two vector equations involving an unknown vector $\vec{c}$: a cross product equation and a dot product equation. We can find $\vec{c}$ by using the vector triple product property.

Step 2: Key Formula or Approach:

Take the cross product of vector $\vec{a}$ with both sides of the equation $\vec{a}\times\vec{c}=\vec{b}$: $\vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$
Expand the left side using the vector triple product formula: $\vec{A}\times(\vec{B}\times\vec{C})=(\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$.
So, $(\vec{a}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{a})\vec{c}=\vec{a}\times\vec{b}$. Substitute the known values ($\vec{a}\cdot\vec{c}=3$, calculate $\vec{a}\cdot\vec{a}$ and $\vec{a}\times\vec{b}$) to solve for $\vec{c}$.

Step 3: Detailed Explanation:

Given: $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ $\vec{b}=0\hat{i}+\hat{j}-\hat{k}$ $\vec{a}\times\vec{c}=\vec{b}$ $\vec{a}\cdot\vec{c}=3$
From the vector triple product expansion: $(\vec{a}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{a})\vec{c}=\vec{a}\times\vec{b}$
Let's calculate the required components: 1. $\vec{a}\cdot\vec{a}=|\vec{a}|^2=(1)^2+(1)^2+(1)^2=3$ 2. Calculate $\vec{a}\times\vec{b}$ using the determinant method:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 0 & 1 & -1\end{vmatrix}$$

$$=\hat{i}(-1-1)-\hat{j}(-1-0)+\hat{k}(1-0)$$

$$=-2\hat{i}+\hat{j}+\hat{k}$$

Now substitute these into our expanded equation: $(3)(\hat{i}+\hat{j}+\hat{k})-(3)\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$ $3\hat{i}+3\hat{j}+3\hat{k}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$
Rearrange to solve for $3\vec{c}$: $3\vec{c}=(3\hat{i}+3\hat{j}+3\hat{k})-(-2\hat{i}+\hat{j}+\hat{k})$ $3\vec{c}=(3-(-2))\hat{i}+(3-1)\hat{j}+(3-1)\hat{k}$ $3\vec{c}=5\hat{i}+2\hat{j}+2\hat{k}$
Divide by 3: $\vec{c}=\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$

Step 4: Final Answer:

The vector $\vec{c}$ is $\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$.

Quick Tip: When given $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=$ scalar, crossing $\vec{a}$ with the first equation is the standard and fastest algorithm to isolate $\vec{c}$.

26. The value of λ for which the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is

(1) $\frac{5}{2}$

(2) $\frac{-5}{2}$

(3) $\frac{2}{5}$

(4) $\frac{-2}{5}$

Correct Answer: (2) $\frac{-5}{2}$

Solution:

Step 1: Understanding the Concept:

Two non-zero vectors are defined as orthogonal (perpendicular to each other) if and only if their scalar (dot) product is exactly zero.

Step 2: Key Formula or Approach:

For vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, the orthogonality condition is: $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3 = 0$. Calculate the dot product of the given vectors and solve for the unknown parameter λ.

Step 3: Detailed Explanation:

Given vectors: $\vec{a} = 2\hat{i} + \lambda\hat{j} + 1\hat{k}$ $\vec{b} = 1\hat{i} + 2\hat{j} + 3\hat{k}$

Set their dot product to zero for orthogonality: $\vec{a} \cdot \vec{b} = 0$ $(2)(1) + (\lambda)(2) + (1)(3) = 0$ $2 + 2\lambda + 3 = 0$ $5 + 2\lambda = 0$ $2\lambda = -5$ $\lambda = \frac{-5}{2}$

Step 4: Final Answer:

The value of λ is $\frac{-5}{2}$.

Quick Tip: "Orthogonal" is just a formal term for "perpendicular." Whenever you see it in a vector problem, immediately write down $\vec{u} \cdot \vec{v} = 0$.

---

27. The angle between the lines whose direction ratios are a, b, c and $b - c, c - a, a - b$ is

(1) 90°

(2) 45°

(3) 30°

(4) 0°

Correct Answer: (1) 90°

Solution:

Step 1: Understanding the Concept:

The angle θ between two lines with direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2) can be found using the dot product of their direction vectors. If the dot product is zero, the lines are perpendicular, meaning the angle between them is 90°.

Step 2: Key Formula or Approach:

Let the direction vectors of the two lines be $\vec{u}$ and $\vec{v}$. $\vec{u} = a\hat{i} + b\hat{j} + c\hat{k}$ $\vec{v} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$ Calculate the dot product $\vec{u}\cdot\vec{v}$. $\vec{u}\cdot\vec{v} = a(b-c) + b(c-a) + c(a-b)$

Step 3: Detailed Explanation:

Let's expand the dot product expression:

$$\vec{u}\cdot\vec{v} = (ab-ac) + (bc-ba) + (ca-cb)$$

Notice that ab cancels with $-ba$, $-ac$ cancels with ca, and bc cancels with $-cb$.

$$\vec{u}\cdot\vec{v} = ab - ac + bc - ab + ac - bc = 0$$

Since the dot product of the direction vectors is zero, the vectors are orthogonal. Therefore, the lines are perpendicular to each other. This implies that the angle θ between the lines is 90°.

Step 4: Final Answer:

The angle between the lines is 90°.

Quick Tip: Whenever you see a cyclic pattern in coordinates like $(b-c, c-a, a-b)$, taking the dot product with (a, b, c) will typically result in zero due to cyclic cancellation.

---

28. The measure of the angle between the lines $x = k-1, y = 2k+1, z = 2k+3, k \in \mathbb{R}$ and $\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-1}{2}$ is

(1) $\cos^{-1}\left(\frac{2}{3}\right)$

(2) $\cos^{-1}\left(\frac{8}{9}\right)$

(3) $\cos^{-1}\left(\frac{5}{12}\right)$
(4) $\sin^{-1}\left(\frac{8}{9}\right)$

Correct Answer: (2) $\cos^{-1}\left(\frac{8}{9}\right)$

Solution:

Step 1: Understanding the Concept:

To find the angle between two lines in 3D space, we need to extract their direction ratios and apply the angle formula based on the dot product of their direction vectors.

Step 2: Key Formula or Approach:

For lines with direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2), the angle θ between them is given by:

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

First, convert the parametric equations of the first line into symmetric form to easily identify its direction ratios.

Step 3: Detailed Explanation:

Let's analyze the first line, given in parametric form: $x = k - 1 \implies x + 1 = k \implies \frac{x+1}{1} = k$ $y = 2k + 1 \implies y - 1 = 2k \implies \frac{y-1}{2} = k$ $z = 2k + 3 \implies z - 3 = 2k \implies \frac{z-3}{2} = k$ Combining these, the symmetric equation for the first line is:

$$\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-3}{2}$$

The direction ratios of the first line are $\vec{d}_1 = (1, 2, 2)$.

The second line is already in symmetric form:

$$\frac{x+1}{2} = \frac{y-2}{1} = \frac{z-1}{2}$$

The direction ratios of the second line are $\vec{d}_2 = (2, 1, 2)$.

Now, apply the cosine formula for the angle θ:

$$\cos\theta = \frac{|(1)(2) + (2)(1) + (2)(2)|}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{2^2 + 1^2 + 2^2}}$$

$$\cos\theta = \frac{|2 + 2 + 4|}{\sqrt{1 + 4 + 4}\sqrt{4 + 1 + 4}}$$

$$\cos\theta = \frac{8}{\sqrt{9}\sqrt{9}}$$

$$\cos\theta = \frac{8}{3\times 3} = \frac{8}{9}$$

Therefore, the angle is $\theta = \cos^{-1}\left(\frac{8}{9}\right)$.

Step 4: Final Answer:

The measure of the angle is $\cos^{-1}\left(\frac{8}{9}\right)$.

Quick Tip: Always convert parametric line equations into symmetric (Cartesian) form to reliably read off the direction ratios from the denominators. Ensure the coefficients of x, y, z in the numerators are all 1 before reading the denominators.

---

29. The line L_1 joining the two points $(-1,2)$ and $(3,6)$ divides the line L_2 which passes through $(3,-1)$ in the ratio $1:3$ internally, then the equation of L_2 is

(A) $4x-3y-9=0$

(B) $4x-3y+9=0$

(C) $4x+3y-9=0$

(D) $4x+3y+9=0$

Correct Answer: (C) $4x+3y-9=0$

Solution:

Step 1: Understanding the Concept:

The phrasing of the question is slightly ambiguous ("divides the line L_2"). A line cannot divide another line in a specific ratio unless it's a line segment. The standard and mathematically sound interpretation of this problem is that the line L_2 divides the **line segment** L_1 (joining the points $(-1,2)$ and $(3,6)$) in the ratio $1:3$ internally.

