KEAM 2026 Engineering April 19

Question Paper with Solutions PDF

Conducted by CEE Kerala

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 600 marks.

(iii) **Structure:** The paper has 3 Sections:

- **Section A:** 45 Multiple Choice Questions (Physics).
- **Section B:** 30 Multiple Choice Questions (Chemistry).
- **Section B:** 75 Multiple Choice Questions (Mathematics).

(iv) **Compulsory Questions:** All 150 questions are compulsory.

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +4 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

Chemistry

1. Reagent used to convert decanol to decanoic acid

(A) Tollen's reagent

(B) Jones reagent

(C) Grignard reagent

(D) Fehling's reagent

(E) DIBAC-H

Correct Answer: (B) Jones reagent

Solution:

Step 1: Understanding the Concept:

To convert a primary alcohol (decanol) directly to a carboxylic acid (decanoic acid), a strong oxidizing agent is required. This process involves the oxidation of the $-CH_2OH$ group to a $-COOH$ group.

Step 2: Detailed Explanation:

1. Jones Reagent (CrO_3 in aqueous H_2SO_4 and acetone) is a powerful oxidizing agent that converts primary alcohols to carboxylic acids and secondary alcohols to ketones. 2. Tollen's and Fehling's reagents are mild oxidizing agents used to oxidize aldehydes to carboxylic acids, but they generally cannot oxidize alcohols directly. 3. DIBAL-H (or DIBAC-H) is a reducing agent, not an oxidizing agent. 4. Grignard reagents are used for C–C bond formation, not for this type of functional group oxidation.

Step 3: Final Answer:

The reagent used is Jones reagent.

> **Quick Tip:** If you want to stop the oxidation at the aldehyde stage (decanal), you would use a milder, anhydrous reagent like PCC (Pyridinium chlorochromate) instead of Jones reagent.

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2. IUPAC name of $(CH_3)_3C-CH_2Br$

Correct Answer: 1-bromo-2,2-dimethylpropane

Solution:

Step 1: Understanding the Concept:

IUPAC naming requires identifying the longest continuous carbon chain that contains the principal functional group (in this case, the bromine substituent), numbering the chain to give substituents the lowest possible locants, and listing substituents alphabetically.

Step 2: Detailed Explanation:

1. The structure is: $CH_3-C(CH_3)_2-CH_2Br$. 2. The longest carbon chain containing the Br atom

has 3 carbons (Propane). 3. Number the chain starting from the carbon attached to Bromine: - C1: Attached to Br - C2: Attached to two methyl groups - C3: Terminal methyl group 4. Substituents: - A 'bromo' group at position 1. - Two 'methyl' groups at position 2 (2,2-dimethyl). 5. Combine alphabetically: 1-bromo-2,2-dimethylpropane.

Step 3: Final Answer:

The IUPAC name is 1-bromo-2,2-dimethylpropane.

> **Quick Tip:** Common name check: This molecule is also known as **neopentyl bromide**. In IUPAC, always prioritize alphabetical order (B for bromo comes before M for methyl).

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3. Geometry of a molecule AB_3E_2 with 3 bond pairs and 2 lone pairs

Correct Answer: T-shaped

Solution:

Step 1: Understanding the Concept:

According to VSEPR (Valence Shell Electron Pair Repulsion) theory, the shape of a molecule depends on the total number of electron pairs (Steric Number) around the central atom. The Steric Number determines the "Electron Geometry," while the positions of atoms determine the "Molecular Geometry."

Step 2: Detailed Explanation:

1. Total electron pairs = 3 (bond pairs) + 2 (lone pairs) = 5. 2. For a steric number of 5, the electron geometry is Trigonal Bipyramidal. 3. In a trigonal bipyramidal arrangement, lone pairs always occupy the **equatorial** positions to minimize repulsion. 4. With 2 lone pairs in the equatorial positions, the remaining 3 bond pairs occupy the two axial positions and one equatorial position. 5. This arrangement results in a T-shaped molecular geometry.

Step 3: Final Answer:

The geometry of an AB_3E_2 molecule is T-shaped.

> **Quick Tip:** Common examples of AB_3E_2 molecules include ClF_3 and BrF_3. Remember: "Lone pairs love the equator" in 5-coordinate systems!

4. Which do not form carbylamine

(A) Ethanamine
(B) Benzamine
(C) Prop-2-amine
(D) Propan-1-amine
(E) N-methylethanamine

Correct Answer: (E) N-methylethanamine

Solution:

Step 1: Understanding the Concept

The Carbylamine reaction (also known as the Hoffmann isocyanide test) is a specific test used to detect the presence of **primary (1°) amines**. In this reaction, a primary amine reacts with chloroform and alcoholic KOH to produce a foul-smelling isocyanide (carbylamine).

Step 2: Detailed Explanation

1. Primary Amines: Ethanamine (1°), Benzamine/Aniline (1°), Prop-2-amine (1°), and Propan-1-amine (1°) all possess the $-NH_2$ group and will give a positive carbylamine test. 2. Secondary and Tertiary Amines: These do not undergo this reaction. 3. N-methylethanamine: The structure is $CH_3-NH-CH_2CH_3$. This is a **secondary (2°) amine** because the nitrogen is attached to two carbon atoms. Therefore, it will not form a carbylamine.

Step 3: Final Answer

The compound that does not form carbylamine is N-methylethanamine.

Quick Tip: To identify a primary amine, look for the suffix "-amine" without any "N-methyl" or "N,N-dimethyl" prefixes. If the nitrogen is bonded to only one carbon, it's primary.

5. Which transition metal has more than one metallic structure at normal temperature?

(A) Cr
(B) Ni

(C) Mn

(D) V

(E) Cu

Correct Answer: (C) Mn

Solution:

Step 1: Understanding the Concept

This question refers to **allotropy** or polymorphism in transition metals. Some metals can exist in different crystalline lattice arrangements (structures) depending on the conditions, even near room temperature or over their stable range.

Step 2: Detailed Explanation

1. Most transition metals like Cr (BCC), Ni (FCC), V (BCC), and Cu (FCC) typically exhibit a single stable metallic structure at standard conditions. 2. Manganese (Mn) is unique among the 3d transition series for its complex structural behavior. It has four allotropic forms $(\alpha, \beta, \gamma, \delta)$. The α-Mn and β-Mn forms are particularly complex and stable at lower temperatures compared to the simpler structures of its neighbors.

Step 3: Final Answer

Manganese (Mn) is the transition metal with more than one metallic structure.

> **Quick Tip:** Manganese often shows "anomalous" physical properties (like lower melting point than expected) compared to its neighbors due to its stable d^5 configuration and complex crystal structures.

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6. An organic compound $C_5H_{10}O$ does not reduces Tollen's reagent but forms addition compound with sodium hydrogen sulphite and gives the Iodoform test. On vigorous oxidation, it gives ethanoic acid and propanoic acid.

Correct Answer: Pentan-2-one

Solution:

Step 1: Understanding the Concept

We use the chemical tests to identify functional groups: 1. Does not reduce Tollen's reagent → It is

not an aldehyde; it must be a **ketone**. 2. Forms addition compound with $NaHSO_3$ → Confirms a **carbonyl group** (usually methyl ketones or aldehydes). 3. Gives Iodoform test → Confirms it is a **methyl ketone** (contains the CH_3CO- group).

Step 2: Detailed Explanation

1. The formula $C_5H_{10}O$ suggests a saturated ketone. Since it is a methyl ketone, the carbonyl must be at the second carbon. Possible structure: **Pentan-2-one** ($CH_3-CO-CH_2-CH_2-CH_3$). 2. Vigorous Oxidation: Ketones undergo cleavage of C–C bonds during vigorous oxidation (Popoff's Rule). 3. Pentan-2-one ($CH_3-CO-C_3H_7$) cleaves such that the carbonyl group stays with the smaller alkyl group. 4. Cleavage results in: - CH_3-COOH (Ethanoic acid) - CH_3-CH_2-COOH (Propanoic acid) 5. This matches the products mentioned in the question.

Step 3: Final Answer

The organic compound is Pentan-2-one.

> **Quick Tip:** The Iodoform test is the "smoking gun" for the CH_3CO- group. If a 5-carbon ketone gives the iodoform test, it must be pentan-2-one, as pentan-3-one is symmetrical and lacks the methyl ketone group.

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10. Spin only magnetic moment given not correct is

(A) Ni^{2+} (4.73)
(B) Fe^{2+} (4.90)
(C) Ti^{2+} (2.84)
(D) CO^{2+} (3.89)
(E) Mg^{2+} (5.92)

Correct Answer: (A) Ni^{2+} (4.73)

Solution:

Step 1: Understanding the Concept:

The "spin-only" magnetic moment (μ) of a transition metal ion is determined by the number of unpaired electrons (n) in its d-orbital.

Step 2: Key Formula or Approach:

$$\mu = \sqrt{n(n+2)} \text{ BM}$$

Step 3: Detailed Explanation:

1. For Ni^{2+} (Atomic No. 28): Configuration is $[Ar]3d^8$. It has 2 unpaired electrons ($n = 2$).

$$\mu = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83 \text{ BM}$$

Option (A) lists 4.73, which is incorrect.

2. For Fe^{2+} (Atomic No. 26): Configuration is $[Ar]3d^6$. It has 4 unpaired electrons ($n = 4$).

$$\mu = \sqrt{4(4+2)} = \sqrt{24} \approx 4.90 \text{ BM}$$

(Correct)

3. For Ti^{2+} (Atomic No. 22): Configuration is $[Ar]3d^2$. It has 2 unpaired electrons ($n = 2$).

$$\mu = \sqrt{2(2+2)} = \sqrt{8} \approx 2.84 \text{ BM}$$

(Correct)

4. For Co^{2+} (Atomic No. 27): Configuration is $[Ar]3d^7$. It has 3 unpaired electrons ($n = 3$).

$$\mu = \sqrt{3(3+2)} = \sqrt{15} \approx 3.87 \text{ BM}$$

(Correct)

5. For Mn^{2+} (Assuming E is Mn instead of Mg based on value): Configuration is $[Ar]3d^5$. It has 5 unpaired electrons ($n = 5$).

$$\mu = \sqrt{5(5+2)} = \sqrt{35} \approx 5.92 \text{ BM}$$

(Correct)

Step 4: Final Answer:

The incorrect magnetic moment is given for Ni^{2+}.

Quick Tip: A quick shortcut: the magnetic moment value always starts with the same digit as the number of unpaired electrons. For example, 2 unpaired electrons $\approx$ 2.8 BM; 3 unpaired electrons $\approx$ 3.8 BM.

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11. Minimum energy required to remove an atom from sodium is 3.313×10^{-19} J. Maximum wavelength of radiation that will get photoelectron

Correct Answer: 6000 Å

Solution:

Step 1: Understanding the Concept:

The minimum energy required to remove an electron is the work function (ϕ_0). The maximum wavelength (threshold wavelength, λ_0) corresponds to this minimum energy, as energy and wavelength are inversely proportional.

Step 2: Key Formula or Approach:

$$E = \frac{hc}{\lambda} \implies \lambda_0 = \frac{hc}{\phi_0}$$

Where $h = 6.626 \times 10^{-34}$ Js and $c = 3 \times 10^8$ m/s.

Step 3: Detailed Explanation:

1. Given Work Function $\phi_0 = 3.313 \times 10^{-19}$ J.

2. Calculate λ_0:

$$\lambda_0 = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{3.313 \times 10^{-19}}$$

3. Notice that $6.626/3.313 = 2$:

$$\lambda_0 = 2 \times 3 \times 10^{-34+8+19}$$

$$\lambda_0 = 6 \times 10^{-7} \text{ m}$$

4. Convert to Angstroms:

$$6 \times 10^{-7} \text{ m} = 6000 \times 10^{-10} \text{ m} = 6000 \text{ Å}$$

Step 4: Final Answer:

The maximum wavelength required is 6000 Å.

Quick Tip: In competitive exams, remember the constant $hc \approx 12400$ eV · Å or 19.8×10^{-26} J · m to speed up calculations.

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12. Find the emf of the reaction at 298K

$$Ni_{(s)} + 2Ag^+_{(2\times10^{-3}M)} \rightarrow Ni^{2+}_{(4\times10^{-2}M)} + 2Ag_{(s)}$$

($E^\circ_{cell} = 1.5$ at 298K)

Correct Answer: 1.38 V

Solution:

Step 1: Understanding the Concept:

The emf of a cell under non-standard conditions is calculated using the Nernst Equation, which accounts for the concentrations of the reactants and products.

Step 2: Key Formula or Approach:

$$E_{cell} = E^\circ_{cell} - \frac{0.0591}{n} \log Q$$

Where n is the number of electrons transferred and Q is the reaction quotient.

Step 3: Detailed Explanation:

1. Identify n: In the reaction $Ni \rightarrow Ni^{2+} + 2e^-$ and $2Ag^+ + 2e^- \rightarrow 2Ag$, $n = 2$.

2. Write Q:

$$Q = \frac{[Ni^{2+}]}{[Ag^+]^2} = \frac{4 \times 10^{-2}}{(2 \times 10^{-3})^2}$$

$$Q = \frac{4 \times 10^{-2}}{4 \times 10^{-6}} = 10^4$$

3. Substitute into Nernst Equation:

$$E_{cell} = 1.5 - \frac{0.0591}{2}\log(10^4)$$

$$E_{cell} = 1.5 - (0.0295 \times 4)$$

$$E_{cell} = 1.5 - 0.118 = 1.382 \text{ V}$$

Step 4: Final Answer:

The emf of the reaction is 1.38 V.

> **Quick Tip:** Always remember to square the concentration of the ion if its stoichiometric coefficient in the balanced equation is 2 (like $[Ag^+]^2$ in this case).

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13. $X_2 + O_2 \rightleftharpoons 2XO$ Concentration of X_2 and O_2 are 4×10^{-3} and 3×10^{-3} respectively. Equilibrium concentration of XO ($K_c = 0.5$)

Correct Answer: 2.45×10^{-3}

Solution:

Step 1: Understanding the Concept:

For a chemical reaction at equilibrium, the equilibrium constant (K_c) is the ratio of the product of the molar concentrations of the products to that of the reactants, each raised to the power of their stoichiometric coefficients.

Step 2: Key Formula or Approach:

For the reaction $A + B \rightleftharpoons 2C$:

$$K_c = \frac{[C]^2}{[A][B]}$$

Step 3: Detailed Explanation:

1. Given equilibrium concentrations: $[X_2] = 4 \times 10^{-3}$ M and $[O_2] = 3 \times 10^{-3}$ M.

2. Given $K_c = 0.5$.

3. Set up the expression for K_c:

$$0.5 = \frac{[XO]^2}{(4 \times 10^{-3})(3 \times 10^{-3})}$$

4. Solve for $[XO]^2$:

$$[XO]^2 = 0.5 \times (12 \times 10^{-6})$$

$$[XO]^2 = 6 \times 10^{-6}$$

5. Calculate $[XO]$:

$$[XO] = \sqrt{6 \times 10^{-6}} = \sqrt{6} \times 10^{-3}$$

$$[XO] \approx 2.45 \times 10^{-3} \text{ M}$$

Step 4: Final Answer:

The equilibrium concentration of XO is 2.45×10^{-3} M.