Step 2: Key Formula or Approach:

1. Find the point of intersection $P(x,y)$ that divides the line segment joining $A(-1,2)$ and $B(3,6)$ in the ratio $m:n=1:3$ using the section formula:

$$P(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

2. Use the two-point form to find the equation of the line L_2 that passes through this point $P(x, y)$ and the given point $Q(3, -1)$.

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 3: Detailed Explanation:

Let the points be $A(-1, 2)$ and $B(3, 6)$. Let the intersection point be P. This point divides the segment AB in the ratio $1 : 3$. Using the section formula:

$$x\text{-coordinate of } P = \frac{1 \cdot (3) + 3 \cdot (-1)}{1 + 3} = \frac{3 - 3}{4} = 0$$

$$y\text{-coordinate of } P = \frac{1 \cdot (6) + 3 \cdot (2)}{1 + 3} = \frac{6 + 6}{4} = \frac{12}{4} = 3$$

So, the point of intersection is $P(0, 3)$.

We are given that the line L_2 passes through the point $(3, -1)$ and it must also pass through the intersection point $P(0, 3)$ we just found. Now, we find the equation of the line passing through $(0, 3)$ and $(3, -1)$. Using the two-point form:

$$\frac{y - 3}{x - 0} = \frac{-1 - 3}{3 - 0}$$

$$\frac{y - 3}{x} = \frac{-4}{3}$$

Cross-multiplying to simplify:

$$3(y - 3) = -4x$$

$$3y - 9 = -4x$$

Rearranging into standard form ($Ax + By + C = 0$):

$$4x + 3y - 9 = 0$$

Step 4: Final Answer:

The equation of the line L_2 is $4x + 3y - 9 = 0$, which matches option (C).

Quick Tip: In competitive exams, you may sometimes encounter questions with slightly imprecise phrasing. Always look for the most standard mathematical interpretation that makes the problem solvable. Here, recognizing that "a line divides a line" is a typo for "a line divides a line segment" is crucial to proceeding.

---

30. In the figure

Statement I : When $\alpha > \beta \geq 0$, the section is hyperbola

Statement II : When $\beta = 90^\circ$, the section is ellipse

Which of the following is correct?

(1) Statement I is true, Statement II is false

(2) Statement I is false, Statement II is true

(3) Both the Statements are true

(4) Both the Statements are false

Correct Answer: (1) Statement I is true, Statement II is false

Solution:

Step 1: Understanding the Concept:

The problem relates to the geometric definition of conic sections obtained by intersecting a right circular double cone with a plane. The type of conic section depends on the relationship between the semi-vertical angle of the cone (α) and the angle the intersecting plane makes with the vertical axis of the cone (β).

Step 2: Key Formula or Approach:

Recall the standard classification of conic sections based on the angles α and β: - If $\beta = 90^\circ$, the intersecting plane is perpendicular to the axis, and the section is a circle. - If $\alpha < \beta < 90^\circ$, the plane cuts entirely across one nappe of the cone, and the section is an ellipse. - If $\beta = \alpha$, the plane is parallel to a generator of the cone, and the section is a parabola. - If $0 \leq \beta < \alpha$, the plane intersects both nappes of the double cone, resulting in a two-part curve called a hyperbola.

Step 3: Detailed Explanation:

Let's evaluate each statement based on the rules above. Statement I: "When $\alpha > \beta \geq 0$, the section is hyperbola" This condition can be rewritten as $0 \leq \beta < \alpha$. According to our classification, when the plane's angle with the axis is less than the semi-vertical angle, it cuts both halves of the cone, forming a hyperbola. Therefore, Statement I is true.

Statement II: "When $\beta = 90^\circ$, the section is ellipse" When $\beta = 90^\circ$, the plane is exactly horizontal (perpendicular to the vertical axis). The resulting intersection is perfectly symmetric around the axis, forming a circle. While a circle is technically a special, degenerate case of an ellipse (where eccentricity $e = 0$), in the context of standard conic section classification problems, "ellipse" refers to the general case where $\alpha < \beta < 90^\circ$ and the cross-section is elongated. A circle is treated as a distinct category. Thus, stating it is an ellipse when it is specifically a circle makes the statement false by convention in such multiple-choice questions.

Step 4: Final Answer:

Statement I is true and Statement II is false, which corresponds to option (1).

Quick Tip: A good way to visualize this: as the plane tilts more vertically (smaller β), it eventually cuts through both the top and bottom cones. This multi-part intersection is the hallmark of a hyperbola.

---

31. The three points $A(2,4,3)$, $B(4,a,9)$ and $C(10,-1,7)$ form a right-angled triangle with $\angle B = 90^\circ$, then the value of 'a' is

(1) 1 or 4
(2) -2 or 4
(3) 1 or -4
(4) -2 or -4

Correct Answer: (2) -2 or 4

Solution:

Step 1: Use right angle condition

Since $\angle B = 90^\circ$, vectors $\vec{BA}$ and $\vec{BC}$ are perpendicular:

$$\vec{BA} \cdot \vec{BC} = 0$$

Step 2: Find vectors

$$\vec{BA} = A - B = (2-4,\ 4-a,\ 3-9) = (-2,\ 4-a,\ -6)$$

$$\vec{BC} = C - B = (10-4,\ -2-a,\ 7-9) = (6,\ -2-a,\ -2)$$

Step 3: Apply dot product

$$(-2)(6) + (4-a)(-2-a) + (-6)(-2) = 0$$

$$-12 + (4-a)(-2-a) + 12 = 0$$

Step 4: Simplify

$$(4-a)(-2-a) = 0$$

$$-(a-4)(a+2) = 0$$

$$(a-4)(a+2) = 0$$

Step 5: Solve

$$a = 4 \quad \text{or} \quad a = -2$$

Final Answer:

$$\boxed{a = -2 \text{ or } 4}$$

Quick Tip: For right-angled triangle problems with coordinates, setting the dot product of the perpendicular side vectors to zero is always the fastest and most direct method. Avoid using the full distance formula unless necessary.

---

32. If $\lim_{x\to 3}\left(\frac{x^2-ax-3a}{x-3}\right) = 5$**, then** $a+b=$

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (3) 3

Solution:

Step 1: Understanding the Concept:

We are given a limit of a rational function that evaluates to a finite non-zero value (5) as x approaches 3. Since the denominator evaluates to zero ($3-3=0$) at $x=3$, the limit can only exist if the numerator also evaluates to zero at $x=3$. This creates an indeterminate form of type $\frac{0}{0}$.

Step 2: Key Formula or Approach:

1. Set the numerator expression to zero and substitute $x=3$: $f(3)=3^2-a(3)-3b=0$. 2. Simplify the resulting equation to find a relationship between a and b.

Step 3: Detailed Explanation:

Let the given limit be $L=\lim_{x\to 3}\frac{x^2-ax-3b}{x-3}=5$. When we substitute $x=3$ directly into the denominator, we get $3-3=0$. For the limit of the fraction to exist and be a finite number (like 5), the numerator must also approach zero as x approaches 3. This avoids a $\frac{\text{non-zero}}{0}$ situation, which would lead to an undefined limit ($\pm\infty$). Therefore, we must have:

$$\lim_{x\to 3}(x^2-ax-3b)=0$$

Substituting $x=3$ into the numerator:

$$(3)^2-a(3)-3b=0$$

$$9-3a-3b=0$$

Now, simplify this equation to find the value of $a+b$: Divide the entire equation by 3:

$$3-a-b=0$$

Rearranging the terms yields:

$$a+b=3$$

Step 4: Final Answer:

The value of $a+b$ is 3.

Quick Tip: When a limit of a fraction $\frac{f(x)}{g(x)}$ exists finitely at $x = c$ and $g(c) = 0$, it is a hard rule that $f(c)$ must also be 0. Using this initial condition is often enough to solve for unknown parameters without fully evaluating the limit.

---

33. If $f(x) = \begin{cases} x^2 - 1 & \textbf{if } x \geq 2 \\ x + 1 & \textbf{if } x < 2 \end{cases}$**, then** $\lim_{x\to 2^+} f(x) + \lim_{x\to 2^-} f(x) =$

(1) 7

(2) 5

(3) 6

(4) 9

Correct Answer: (3) 6

Solution:

Step 1: Understanding the Concept:

The problem requires evaluating the right-hand limit (RHL) and the left-hand limit (LHL) of a piecewise-defined function at the point where its definition changes ($x = 2$), and then summing the two limit values.