> **Quick Tip:** Always ensure the units are consistent and the concentration of products is squared in the numerator if the stoichiometric coefficient is 2.

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14. Which pairs have ability to form p-p multiple bonds

(A) C and O

(B) B and N

(C) N and P

(D) F and Cl

(E) C and Si

Correct Answer: (A) C and O

Solution:

Step 1: Understanding the Concept:

The ability to form $p\pi - p\pi$ multiple bonds is primarily found in second-period elements (C, N, O). This is because their p-orbitals are small and can overlap effectively sideways. Higher period elements have larger, more diffuse orbitals, leading to poor overlap.

Step 2: Detailed Explanation:

1. Carbon and Oxygen (A): Both are second-period elements. They readily form stable double and triple bonds (e.g., in CO_2, CO, and carbonyl groups) via $p\pi - p\pi$ overlap.
2. Boron and Nitrogen (B): While they can form bonds, B is often electron-deficient, and they typically form coordinate covalent bonds rather than standard stable $p\pi - p\pi$ multiple bonds in the same way C and O do.
3. Others (C, D, E): Pairs like N and P or C and Si involve a third-period element (P, Si). The larger size of 3p orbitals makes $p\pi - p\pi$ bonding much weaker and less common.

Step 3: Final Answer:

The pair C and O has the best ability to form $p\pi - p\pi$ multiple bonds.

> **Quick Tip:** Remember that $p\pi - p\pi$ bonding is a "small atom" game. Elements from the 3rd period and below generally prefer single bonds or involve d-orbitals ($p\pi - d\pi$) if multiple bonding occurs.

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15. Pyridinium chlorochromate is a complex of ....

Correct Answer: Chromium(VI) oxide, Pyridine, and HCl

Solution:

Step 1: Understanding the Concept:

Pyridinium chlorochromate, commonly known as PCC, is a standard reagent used in organic chemistry for the controlled oxidation of alcohols.

Step 2: Detailed Explanation:

1. PCC is prepared by the reaction of pyridine (C_5H_5N), chromium trioxide (CrO_3), and concentrated hydrochloric acid (HCl).
2. Chemically, it consists of a pyridinium cation $[C_5H_5NH]^+$ and a chlorochromate anion $[CrO_3Cl]^-$.
3. Its primary use is to oxidize primary alcohols to aldehydes without further oxidation to carboxylic acids, and secondary alcohols to ketones.

Step 3: Final Answer:

Pyridinium chlorochromate is a complex of Chromium(VI) oxide, Pyridine, and Hydrochloric acid.

> **Quick Tip:** PCC is favored because it is soluble in organic solvents like dichloromethane (DCM), allowing for oxidation under anhydrous conditions.

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16. In Chemotherapy, ligand used to remove excess of Cu

Correct Answer: D-Penicillamine

Solution:

Step 1: Understanding the Concept:

The removal of toxic excess metals from the body is achieved through chelation therapy. A chelating agent (ligand) binds to the metal ion to form a stable, water-soluble complex that can be excreted safely.

Step 2: Detailed Explanation:

1. Excess copper in the body is associated with conditions like Wilson's disease.

2. D-Penicillamine is the specific ligand used for this purpose. It contains amino, carboxyl, and thiol ($-SH$) groups that coordinate with Cu^{2+} ions.

3. Other common chelating agents include EDTA (for Lead poisoning) and Desferrioxamine B (for Iron excess).

Step 3: Final Answer:

The ligand used to remove excess Copper is D-Penicillamine.

> **Quick Tip:** Remember: Penicillamine for Copper, EDTA for Lead, and BAL (British Anti-Lewisite) for Mercury/Arsenic.

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17. IUPAC name of

$$CH_3-CH(OH)-CH_2-CH(OH)-CH(C_2H_5)-CH_2-CH_3$$

Correct Answer: 5-ethylheptane-2,4-diol

Solution:

Step 1: Understanding the Concept:

IUPAC nomenclature for polyfunctional alcohols requires identifying the longest carbon chain containing the $-OH$ groups and numbering it to give the hydroxyl groups the lowest possible locants.

Step 2: Detailed Explanation:

1. Identify the Longest Chain: The chain containing both hydroxyl groups and the ethyl branch. - Starting from the left: $C_1(H_3)-C_2H(OH)-C_3H_2-C_4H(OH)-C_5H(C_2H_5)-C_6H_2-C_7H_3$. - This is a 7-carbon chain (Heptane).

2. Numbering: Numbering from the left gives the $-OH$ groups positions 2 and 4. Numbering from the right would give them 4 and 6. Therefore, we number from the left.

3. Substituents: There is an ethyl group ($-C_2H_5$) at position 5.

4. Assembly: 5-ethyl + heptane + 2,4-diol = 5-ethylheptane-2,4-diol.

Step 3: Final Answer:

The IUPAC name is 5-ethylheptane-2,4-diol.

> **Quick Tip:** When multiple $-OH$ groups are present, keep the "e" in the alkane name (e.g., heptan**e**diol) and use numerical prefixes like diol or triol.

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18. Which are carcinogenic hydrocarbon

(i) 1,2-Benzanthracene

(ii) pent-1-yne

(iii) 1,2-Benzpyrene

(iv) cyclohexane

(v) 3-methylcholanthrene

Correct Answer: (i), (iii), (v)

Solution:

Step 1: Understanding the Concept:

Carcinogenic hydrocarbons are substances that can cause cancer. These are typically Polynuclear Aromatic Hydrocarbons (PAHs) formed during the incomplete combustion of organic matter like tobacco, coal, and petroleum.

Step 2: Detailed Explanation:

1. Polynuclear Aromatic Hydrocarbons (PAHs): Molecules containing multiple fused benzene rings are often toxic and carcinogenic. - (i) 1,2-Benzanthracene: A classic PAH known for carcinogenic properties. - (iii) 1,2-Benzpyrene: Found in cigarette smoke and coal tar; highly carcinogenic. - (v) 3-methylcholanthrene: A potent laboratory carcinogen. 2. Other Hydrocarbons: Simple alkanes (cyclohexane) and alkynes (pent-1-yne) are generally not considered carcinogenic in this context.

Step 3: Final Answer:

The carcinogenic hydrocarbons are 1,2-Benzanthracene, 1,2-Benzpyrene, and 3-methylcholanthrene.

> **Quick Tip:** Most carcinogenic hydrocarbons mentioned in standard chemistry textbooks are those with four or more fused benzene rings.

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19. Metals used in preparation of dihydrogen in lab

Correct Answer: Granulated Zinc

Solution:

Step 1: Understanding the Concept:

In the laboratory, dihydrogen gas (H_2) is usually prepared by the reaction of dilute acids with metals that are more reactive than hydrogen in the electrochemical series.

Step 2: Key Formula or Approach:

The general reaction is: $\text{Metal} + \text{Dilute Acid} \rightarrow \text{Salt} + H_2(g)$

Step 3: Detailed Explanation:

1. Granulated Zinc is the most preferred metal for this reaction. It is reacted with dilute hydrochloric

acid (HCl) or dilute sulphuric acid (H_2SO_4).

$$Zn(s) + 2HCl(aq) \rightarrow ZnCl_2(aq) + H_2(g)$$

2. Zinc is chosen because it reacts at a moderate, controllable rate. 3. Pure Zinc is usually not used because the reaction is too slow; the "granulated" form contains small amounts of impurities (like copper) which act as a catalyst to speed up the reaction.

Step 4: Final Answer:

Granulated Zinc is the metal most commonly used in the lab preparation of dihydrogen.

> **Quick Tip:** Do not use highly reactive metals like Sodium or Potassium in the lab to make H_2, as the reaction is explosive. Similarly, avoid metals like Copper or Silver as they do not react with dilute acids to release H_2.

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20. Volume of methanol to make 2L of 0.4M solution (density = 0.64 kg/L, Molar Mass = 32 g/mol)

Correct Answer: 40 mL

Solution:

Step 1: Understanding the Concept:

To find the volume of a pure liquid required to make a solution, we first calculate the required mass of the solute using molarity and then convert that mass to volume using the given density.

Step 2: Key Formula or Approach:

1. Moles = Molarity × Volume (L)

2. Mass = Moles × Molar Mass

3. Volume of solute = $\frac{\text{Mass}}{\text{Density}}$

Step 3: Detailed Explanation:

1. Calculate Moles of Methanol needed:

$$n = 0.4 \text{ mol/L} \times 2 \text{ L} = 0.8 \text{ moles}$$

2. Calculate Mass of Methanol needed:

$$\text{Mass} = 0.8 \text{ mol} \times 32 \text{ g/mol} = 25.6 \text{ g}$$

3. Convert Density to g/mL:

$$\text{Density} = 0.64 \text{ kg/L} = 640 \text{ g/L} = 0.64 \text{ g/mL}$$

4. Calculate Volume of Methanol:

$$V = \frac{25.6 \text{ g}}{0.64 \text{ g/mL}} = \frac{2560}{64} = 40 \text{ mL}$$

Step 4: Final Answer:

The volume of methanol required is 40 mL.

> **Quick Tip:** Always check units for density. 0.64 kg/L is the same as 0.64 g/mL. Recognizing this immediately saves a lot of time in decimal shifts!

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21. Enthalpy of combustion of benzene, graphite, dihydrogen are -3260, -390 and -290 kJ/mol. Find the enthalpy of formation of benzene

Correct Answer: +50 kJ/mol

Solution:

Step 1: Understanding the Concept:

According to Hess's Law, the enthalpy of formation of a compound can be calculated from the enthalpies of combustion of its constituent elements and the compound itself.

Step 2: Key Formula or Approach:

For the formation reaction: $6C(graphite) + 3H_2(g) \rightarrow C_6H_6(l)$

$$\Delta_f H^\circ = \sum \Delta_c H^\circ(\text{reactants}) - \sum \Delta_c H^\circ(\text{products})$$

Step 3: Detailed Explanation:

1. List given enthalpies of combustion ($\Delta_c H^\circ$): - $C(graphite) : -390$ kJ/mol - $H_2(g) : -290$ kJ/mol - $C_6H_6(l) : -3260$ kJ/mol 2. Apply the formula for formation of 1 mole of Benzene:

$$\Delta_f H^\circ = [6 \times (-390) + 3 \times (-290)] - [-3260]$$

3. Calculate the values: - $6 \times -390 = -2340$ - $3 \times -290 = -870$ - Sum of reactant combustion: $-2340 - 870 = -3210$ 4. Final Calculation:

$$\Delta_f H^\circ = -3210 + 3260 = +50 \text{ kJ/mol}$$

Step 4: Final Answer:

The enthalpy of formation of benzene is $+50$ kJ/mol.

Quick Tip: Remember: $\Delta H = \text{Combustion of Reactants} - \text{Combustion of Products}$. This is the **reverse** of the standard formula used for Enthalpy of Formation $(\text{Prod} - \text{React})$.

MATHEMATICS

1. If $9P_5 = 504\ (6P_\lambda)$**. Find** λ

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:

This problem involves permutations, which represent the number of ways to arrange a subset of items from a larger set. To find the unknown value λ, we need to evaluate the known permutation on the left side and solve the resulting algebraic equation.

Step 2: Key Formula or Approach:

The formula for permutations of n items taken r at a time is:

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

Step 3: Detailed Explanation:

1. First, evaluate the left-hand side of the equation, $9P_5$:

$$9P_5 = 9 \times 8 \times 7 \times 6 \times 5 = 15120$$

2. Substitute this value back into the original equation:

$$15120 = 504 \times (6P_\lambda)$$

3. Divide both sides by 504 to isolate the permutation term:

$$6P_\lambda = \frac{15120}{504} = 30$$

4. Determine the value of λ by expanding the permutation $6P_\lambda$ until the product reaches 30: - If $\lambda = 1$, $6P_1 = 6$ - If $\lambda = 2$, $6P_2 = 6 \times 5 = 30$ Since the product matches, $\lambda = 2$.

Step 4: Final Answer:

The value of λ is 2.

Quick Tip: In permutation equations like $nP_r = X$, you can find r by starting with n and multiplying by descending integers $(n-1, n-2, \text{etc.})$ until the product equals X. The number of integers used is your answer for r.

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2. If $\vec{a} = 2\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b} = -5\hat{i} - \hat{j} + 8\hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - \lambda\hat{k}$. If $\vec{a} + \vec{b} + \vec{c}$ is perpendicular to $\vec{a} - \vec{b} + \vec{c}$, Find λ

Correct Answer: 12 or -4

Solution:

Step 1: Understanding the Concept:

Two non-zero vectors are perpendicular if and only if their dot product (scalar product) is zero. We must compute the two resultant vectors resulting from the addition and subtraction operations and then apply the condition for perpendicularity.

Step 2: Key Formula or Approach:

1. Vector addition/subtraction: Perform the operation component-wise. 2. Perpendicularity: $\vec{u}\cdot\vec{v}=0$.
3. Dot product: $\vec{u}\cdot\vec{v}=u_x v_x+u_y v_y+u_z v_z$.

Step 3: Detailed Explanation:

1. Find $\vec{u}=\vec{a}+\vec{b}+\vec{c}$:

$$\vec{u}=(2-5+3)\hat{i}+(-2-1+1)\hat{j}+(4+8-\lambda)\hat{k}$$

$$\vec{u}=0\hat{i}-2\hat{j}+(12-\lambda)\hat{k}$$

2. Find $\vec{v}=\vec{a}-\vec{b}+\vec{c}$:

$$\vec{v}=(2-(-5)+3)\hat{i}+(-2-(-1)+1)\hat{j}+(4-8-\lambda)\hat{k}$$

$$\vec{v}=10\hat{i}+0\hat{j}+(-4-\lambda)\hat{k}$$

3. Set the dot product $\vec{u}\cdot\vec{v}=0$:

$$(0)(10)+(-2)(0)+(12-\lambda)(-4-\lambda)=0$$

$$(12-\lambda)(-4-\lambda)=0$$

4. Solve the resulting equation:

$$-48-12\lambda+4\lambda+\lambda^2=0$$

$$\lambda^2-8\lambda-48=0$$

$$(\lambda-12)(\lambda+4)=0$$

The roots are $\lambda=12$ and $\lambda=-4$.

Step 4: Final Answer:

The values of λ are 12 and -4.

Quick Tip: Remember the algebraic identity $(A-B)(A+B)=A^2-B^2$. If you rearrange the vectors as $(\vec{a}+\vec{c})+\vec{b}$ and $(\vec{a}+\vec{c})-\vec{b}$, the dot product simplifies quickly to $|\vec{a}+\vec{c}|^2-|\vec{b}|^2=0$.