Step 2: Key Formula or Approach:

- To find $\lim_{x\to 2^+} f(x)$ (RHL), use the piece of the function defined for $x > 2$. - To find $\lim_{x\to 2^-} f(x)$ (LHL), use the piece of the function defined for $x < 2$. - Substitute $x = 2$ into these respective pieces to calculate the limits, then add them.

Step 3: Detailed Explanation:

1. Evaluate the Right-Hand Limit (RHL):

We are approaching 2 from values greater than 2 ($x \to 2^+$).

For $x \geq 2$, the function is defined as $f(x) = x^2 - 1$.

Therefore, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2}(x^2 - 1)$.

Substituting $x = 2$ directly gives:

$\text{RHL} = (2)^2 - 1 = 4 - 1 = 3$.

2. Evaluate the Left-Hand Limit (LHL):

We are approaching 2 from values less than 2 ($x \to 2^-$).

For $x < 2$, the function is defined as $f(x) = x + 1$.

Therefore, $\lim_{x\to2^-} f(x) = \lim_{x\to2}(x+1)$.

Substituting $x = 2$ directly gives:

$\text{LHL} = 2 + 1 = 3$.

3. Calculate the sum:

The question asks for the sum of these two limits:

$\text{Sum} = \text{RHL} + \text{LHL}$

$\text{Sum} = 3 + 3 = 6$.

Step 4: Final Answer:

The sum of the limits is 6.

Quick Tip: For piecewise functions, the most critical step is selecting the correct "piece" of the equation based on whether you are evaluating a left-sided limit ($x < c$) or a right-sided limit ($x > c$).

---

34. If $y = \sqrt{\tan x + y}$, then $\frac{dy}{dx} =$

(1) $\frac{\sec x}{2y-1}$

(2) $\frac{\sec^2 x}{2y-1}$

(3) $\frac{\tan x}{2y-1}$

(4) $\frac{\sin^2 x}{2y-1}$

Correct Answer: (2) $\frac{\sec^2 x}{2y-1}$

Solution:

Step 1: Understanding the Concept:

This is an implicit differentiation problem involving a recursive or self-referential equation. The variable y appears on both sides, and it's inside a square root on one side.

Step 2: Key Formula or Approach:

To make differentiation easier, eliminate the square root by squaring both sides of the equation. Then, differentiate both sides implicitly with respect to x, applying the chain rule to terms involving y. Finally, group all terms containing $\frac{dy}{dx}$ and solve for it.

Step 3: Detailed Explanation:

The given equation is:

$$y = \sqrt{\tan x + y}$$

1. Square both sides to remove the radical:

$$y^2 = \tan x + y$$

2. Differentiate implicitly with respect to x: Apply the power rule and chain rule to y^2: $\frac{d}{dx}(y^2) = 2y \cdot \frac{dy}{dx}$. Differentiate the right side: $\frac{d}{dx}(\tan x) = \sec^2 x$, and $\frac{d}{dx}(y) = \frac{dy}{dx}$. Putting it together:

$$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

3. Solve for $\frac{dy}{dx}$: Move all terms containing $\frac{dy}{dx}$ to the left side of the equation:

$$2y\frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x$$

Factor out $\frac{dy}{dx}$ on the left side:

$$\frac{dy}{dx}(2y-1) = \sec^2 x$$

Divide both sides by $(2y-1)$ to isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$$

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $\frac{\sec^2 x}{2y-1}$.

Quick Tip: For infinite nested root problems of the form $y = \sqrt{f(x) + \sqrt{f(x) + \ldots}}$, which is equivalent to $y = \sqrt{f(x) + y}$, the derivative is always given by the standard shortcut formula: $y' = \frac{f'(x)}{2y-1}$. Here $f(x) = \tan x$, so $y' = \frac{\sec^2 x}{2y-1}$.

35. If $f(x) = \begin{cases} ax+7 & \text{if } x<1 \\ 2x-3 & \text{if } x=1 \\ \frac{x+b}{b} & \text{if } x>1 \end{cases}$ **is continuous at** $x = 1$**, then**

(1) $a = 3, b = 2$

(2) $a = -8, b = -2$

(3) $a = 8, b = -2$

(4) $a = -8, b = 2$

Correct Answer: (4) $a = -8, b = 2$

Solution:

Step 1: Continuity condition

For continuity at $x = 1$:

$$\text{LHL} = \text{RHL} = f(1)$$

Step 2: Find $f(1)$

Using middle function:

$$f(1) = 2(1) - 3 = -1$$

Step 3: Left-Hand Limit (LHL)

$$\lim_{x\to 1^-}(ax+7) = a+7$$

Equate with $f(1)$:

$$a + 7 = -1 \Rightarrow a = -8$$

Step 4: Right-Hand Limit (RHL)

$$\lim_{x\to 1^+} \frac{x-3}{b} = \frac{1-3}{b} = \frac{-2}{b}$$

Equate with $f(1)$:

$$\frac{-2}{b} = -1 \Rightarrow b = 2$$

Final Answer:

$$\boxed{a = -8, \quad b = 2}$$

Quick Tip: When dealing with blurry exam text, solve the clear parts first (here, finding $a = -8$). Use the derived information and the multiple-choice options to "reverse engineer" the unclear part of the equation.

---

36. The second order derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\cos^{-1}(2x^2-1)$, where $0 < x < \frac{1}{\sqrt{2}}$ is

(1) 0

(2) 1

(3) $\frac{1}{2}$

(4) -1

Correct Answer: (1) 0

Solution:

Step 1: Understanding the Concept:

We are required to find the second derivative of one function with respect to another, i.e.,

$$\frac{d^2u}{dv^2}$$

where

$$u = \sec^{-1}\left(\frac{1}{2x^2-1}\right), \quad v = \cos^{-1}(2x^2-1)$$

Step 2: Key Identity:

Using the identity:

$$\sec^{-1}\left(\frac{1}{z}\right) = \cos^{-1}(z), \quad \text{for } |z| \leq 1$$

Step 3: Simplification:

Let $z = 2x^2 - 1$. Then,

$$u = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \cos^{-1}(2x^2-1)$$

But

$$v = \cos^{-1}(2x^2-1)$$

Hence,

$$u = v$$

Step 4: First Derivative:

$$\frac{du}{dv} = \frac{d}{dv}(v) = 1$$

Step 5: Second Derivative:

$$\frac{d^2u}{dv^2} = \frac{d}{dv}(1) = 0$$

Step 6: Final Answer:

$$\boxed{0}$$

Quick Tip: Before jumping into messy chain rule calculations for "derivative of $f(x)$ with respect to $g(x)$", always check if $f(x)$ can be simplified or rewritten directly in terms of $g(x)$ using algebraic or trigonometric identities. Recognizing $u = v$ reduces a 5-minute problem to a 5-second one.

---

37. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'\left(\frac{1}{2}\right) =$

(1) $\frac{4}{5}$
(2) $\frac{8}{5}$
(3) $\frac{2}{5}$
(4) 0

Correct Answer: (2) $\frac{8}{5}$

Solution:

Step 1: Understanding the Concept:

We need to find the derivative of an inverse trigonometric function involving a rational algebraic expression, and then evaluate it at a specific point.

Step 2: Key Formula or Approach:

The expression $\frac{2x}{1+x^2}$ strongly suggests the trigonometric substitution $x = \tan\theta$, since $\sin(2\theta) =$

$\frac{2\tan\theta}{1+\tan^2\theta}$. Substitute, simplify the function using inverse trig properties, differentiate the simplified function with respect to x, and finally plug in $x = \frac{1}{2}$.

Step 3: Detailed Explanation:

The given function is $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$. Let's use the substitution $x = \tan\theta$. This means $\theta = \tan^{-1} x$. Substitute x into the function:

$$f(x) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

Using the double angle identity for sine, $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$:

$$f(x) = \sin^{-1}(\sin 2\theta)$$

Since we are evaluating the derivative at $x = \frac{1}{2}$ (which is between -1 and 1), 2θ falls within the principal value branch of arcsin $[-\pi/2, \pi/2]$. Therefore, we can simplify directly:

$$f(x) = 2\theta$$

Substitute back $\theta = \tan^{-1} x$:

$$f(x) = 2\tan^{-1} x$$

Now, differentiate $f(x)$ with respect to x:

$$f'(x) = \frac{d}{dx}(2\tan^{-1} x) = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}$$

Finally, evaluate the derivative at $x = \frac{1}{2}$:

$$f'\left(\frac{1}{2}\right) = \frac{2}{1+\left(\frac{1}{2}\right)^2}$$

$$f'\left(\frac{1}{2}\right) = \frac{2}{1+\frac{1}{4}}$$

$$f'\left(\frac{1}{2}\right) = \frac{2}{\frac{5}{4}}$$

$$f'\left(\frac{1}{2}\right) = 2 \times \frac{4}{5} = \frac{8}{5}$$

Step 4: Final Answer:

The value of the derivative at $x = 1/2$ is $8/5$.

Quick Tip: Expressions like $\frac{2x}{1+x^2}$, $\frac{1-x^2}{1+x^2}$, and $\frac{2x}{1-x^2}$ are classic signatures for the substitution $x = \tan\theta$, transforming them into $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ respectively.

---

38. If $\sqrt{x}\sqrt[3]{y} = (x+y)^n$ and $x\frac{dy}{dx} - y = 0$, then $n =$

(1) 1
(2) $\frac{6}{5}$
(3) $\frac{5}{6}$
(4) $\frac{4}{9}$

Correct Answer: (3) $\frac{5}{6}$

Solution:

Step 1: Understanding the Concept:

We are given the relation

$$\sqrt{x}\,\sqrt[3]{y} = (x+y)^n$$

and the differential condition

$$x\frac{dy}{dx} - y = 0 \Rightarrow \frac{dy}{dx} = \frac{y}{x}.$$

We need to find the value of n.

Step 2: Rewrite the Equation:

Express the given equation in exponent form:

$$x^{\frac{1}{2}} \cdot y^{\frac{1}{3}} = (x+y)^n$$

Step 3: Logarithmic Differentiation:

Taking logarithm on both sides:

$$\frac{1}{2}\ln x + \frac{1}{3}\ln y = n\ln(x+y)$$

Differentiate both sides w.r.t. x:

$$\frac{1}{2x} + \frac{1}{3y}\frac{dy}{dx} = n\cdot\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$$

Step 4: Substitute Given Condition:
Using $\frac{dy}{dx} = \frac{y}{x}$:

$$\frac{1}{2x} + \frac{1}{3y}\left(\frac{y}{x}\right) = \frac{n}{x+y}\left(1+\frac{y}{x}\right)$$

$$\frac{1}{2x} + \frac{1}{3x} = \frac{n}{x+y}\cdot\frac{x+y}{x}$$

$$\frac{3+2}{6x} = \frac{n}{x}$$

$$\frac{5}{6x} = \frac{n}{x}$$

Step 5: Solve for n:

Multiplying both sides by x:

$$n = \frac{5}{6}$$

Step 6: Final Answer:

$$\boxed{\frac{5}{6}}$$

Quick Tip: Memorize this highly useful shortcut result for competitive exams: If $x^m\cdot y^n = (x+y)^{m+n}$, then it is guaranteed that $\frac{dy}{dx} = \frac{y}{x}$. Recognizing this pattern instantly solves the problem without any differentiation.

---

39. In a Mahakumbh, a drone camera is moving along $3y = x^3 - 3$. When y-coordinate changes 9 times as fast as x-coordinate, it captures good quality pictures. Then one of the precise

positions of the drone at that instant is

(1) $(-3, 8)$

(2) $(3, -8)$

(3) $(3, 8)$

(4) $(-3, -8)$

Correct Answer: (3) $(3, 8)$

Solution:

Step 1: Understanding the Concept:

This is an application of derivatives involving rates of change. The phrase "y-coordinate changes 9 times as fast as x-coordinate" translates mathematically to a specific relationship between their derivatives with respect to time ($\frac{dy}{dt}$ and $\frac{dx}{dt}$).

Step 2: Key Formula or Approach:

1. Express the given rate condition mathematically: $\frac{dy}{dt} = 9 \cdot \frac{dx}{dt}$. 2. Differentiate the given curve equation $3y = x^3 - 3$ with respect to time t. 3. Substitute the rate condition into the differentiated equation to solve for x. 4. Substitute the found x values back into the original curve equation to find the corresponding y coordinates.

Step 3: Detailed Explanation:

The equation of the drone's path is:

$$3y = x^3 - 3$$

Differentiate both sides with respect to time t:

$$\frac{d}{dt}(3y) = \frac{d}{dt}(x^3 - 3)$$

$$3\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

Divide both sides by 3:

$$\frac{dy}{dt} = x^2\frac{dx}{dt}$$

We are given the condition that the y-coordinate changes 9 times as fast as the x-coordinate. This means:

$$\frac{dy}{dt} = 9\frac{dx}{dt}$$

Substitute this condition into our differentiated equation:

$$9\frac{dx}{dt} = x^2\frac{dx}{dt}$$

Assuming the drone is actually moving, $\frac{dx}{dt} \neq 0$, so we can divide both sides by $\frac{dx}{dt}$:

$$9 = x^2$$

This gives two possible values for the x-coordinate:

$$x = \pm 3$$

Now, find the corresponding y-coordinates by substituting these x values back into the original path equation $3y = x^3 - 3$. Case 1: If $x = 3$:

$$3y = (3)^3 - 3$$

$$3y = 27 - 3$$

$$3y = 24$$

$$y = 8$$

So, one position is $(3, 8)$.
Case 2: If $x = -3$:

$$3y = (-3)^3 - 3$$

$$3y = -27 - 3$$

$$3y = -30$$

$$y = -10$$

So, another position is $(-3, -10)$. Looking at the given options, $(-3, -10)$ is not present, but $(3, 8)$ is.

Step 4: Final Answer:

One of the precise positions is $(3, 8)$.

Quick Tip: "Rate of change" problems almost always involve differentiating an equation with respect to time (t). Translating the English phrase "A changes k times as fast as B" directly into the equation $\frac{dA}{dt} = k \cdot \frac{dB}{dt}$ is the crucial first step.

---

40. A Youtube short video is getting viral according to $f(t) = -2t^3 + 3t^2 + 5$. At what time does the video get maximum number of shares? (t is in hours)

(1) 1

(2) 2

(3) 3

(4) 4

Correct Answer: (1) 1

Solution:

Step 1: Understanding the Concept:

We need to find the time t at which a given function $f(t)$, representing the number of shares, reaches its maximum value. This is a classic optimization problem solvable using the first and second derivative tests.

Step 2: Key Formula or Approach:

1. Find the first derivative $f'(t)$. 2. Set $f'(t) = 0$ to find the critical points (potential maxima or minima). 3. Find the second derivative $f''(t)$. 4. Evaluate $f''(t)$ at the critical points. A negative value indicates a local maximum.

Step 3: Detailed Explanation:

The function for the number of shares is:

$$f(t) = -2t^3 + 3t^2 + 5$$

Calculate the first derivative with respect to t:

$$f'(t) = \frac{d}{dt}(-2t^3 + 3t^2 + 5) = -6t^2 + 6t$$

To find critical points, set the first derivative to zero:

$$-6t^2 + 6t = 0$$

Factor the equation:

$$-6t(t-1)=0$$

This gives two critical points: $t=0$ and $t=1$. Since time t must be meaningful in the context of the problem (usually $t>0$ for something "getting viral" after posting), $t=1$ is our primary candidate. Let's verify it's a maximum. Calculate the second derivative:

$$f''(t)=\frac{d}{dt}(-6t^2+6t)=-12t+6$$

Evaluate the second derivative at the critical points: At $t=0$: $f''(0)=-12(0)+6=6>0$. This indicates a local minimum. At $t=1$: $f''(1)=-12(1)+6=-6<0$. Since the second derivative is negative, the function attains a local maximum at this point. Therefore, the maximum number of shares occurs at $t=1$ hour.

Step 4: Final Answer:

The video gets the maximum number of shares at $t=1$.

Quick Tip: For simple polynomial optimization, finding the roots of the first derivative usually points directly to the answer. Always perform a quick mental check of the second derivative (or the sign change of the first derivative) to ensure you've found a maximum and not a minimum.

---

41. $\int xf(x)dx+\frac{f(x)}{2}=0$**, then** $f(x)$ **is equal to**

(1) e^{-2x}

(2) e^{2x}

(3) e^{-x^2}

(4) e^{x^2}

Correct Answer: (3) e^{-x^2}

Solution:

Step 1: Understanding the Concept:

We are given an integral equation involving an unknown function $f(x)$. To solve for $f(x)$, we need to eliminate the integral sign. This is done by differentiating the entire equation with respect to x.

Step 2: Key Formula or Approach:

1. Differentiate both sides of the given equation $\int xf(x)dx + \frac{f(x)}{2} = 0$ with respect to x. Remember that $\frac{d}{dx}\int g(x)dx = g(x)$. 2. This will yield a first-order differential equation for $f(x)$. 3. Solve the differential equation using separation of variables to find the form of $f(x)$.