---

3. Find $\begin{vmatrix} 11 & 1 & 1 \\ 1 & 21 & 1 \\ 1 & 1 & 31 \end{vmatrix}$

Correct Answer: 7100

Solution:

Step 1: Understanding the Concept:

The determinant of a matrix represents a unique scalar value associated with that matrix. For a 3×3 determinant, we can calculate the value by expanding along any row or column.

Step 2: Key Formula or Approach:

Expansion by the first row:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei-fh)-b(di-fg)+c(dh-eg)$$

Step 3: Detailed Explanation:

1. Expand the determinant along Row 1:

$$D = 11\begin{vmatrix} 21 & 1 \\ 1 & 31 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 31 \end{vmatrix} + 1\begin{vmatrix} 1 & 21 \\ 1 & 1 \end{vmatrix}$$

2. Evaluate the 2×2 minors: - Minor 1: $(21\times31)-(1\times1)=651-1=650$ - Minor 2: $(1\times31)-(1\times1)=31-1=30$ - Minor 3: $(1\times1)-(21\times1)=1-21=-20$ 3. Substitute these values back into the expanded expression:

$$D = 11(650)-1(30)+1(-20)$$

$$D = 7150-30-20$$

$$D = 7100$$

Step 4: Final Answer:

The value of the determinant is 7100.

> **Quick Tip:** To simplify determinants with large numbers, apply row or column operations first. For example, $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$ will create zeros and smaller integers, making the expansion much faster.

---

4. Find the eqn of the line passing through (-1, 2, -4) and parallel to $\frac{-x-1}{4} = \frac{2y+1}{-1} = \frac{-z+4}{3}$

Correct Answer: $\frac{x+1}{-4} = \frac{y-2}{-1/2} = \frac{z+4}{-3}$

Solution:

Step 1: Understanding the Concept:

A line in 3D space is uniquely determined by a point through which it passes and a direction vector. If two lines are parallel, they share the same direction vector (or proportional vectors). We must first convert the given line equation into standard form to identify its true direction ratios.

Step 2: Key Formula or Approach:

1. Standard form of a line: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$, where (a, b, c) is the direction vector.
2. Equation of a line passing through (x_0, y_0, z_0) with direction ratios (a, b, c): $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$.

Step 3: Detailed Explanation:

1. Convert the given line equation to standard form:

$$\frac{-(x+1)}{4} = \frac{2(y+1/2)}{-1} = \frac{-(z-4)}{3}$$

2. Simplify by moving coefficients of x, y, z to the denominator:

$$\frac{x+1}{-4} = \frac{y+1/2}{-1/2} = \frac{z-4}{-3}$$

3. The direction ratios of the given line are $(-4, -1/2, -3)$. Since our required line is parallel, it shares these direction ratios.

4. The required line passes through $(-1, 2, -4)$. Substitute these into the point-direction form:

$$\frac{x-(-1)}{-4} = \frac{y-2}{-1/2} = \frac{z-(-4)}{-3}$$

$$\frac{x+1}{-4} = \frac{y-2}{-1/2} = \frac{z+4}{-3}$$

Step 4: Final Answer:

The equation of the line is $\frac{x+1}{-4} = \frac{y-2}{-1/2} = \frac{z+4}{-3}$.

> **Quick Tip:** Always ensure the coefficients of x, y, and z in the numerator are exactly +1 before identifying the direction ratios from the denominators.

---

5. Find the minimum value of $f(x) = \frac{x^{100}-1}{x^{100}+1}$

Correct Answer: -1

Solution:

Step 1: Understanding the Concept:

To find the minimum value of a function, we look at its behavior and range. For algebraic fractions involving even powers of x, we can use the fact that $x^{2n} \geq 0$ for all real numbers x.

Step 2: Key Formula or Approach:

Let $t = x^{100}$. Since the exponent 100 is even, $t \geq 0$. We can then analyze the function $g(t) = \frac{t-1}{t+1}$ for $t \in [0, \infty)$.

Step 3: Detailed Explanation:

1. Let $t = x^{100}$. The function becomes $f(t) = \frac{t-1}{t+1}$.

2. Rewrite the function to isolate the variable in the denominator:

$$f(t) = \frac{t+1-2}{t+1} = 1 - \frac{2}{t+1}$$

3. Since $t \geq 0$, the denominator $t+1$ is at its minimum when $t=0$. 4. When $t=0$, the term $\frac{2}{t+1}$ is at its **maximum** value: $\frac{2}{0+1}=2$. 5. Consequently, $f(t)=1-$(maximum value) will give the **minimum** value:

$$f(0)=1-2=-1$$

6. As $t \to \infty$, the term $\frac{2}{t+1} \to 0$, so $f(t) \to 1$. Therefore, -1 is indeed the absolute minimum.

Step 4: Final Answer:

The minimum value of the function is -1.

> **Quick Tip:** For any function of the form $\frac{u-k}{u+k}$ where $u \geq 0$, the minimum value occurs at the minimum possible value of u.

---

6. If $y=\frac{3x^3-2x^2+x}{|x|}$, $x \neq 0$ find $\frac{dy}{dx}$ at $x=-2$

Correct Answer: 12

Solution:

Step 1: Understanding the Concept:

The absolute value function $|x|$ is defined piece-wise. Since we are asked for the derivative at $x=-2$, we only need to consider the behavior of the function for negative values of x, where $|x|=-x$.

Step 2: Key Formula or Approach:

1. If $x<0$, then $|x|=-x$.

2. Use basic differentiation rules: $\frac{d}{dx}(x^n)=nx^{n-1}$.

Step 3: Detailed Explanation:

1. For $x=-2$, the value is negative. Thus, we substitute $|x|=-x$ into the equation:

$$y=\frac{3x^3-2x^2+x}{-x}$$

2. Simplify the expression by dividing each term in the numerator by $-x$:

$$y=-3x^2+2x-1$$

3. Now, differentiate y with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(-3x^2 + 2x - 1)$$

$$\frac{dy}{dx} = -6x + 2$$

4. Evaluate the derivative at $x = -2$:

$$\left.\frac{dy}{dx}\right|_{x=-2} = -6(-2) + 2$$

$$\frac{dy}{dx} = 12 + 2 = 14$$

Step 4: Final Answer:

The value of $\frac{dy}{dx}$ at $x = -2$ is 14.

> **Quick Tip:** Always simplify the function using the specific domain constraints (in this case $x < 0$) before differentiating. It turns a complicated absolute value fraction into a simple polynomial.

---

7. $\int \frac{\sin(\cot^{-1} x)}{1+x^2}\, dx$

Correct Answer: $-\sqrt{\frac{1}{1+x^2}} + C$ (or $\cos(\cot^{-1} x) + C$)

Solution:

Step 1: Understanding the Concept:

This is an integration problem involving inverse trigonometric functions. We can use the method of substitution because the derivative of the inner function ($\cot^{-1} x$) is present in the integrand.

Step 2: Key Formula or Approach:

1. Derivative of inverse cotangent: $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$.
2. Standard integral: $\int \sin(u)\, du = -\cos(u) + C$.

Step 3: Detailed Explanation:

1. Let $t = \cot^{-1} x$. 2. Differentiating both sides with respect to x:

$$dt = -\frac{1}{1+x^2}dx \implies -dt = \frac{1}{1+x^2}dx$$

3. Substitute these into the integral:

$$\int \sin(t)(-dt) = -\int \sin(t)dt$$

4. Integrate:

$$-(-\cos t) + C = \cos t + C$$

5. Substitute back $t = \cot^{-1} x$:

$$\text{Final Result} = \cos(\cot^{-1} x) + C$$

6. To simplify further, if $\cot\theta = x$, then $\cos\theta = \frac{x}{\sqrt{1+x^2}}$.

$$\text{Result} = \frac{x}{\sqrt{1+x^2}} + C$$

Step 4: Final Answer:

The integral is $\frac{x}{\sqrt{1+x^2}} + C$.

Quick Tip: Whenever you see an inverse trig function alongside its derivative (like $\frac{1}{1+x^2}$ for $\tan^{-1}$ or $\cot^{-1}$), substitution is almost always the fastest path to the solution.

---

8. Find the number of terms in 2, 6, 18...1458

Correct Answer: 7

Solution:

Step 1: Understanding the Concept:

The given sequence is a Geometric Progression (G.P.) because the ratio between consecutive terms is constant. We need to find the total count of terms (n) using the formula for the general term.

Step 2: Key Formula or Approach:

The n^{th} term of a G.P. is given by:

$$a_n = a \cdot r^{n-1}$$

where a is the first term and r is the common ratio.

Step 3: Detailed Explanation:

1. Identify the parameters: - First term $(a) = 2$. - Common ratio $(r) = 6/2 = 3$. - Last term $(a_n) = 1458$. 2. Substitute into the formula:

$$1458 = 2 \cdot 3^{n-1}$$

3. Divide both sides by 2:

$$729 = 3^{n-1}$$

4. Express 729 as a power of 3: - $3^1 = 3, 3^2 = 9, 3^3 = 27, 3^4 = 81, 3^5 = 243, 3^6 = 729$.

$$3^6 = 3^{n-1}$$

5. Equating the exponents:

$$6 = n - 1 \implies n = 7$$

Step 4: Final Answer:

The number of terms in the sequence is 7.

> **Quick Tip:** To quickly find the power of 3 for numbers like 729, remember that $3^3 = 27$ and $27 \times 27 = 729$. Therefore, $3^3 \times 3^3 = 3^6$.

---

9. Find the domain of $\frac{\log(x-5)}{x^2+3x-4}$

Correct Answer: $(5, \infty)$

Solution:

Step 1: Understanding the Concept:

The domain of a function is the set of all real values of x for which the function is defined. For this function, two conditions must be met: the argument of the logarithm must be positive, and the denominator must not be zero.

Step 2: Detailed Explanation:

1. Condition 1 (Logarithm): The value inside $\log(x-5)$ must be greater than zero.

$$x-5>0 \implies x>5$$

This gives the interval $(5,\infty)$.

2. Condition 2 (Denominator): The denominator x^2+3x-4 cannot be zero. - Factor the quadratic: $(x+4)(x-1)\neq 0$. - This means $x\neq -4$ and $x\neq 1$.

3. Intersection of Conditions: - From condition 1, we need $x>5$. - From condition 2, we must exclude $x=-4$ and $x=1$. - Since both -4 and 1 are already outside the range $x>5$, they do not impose additional restrictions on the interval $(5,\infty)$.

Step 3: Final Answer:

The domain of the function is $(5,\infty)$.

> **Quick Tip:** When finding domains of fractions with logs, always check the log condition first. Often, the log's restriction is so strong that it automatically excludes the values that make the denominator zero.

---

10. $\lim_{x\to 0}\left[\frac{\sin^2 x}{1-\cos x}\right]$

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:

To evaluate this limit, we recognize that substituting $x=0$ directly results in an indeterminate form of $\frac{0}{0}$. We can solve this by using trigonometric identities to simplify the expression or by applying L'Hôpital's Rule.

Step 2: Key Formula or Approach:

1. Trigonometric identity: $\sin^2 x = 1 - \cos^2 x$.

2. Difference of squares: $a^2 - b^2 = (a-b)(a+b)$.

Step 3: Detailed Explanation:

1. Substitute $\sin^2 x$ with $1 - \cos^2 x$:

$$\lim_{x\to 0} \frac{1-\cos^2 x}{1-\cos x}$$

2. Apply the difference of squares formula to the numerator:

$$\lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{1-\cos x}$$

3. Cancel the common term $(1-\cos x)$ from the numerator and denominator (since $x \to 0$, $\cos x \neq 1$):

$$\lim_{x\to 0}(1+\cos x)$$

4. Substitute $x = 0$:

$$1 + \cos(0) = 1 + 1 = 2$$

Step 4: Final Answer:

The value of the limit is 2.

Quick Tip: When you see $1-\cos x$ in a limit, consider multiplying the numerator and denominator by its conjugate $1+\cos x$. This often transforms the denominator into $\sin^2 x$, allowing for easy simplification.

---

11. If $\tan\alpha = \frac{5}{6}$, $\tan\beta = \frac{1}{11}$ $(0 < \alpha, \beta < \frac{\pi}{2})$, find $\alpha + \beta$

Correct Answer: $\frac{\pi}{4}$ (or 45°)

Solution:

Step 1: Understanding the Concept:

To find the sum of two angles when their tangents are known, we use the compound angle formula for tangent. This allows us to find $\tan(\alpha+\beta)$ and subsequently determine the angle.

Step 2: Key Formula or Approach:

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Step 3: Detailed Explanation:

1. Substitute the given values $\tan\alpha = 5/6$ and $\tan\beta = 1/11$ into the formula:

$$\tan(\alpha+\beta) = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \left(\frac{5}{6}\cdot\frac{1}{11}\right)}$$

2. Simplify the numerator:

$$\text{Numerator} = \frac{55+6}{66} = \frac{61}{66}$$

3. Simplify the denominator:

$$\text{Denominator} = 1 - \frac{5}{66} = \frac{66-5}{66} = \frac{61}{66}$$

4. Calculate the ratio:

$$\tan(\alpha+\beta) = \frac{61/66}{61/66} = 1$$

5. Since $0 < \alpha, \beta < \pi/2$, the sum $\alpha+\beta$ must result in an angle where the tangent is 1.

$$\alpha+\beta = \tan^{-1}(1) = \frac{\pi}{4}$$

Step 4: Final Answer:

The sum $\alpha+\beta$ is $\frac{\pi}{4}$.

Quick Tip: If the product of the numerators plus the product of the denominators equals the same value across terms in a tan addition problem, the result is often 1, which points toward an answer of 45°.

---

12. Find the sum of all 3 digit numbers using the digits 1, 2, 3, 4 without repetition

Correct Answer: 6660

Solution:

Step 1: Understanding the Concept:

When forming numbers with a set of digits, each digit appears in the units, tens, and hundreds places an equal number of times. We can calculate the sum by finding how often each digit contributes to each place value.

Step 2: Key Formula or Approach:

1. Total numbers possible: ${}^nP_r$. For 4 digits taking 3 at a time, $4 \times 3 \times 2 = 24$ numbers.
2. Frequency of each digit in each position: $\frac{\text{Total numbers}}{\text{Total digits}} = \frac{24}{4} = 6$ times.
3. Sum formula: (Sum of digits) $\times$ (Frequency) $\times$ (111...r times).

Step 3: Detailed Explanation:

1. Sum of the given digits: $1 + 2 + 3 + 4 = 10$.
2. Each digit appears in the hundreds, tens, and units place exactly 6 times. 3. Sum of units place: $6 \times 10 = 60$. 4. Sum of tens place: $6 \times 10 \times 10 = 600$. 5. Sum of hundreds place: $6 \times 10 \times 100 = 6000$.
6. Total sum: $6000 + 600 + 60 = 6660$.

Step 4: Final Answer:

The sum of all such 3-digit numbers is 6660.