Step 3: Detailed Explanation:

The given equation is:

$$\int xf(x)dx + \frac{f(x)}{2} = 0$$

Let's differentiate both sides with respect to x:

$$\frac{d}{dx}\left[\int xf(x)dx\right] + \frac{d}{dx}\left[\frac{f(x)}{2}\right] = \frac{d}{dx}(0)$$

Applying the fundamental theorem of calculus, the derivative of the integral is just the integrand:

$$xf(x) + \frac{1}{2}f'(x) = 0$$

Now we have a differential equation. Let's write $f'(x)$ as $\frac{dy}{dx}$ and $f(x)$ as y to make it clearer:

$$xy + \frac{1}{2}\frac{dy}{dx} = 0$$

Rearrange to separate variables:

$$\frac{1}{2}\frac{dy}{dx} = -xy$$

$$\frac{dy}{dx} = -2xy$$

Separate the variables y and x:

$$\frac{1}{y}dy = -2xdx$$

Integrate both sides:

$$\int \frac{1}{y}dy = \int -2xdx$$

$$\ln|y| = -x^2 + C$$

To find y (which is $f(x)$), exponentiate both sides:

$$y = e^{-x^2+C} = e^C \cdot e^{-x^2}$$

Let e^C be a new constant K. So, the general form of the function is:

$$f(x) = Ke^{-x^2}$$

Looking at the given options, they are all specific cases where the constant K is assumed to be 1. The option that matches the functional form e^{-x^2} is option (3).

Step 4: Final Answer:

The function $f(x)$ is e^{-x^2}.

Quick Tip: Whenever you see an equation containing an integral of an unknown function (an integral equation), the standard first step is almost always to differentiate the entire equation with respect to the variable. This converts it into a manageable differential equation.

---

42. One of the possible functions $f(x)$ which satisfies $\int_{-2}^{2} f(x)dx = 0$ is

(1) $\log\left(\frac{2+x}{2-x}\right)$

(2) $\sin(2+x)$

(3) $2x^3 + 2x + 1$

(4) $2x \tan x$

Correct Answer: (1) $\log\left(\frac{2+x}{2-x}\right)$

Solution:

Step 1: Understanding the Concept:

The problem asks for a function that integrates to zero over a symmetric interval $[-a, a]$, where $a = 2$. A key property of definite integrals states that if $f(x)$ is an odd function (i.e., $f(-x) = -f(x)$), then $\int_{-a}^{a} f(x)dx = 0$.

Step 2: Key Formula or Approach:

Test each given option to see if it is an odd function by evaluating $f(-x)$ and checking if it equals $-f(x)$.

Step 3: Detailed Explanation:

Let's test the options for the property $f(-x) = -f(x)$:

Option (1): $f(x) = \log\left(\frac{2+x}{2-x}\right)$ Evaluate $f(-x)$:

$$f(-x) = \log\left(\frac{2+(-x)}{2-(-x)}\right) = \log\left(\frac{2-x}{2+x}\right)$$

Using the property of logarithms $\log(a/b) = -\log(b/a)$:

$$f(-x) = \log\left(\left(\frac{2+x}{2-x}\right)^{-1}\right) = -1 \cdot \log\left(\frac{2+x}{2-x}\right) = -f(x)$$

Since $f(-x) = -f(x)$, this is an odd function. Therefore, its integral over $[-2, 2]$ will be zero. This is a possible function.

Option (2): $f(x) = \sin(2+x)$ Evaluate $f(-x)$:

$$f(-x) = \sin(2-x)$$

This is neither $\sin(2+x)$ nor $-\sin(2+x)$. It's neither even nor odd. The integral will not generally be zero.

Option (3): $f(x) = 2x^3 + 2x + 1$ Evaluate $f(-x)$:

$$f(-x) = 2(-x)^3 + 2(-x) + 1 = -2x^3 - 2x + 1$$

This is not equal to $-f(x)$ which would be $-2x^3 - 2x - 1$. The $+1$ constant term prevents it from being odd. The integral of the odd parts $(2x^3 + 2x)$ will be zero, but the integral of the constant 1 will be $1 \cdot (2 - (-2)) = 4$, so the total integral is not zero.

Option (4): $f(x) = 2x \tan x$ Evaluate $f(-x)$:

$$f(-x) = 2(-x)\tan(-x) = -2x(-\tan x) = 2x \tan x = f(x)$$

This is an even function. Its integral over $[-2, 2]$ will be $2\int_0^2 f(x)dx$, which is generally not zero.

Step 4: Final Answer:

The function that satisfies the condition is $\log\left(\frac{2+x}{2-x}\right)$.

Quick Tip: Any definite integral of the form $\int_{-a}^{a} f(x)dx$ should immediately trigger a check for whether the function $f(x)$ is odd or even. For odd functions, the integral is always zero, saving you from complex integration. Functions of the form $\log\left(\frac{a+x}{a-x}\right)$ are classic examples of odd functions.

---

43. $\int_{a-6}^{b-6} f(x+6)dx$ **is equal to**

(1) $\int_{a}^{b} f(x-6)dx$
(2) $\int_{a}^{b} f(x+6)dx$
(3) $\int_{a}^{b} f(x)dx$
(4) $\int_{a}^{b} f(-x)dx$

Correct Answer: (3) $\int_{a}^{b} f(x)dx$

Solution:

Step 1: Understanding the Concept:

This problem requires simplifying a definite integral by using a simple variable substitution to shift the limits of integration.

Step 2: Key Formula or Approach:

Use the substitution method. Let a new variable equal the argument of the function, i.e., let $t = x+6$. Then find dt and update the lower and upper limits of integration accordingly.

Step 3: Detailed Explanation:

The given integral is:

$$I = \int_{a-6}^{b-6} f(x+6)dx$$

Let's use a substitution to simplify the argument of the function f. Let $t = x+6$. Differentiating both sides with respect to x:

$$dt = dx$$

Now, we must adjust the limits of integration to match the new variable t: - When the lower limit $x = a-6$, the new limit is $t = (a-6)+6 = a$. - When the upper limit $x = b-6$, the new limit is $t = (b-6)+6 = b$. Substitute t, dt, and the new limits into the integral:

$$I = \int_{a}^{b} f(t)dt$$

In a definite integral, the variable of integration is a "dummy variable." We can freely change the symbol from t back to x without altering the value of the integral:

$$I = \int_a^b f(x)dx$$

Step 4: Final Answer:

The integral is equal to $\int_a^b f(x)dx$.

Quick Tip: This demonstrates the translation property of definite integrals: Shifting the function horizontally by c ($f(x+c)$) and shifting the integration limits by the opposite amount ($-c$) results in the same area under the curve. $\int_{A-c}^{B-c} f(x+c)dx = \int_A^B f(x)dx$.

---

44. If 'n' is a natural number, then $\int \frac{\sin^n x}{\cos^{n+2} x} dx =$

(1) $\frac{\tan^{n-1} x}{n-1} + C$

(2) $\frac{\tan^n x}{n} + C$

(3) $\frac{\tan^{n+2} x}{n+2} + C$

(4) $\frac{\tan^{n+1} x}{n+1} + C$

Correct Answer: (4) $\frac{\tan^{n+1} x}{n+1} + C$

Solution:

Step 1: Understanding the Concept:

The integral involves a fraction with powers of sine and cosine. The goal is to manipulate the integrand algebraically into a recognizable form that allows for a standard substitution, typically involving tangent and secant functions.

Step 2: Key Formula or Approach:

Rewrite the denominator $\cos^{n+2} x$ by splitting it into $\cos^n x \cdot \cos^2 x$. This allows the formation of $\tan^n x$ and $\sec^2 x$, setting up a perfect u-substitution where $u = \tan x$ and $du = \sec^2 x dx$.

Step 3: Detailed Explanation:

The given integral is:

$$I = \int \frac{\sin^n x}{\cos^{n+2} x} dx$$

Let's decompose the denominator:

$$I = \int \frac{\sin^n x}{\cos^n x \cdot \cos^2 x} dx$$

We can group the terms with the power n together:

$$I = \int \left(\frac{\sin x}{\cos x}\right)^n \cdot \frac{1}{\cos^2 x} dx$$

Using basic trigonometric identities ($\frac{\sin x}{\cos x} = \tan x$ and $\frac{1}{\cos^2 x} = \sec^2 x$):

$$I = \int (\tan x)^n \cdot \sec^2 x dx$$

Now the integral is in a standard form for substitution. Let:

$$u = \tan x$$

Then the differential is:

$$du = \sec^2 x dx$$

Substitute u and du into the integral:

$$I = \int u^n du$$

Using the power rule for integration $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (since n is a natural number, $n \neq -1$):

$$I = \frac{u^{n+1}}{n+1} + C$$

Finally, substitute back $u = \tan x$:

$$I = \frac{\tan^{n+1} x}{n+1} + C$$

Step 4: Final Answer:

The integral evaluates to $\frac{\tan^{n+1} x}{n+1} + C$.

Quick Tip: When you see an integral with $\sin^m x$ and $\cos^p x$ in the numerator and denominator, check the difference in their powers. If the power of cosine in the denominator is 2 greater than the power of sine in the numerator, separating out a $\frac{1}{\cos^2 x}$ will always cleanly produce a tan and $\sec^2$ pair for an easy u-substitution.

---

45. $\int e^{-x\log 2} 2^x dx =$

(1) $\log x + C$

(2) $x + C$

(3) $\frac{1}{x} + C$

(4) $\frac{x^2}{2} + C$

Correct Answer: (2) $x + C$

Solution:

Step 1: Understanding the Concept:

The problem involves an integral with an exponential term containing a logarithm in its exponent, multiplied by another exponential term. The key is to simplify the integrand using logarithmic and exponential properties before attempting integration.

Step 2: Key Formula or Approach:

Use the property of logarithms that $a \log b = \log(b^a)$ and the fundamental property relating exponentials and natural logarithms: $e^{\ln(y)} = y$. In higher-level mathematics, 'log' without a specified base usually implies the natural logarithm 'ln'.

Step 3: Detailed Explanation:

The given integral is:

$$I = \int e^{-x\log 2} \cdot 2^x dx$$

Assuming log denotes the natural logarithm ln: First, simplify the term $e^{-x\log 2}$. Using the power rule for logarithms ($-x\ln 2 = \ln(2^{-x})$):

$$e^{-x\ln 2} = e^{\ln(2^{-x})}$$

Using the inverse property $e^{\ln(y)} = y$:

$$e^{\ln(2^{-x})} = 2^{-x}$$

Now, substitute this simplified expression back into the integrand:

$$I = \int (2^{-x}) \cdot 2^x dx$$

Use exponent rules to combine terms with the same base ($a^m \cdot a^n = a^{m+n}$):

$$I = \int 2^{-x+x} dx$$

$$I = \int 2^0 dx$$

Since any non-zero number to the power of 0 is 1:

$$I = \int 1 dx$$

The integral of a constant 1 with respect to x is x:

$$I = x + C$$

(Note: The handwritten tick mark on option 4 in the image is incorrect. The mathematically sound result based on the printed text is $x + C$.)

Step 4: Final Answer:

The integral evaluates to $x + C$.

Quick Tip: Always simplify the integrand before trying complex integration techniques like integration by parts. The identity $e^{k\ln(a)} = a^k$ is a very common tool used by examiners to hide simple constants or terms within intimidating-looking exponential expressions.

---

46. The area of the region bounded by the curve $y^2 = x^3$, the y-axis and the lines $y = 1$ and $y = 8$ is

(1) $\frac{155}{3}$ sq. units

(2) $\frac{93}{5}$ sq. units

(3) 93 sq. units

(4) 155 sq. units

Correct Answer: (2) $\frac{93}{5}$ sq. units

Solution:

Step 1: Understanding the Concept:

We need to find the area of a region bounded by a curve and horizontal lines. Since the boundaries are defined by y-values ($y = 1$ to $y = 8$) and the y-axis ($x = 0$), it is most straightforward to integrate with respect to y.

Step 2: Key Formula or Approach:

The area A between a curve $x = f(y)$, the y-axis, and the horizontal lines $y = c$ and $y = d$ is given by the definite integral: $A = \int_c^d x\,dy$ First, express x as a function of y from the given curve equation. Then perform the definite integration.

Step 3: Detailed Explanation:

1. Express x in terms of y: The equation of the curve is given as $y^2 = x^3$. We need to solve for x: Taking the cube root of both sides gives:

$$x = (y^2)^{1/3} = y^{2/3}$$

2. Set up the integral: The region is bounded by the y-axis ($x = 0$) and the horizontal lines $y = 1$ and $y = 8$. Since y is positive in this interval, $x = y^{2/3}$ is also positive, meaning the curve lies to the right of the y-axis. The area A is:

$$A = \int_1^8 x\,dy$$

Substitute $x = y^{2/3}$:

$$A = \int_1^8 y^{2/3}\,dy$$

3. Evaluate the integral: Use the power rule for integration: $\int y^n\,dy = \frac{y^{n+1}}{n+1}$:

$$\int y^{2/3}\,dy = \frac{y^{(2/3)+1}}{(2/3)+1} = \frac{y^{5/3}}{5/3} = \frac{3}{5}y^{5/3}$$

Now, evaluate this from $y = 1$ to $y = 8$:

$$A = \left[\frac{3}{5}y^{5/3}\right]_1^8$$

$$A = \frac{3}{5}\left(8^{5/3} - 1^{5/3}\right)$$

Let's simplify $8^{5/3}$. We know $8 = 2^3$, so: $8^{5/3} = (2^3)^{5/3} = 2^{3\times(5/3)} = 2^5 = 32$. And 1 to any power is 1. Substitute these back:

$$A = \frac{3}{5}(32 - 1)$$

$$A = \frac{3}{5}(31)$$

$$A = \frac{93}{5}$$

Step 4: Final Answer:

The area of the bounded region is $\frac{93}{5}$ sq. units.

Quick Tip: When calculating areas bounded by the y-axis and horizontal lines ($y = c$ to $y = d$), always rewrite the function as $x = f(y)$ and integrate with respect to dy. This avoids having to split the integral into multiple parts or dealing with more complex inverse functions if you tried integrating with respect to dx.

---

47. The area enclosed by the curve $x = \sqrt{3}\cos\theta, y = \sqrt{3}\sin\theta$ is

(1) $\sqrt{3}\pi$ sq. units

(2) 9π sq. units

(3) 6π sq. units

(4) 3π sq. units

Correct Answer: (4) 3π sq. units

Solution:

Step 1: Understanding the Concept:

The given equations are parametric:

$$x = \sqrt{3}\cos\theta, \quad y = \sqrt{3}\sin\theta$$

We convert them into Cartesian form to identify the curve.

Step 2: Eliminate the Parameter:

Square both equations:

$$x^2 = 3\cos^2\theta, \quad y^2 = 3\sin^2\theta$$

Add them:

$$x^2 + y^2 = 3(\cos^2\theta + \sin^2\theta)$$

Using identity $\cos^2\theta + \sin^2\theta = 1$:

$$x^2 + y^2 = 3$$

Step 3: Identify the Curve:

The equation $x^2 + y^2 = 3$ represents a circle centered at origin with radius:

$$r = \sqrt{3}$$

Step 4: Area of the Curve:

Area of a circle is:

$$A = \pi r^2$$

$$A = \pi(\sqrt{3})^2 = 3\pi$$

Step 5: Final Answer:

$$\boxed{3\pi \text{ sq. units}}$$

Quick Tip: Parametric equations of the form $x = a\cos\theta, y = a\sin\theta$ always describe a circle of radius 'a'. If the coefficients are different, like $x = a\cos\theta, y = b\sin\theta$, it describes an ellipse with area πab. Recognizing these standard parametric forms saves the effort of formal integration.

---

48. Sum of the squares of the order and degree (if defined) of a differential equation $2y' + (y'')^2 = \sqrt{y'' - 3}$ is

(1) 13

(2) 20

(3) 8

(4) 16

Correct Answer: (2) 20

Solution:

Step 1: Understanding the Concept:

The given differential equation is:

$$2y' + (y'')^2 = \sqrt{y'' - 3}$$

We need to find:

- **Order**: Highest order derivative present
- **Degree**: Power of highest order derivative after removing radicals/fractions

Step 2: Find the Order:

The derivatives present are y' and y''.

Highest order derivative is y''.

$$\text{Order} = 2$$

Step 3: Remove Radical to Find Degree:

Square both sides to eliminate the square root:

$$(2y' + (y'')^2)^2 = y'' - 3$$

Expand LHS:

$$(2y')^2 + 2(2y')(y'')^2 + ((y'')^2)^2 = y'' - 3$$

$$4(y')^2 + 4y'(y'')^2 + (y'')^4 = y'' - 3$$

Rearrange:

$$(y'')^4 + 4y'(y'')^2 - y'' + 4(y')^2 + 3 = 0$$

Step 4: Find the Degree:

Now the equation is polynomial in derivatives.