> **Quick Tip:** The general formula for the sum of all r-digit numbers formed using n non-zero digits is:
>
> $$(n-1)P(r-1) \times (\text{Sum of digits}) \times \left(\frac{10^r - 1}{9}\right)$$

---

13. $\int_0^1 \left[\tan^{-1}\left(\frac{1}{1+x+x^2+x^3}\right) + \tan^{-1}(1+x+x^2+x^3)\right] dx$

Correct Answer: $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

This problem uses a fundamental property of inverse trigonometric functions. The expression inside the integral is a sum of the form $\tan^{-1}(1/u) + \tan^{-1}(u)$.

Step 2: Key Formula or Approach:

For any $u > 0$:

$$\tan^{-1}(u) + \tan^{-1}\left(\frac{1}{u}\right) = \frac{\pi}{2}$$

Step 3: Detailed Explanation:

1. Let $u = 1 + x + x^2 + x^3$. Since x ranges from 0 to 1, u is clearly positive. 2. The integrand simplifies directly:

$$\tan^{-1}\left(\frac{1}{u}\right) + \tan^{-1}(u) = \frac{\pi}{2}$$

3. Now, evaluate the integral:

$$\int_0^1 \frac{\pi}{2} dx = \frac{\pi}{2}[x]_0^1$$

$$\frac{\pi}{2}(1-0) = \frac{\pi}{2}$$

Step 4: Final Answer:

The value of the integral is $\frac{\pi}{2}$.

> **Quick Tip:** When an integrand looks extremely complex with algebraic terms nested inside inverse trig functions, check for identities like $\tan^{-1} u + \cot^{-1} u = \pi/2$ or $\tan^{-1} u + \tan^{-1}(1/u) = \pi/2$ first.

---

14. Find the differential equation of $y = Ae^x + Be^{-2x}$

Correct Answer: $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

Solution:

Step 1: Understanding the Concept:

To form a differential equation from a general solution, we differentiate the equation as many times as there are arbitrary constants (in this case, two: A and B) and then eliminate those constants.

Step 2: Key Formula or Approach:

1. First derivative: y'
2. Second derivative: y''
3. Elimination of A and B.

Step 3: Detailed Explanation:

1. Given: $y = Ae^x + Be^{-2x}$ (Eq. 1) 2. Differentiate once:

$$\frac{dy}{dx} = Ae^x - 2Be^{-2x} \quad \text{(Eq. 2)}$$

3. Differentiate again:

$$\frac{d^2y}{dx^2} = Ae^x + 4Be^{-2x} \quad \text{(Eq. 3)}$$

4. Now, solve for constants or use substitution. From (Eq. 1), $Ae^x = y - Be^{-2x}$. Substitute this into (Eq. 2):

$$y' = (y - Be^{-2x}) - 2Be^{-2x} \implies y' = y - 3Be^{-2x} \implies 3Be^{-2x} = y - y'$$

5. Substitute Ae^x and Be^{-2x} into (Eq. 3):

$$y'' = (y - Be^{-2x}) + 4Be^{-2x} = y + 3Be^{-2x}$$

6. Replace $3Be^{-2x}$ with $(y - y')$:

$$y'' = y + (y - y') \implies y'' + y' - 2y = 0$$

Step 4: Final Answer:

The differential equation is $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$.

Quick Tip: For solutions of the form $y = Ae^{m_1x} + Be^{m_2x}$, the differential equation is always $y'' - (m_1 + m_2)y' + (m_1m_2)y = 0$. Here $m_1 = 1, m_2 = -2$, so $y'' - (-1)y' + (-2)y = 0$.

---

15. Solve $5 < |x-1| < 15$

Correct Answer: $x \in (-14, -4) \cup (6, 16)$

Solution:

Step 1: Understanding the Concept:

An absolute value inequality $a < |u| < b$ describes two separate regions on the number line. It translates to saying that the distance of u from zero is between a and b.

Step 2: Key Formula or Approach:

The inequality $a < |u| < b$ is equivalent to:

$$u \in (-b, -a) \cup (a, b)$$

Step 3: Detailed Explanation:

1. Let $u = x - 1$. Our inequality is $5 < |u| < 15$. 2. Case 1: Positive range

$$5 < x - 1 < 15$$

Add 1 to all parts:

$$6 < x < 16 \implies x \in (6, 16)$$

3. Case 2: Negative range

$$-15 < x - 1 < -5$$

Add 1 to all parts:

$$-14 < x < -4 \implies x \in (-14, -4)$$

4. Combine both sets of solutions using the union symbol.

Step 4: Final Answer:

The solution is $x \in (-14, -4) \cup (6, 16)$.

Quick Tip: Think of $|x-1|$ as "the distance of x from 1". The question is asking for all points whose distance from 1 is more than 5 but less than 15. This naturally creates two "belts" on either side of 1.

---

16. Find the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$

Correct Answer: $\frac{24}{25}$

Solution:

Step 1: Understanding the Concept:

This problem requires the use of the double-angle identity for sine. We treat the inverse sine function as an angle θ, find the corresponding trigonometric values, and then apply the identity.

Step 2: Key Formula or Approach:

1. Let $\theta = \sin^{-1}\frac{3}{5}$, which implies $\sin\theta = \frac{3}{5}$.
2. Identity: $\sin(2\theta) = 2\sin\theta\cos\theta$.

Step 3: Detailed Explanation:

1. Since $\sin\theta = \frac{3}{5}$ and the principal value range for $\sin^{-1}$ is $[-\pi/2, \pi/2]$, $\cos\theta$ must be positive. 2. Using the identity $\cos^2\theta = 1-\sin^2\theta$:

$$\cos\theta = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

3. Substitute $\sin\theta$ and $\cos\theta$ into the double-angle formula:

$$\sin(2\theta) = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$

$$\sin(2\theta) = \frac{24}{25}$$

Step 4: Final Answer:

The value is $\frac{24}{25}$.

Quick Tip: For any $\sin^{-1}x$, the term $\sin(2\sin^{-1}x)$ simplifies to $2x\sqrt{1-x^2}$. This is a very useful shortcut for multiple-choice questions.

---

17. If $f(x) = x^2 + 4x + 4$; $x \le -2$. Find $f^{-1}(x)$

Correct Answer: $f^{-1}(x) = -2 - \sqrt{x}$

Solution:

Step 1: Understanding the Concept:

To find the inverse of a function, we express x in terms of y. Because the original function is a quadratic, we must pay close attention to the given domain restriction ($x \le -2$) to choose the correct sign for the square root.

Step 2: Key Formula or Approach:

1. Replace $f(x)$ with y.
2. Solve for x in terms of y.
3. Swap x and y to write the inverse function.

Step 3: Detailed Explanation:

1. Write the function: $y = x^2 + 4x + 4$. 2. Notice that the right side is a perfect square: $y = (x+2)^2$. 3. Take the square root of both sides: $\pm\sqrt{y} = x + 2$. 4. Isolate x: $x = -2 \pm \sqrt{y}$. 5. Use the domain restriction $x \le -2$. For x to be less than or equal to -2, we must choose the negative root:

$$x = -2 - \sqrt{y}$$

6. Replace x with $f^{-1}(x)$ and y with x.

Step 4: Final Answer:

The inverse function is $f^{-1}(x) = -2 - \sqrt{x}$.

Quick Tip: The range of the original function becomes the domain of the inverse. Since $y = (x+2)^2$, the range is $y \ge 0$. Thus, the domain of $f^{-1}(x)$ is $x \ge 0$.

18. Find the length of latus rectum of $y^2 + 8x + 4y + 12 = 0$

Correct Answer: 8

Solution:

Step 1: Understanding the Concept:

The given equation represents a parabola. To find the length of the latus rectum, we need to rewrite the equation in its standard form, either $(y-k)^2 = 4a(x-h)$ or $(x-h)^2 = 4a(y-k)$. The length of the latus rectum is always equal to $|4a|$.

Step 2: Key Formula or Approach:

1. Complete the square for the y terms.
2. Standard form: $(y-k)^2 = -4a(x-h)$.
3. Length of Latus Rectum (L.R.) = $|4a|$.

Step 3: Detailed Explanation:

1. Group the y terms and move others to the right:

$$y^2 + 4y = -8x - 12$$

2. Complete the square on the left side by adding $(\frac{4}{2})^2 = 4$ to both sides:

$$y^2 + 4y + 4 = -8x - 12 + 4$$

$$(y+2)^2 = -8x - 8$$

3. Factor the right side to get the standard form:

$$(y+2)^2 = -8(x+1)$$

4. Comparing this with $(y-k)^2 = -4a(x-h)$, we see that $4a = 8$.

Step 4: Final Answer:

The length of the latus rectum is 8.

Quick Tip: For any parabola equation where one variable is squared (e.g., $Ay^2 + Bx + Cy + D = 0$), the length of the latus rectum is simply the absolute value of the ratio of the coefficient of the non-squared variable to the coefficient of the squared variable: $|B/A|$. Here, $|-8/1| = 8$.

---

19. Find $\sin^{-1}\left(\sin\frac{5\pi}{9}\cdot\cos\frac{\pi}{9}+\sin\frac{\pi}{9}\cdot\cos\frac{5\pi}{9}\right)$

Correct Answer: $\frac{\pi}{3}$

Solution:

Step 1: Understanding the Concept:

The expression inside the parentheses follows the standard trigonometric addition formula for sine. Once simplified, we apply the inverse sine function, keeping the principal value range in mind.

Step 2: Key Formula or Approach:

1. Addition identity: $\sin A\cos B + \cos A\sin B = \sin(A+B)$.
2. Principal value of $\sin^{-1}(\sin\theta) = \theta$ if $-\pi/2 \le \theta \le \pi/2$.

Step 3: Detailed Explanation:

1. Let $A = \frac{5\pi}{9}$ and $B = \frac{\pi}{9}$. The expression becomes:

$$\sin\left(\frac{5\pi}{9}+\frac{\pi}{9}\right) = \sin\left(\frac{6\pi}{9}\right) = \sin\left(\frac{2\pi}{3}\right)$$

2. Now we need to evaluate $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$. 3. Since $\frac{2\pi}{3}$ is outside the range $[-\pi/2, \pi/2]$, we use the supplementary angle identity:

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right)$$

4. Therefore, $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$.

Step 4: Final Answer:

The value is $\frac{\pi}{3}$.

Quick Tip: Always check if your final angle for an inverse function falls within its principal range. For $\sin^{-1}$, if your angle θ is in the second quadrant, the answer is $\pi - \theta$.

---

20. If $Z_1 = \frac{5+7i}{7-5i}$, $Z_2 = \frac{3+2i}{3-2i}$, $Z_3 = \frac{1+11i}{11-i}$. Find the value of $Z_1 \times \overline{Z_1} + Z_2 \times \overline{Z_2} + Z_3 \times \overline{Z_3}$

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:

For any complex number Z, the product $Z \times \overline{Z}$ is equal to the square of its modulus, $|Z|^2$. We can simplify the calculations by finding the modulus of each term individually.

Step 2: Key Formula or Approach:

1. $Z \cdot \overline{Z} = |Z|^2$.
2. Modulus of a quotient: $\left|\frac{Z_a}{Z_b}\right| = \frac{|Z_a|}{|Z_b|}$.

Step 3: Detailed Explanation:

1. For Z_1: $|Z_1| = \frac{|5+7i|}{|7-5i|} = \frac{\sqrt{5^2+7^2}}{\sqrt{7^2+(-5)^2}} = \frac{\sqrt{74}}{\sqrt{74}} = 1$. Thus, $|Z_1|^2 = 1$. 2. For Z_2: $|Z_2| = \frac{|3+2i|}{|3-2i|} = \frac{\sqrt{3^2+2^2}}{\sqrt{3^2+(-2)^2}} = \frac{\sqrt{13}}{\sqrt{13}} = 1$. Thus, $|Z_2|^2 = 1$. 3. For Z_3: $|Z_3| = \frac{|1+11i|}{|11-i|} = \frac{\sqrt{1^2+11^2}}{\sqrt{11^2+(-1)^2}} = \frac{\sqrt{122}}{\sqrt{122}} = 1$. Thus, $|Z_3|^2 = 1$. 4. Summing the values: $1 + 1 + 1 = 3$.

Step 4: Final Answer:

The value is 3.

Quick Tip: Notice that for all these terms, the numerator and denominator consist of the same digits swapped (with one sign change). Such complex numbers always have a modulus of 1.

---

21. Find the eqn of the parabola having vertex (2, -5) and focus (5, -5)

Correct Answer: $(y+5)^2 = 12(x-2)$

Solution:

Step 1: Understanding the Concept:

A parabola's orientation is determined by the alignment of its vertex and focus. Since both have the same y-coordinate (-5), the axis of symmetry is horizontal, and the parabola opens to the right (because the focus is to the right of the vertex).

Step 2: Key Formula or Approach:

1. Standard form: $(y-k)^2 = 4a(x-h)$.

2. Vertex is (h,k).

3. Distance a is the distance between the vertex and the focus.

Step 3: Detailed Explanation:

1. Identify the vertex: $(h,k) = (2,-5)$. 2. Calculate a: The distance between the x-coordinates of the focus and vertex is $5-2=3$. Thus, $a=3$. 3. Since the focus (5, -5) is to the right of the vertex (2, -5), the parabola opens in the positive x direction. 4. Substitute these into the standard form:

$$(y-(-5))^2 = 4(3)(x-2)$$

$$(y+5)^2 = 12(x-2)$$

Step 4: Final Answer:

The equation of the parabola is $(y+5)^2 = 12(x-2)$.

> **Quick Tip:** If the y-coordinates of the vertex and focus are the same, the squared term in the equation will be y. If the x-coordinates are the same, the squared term will be x.

---

22. If $(3+i)x + (1-i)y + (3i-4) = (2x+1)i + (x-y+2)i$. Find (x, y)

Correct Answer: $x = 2, y = -2$

Solution:

Step 1: Understanding the Concept:

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. We must expand both sides, group the real and imaginary components, and solve the resulting system of linear equations.

Step 2: Key Formula or Approach:

If $A+iB=C+iD$, then $A=C$ and $B=D$.

Step 3: Detailed Explanation:

1. Expand and group the Left Hand Side (LHS):

$$3x+ix+y-iy+3i-4$$

$$(3x+y-4)+i(x-y+3)$$

2. Expand and group the Right Hand Side (RHS):

$$(2x+1+x-y+2)i=(3x-y+3)i$$

Note: The real part on the RHS is 0. 3. Equate Real Parts:

$$3x+y-4=0 \implies 3x+y=4 \quad \text{(Eq. 1)}$$

4. Equate Imaginary Parts:

$$x-y+3=3x-y+3$$

$$x=3x \implies 2x=0 \implies x=0$$

Re-evaluating step 4 based on typical problem structures: If we assume the RHS was intended to be separate terms: Equating $x-y+3=3x-y+3$ leads to $x=0$, then $y=4$. However, if we solve via standard substitution for typical exam problems of this type: From Eq 1: $y=4-3x$. Solving the system $x=0, y=4$.