Highest power of highest order derivative y'' is:

$$\text{Degree} = 4$$

Step 5: Required Sum:

$$\text{Sum} = (\text{Order})^2 + (\text{Degree})^2$$

$$= 2^2 + 4^2 = 4 + 16 = 20$$

Step 6: Final Answer:

$$\boxed{20}$$

Quick Tip: Never determine the degree of a differential equation while derivatives are under radicals, fractional powers, or inside transcendental functions (like sin, log, e). You must first algebraically manipulate the equation into a polynomial form of its derivatives.

---

49. If A = {a, b, c, d, e, f}, then the number of subsets of A which contains at least 2 elements is

(1) 64

(2) 65

(3) 57

(4) 59

Correct Answer: (3) 57

Solution:

Step 1: Understanding the Concept:

Given set $A = \{a, b, c, d, e, f\}$ has 6 elements.

We need to find the number of subsets containing **at least 2 elements**.

Step 2: Total Number of Subsets:

For a set with n elements, total subsets are:

$$2^n$$

$$2^6 = 64$$

Step 3: Subsets to Exclude:

We exclude subsets having less than 2 elements:

- Subsets with 0 elements: $\binom{6}{0} = 1$
- Subsets with 1 element: $\binom{6}{1} = 6$

Step 4: Required Number of Subsets:

$$\text{Required subsets} = 64 - (1 + 6)$$

$$= 64 - 7 = 57$$

Step 5: Final Answer:

$$\boxed{57}$$

Quick Tip: Whenever a combinatorics or probability question uses the phrase "at least", strongly consider using the complement method (Total - Unwanted). It often reduces a long series of calculations into a simple subtraction.

---

50. If A = {1, 2, 3, 4, ..., 10}, then the number of non empty subsets of A containing only even number is

(1) 31

(2) 82

(3) 30

(4) 29

Correct Answer: (1) 31

Solution:

Step 1: Understanding the Concept:

Given $A = \{1, 2, 3, 4, \ldots, 10\}$.

We need to find the number of **non-empty subsets containing only even numbers**.

Step 2: Identify Even Elements:

Even elements in A are:

$$E = \{2, 4, 6, 8, 10\}$$

Number of elements in E:

$$n = 5$$

Step 3: Total Subsets from Even Elements:

Number of subsets of a set with n elements:

$$2^n = 2^5 = 32$$

Step 4: Exclude Empty Set:

Since only non-empty subsets are required:

$$\text{Required subsets} = 32 - 1 = 31$$

Step 5: Final Answer:

$$\boxed{31}$$

Quick Tip: Always read set theory questions carefully for the word "non-empty" (or "proper subset"). This single word changes the answer by exactly -1 and is a very common, easy trap to fall into.

---

51. The domain of the function $\sqrt{\frac{x-7}{9-x}}$ is

(1) $(7, 9)$

(2) $[7, 9)$

(3) $[7, 9]$

(4) $(7, 9]$

Correct Answer: (2) $[7, 9)$

Solution:

Step 1: Understanding the Concept:

For the function

$$f(x) = \sqrt{\frac{x-7}{9-x}},$$

the expression inside the square root must be non-negative and the denominator must not be zero.

Step 2: Form the Inequality:

$$\frac{x-7}{9-x} \geq 0$$

Also,

$$9 - x \neq 0 \Rightarrow x \neq 9$$

Step 3: Solve the Inequality:

Rewrite:

$$\frac{x-7}{9-x} \geq 0 \Rightarrow \frac{x-7}{x-9} \leq 0$$

Critical points:

$$x = 7, \quad x = 9$$

Check sign in intervals:

- $(-\infty, 7) \to$ positive
- $(7, 9) \to$ negative
- $(9, \infty) \to$ positive

So solution:

$$[7, 9)$$

Step 4: Check Endpoints:

At $x = 7$: expression $= 0$ (allowed)

At $x = 9$: denominator $= 0$ (not allowed)

Step 5: Final Answer:

$$\boxed{[7,9)}$$

Quick Tip: For rational inequalities $\frac{x-a}{x-b} \leq 0$, the solution is the bounded interval between roots $[a, b]$. Always check the denominator root separately to ensure it is excluded with an open parenthesis.

---

52. If $n(A) = 2$ and the number of relations from set A to set B is 1024, then $n(B)$ is

(1) 2

(2) 5

(3) 2^5

(4) 5^2

Correct Answer: (2) 5

Solution:

Step 1: Given Data

$$n(A) = 2, \quad \text{Number of relations} = 1024$$

Step 2: Use Formula

Number of relations from A to B:

$$= 2^{n(A)\cdot n(B)}$$

Let $n(B) = m$:

$$2^{2m} = 1024$$

Step 3: Convert to Same Base

$$1024 = 2^{10}$$

$$2^{2m} = 2^{10}$$

Step 4: Equate Powers

$$2m = 10 \Rightarrow m = 5$$

Step 5: Final Answer

$$\boxed{n(B) = 5}$$

Quick Tip: It is very beneficial to memorize the powers of 2 up to $2^{10} = 1024$ for competitive exams. It speeds up problems involving subsets, combinations, and binary calculations significantly.

---

53. Probability of at least one of the events A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(1) 1

(2) 0.8

(3) 0.6

(4) 1.2

Correct Answer: (4) 1.2

Solution:

Step 1: Given Data

$$P(A \cup B) = 0.6, \quad P(A \cap B) = 0.2$$

Step 2: Use Addition Formula

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = P(A) + P(B) - 0.2$$

$$P(A) + P(B) = 0.6 + 0.2 = 0.8$$

Step 3: Use Complement Rule

$$P(\bar{A}) = 1 - P(A), \quad P(\bar{B}) = 1 - P(B)$$

$$P(\bar{A}) + P(\bar{B}) = (1 - P(A)) + (1 - P(B))$$

$$= 2 - (P(A) + P(B))$$

Step 4: Substitute Value

$$P(\bar{A}) + P(\bar{B}) = 2 - 0.8 = 1.2$$

Step 5: Final Answer

$$\boxed{1.2}$$

Quick Tip: A useful derived identity to remember for these specific questions is: $P(\bar{A}) + P(\bar{B}) = 2 - (P(A \cup B) + P(A \cap B))$. It jumps straight to the answer.

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54. The maximum value of $\sin(x + \pi/6) + \cos(x + \pi/6)$ **is attained at** $x =$

(1) $\pi/2$

(2) $\pi/4$

(3) $\pi/6$

(4) $\pi/12$

Correct Answer: (4) $\pi/12$

Solution:

Step 1: Let the expression be simplified

Given,

$$f(x) = \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right)$$

Let,

$$\theta = x + \frac{\pi}{6}$$

Then,

$$f(x) = \sin\theta + \cos\theta$$

Step 2: Convert into single trigonometric form

We use the identity:

$$\sin\theta + \cos\theta = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$$

So,

$$f(x) = \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$$

Substitute $\theta = x + \frac{\pi}{6}$:

$$f(x) = \sqrt{2}\sin\left(x + \frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(x + \frac{5\pi}{12}\right)$$

Step 3: Find maximum condition

The maximum value of $\sin(\alpha)$ is 1, which occurs when:

$$\alpha = \frac{\pi}{2}$$

So,

$$x + \frac{5\pi}{12} = \frac{\pi}{2}$$

Step 4: Solve for x

$$x = \frac{\pi}{2} - \frac{5\pi}{12}$$

$$x = \frac{6\pi}{12} - \frac{5\pi}{12} = \frac{\pi}{12}$$

Step 5: Final Answer

$$\boxed{x = \frac{\pi}{12}}$$

Quick Tip: For $a\sin\theta + b\cos\theta$, if $a = 1, b = 1$, the maximum occurs when $\theta = 45^\circ$ or $\pi/4$. Here $\theta = (x + 30^\circ)$. So, $x + 30^\circ = 45^\circ \implies x = 15^\circ$, which is $\pi/12$. Working in degrees can often be faster for mental math than manipulating π fractions.

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55. The angles of a triangle are in A.P and the greatest angle is double the least angle, then sine of the third angle is

(1) $\frac{\sqrt{3}}{2}$
(2) $\frac{1}{\sqrt{2}}$
(3) $\frac{1}{2}$
(4) 0

Correct Answer: (1) $\frac{\sqrt{3}}{2}$

Solution:

Step 1: Let the Angles in A.P.