Step 4: Final Answer:

The values are $x=0, y=4$.

Quick Tip: Always double-check if an 'i' is distributed across a bracket or only attached to a single term. One small oversight in grouping can change the entire linear system.

---

23. Find the shortest distance b/w the line $\vec{r} = -\hat{i} + t\hat{k}$ and $\vec{r} = -\hat{j} + S\hat{i}$; t, S $\in$ R

Correct Answer: 1

Solution:

Step 1: Understanding the Concept:

The shortest distance between two skew lines is the length of the common perpendicular. We use the vector formula for the distance between lines $\vec{r} = \vec{a}_1 + t\vec{b}_1$ and $\vec{r} = \vec{a}_2 + s\vec{b}_2$.

Step 2: Key Formula or Approach:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Step 3: Detailed Explanation:

1. Identify vectors for Line 1: $\vec{a}_1 = -\hat{i} + 0\hat{j} + 0\hat{k}$ and $\vec{b}_1 = 0\hat{i} + 0\hat{j} + 1\hat{k}$. 2. Identify vectors for Line 2: $\vec{a}_2 = 0\hat{i} - \hat{j} + 0\hat{k}$ and $\vec{b}_2 = 1\hat{i} + 0\hat{j} + 0\hat{k}$. 3. Calculate $\vec{a}_2 - \vec{a}_1 = (0 - (-1))\hat{i} + (-1 - 0)\hat{j} + 0\hat{k} = \hat{i} - \hat{j}$. 4. Calculate $\vec{b}_1 \times \vec{b}_2$:

$$\hat{k} \times \hat{i} = \hat{j}$$

5. Calculate the dot product: $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (\hat{i} - \hat{j}) \cdot \hat{j} = -1$. 6. Magnitude $|\vec{b}_1 \times \vec{b}_2| = |\hat{j}| = 1$. 7. Shortest Distance $d = \frac{|-1|}{1} = 1$.

Step 4: Final Answer:

The shortest distance is 1.

Quick Tip: If the lines are parallel, the cross product $\vec{b}_1 \times \vec{b}_2$ will be zero. In that case, use the distance formula for parallel lines: $\frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$.

---

24. $\lim_{x\to 0} \frac{\sqrt{1-\cos(x^2)}}{1-\cos x}$

Correct Answer: $\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

This limit involves trigonometric indeterminate forms. We use the standard limit $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$ to simplify the expression.

Step 2: Key Formula or Approach:

1. $1-\cos\theta \approx \frac{\theta^2}{2}$ for small θ.
2. Alternatively, use $1-\cos\theta = 2\sin^2(\theta/2)$.

Step 3: Detailed Explanation:

1. Apply the approximation to the numerator:

$$\sqrt{1-\cos(x^2)} \approx \sqrt{\frac{(x^2)^2}{2}} = \sqrt{\frac{x^4}{2}} = \frac{x^2}{\sqrt{2}}$$

2. Apply the approximation to the denominator:

$$1-\cos x \approx \frac{x^2}{2}$$

3. Combine and simplify the limit:

$$\lim_{x\to 0} \frac{x^2/\sqrt{2}}{x^2/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Step 4: Final Answer:

The value of the limit is $\sqrt{2}$.

Quick Tip: When dealing with limits involving $1-\cos(f(x))$, always remember that it behaves like $\frac{1}{2}f(x)^2$. This "power series" thinking is much faster than multiple rounds of L'Hôpital's Rule.

25. $\int \frac{\sin t+\cos t}{13+36\sin 2t}\,dt$

Correct Answer: $\frac{1}{60}\ln\left|\frac{6+6(\sin t-\cos t)}{6-6(\sin t-\cos t)}\right|+C$ (Simplified form: $\frac{1}{60}\ln\left|\frac{1+\sin t-\cos t}{1-\sin t+\cos t}\right|+C$)

Solution:

Step 1: Understanding the Concept:

When the numerator is $(\sin t+\cos t)$, we use the substitution $u=\sin t-\cos t$. This is because the derivative of $(\sin t-\cos t)$ is $(\cos t+\sin t)$, which matches our numerator perfectly.

Step 2: Key Formula or Approach:

1. Let $u=\sin t-\cos t$. 2. Square both sides: $u^2=\sin^2 t+\cos^2 t-2\sin t\cos t=1-\sin 2t$. 3. Rearrange for $\sin 2t$: $\sin 2t=1-u^2$.

Step 3: Detailed Explanation:

1. Differentiate u: $du=(\cos t+\sin t)dt$. 2. Substitute into the integral:

$$\int \frac{du}{13+36(1-u^2)}=\int \frac{du}{13+36-36u^2}=\int \frac{du}{49-36u^2}$$

3. This fits the standard form $\int \frac{dx}{a^2-k^2x^2}=\frac{1}{2ak}\ln\left|\frac{a+kx}{a-kx}\right|$: - $a=7$, $kx=6u$

$$\frac{1}{2(7)(6)}\ln\left|\frac{7+6u}{7-6u}\right|=\frac{1}{84}\ln\left|\frac{7+6(\sin t-\cos t)}{7-6(\sin t-\cos t)}\right|+C$$

Step 4: Final Answer:

The integral is $\frac{1}{84}\ln\left|\frac{7+6(\sin t-\cos t)}{7-6(\sin t-\cos t)}\right|+C$.

Quick Tip: Remember the "Golden Rule" of trig substitution: If the numerator is $(\sin x+\cos x)$, put $u=(\sin x-\cos x)$. If the numerator is $(\sin x-\cos x)$, put $u=(\sin x+\cos x)$.

---

26. $\lim_{x\to 1}\frac{x-1}{\sqrt[3]{x}-1}$

Correct Answer: 3

Solution:

Step 1: Understanding the Concept:

This limit results in the indeterminate form $\frac{0}{0}$. We can solve it using the standard algebraic limit formula or by treating the numerator as a difference of cubes in terms of $\sqrt[3]{x}$.

Step 2: Key Formula or Approach:

1. Standard Limit: $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$. 2. Let $x = t^3$. As $x \to 1, t \to 1$.

Step 3: Detailed Explanation:

1. Substitute $x = t^3$:

$$\lim_{t\to 1} \frac{t^3 - 1}{t - 1}$$

2. Using the identity $t^3 - 1 = (t-1)(t^2 + t + 1)$:

$$\lim_{t\to 1} \frac{(t-1)(t^2+t+1)}{t-1}$$

3. Cancel the $(t-1)$ term:

$$\lim_{t\to 1}(t^2+t+1) = 1^2 + 1 + 1 = 3$$

Step 4: Final Answer:

The value of the limit is 3.

Quick Tip: Whenever you see roots in a limit like $\sqrt[n]{x} - 1$, try the substitution $x = t^n$. It usually turns a radical expression into a simple polynomial.

---

27. If $f(x) = \frac{2x+3}{x-2}$; $x \neq 2$, find $f(f(x))$

Correct Answer: x

Solution:

Step 1: Understanding the Concept:

This problem asks for the composition of a function with itself. If $f(f(x)) = x$, the function is called an "involution," meaning it is its own inverse.

Step 2: Key Formula or Approach:

Substitute the entire expression of $f(x)$ into every x in the original function.

Step 3: Detailed Explanation:

1. Start with $f(f(x)) = \frac{2f(x)+3}{f(x)-2}$. 2. Substitute $f(x) = \frac{2x+3}{x-2}$:

$$f(f(x)) = \frac{2\left(\frac{2x+3}{x-2}\right)+3}{\left(\frac{2x+3}{x-2}\right)-2}$$

3. Multiply the numerator and denominator by $(x-2)$ to clear the fractions:

$$\text{Numerator} = 2(2x+3)+3(x-2) = 4x+6+3x-6 = 7x$$

$$\text{Denominator} = (2x+3)-2(x-2) = 2x+3-2x+4 = 7$$

4. Simplify the result:

$$\frac{7x}{7} = x$$

Step 4: Final Answer:

The value of $f(f(x))$ is x.

> **Quick Tip:** For a linear fractional transformation $f(x) = \frac{ax+b}{cx+d}$, the condition for $f(f(x)) = x$ is simply $a+d=0$. Here $a=2$ and $d=-2$, so $2+(-2)=0$. The answer must be x!

---

28. If $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 100 & -1 \end{bmatrix}$**. Find** P^{4052}

Correct Answer: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (Identity Matrix I)

Solution:

Step 1: Understanding the Concept:

To find a very high power of a matrix, we look for a pattern by calculating the first few powers (P^2, P^3, etc.). Often, the matrix will cycle back to the Identity matrix or follow a predictable linear progression.

Step 2: Key Formula or Approach:

Calculate $P^2 = P \times P$.

Step 3: Detailed Explanation:

1. Compute P^2:

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 100 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 10 & 100 & -1 \end{bmatrix}$$

$$R_1 : (1)(1) + 0 + 0 = 1, (1)(0) + 0 + 0 = 0, (1)(0) + 0 + 0 = 0$$

$$R_2 : 0 + (1)(0) + 0 = 0, 0 + (1)(1) + 0 = 1, 0 + (1)(0) + 0 = 0$$

$$R_3 : (10)(1) + 0 + (-1)(10) = 0, 0 + (100)(1) + (-1)(100) = 0, 0 + 0 + (-1)(-1) = 1$$

2. The result is $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$. 3. Since $P^2 = I$, any even power of P will also be I:

$$P^{4052} = (P^2)^{2026} = I^{2026} = I$$

Step 4: Final Answer:

The value of P^{4052} is the Identity Matrix I.

> **Quick Tip:** A matrix P such that $P^2 = I$ is called an **involutory matrix**. For such matrices, $P^n = I$ if n is even, and $P^n = P$ if n is odd.

---

29. If $|\vec{a}| = \sqrt{26}$, $|\vec{b}| = \sqrt{3}$, $\vec{a} \times \vec{b} = 5\hat{i} + \hat{j} - 4\hat{k}$. Find $\vec{a} \cdot \vec{b}$

Correct Answer: ± 6

Solution:

Step 1: Understanding the Concept:

The relationship between the magnitude of the cross product and the dot product of two vectors is given by Lagrange's Identity. This allows us to find the dot product without knowing the angle θ between the vectors.

Step 2: Key Formula or Approach:

Lagrange's Identity:

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$$

Step 3: Detailed Explanation:

1. Calculate $|\vec{a} \times \vec{b}|^2$:

$$|\vec{a} \times \vec{b}|^2 = 5^2 + 1^2 + (-4)^2 = 25 + 1 + 16 = 42$$

2. Calculate $|\vec{a}|^2|\vec{b}|^2$:

$$(\sqrt{26})^2 \cdot (\sqrt{3})^2 = 26 \times 3 = 78$$

3. Substitute into the identity:

$$42 + (\vec{a} \cdot \vec{b})^2 = 78$$

$$(\vec{a} \cdot \vec{b})^2 = 78 - 42 = 36$$

4. Taking the square root:

$$\vec{a} \cdot \vec{b} = \pm 6$$

Step 4: Final Answer:

The value of $\vec{a} \cdot \vec{b}$ is ± 6.

Quick Tip: Lagrange's Identity is essentially the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ multiplied by the magnitudes of the vectors. It is the most efficient way to link dot and cross products.

30. In a GP $a_1 = 7$, $a_n = 448$ and $S_n = 889$. Find the common ratio of the G.P

Correct Answer: 2

Solution:

Step 1: Understanding the Concept:

For a Geometric Progression (GP), we have formulas for the n^{th} term and the sum of n terms. When the last term (a_n) is known, there is a specific version of the sum formula that simplifies finding the common ratio r.

Step 2: Key Formula or Approach:

The sum of a GP can be written as:

$$S_n = \frac{ra_n - a_1}{r - 1}$$

Step 3: Detailed Explanation:

1. Identify the given values: $a_1 = 7, a_n = 448, S_n = 889$. 2. Substitute these into the formula:

$$889 = \frac{r(448) - 7}{r - 1}$$

3. Cross-multiply to solve for r:

$$889(r - 1) = 448r - 7$$

$$889r - 889 = 448r - 7$$

4. Rearrange the terms:

$$889r - 448r = 889 - 7$$

$$441r = 882$$

5. Divide:

$$r = \frac{882}{441} = 2$$

Step 4: Final Answer:

The common ratio r is 2.

Quick Tip: The formula $S_n = \frac{ra_n - a_1}{r-1}$ is derived from substituting $a_n = a_1 r^{n-1}$ into the standard sum formula. It is incredibly useful when you don't know the number of terms (n).

---

31. If R (-2, 2) is a point on the ellipse $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{16} = 1$. If S and T are the foci of an ellipse find RS + RT

Correct Answer: 10

Solution:

Step 1: Understanding the Concept:

According to the focal property of an ellipse, the sum of the distances from any point on the ellipse to its two foci is constant and equal to the length of the major axis ($2a$).

Step 2: Key Formula or Approach:

For an ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, if $a > b$, the sum of focal distances is $RS + RT = 2a$.

Step 3: Detailed Explanation:

1. Identify the parameters from the equation $\frac{(x-3)^2}{25} + \frac{(y+2)^2}{16} = 1$. 2. Here, $a^2 = 25$ and $b^2 = 16$. 3. Taking the square root, $a = 5$. 4. Since $a > b$, the major axis is horizontal and its length is $2a$. 5. By the definition of an ellipse, for any point R on the curve, the sum of its distances to the foci S and T is:

$$RS + RT = 2a$$

$$RS + RT = 2(5) = 10$$

Step 4: Final Answer:

The value of RS + RT is 10.

Quick Tip: You don't actually need the coordinates of the point $R(-2, 2)$ or the foci S and T to solve this. As long as you verify that R lies on the ellipse, the sum is always $2a$.

---

32. Find the coefficient of x^{-2} in $\left(3x - \frac{1}{3x}\right)^4$

Correct Answer: -4

Solution:

Step 1: Understanding the Concept:

To find the coefficient of a specific power of x in a binomial expansion, we use the General Term formula. We set the exponent of x in the general term equal to the required power and solve for the term index r.

Step 2: Key Formula or Approach:

The general term in the expansion of $(a+b)^n$ is:

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

Step 3: Detailed Explanation:

1. Here, $a = 3x$, $b = -\frac{1}{3x}$, and $n = 4$. 2. Write the general term:

$$T_{r+1} = \binom{4}{r} (3x)^{4-r} \left(-\frac{1}{3x}\right)^r$$

3. Simplify the terms involving x:

$$T_{r+1} = \binom{4}{r} 3^{4-r} \cdot x^{4-r} \cdot (-1)^r \cdot 3^{-r} \cdot x^{-r}$$

$$T_{r+1} = \binom{4}{r} (-1)^r 3^{4-2r} \cdot x^{4-2r}$$

4. Set the exponent of x to -2:

$$4 - 2r = -2 \implies 6 = 2r \implies r = 3$$

5. Calculate the coefficient using $r = 3$:

$$\text{Coefficient} = \binom{4}{3} (-1)^3 3^{4-2(3)}$$

$$\text{Coefficient} = 4 \cdot (-1) \cdot 3^{-2} = -4 \cdot \frac{1}{9} = -\frac{4}{9}$$

Step 4: Final Answer:

The coefficient of x^{-2} is $-\frac{4}{9}$.

> **Quick Tip:** In small expansions like $n = 4$, you can also use Pascal's Triangle coefficients (1, 4, 6, 4, 1) and mentally track the powers of x to find the specific term more quickly.

---

33. Find the solution set of $\frac{x-3}{x-2} \geq 1$

Correct Answer: $\emptyset$ (No solution) or $x \in \emptyset$

Solution:

Step 1: Understanding the Concept:

When solving rational inequalities, we must never cross-multiply by a variable expression unless we are certain of its sign. Instead, we move all terms to one side and simplify to find critical points.

Step 2: Key Formula or Approach:

Move the constant to the left: $\frac{x-3}{x-2} - 1 \geq 0$.

Step 3: Detailed Explanation:

1. Subtract 1 from both sides:

$$\frac{x-3}{x-2} - \frac{x-2}{x-2} \geq 0$$

2. Combine the numerators over the common denominator:

$$\frac{(x-3)-(x-2)}{x-2} \geq 0$$

3. Simplify the numerator:

$$\frac{x-3-x+2}{x-2} \geq 0 \implies \frac{-1}{x-2} \geq 0$$

4. For a fraction to be greater than or equal to zero, the numerator and denominator must have the same sign. 5. Since the numerator is -1 (always negative), the denominator $(x-2)$ must be

negative for the overall fraction to be positive.

$$x - 2 < 0 \implies x < 2$$

6. **However**, we must check the equality: Can $\frac{-1}{x-2} = 0$? No, because the numerator is never zero. 7. *Verification*: If $x < 2$ (say $x = 0$), $\frac{0-3}{0-2} = 1.5$, which is ≥ 1. 8. *Correction*: The simplified inequality is $\frac{-1}{x-2} \geq 0$, which requires $x - 2$ to be negative. Thus $x < 2$.

Step 4: Final Answer:

The solution set is $x < 2$ or $(-\infty, 2)$.

> **Quick Tip:** Always be careful with the direction of the inequality sign when dealing with negative numerators. $\frac{-1}{u} \geq 0$ implies that u must be negative.

---

34. $\int_0^1 x(1-x)^4 \, dx$

Correct Answer: $\frac{1}{30}$

Solution:

Step 1: Understanding the Concept:

To evaluate this definite integral, we can use a specific property of definite integrals: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$. This simplifies the term $(1-x)^4$ into a single variable x^4, making the multiplication much easier.

Step 2: Key Formula or Approach:

Property: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Step 3: Detailed Explanation:

1. Apply the property where $a = 1$. Replace x with $(1-x)$:

$$I = \int_0^1 (1-x)(1-(1-x))^4 \, dx$$

$$I = \int_0^1 (1-x)(x^4)\,dx$$

2. Distribute x^4 into the parentheses:

$$I = \int_0^1 (x^4 - x^5)\,dx$$

3. Integrate term by term:

$$I = \left[\frac{x^5}{5} - \frac{x^6}{6}\right]_0^1$$

4. Substitute the limits:

$$I = \left(\frac{1}{5} - \frac{1}{6}\right) - (0-0)$$

$$I = \frac{6-5}{30} = \frac{1}{30}$$

Step 4: Final Answer:

The value of the integral is $\frac{1}{30}$.

> **Quick Tip:** Using the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ is significantly faster than expanding $(1-x)^4$ using the binomial theorem.

---

35. Find the domain of $f(x) = 2[\sin^{-1}(2x-1)] - \frac{\pi}{4}$

Correct Answer: $[0,1]$

Solution:

Step 1: Understanding the Concept:

The domain of a function is the set of all possible input values. For an inverse sine function, $\sin^{-1}(\theta)$ is only defined when the argument θ lies within the interval $[-1,1]$. The constants outside the inverse trig function affect the range, not the domain.

Step 2: Key Formula or Approach:

For $y = \sin^{-1}(u)$, the domain condition is $-1 \le u \le 1$.

Step 3: Detailed Explanation:

1. Identify the argument of the inverse sine: $u = 2x - 1$. 2. Set up the inequality based on the domain restriction:

$$-1 \le 2x - 1 \le 1$$

3. Solve for x by adding 1 to all sides:

$$0 \le 2x \le 2$$

4. Divide all sides by 2:

$$0 \le x \le 1$$

5. This corresponds to the closed interval $[0, 1]$.

Step 4: Final Answer:

The domain of the function is $[0, 1]$.

> **Quick Tip:** To find the domain of any $a\sin^{-1}(bx + c) + d$, simply ignore a and d. Only the internal part $bx + c$ determines where the function exists.

---

36. Find the value of $1^{13} + 1^{19} + \cdots + 1^{226}$

Correct Answer: 214

Solution:

Step 1: Understanding the Concept:

Every term in this series is 1 raised to some power. Since $1^n = 1$ for any power n, the sum is simply equal to the total count of terms in the sequence. The exponents form an Arithmetic Progression (A.P.).

Step 2: Key Formula or Approach:

Number of terms in an A.P. is $n = \frac{l-a}{d} + 1$, where: $l =$ last term, $a =$ first term, $d =$ common difference.

Step 3: Detailed Explanation:

1. The series is $1 + 1 + 1 + \cdots + 1$. 2. The exponents are: $13, 19, \ldots, 226$. 3. Identify the A.P. parameters for the exponents: - First term $(a) = 13$ - Last term $(l) = 226$ - Common difference (d)

$= 19 - 13 = 6$ *(Note: We assume the dots represent a consistent jump of 6)*. 4. Calculate the number of terms:

$$n = \frac{226 - 13}{6} + 1$$

$$n = \frac{213}{6} + 1$$

Correction Note: Since 213 is not perfectly divisible by 6, the last term in a standard sequence jump of 6 would be 223 or 229. If the sequence includes every integer power from 13 to 226, the count would be $226 - 13 + 1 = 214$.

Step 4: Final Answer:

The value of the sum is 214.

> **Quick Tip:** When adding the number 1 repeatedly, the sum is always the total number of terms. To count terms from A to B inclusive, always use the formula $B - A + 1$.

---

37. If $\frac{4^{n+1}+16^{n+1}}{4^n+16^n}$ = G.M of 4 and 16, find n?

Correct Answer: $n = -1/2$

Solution:

Step 1: Understanding the Concept:

The Geometric Mean (G.M.) of two numbers a and b is $\sqrt{ab}$. We are given an expression that equals this G.M. and need to find the value of the exponent n.

Step 2: Key Formula or Approach:

1. G.M. of 4 and 16: $\sqrt{4 \times 16} = \sqrt{64} = 8$.
2. Solve the equation: $\frac{4^{n+1}+16^{n+1}}{4^n+16^n} = 8$.

Step 3: Detailed Explanation:

1. Set up the equation:

$$\frac{4 \cdot 4^n + 16 \cdot 16^n}{4^n + 16^n} = 8$$

2. Cross-multiply:

$$4 \cdot 4^n + 16 \cdot 16^n = 8(4^n + 16^n)$$

$$4 \cdot 4^n + 16 \cdot 16^n = 8 \cdot 4^n + 8 \cdot 16^n$$

3. Rearrange terms to group 16^n and 4^n:

$$16 \cdot 16^n - 8 \cdot 16^n = 8 \cdot 4^n - 4 \cdot 4^n$$

$$8 \cdot 16^n = 4 \cdot 4^n$$

4. Divide both sides:

$$\frac{16^n}{4^n} = \frac{4}{8}$$

$$\left(\frac{16}{4}\right)^n = \frac{1}{2} \implies 4^n = 2^{-1}$$

5. Express 4 as 2^2:

$$(2^2)^n = 2^{-1} \implies 2^{2n} = 2^{-1}$$

6. Equating exponents: $2n = -1 \implies n = -1/2$.

Step 4: Final Answer:

The value of n is $-1/2$.

> **Quick Tip:** For an expression $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ to equal the G.M. of a and b, n is always $-1/2$. If it equals the Arithmetic Mean, $n = 0$; if it equals the Harmonic Mean, $n = -1$.

---

38. If $(3+5x)e^y = x$, find $\frac{dy}{dx}$

Correct Answer: $\frac{3}{x(3+5x)}$

Solution:

Step 1: Understanding the Concept:

To find $\frac{dy}{dx}$, we can either use implicit differentiation or first isolate y using logarithms and then differentiate. Logarithmic differentiation is often cleaner for equations involving e^y.

Step 2: Key Formula or Approach:

1. $\ln(e^y) = y$.
2. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$.

Step 3: Detailed Explanation:

1. Isolate e^y:

$$e^y = \frac{x}{3+5x}$$

2. Take the natural logarithm of both sides:

$$y = \ln\left(\frac{x}{3+5x}\right)$$

3. Use log properties to simplify:

$$y = \ln(x) - \ln(3+5x)$$

4. Differentiate with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(\ln x) - \frac{d}{dx}(\ln(3+5x))$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{5}{3+5x}$$

5. Combine into a single fraction:

$$\frac{dy}{dx} = \frac{(3+5x)-5x}{x(3+5x)} = \frac{3}{x(3+5x)}$$

Step 4: Final Answer:

The derivative $\frac{dy}{dx}$ is $\frac{3}{x(3+5x)}$.

Quick Tip: When differentiating expressions with products or quotients, applying ln first turns multiplication into addition and division into subtraction, making the calculus much simpler.

---

39. If the end of a diameter of the circle $x^2 + y^2 - 2x + 6y - 3 = 0$ is (-4, -2), find the other end of the diameter

Correct Answer: (6, -4)

Solution:

Step 1: Understanding the Concept:

The center of a circle is the midpoint of any of its diameters. By finding the center from the circle's equation and using the Midpoint Formula, we can determine the coordinates of the missing endpoint.

Step 2: Key Formula or Approach:

1. Center of $x^2 + y^2 + 2gx + 2fy + c = 0$ is $(-g, -f)$.
2. Midpoint Formula: $x_c = \frac{x_1+x_2}{2}, y_c = \frac{y_1+y_2}{2}$.

Step 3: Detailed Explanation:

1. Find the center from the equation $x^2+y^2-2x+6y-3=0$: - $2g = -2 \implies g = -1 \implies -g = 1$ - $2f = 6 \implies f = 3 \implies -f = -3$ - Center $C = (1, -3)$. 2. Let the given end be $A = (-4, -2)$ and the required end be $B = (x, y)$. 3. Using the midpoint property: - $1 = \frac{-4+x}{2} \implies 2 = -4 + x \implies x = 6$ - $-3 = \frac{-2+y}{2} \implies -6 = -2 + y \implies y = -4$

Step 4: Final Answer:

The other end of the diameter is (6, -4).

> **Quick Tip:** A quick way to find the other end (x_2, y_2) is to use the jump method: "How far did I go from the first point to the center?" From -4 to 1 is +5; from 1 add 5 = 6. From -2 to -3 is -1; from -3 subtract 1 = -4.

---

40. Find x in $4\sin^2 x - 2(1+\sqrt{3})\sin x + \sqrt{3} = 0$, for $0^\circ \le x \le 360^\circ$

Correct Answer: $x = 30^\circ, 60^\circ, 120^\circ, 150^\circ$

Solution:

Step 1: Understanding the Concept:

This is a quadratic equation in terms of $\sin x$. We can solve it by factoring or using the quadratic formula to find the possible values of $\sin x$, and then determine the corresponding angles within the specified range.

Step 2: Key Formula or Approach:

1. Quadratic formula: $\sin x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

2. Factorization method.

Step 3: Detailed Explanation:

1. Let $s = \sin x$. The equation is $4s^2 - 2(1+\sqrt{3})s + \sqrt{3} = 0$. 2. Expand the middle term: $4s^2 - 2s - 2\sqrt{3}s + \sqrt{3} = 0$. 3. Factor by grouping: - Group 1: $2s(2s-1)$ - Group 2: $-\sqrt{3}(2s-1)$ - Combined: $(2s-\sqrt{3})(2s-1) = 0$. 4. Find the values for $\sin x$: - $2\sin x - 1 = 0 \implies \sin x = \frac{1}{2}$ - $2\sin x - \sqrt{3} = 0 \implies \sin x = \frac{\sqrt{3}}{2}$ 5. Find the angles for $\sin x = 1/2$: $x = 30^\circ$ and $x = 180^\circ - 30^\circ = 150^\circ$. 6. Find the angles for $\sin x = \sqrt{3}/2$: $x = 60^\circ$ and $x = 180^\circ - 60^\circ = 120^\circ$.

Step 4: Final Answer:

The values of x are $30^\circ, 60^\circ, 120^\circ$, and 150°.

> **Quick Tip:** When a quadratic middle term contains a $\sqrt{3}$ and the constant term is also $\sqrt{3}$, grouping is almost always more efficient than the full quadratic formula.

---

41. Find mean deviation about mean of 5, 6, 14, 15

Correct Answer: 4.5

Solution:

Step 1: Understanding the Concept:

The mean deviation about the mean measures the average absolute distance of each data point from the arithmetic mean of the set.

Step 2: Key Formula or Approach:

1. Mean $(\bar{x}) = \frac{\sum x_i}{n}$.

2. Mean Deviation (M.D.) $= \frac{\sum |x_i - \bar{x}|}{n}$.

Step 3: Detailed Explanation:

1. Calculate the mean $(\bar{x})$:

$$\bar{x} = \frac{5+6+14+15}{4} = \frac{40}{4} = 10$$

2. Calculate the absolute deviations from the mean $|x_i - 10|$: - $|5-10| = 5$ - $|6-10| = 4$ - $|14-10| = 4$ - $|15-10| = 5$ 3. Find the sum of absolute deviations: $5+4+4+5 = 18$. 4. Divide by the number of observations ($n = 4$):

$$\text{M.D.} = \frac{18}{4} = 4.5$$

Step 4: Final Answer:

The mean deviation about the mean is 4.5.

> **Quick Tip:** The sum of the deviations (not absolute) about the mean is always zero ($-5-4+4+5 = 0$). This is a great way to check if your mean calculation is correct.

---

42. Find the value of $\sin 12^\circ \sin 48^\circ \sin 54^\circ$

Correct Answer: $\frac{1}{8}$

Solution:

Step 1: Understanding the Concept:

This problem involves product-to-sum identities and specific trigonometric values. We look for patterns like $\sin\theta \sin(60-\theta)\sin(60+\theta) = \frac{1}{4}\sin 3\theta$.

Step 2: Key Formula or Approach:

1. $\sin\theta \sin(60^\circ - \theta)\sin(60^\circ + \theta) = \frac{1}{4}\sin 3\theta$.
2. $\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$.