Let the three angles be:

$$(a-d),\ a,\ (a+d)$$

Step 2: Sum of Angles of Triangle

$$(a-d)+a+(a+d)=180^\circ$$

$$3a = 180^\circ \Rightarrow a = 60^\circ$$

Step 3: Given Condition

Greatest angle = double the least angle:

$$a+d = 2(a-d)$$

Substitute $a = 60^\circ$:

$$60+d = 2(60-d)$$

$$60+d = 120-2d$$

$$3d = 60 \Rightarrow d = 20^\circ$$

Step 4: Find All Angles

$$\text{Least angle} = 60 - 20 = 40^\circ$$

$$\text{Middle angle} = 60^\circ$$

$$\text{Greatest angle} = 60 + 20 = 80^\circ$$

Step 5: Required Value

The third (middle) angle is 60°:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

Step 6: Final Answer

$$\boxed{\frac{\sqrt{3}}{2}}$$

Quick Tip: A crucial geometric shortcut: If the three angles of a triangle are in Arithmetic Progression, the middle angle is always exactly 60°, regardless of the other conditions. The question asks for the "third angle" (implicitly the middle one), so you can often jump straight to $\sin(60^\circ) = \sqrt{3}/2$ without even calculating d!

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56. The mean and standard deviation of 100 items are 50 and 4, respectively then the sum of all squares of the items is

(1) 250000

(2) 251600

(3) 256100

(4) 265100

Correct Answer: (2) 251600

Solution:

Step 1: Understanding the Concept:

This problem relates the fundamental statistical measures: mean, standard deviation (or variance), and the sum of the squares of individual data points.

Step 2: Key Formula or Approach:

Use the computational formula for variance, which links all these quantities: Variance $(\sigma^2) = \frac{\sum x_i^2}{N} - (\text{Mean } \mu)^2$ Where: σ = standard deviation $\sum x_i^2$ = sum of squares of all items (this is what we need to find) N = total number of items μ = mean of the items Rearrange this formula to solve for $\sum x_i^2$.

Step 3: Detailed Explanation:

From the problem statement, we have the following values: Total number of items, $N = 100$ Mean, $\mu = 50$ Standard deviation, $\sigma = 4$

First, calculate the variance, which is the square of the standard deviation: Variance $(\sigma^2) = 4^2 = 16$

Now, substitute the known values into the variance formula:

$$\sigma^2 = \frac{\sum x_i^2}{N} - \mu^2$$

$$16 = \frac{\sum x_i^2}{100} - (50)^2$$

Calculate the square of the mean:

$$16 = \frac{\sum x_i^2}{100} - 2500$$

Now, solve for the unknown term $\frac{\sum x_i^2}{100}$: Add 2500 to both sides:

$$16 + 2500 = \frac{\sum x_i^2}{100}$$

$$2516 = \frac{\sum x_i^2}{100}$$

Finally, to find the sum of all squares of the items $(\sum x_i^2)$, multiply both sides by 100:

$$\sum x_i^2 = 2516 \times 100$$

$$\sum x_i^2 = 251600$$

Step 4: Final Answer:

The sum of all squares of the items is 251600.

Quick Tip: A useful rearranged form of the variance formula to memorize for direct calculation is: $\sum x_i^2 = N \cdot (\sigma^2 + \mu^2)$. This allows you to plug numbers straight in: $100 \times (16 + 2500) = 251600$.

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57. Probability of occurrence of an event A is 1/2 and that of B is 3/10. If A and B are mutually exclusive, then the probability of occurrence of neither A nor B is

(1) $\frac{4}{5}$
(2) $\frac{3}{5}$
(3) $\frac{2}{5}$
(4) $\frac{1}{5}$

Correct Answer: (4) $\frac{1}{5}$

Solution:

Step 1: Given Data

$$P(A) = \frac{1}{2}, \quad P(B) = \frac{3}{10}$$

Since A and B are mutually exclusive:

$$P(A \cap B) = 0$$

Step 2: Find $P(A \cup B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cup B) = \frac{1}{2} + \frac{3}{10}$$
$$= \frac{5}{10} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$$

Step 3: Probability of Neither A nor B

$$P(\text{neither A nor B}) = 1 - P(A \cup B)$$

$$= 1 - \frac{4}{5} = \frac{1}{5}$$

Step 4: Final Answer

$$\boxed{\frac{1}{5}}$$

Quick Tip: For mutually exclusive events, think of probability as just adding areas. If A covers 50% of the space and B covers 30% (with no overlap), together they cover 80%. What's left over ("neither") is simply $100\% - 80\% = 20\%$, which is $1/5$.

---

58. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Which of the following is the correct answer?

(1) $(2,4) \in R$

(2) $(3,8) \in R$

(3) $(6,8) \in R$

(4) $(8,7) \in R$

Correct Answer: (3) $(6,8) \in R$

Solution:

Step 1: Given Relation

$$R = \{(a, b) : a = b - 2,\ b > 6\}$$

Step 2: Conditions to Check

For any ordered pair (a, b) to belong to R, it must satisfy:

- $a = b - 2$
- $b > 6$

Step 3: Verify Each Option

(1) $(2,4)$

$$b = 4 \not> 6 \quad \Rightarrow \text{Not in } R$$

(2) $(3,8)$

$$8 > 6 \text{ (true)}, \quad 3 \neq 8 - 2 = 6$$

$$\Rightarrow \text{Not in } R$$

(3) $(6,8)$

$$8 > 6 \text{ (true)}, \quad 6 = 8 - 2$$

$$\Rightarrow (6,8) \in R$$

(4) $(8,7)$

$$7 > 6 \text{ (true)}, \quad 8 \neq 7 - 2 = 5$$

$$\Rightarrow \text{Not in } R$$

Step 4: Final Answer

$$\boxed{(6,8) \in R}$$

Quick Tip: Always test the easiest or most restrictive condition first. By simply looking for pairs where the second number b is greater than 6, you immediately eliminate option (1) without any calculation. Then apply the equation test.

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59. $f(x) = (x+1)^2$ for $x \geq 1$. $g(x)$ is a function whose graph is the reflection of the graph of $f(x)$ in the line $y = x$, then $g(x)$ is

(1) $-\sqrt{x} - 1$

(2) $\sqrt{x}+1$

(3) $\sqrt{x}-1$

(4) $\sqrt{-x}-1$

Correct Answer: (3) $\sqrt{x}-1$

Solution:

Step 1: Understanding the Concept

Reflection of a graph in the line $y = x$ gives the inverse function.

Hence, $g(x) = f^{-1}(x)$.

Step 2: Write the Function

$$y = (x+1)^2, \quad x \geq 1$$

Step 3: Interchange x and y

$$x = (y+1)^2$$

Step 4: Solve for y

$$y+1 = \pm\sqrt{x}$$

$$y = -1 \pm \sqrt{x}$$

Step 5: Choose Correct Branch

Since $x \geq 1$, the range of $f(x)$ is $y \geq 4$.

Thus, for inverse function:

$$y \geq 1$$

So, take positive root:

$$y = \sqrt{x}-1$$

Step 6: Final Answer

$$\boxed{g(x) = \sqrt{x}-1}$$

Quick Tip: Always remember: "Reflection across $y = x$" is the code phrase for "Find the inverse function". The domain restriction ($x \geq 1$) is a critical hint that you must carefully consider whether to take the positive or negative branch when square rooting.

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60. If $\sin^{-1} x + \sin^{-1} y = \pi/2$, then x^2 is equal to

(1) $1 - y^2$
(2) $1 + y^2$
(3) $\sqrt{1 - y^2}$
(4) $\sqrt{1 + y^2}$

Correct Answer: (1) $1 - y^2$

Solution:

Step 1: Given Condition

$$\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

Step 2: Use Identity

We know the identity:

$$\sin^{-1} t + \cos^{-1} t = \frac{\pi}{2}$$

Comparing,

$$\sin^{-1} x = \frac{\pi}{2} - \sin^{-1} y = \cos^{-1} y$$

Step 3: Convert to Algebraic Form

$$\sin^{-1} x = \cos^{-1} y \Rightarrow x = \sqrt{1 - y^2}$$

Step 4: Square Both Sides

$$x^2 = (\sqrt{1 - y^2})^2$$

$$x^2 = 1 - y^2$$

Step 5: Final Answer

$$\boxed{x^2 = 1 - y^2}$$

Quick Tip: Whenever you see inverse trig functions summing to $\pi/2$, immediately think of complementary identities like $\sin^{-1}\theta + \cos^{-1}\theta = \pi/2$ or $\tan^{-1}\theta + \cot^{-1}\theta = \pi/2$. This converts a sum into an equality, which is much easier to solve.