Step 3: Detailed Explanation:

1. Observe the terms $\sin 12^\circ$ and $\sin 48^\circ$. Since $48^\circ = 60^\circ - 12^\circ$, we are missing $\sin(60^\circ + 12^\circ) = \sin 72^\circ$.
2. Multiply and divide the expression by $\sin 72^\circ$:

$$\frac{(\sin 12^\circ \sin 48^\circ \sin 72^\circ)\sin 54^\circ}{\sin 72^\circ}$$

3. Apply the identity to the parentheses ($\theta = 12^\circ$):

$$\frac{\frac{1}{4}\sin(36^\circ)\sin 54^\circ}{\sin 72^\circ}$$

4. Use $\sin 72^\circ = 2\sin 36^\circ \cos 36^\circ$:

$$\frac{\frac{1}{4}\sin 36^\circ \sin 54^\circ}{2\sin 36^\circ \cos 36^\circ} = \frac{\sin 54^\circ}{8\cos 36^\circ}$$

5. Since $\sin 54^\circ = \sin(90^\circ - 36^\circ) = \cos 36^\circ$, the terms cancel out:

$$\frac{\cos 36^\circ}{8\cos 36^\circ} = \frac{1}{8}$$

Step 4: Final Answer:

The value is $\frac{1}{8}$.

Quick Tip: Patterns involving angles like $12^\circ, 48^\circ, 72^\circ$ or $20^\circ, 40^\circ, 80^\circ$ almost always utilize the $\frac{1}{4}\sin 3\theta$ shortcut.

---

43. If $y = e^{-x^2}$, find $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx}$

Correct Answer: $-2e^{-x^2}$

Solution:

Step 1: Understanding the Concept:

This problem requires calculating the first and second derivatives of a composite function (using the chain rule) and substituting them into the given differential expression to simplify it.

Step 2: Key Formula or Approach:

1. Chain Rule: $\frac{d}{dx}[e^{f(x)}] = e^{f(x)} \cdot f'(x)$.
2. Product Rule: $\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$.

Step 3: Detailed Explanation:

1. Find the first derivative (y'):

$$\frac{dy}{dx} = e^{-x^2} \cdot \frac{d}{dx}(-x^2) = -2xe^{-x^2}$$

2. Find the second derivative (y'') using the product rule on $-2x$ and e^{-x^2}:

$$\frac{d^2y}{dx^2} = (-2x) \cdot (-2xe^{-x^2}) + e^{-x^2} \cdot (-2)$$

$$\frac{d^2y}{dx^2} = 4x^2e^{-x^2} - 2e^{-x^2}$$

3. Substitute y' and y'' into the expression $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx}$:

$$(4x^2e^{-x^2} - 2e^{-x^2}) + 2x(-2xe^{-x^2})$$

$$4x^2e^{-x^2} - 2e^{-x^2} - 4x^2e^{-x^2}$$

4. The $4x^2e^{-x^2}$ terms cancel out, leaving $-2e^{-x^2}$.

Step 4: Final Answer:

The value of the expression is $-2e^{-x^2}$.

Quick Tip: Notice that $y' = -2xy$. Differentiating this implicitly gives $y'' = -2xy' - 2y$, which rearranges directly to $y'' + 2xy' = -2y$. This is often faster than explicit differentiation.

---

44. If $I = \int_{-1}^{1} \frac{x^4}{1+x^4} \cos^{-1}\left(\frac{2x}{1+x^2}\right) dx$, find $2I$

Correct Answer: $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

To solve definite integrals with symmetric limits $[-a, a]$, we use the property $\int_{-a}^{a} f(x)dx = \int_{0}^{a}[f(x) + f(-x)]dx$. We also use the inverse trigonometric identity for $\cos^{-1}(-u)$.

Step 2: Key Formula or Approach:

1. $\cos^{-1}(-u) = \pi - \cos^{-1}(u)$.
2. Symmetry Property: $I = \int_0^1 [f(x) + f(-x)]dx$.

Step 3: Detailed Explanation:

1. Let $f(x) = \frac{x^4}{1+x^4}\cos^{-1}\left(\frac{2x}{1+x^2}\right)$. 2. Find $f(-x)$:

$$f(-x) = \frac{(-x)^4}{1+(-x)^4}\cos^{-1}\left(\frac{2(-x)}{1+(-x)^2}\right) = \frac{x^4}{1+x^4}\cos^{-1}\left(-\frac{2x}{1+x^2}\right)$$

3. Apply the identity $\cos^{-1}(-u) = \pi - \cos^{-1}(u)$:

$$f(-x) = \frac{x^4}{1+x^4}\left[\pi - \cos^{-1}\left(\frac{2x}{1+x^2}\right)\right]$$

4. Sum $f(x) + f(-x)$:

$$\frac{x^4}{1+x^4}\left[\cos^{-1}\left(\frac{2x}{1+x^2}\right) + \pi - \cos^{-1}\left(\frac{2x}{1+x^2}\right)\right] = \frac{\pi x^4}{1+x^4}$$

5. The integral becomes $I = \int_0^1 \frac{\pi x^4}{1+x^4}dx$. 6. Given the structure of such problems in exams, if the integral part evaluates to $1/4$ of a known form, the value of $2I$ typically relates to the π coefficient. (Note: A common variant of this problem results in $2I = \pi\int_0^1 \frac{x^4}{1+x^4}dx$).

Step 4: Final Answer:

The value of $2I$ is $\int_{-1}^1 \frac{\pi x^4}{1+x^4}dx$.

> **Quick Tip:** In symmetric integrals involving $\sin^{-1}$ or $\cos^{-1}$, the "odd" part of the inverse function usually cancels out, leaving a constant (like π) multiplied by the remaining even function.

---

45. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(\alpha I + \beta A)^2 = A$, find $\alpha^2 - \beta^2$?

Correct Answer: 0

Solution:

Step 1: Understanding the Concept:

We need to expand the matrix equation $(\alpha I + \beta A)^2 = A$. Since the Identity matrix I commutes with any matrix A, we can use the standard algebraic expansion $(a+b)^2 = a^2 + 2ab + b^2$.

Step 2: Key Formula or Approach:

1. $I^2 = I$.
2. $IA = AI = A$.
3. Calculate A^2.

Step 3: Detailed Explanation:

1. Calculate A^2:

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

2. Expand the given equation:

$$(\alpha I + \beta A)^2 = \alpha^2 I^2 + 2\alpha\beta IA + \beta^2 A^2$$

$$\alpha^2 I + 2\alpha\beta A + \beta^2(-I) = A$$

3. Group the terms for I and A:

$$(\alpha^2 - \beta^2)I + (2\alpha\beta)A = 0 \cdot I + 1 \cdot A$$

4. Compare the coefficients of I on both sides:

$$\alpha^2 - \beta^2 = 0$$

Step 4: Final Answer:

The value of $\alpha^2 - \beta^2$ is 0.

Quick Tip: The matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ behaves exactly like the imaginary unit i in complex numbers because $A^2 = -I$. This means the problem is equivalent to solving $(\alpha + \beta i)^2 = i$.

46. If $\sin\theta\cos\theta > 0$, then θ lies in which quadrant(s)?

Correct Answer: Quadrant I or Quadrant III

Solution:

Step 1: Understanding the Concept:

For the product of two numbers to be positive (> 0), both numbers must have the same sign. In trigonometry, this means $\sin\theta$ and $\cos\theta$ must be both positive or both negative.

Step 2: Key Formula or Approach:

1. $\sin\theta > 0$ and $\cos\theta > 0$ (Same sign: Positive).

2. $\sin\theta < 0$ and $\cos\theta < 0$ (Same sign: Negative).

Step 3: Detailed Explanation:

1. **Quadrant I:** All trigonometric ratios $(\sin, \cos, \tan)$ are positive. Therefore, $(+)(+) = (+)$. 2. **Quadrant II:** sin is positive, but cos is negative. Therefore, $(+)(-) = (-)$. 3. **Quadrant III:** Both sin and cos are negative. Therefore, $(-)(-) = (+)$. 4. **Quadrant IV:** cos is positive, but sin is negative. Therefore, $(-)(+) = (-)$. 5. Thus, the product is positive only in the first and third quadrants.

Step 4: Final Answer:

θ lies in Quadrant I or Quadrant III.

Quick Tip: The expression $\sin\theta\cos\theta$ can be written as $\frac{1}{2}\sin 2\theta$. If $\sin 2\theta > 0$, then 2θ is in the 1st or 2nd quadrant, which means θ is in the 1st or 3rd quadrant.

---

47. Find the product: $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

Correct Answer: $\begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix}$

Solution:

Step 1: Understanding the Concept:

Matrix multiplication involves multiplying the elements of each row of the first matrix by the corresponding elements of the column of the second matrix and summing the results.

Step 2: Key Formula or Approach:

For a 3×3 matrix multiplied by a 3×1 matrix, the result is a 3×1 matrix.

Step 3: Detailed Explanation:

1. Row 1 multiplication: $(1 \times 2) + (1 \times 3) + (1 \times 1) = 2 + 3 + 1 = 6$. 2. Row 2 multiplication: $(-1 \times 2) + (0 \times 3) + (1 \times 1) = -2 + 0 + 1 = -1$. 3. Row 3 multiplication: $(1 \times 2) + (0 \times 3) + (-1 \times 1) = 2 + 0 - 1 = 1$.

Step 4: Final Answer:

The resulting matrix is $\begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix}$.

> **Quick Tip:** To avoid mistakes, always verify the dimensions first. A $(m \times n)$ matrix times a $(n \times p)$ matrix will always result in a $(m \times p)$ matrix.

---

48. Solve $(x + 2y)dx + (2x - y)dy = 0$

Correct Answer: $\frac{x^2}{2} + 2xy - \frac{y^2}{2} = C$

Solution:

Step 1: Understanding the Concept:

This is a first-order differential equation of the form $Mdx + Ndy = 0$. We first check if the equation is "exact" by testing if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Step 2: Key Formula or Approach:

1. $M = x+2y, N = 2x-y$. 2. Test for exactness: $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$. 3. General solution: $\int Mdx$(treating y as constan $\int$(terms of N not containing x)$dy = C$.

Step 3: Detailed Explanation:

1. Calculate partial derivatives: - $\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(x+2y) = 2$ - $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2x-y) = 2$ 2. Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, the equation is exact. 3. Integrate M with respect to x:

$$\int (x+2y)dx = \frac{x^2}{2} + 2xy$$

4. Integrate terms in N that do not contain x with respect to y:

$$\int (-y)dy = -\frac{y^2}{2}$$

5. Combine the results and equate to a constant C.

Step 4: Final Answer:

The solution is $\frac{x^2}{2} + 2xy - \frac{y^2}{2} = C$.

Quick Tip: If an equation is exact, you can also solve it by just integrating Mdx and Ndy and combining all unique terms found in both results into one equation equal to C.

---

49. Find the interval where $f(x) = 1 + x\log\left(x + \sqrt{x^2+1}\right) - \sqrt{x^2+1}$ is strictly increasing for $x \geq 0$.

Correct Answer: $(0, \infty)$

Solution:

Step 1: Understanding the Concept:

A function is strictly increasing in an interval if its first derivative $f'(x)$ is greater than zero for all points in that interval. We need to differentiate the function and analyze the sign of the resulting expression.

Step 2: Key Formula or Approach:

1. Product Rule: $(uv)' = u'v + uv'$.
2. Chain Rule: $\frac{d}{dx}\ln(x+\sqrt{x^2+1}) = \frac{1}{\sqrt{x^2+1}}$.

Step 3: Detailed Explanation:

1. Differentiate the terms: - Derivative of 1 is 0. - Derivative of $x\log(x+\sqrt{x^2+1})$ using product rule:

$$(1)\log(x+\sqrt{x^2+1})+x\left(\frac{1}{\sqrt{x^2+1}}\right)$$

- Derivative of $-\sqrt{x^2+1}$:

$$-\frac{1}{2\sqrt{x^2+1}}\cdot 2x=-\frac{x}{\sqrt{x^2+1}}$$

2. Combine the derivatives to find $f'(x)$:

$$f'(x)=\log(x+\sqrt{x^2+1})+\frac{x}{\sqrt{x^2+1}}-\frac{x}{\sqrt{x^2+1}}$$

$$f'(x)=\log(x+\sqrt{x^2+1})$$

3. For $x>0$, the term $x+\sqrt{x^2+1}>1$. Since $\log(u)>0$ for $u>1$, it follows that $f'(x)>0$ for all $x>0$.

Step 4: Final Answer:

The function is strictly increasing in the interval $(0,\infty)$.

Quick Tip: The expression $\log(x+\sqrt{x^2+1})$ is the inverse hyperbolic sine, $\sinh^{-1}(x)$. Its derivative is a standard result: $1/\sqrt{x^2+1}$. Recognizing this saves significant time in the product rule step.

---

50. $\int\left[\frac{1}{1+x^2}-\frac{2x}{(1+x^2)^2}\right]e^x\,dx$

Correct Answer: $\mathrm{e}^x\frac{}{1+x^2+C}$

Solution:

Step 1: Understanding the Concept:

This integral follows the special form $\int e^x[f(x)+f'(x)]\,dx$, which always evaluates to $e^xf(x)+C$. We need to identify which part is the function and which is its derivative.

Step 2: Key Formula or Approach:

Identify $f(x)$ such that the integrand is $e^x[f(x)+f'(x)]$.

Step 3: Detailed Explanation:

1. Let $f(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$. 2. Calculate $f'(x)$ using the chain rule:

$$f'(x) = -1(1+x^2)^{-2} \cdot (2x) = -\frac{2x}{(1+x^2)^2}$$

3. The given integral matches the pattern:

$$\int e^x \left[\frac{1}{1+x^2} + \left(-\frac{2x}{(1+x^2)^2} \right) \right] dx$$

4. Therefore, the result is $e^x f(x) + C$.

Step 4: Final Answer:

The integral is $\frac{e^x}{1+x^2} + C$.

> **Quick Tip:** Whenever you see e^x multiplied by a sum of two terms in an integral, always check if one term is the derivative of the other. It is a very common shortcut in calculus exams.

---

51. A perpendicular drawn from the origin to the straight line $\sqrt{3}x + y - 24 = 0$ makes an angle α with the positive direction of the X-axis. Find α.

Correct Answer: $30°$ or $\pi/6$

Solution:

Step 1: Understanding the Concept:

We need to convert the general form of the line equation $Ax + By + C = 0$ into the "Normal Form" (or Perpendicular Form): $x\cos\alpha + y\sin\alpha = p$, where p is the distance from the origin and α is the angle requested.

Step 2: Key Formula or Approach:

Divide the equation $Ax + By = -C$ by $\sqrt{A^2 + B^2}$ to normalize it.

Step 3: Detailed Explanation:

1. Rewrite the equation: $\sqrt{3}x + y = 24$. 2. Calculate $\sqrt{A^2 + B^2}$:

$$\sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = 2$$

3. Divide the entire equation by 2:

$$\frac{\sqrt{3}}{2}x + \frac{1}{2}y = \frac{24}{2}$$

$$\left(\frac{\sqrt{3}}{2}\right)x + \left(\frac{1}{2}\right)y = 12$$

4. Compare this to $x\cos\alpha + y\sin\alpha = p$: - $\cos\alpha = \frac{\sqrt{3}}{2}$ - $\sin\alpha = \frac{1}{2}$ 5. The angle α that satisfies both is $30°$.

Step 4: Final Answer:

The angle α is $30°$.

Quick Tip: If the constant term in your normalized equation is negative, multiply the whole equation by -1 first. The distance p must always be positive in the Normal Form.

---

52. Which of the following is not true?

(a) $f(x) = x|x|$ is differentiable in $(-1, 1)$

(b) $g(x) = \sqrt{|x|}$ is differentiable in $(4, 5)$

(c) $h(x) = |x-2| + |x-3|$ is differentiable in $(2, 3)$

(d) $k(x) = |x+1| + |x-6|$ is differentiable in $(-1, 6)$

(e) $f(x) = x + [x]$ is differentiable at $x = 2$

Correct Answer: (e) $f(x) = x + [x]$ is differentiable at $x = 2$

Solution:

Step 1: Understanding the Concept:

A function is not differentiable at points where its graph has a "sharp corner" (like absolute value functions at their roots) or where the function is discontinuous (like the Greatest Integer Function at integers).

Step 2: Key Formula or Approach:

1. $|x-a|$ is non-differentiable at $x=a$. 2. $[x]$ (Greatest Integer Function) is discontinuous at all integers $n \in \mathbb{Z}$.

Step 3: Detailed Explanation:

1. **Option (a):** $x|x|$ becomes x^2 for $x \geq 0$ and $-x^2$ for $x < 0$. The derivative at 0 is 0 from both sides, so it is differentiable. 2. **Option (b):** In $(4,5)$, $|x|$ is just x. $\sqrt{x}$ is smooth and differentiable here. 3. **Option (d):** In the open interval $(-1,6)$, the "sharp corners" at $x=-1$ and $x=6$ are excluded, so the function is a simple linear sum and is differentiable. 4. **Option (e):** $[x]$ is discontinuous at $x=2$. Since continuity is a prerequisite for differentiability, $x+[x]$ cannot be differentiable at $x=2$.

Step 4: Final Answer:

The statement (e) is not true.

> **Quick Tip:** Always check for continuity first. If a function like $[x]$ or $\text{sgn}(x)$ jumps at a point, you can immediately conclude it is not differentiable there without doing any calculus.

---

53. If $f(x) = \begin{cases} \frac{2x^2+3x-5}{x-1}, & x \neq 1 \\ k, & x = 1 \end{cases}$ **is continuous at** $x=1$**, then find** k**.**

Correct Answer: 7

Solution:

Step 1: Understanding the Concept:

For a function to be continuous at a point $x=a$, the limit of the function as x approaches a must exist and be equal to the value of the function at that point, i.e., $\lim_{x\to 1} f(x) = f(1)$.

Step 2: Key Formula or Approach:

1. $f(1) = k$. 2. Evaluate $\lim_{x\to 1} \frac{2x^2+3x-5}{x-1}$.

Step 3: Detailed Explanation:

1. The limit results in a $\frac{0}{0}$ form at $x = 1$. Factor the numerator:

$$2x^2 + 3x - 5 = 2x^2 + 5x - 2x - 5$$

$$= x(2x+5) - 1(2x+5) = (x-1)(2x+5)$$

2. Substitute the factored form into the limit:

$$\lim_{x \to 1} \frac{(x-1)(2x+5)}{x-1}$$

3. Cancel the $(x-1)$ term:

$$\lim_{x \to 1}(2x+5) = 2(1) + 5 = 7$$

4. For continuity, k must equal the limit value. Therefore, $k = 7$.

Step 4: Final Answer:

The value of k is 7.

Quick Tip: For $\frac{0}{0}$ limits in continuity problems, you can also use L'Hôpital's Rule: differentiate the top $(4x+3)$ and the bottom (1). Plugging in $x = 1$ gives $4(1) + 3 = 7$.

PHYSICS

1. A spherical conductor contains 5×10^6 electrons. If the radius of the sphere is 10cm, find the electric field at its surface.

Correct Answer: 7.2×10^{-1} N/C (pointing towards the center)

Solution:

Step 1: Understanding the Concept:

For a spherical conductor, the charge resides on the surface. The electric field at the surface is calculated as if the entire charge were concentrated at the center. Because electrons are negatively charged, the field will be directed radially inward.

Step 2: Key Formula or Approach:

1. Total charge: $Q = n \cdot e$, where $e = -1.6 \times 10^{-19}$ C.
2. Electric Field: $E = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{R^2}$.
3. Constant: $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2$.

Step 3: Detailed Explanation:

1. Convert radius to meters: $R = 10\,\text{cm} = 0.1\,\text{m}$. 2. Calculate total charge Q:

$$Q = (5 \times 10^6) \times (1.6 \times 10^{-19}\,\text{C}) = 8 \times 10^{-13}\,\text{C}$$

3. Calculate Electric Field E:

$$E = \frac{(9 \times 10^9) \times (8 \times 10^{-13})}{(0.1)^2}$$

$$E = \frac{72 \times 10^{-4}}{0.01} = 72 \times 10^{-2} = 0.72\,\text{N/C}$$

Step 4: Final Answer:

The electric field at the surface is $0.72\,\text{N/C}$.

Quick Tip: Inside a solid spherical conductor, the electric field is always zero. The formula used above only applies to the surface and points outside the sphere.

---

2. Find the total energy released when 235g of ^{235}U undergoes complete fission. Assume that the energy released per fission is about 200 MeV.

Correct Answer: 1.92×10^{13} J

Solution:

Step 1: Understanding the Concept:

The total energy released is the product of the number of nuclei in the sample and the energy released per single fission event. We first find the number of atoms using Avogadro's constant.

Step 2: Key Formula or Approach:

1. Number of nuclei $(N) = \frac{\text{mass}}{\text{molar mass}} \times N_A$.
2. Total Energy $= N \times$ Energy per fission.
3. Conversion: $1\,\text{MeV} = 1.6 \times 10^{-13}$ J.

Step 3: Detailed Explanation:

1. Calculate the number of atoms (N) in 235g of Uranium-235:

$$N = \frac{235}{235} \times 6.023 \times 10^{23} = 6.023 \times 10^{23} \text{ nuclei}$$

2. Energy per fission in Joules:

$$200\,\text{MeV} = 200 \times 1.6 \times 10^{-13}\,\text{J} = 3.2 \times 10^{-11}\,\text{J}$$

3. Calculate Total Energy:

$$E_{total} = (6.023 \times 10^{23}) \times (3.2 \times 10^{-11}\,\text{J})$$

$$E_{total} \approx 19.27 \times 10^{12} = 1.927 \times 10^{13}\,\text{J}$$

Step 4: Final Answer:

The total energy released is approximately 1.93×10^{13} J.

Quick Tip: Fission of just 1 kg of Uranium provides roughly as much energy as burning 3 million kg of coal. This high energy density is why nuclear power is so efficient.

---

3. Transverse wave in a string is given by $y = 3\sin 2\pi(25t + 0.4x)$ m. What is the velocity of the wave?

Correct Answer: $62.5\,\text{m/s}$

Solution:

Step 1: Understanding the Concept:

The standard equation for a traveling wave is $y = A\sin(\omega t + kx)$. The velocity of the wave is the ratio of the angular frequency (ω) to the wave number (k).

Step 2: Key Formula or Approach:

1. Wave velocity $v = \frac{\omega}{k}$.
2. Alternatively, $v = \frac{\text{Coefficient of } t}{\text{Coefficient of } x}$.

Step 3: Detailed Explanation:

1. Rewrite the given equation by distributing the 2π:

$$y = 3\sin(50\pi t + 0.8\pi x)$$

2. Identify the constants: - Angular frequency $\omega = 50\pi$ - Wave number $k = 0.8\pi$ 3. Calculate velocity v:

$$v = \frac{50\pi}{0.8\pi} = \frac{50}{0.8} = \frac{500}{8}$$

$$v = 62.5\,\text{m/s}$$

Step 4: Final Answer:

The velocity of the wave is $62.5\,\text{m/s}$.

Quick Tip: If the signs of the t and x terms are the same (both positive or both negative), the wave travels in the **negative** x-direction. If the signs are opposite, it travels in the **positive** x-direction.

---

4. A ball of mass 200g strikes a wall with a speed 5m/s and rebounds with same speed in the opposite direction. If the average force exerted on the ball is 5N, find the time of contact between the ball and wall.

Correct Answer: $0.4\,\text{s}$

Solution:

Step 1: Understanding the Concept:

According to Newton's Second Law of Motion, the average force exerted on an object is equal to the rate of change of its momentum. Since velocity is a vector, we must account for the change in direction during the rebound.

Step 2: Key Formula or Approach:

1. Momentum $(p) = m \times v$.
2. Change in momentum $(\Delta p) = m(v_f - v_i)$.
3. Force $(F) = \frac{\Delta p}{\Delta t} \implies \Delta t = \frac{\Delta p}{F}$.

Step 3: Detailed Explanation:

1. Identify the given values and convert to SI units: - Mass $(m) = 200\,\text{g} = 0.2\,\text{kg}$. - Initial velocity $(v_i) = 5\,\text{m/s}$. - Final velocity $(v_f) = -5\,\text{m/s}$ (opposite direction). - Force $(F) = 5\,\text{N}$. 2. Calculate the change in momentum (Δp):

$$\Delta p = m(v_f - v_i) = 0.2(-5-5) = 0.2(-10) = -2\,\text{kg}\cdot\text{m/s}$$

3. The magnitude of the impulse is $2\,\text{N}\cdot\text{s}$. Calculate time (Δt):

$$\Delta t = \frac{|\Delta p|}{F} = \frac{2}{5} = 0.4\,\text{s}$$

Step 4: Final Answer:

The time of contact between the ball and the wall is $0.4\,\text{s}$.

Quick Tip: Always remember that Δv for a rebound is $2v$, not zero. If you assume the change in speed is zero, you'll mistakenly calculate the force or time as zero!

---

5. Two identical cells, each of emf 2V and internal resistance 0.1Ω, are connected in parallel. Find effective emf and the effective internal resistance of the combination.

Correct Answer: $E_{eff} = 2\,\text{V}, r_{eff} = 0.05\,\Omega$

Solution:

Step 1: Understanding the Concept:

When identical cells are connected in parallel, the effective electromotive force (emf) remains equal to the emf of a single cell, while the internal resistances combine like resistors in parallel.

Step 2: Key Formula or Approach:

For n identical cells in parallel: 1. $E_{eff} = E$. 2. $r_{eff} = \frac{r}{n}$.

Step 3: Detailed Explanation:

1. Given: $E = 2\,\text{V}$, $r = 0.1\,\Omega$, and $n = 2$. 2. Calculate effective emf:

$$E_{eff} = 2\,\text{V}$$

3. Calculate effective internal resistance:

$$r_{eff} = \frac{0.1}{2} = 0.05\,\Omega$$

Step 4: Final Answer:

The effective emf is $2\,\text{V}$ and the effective internal resistance is $0.05\,\Omega$.

Quick Tip: Connecting cells in parallel doesn't increase the voltage, but it does allow the battery to provide a higher current and reduces the overall internal power loss.

---

6. A copper wire of cross sectional area 2mm^2 carries a current I and has drift velocity V$_1$. Another copper wire of cross-sectional area 1.5mm^2 carries a current 2I and has drift velocity V$_2$. Find ratio $\frac{V_1}{V_2}$

Correct Answer: 3 : 8

Solution:

Step 1: Understanding the Concept:

Drift velocity (V_d) is the average velocity attained by charged particles (electrons) in a material due to an electric field. It is related to the current and the physical properties of the conductor.

Step 2: Key Formula or Approach:

The relation between current (I) and drift velocity (V_d) is:

$$I = nAeV_d \implies V_d = \frac{I}{nAe}$$

Since both wires are copper, the electron density (n) and charge (e) are constant. Thus, $V_d \propto \frac{I}{A}$.

Step 3: Detailed Explanation:

1. For Wire 1: $V_1 = k\frac{I}{2}$ (where $k = \frac{1}{ne}$). 2. For Wire 2: $V_2 = k\frac{2I}{1.5}$. 3. Form the ratio:

$$\frac{V_1}{V_2} = \frac{I/2}{2I/1.5}$$

4. Simplify the fraction:

$$\frac{V_1}{V_2} = \frac{1}{2} \times \frac{1.5}{2} = \frac{1.5}{4}$$

5. Multiply numerator and denominator by 2 to get integers:

$$\frac{1.5 \times 2}{4 \times 2} = \frac{3}{8}$$

Step 4: Final Answer:

The ratio $\frac{V_1}{V_2}$ is 3 : 8.

> **Quick Tip:** Drift velocity is surprisingly slow (usually millimeters per second), even though the electric signal itself travels at nearly the speed of light.

---

7. A solid sphere of radius 20cm has the same mass as a solid cylinder. If their moments of inertia about their respective central axes are equal, find the radius of the cylinder.

Correct Answer: $4\sqrt{10}\,\text{cm} \approx 12.65\,\text{cm}$

Solution:

Step 1: Understanding the Concept:

Moment of inertia depends on the distribution of mass relative to the axis of rotation. For different geometric shapes, we use specific formulas that relate mass and radius to the inertia. We are given that these values are equal for two different shapes of the same mass.

Step 2: Key Formula or Approach:

1. M.I. of a solid sphere $(I_s) = \frac{2}{5}MR_s^2$.
2. M.I. of a solid cylinder $(I_c) = \frac{1}{2}MR_c^2$.
3. Set $I_s = I_c$ and solve for R_c.

Step 3: Detailed Explanation:

1. Given: $R_s = 20\,\text{cm}$ and masses are equal (M). 2. Equate the formulas:

$$\frac{2}{5}M(20)^2 = \frac{1}{2}MR_c^2$$

3. Cancel M from both sides and simplify:

$$\frac{2}{5} \times 400 = \frac{1}{2}R_c^2$$

$$2 \times 80 = \frac{1}{2}R_c^2$$

$$160 = \frac{1}{2}R_c^2$$

4. Multiply by 2:

$$R_c^2 = 320$$

5. Take the square root:

$$R_c = \sqrt{320} = \sqrt{64 \times 5} = 8\sqrt{5}\,\text{cm} \approx 17.89\,\text{cm}$$

(Correction: Recalculating $8\sqrt{5} \approx 8 \times 2.236 = 17.888$*)*

Step 4: Final Answer:

The radius of the cylinder is $8\sqrt{5}\,\text{cm}$.

> **Quick Tip:** The sphere's factor is 0.4 $(2/5)$ and the cylinder's is 0.5 $(1/2)$. Because the sphere has more of its mass concentrated toward the center, it needs a larger radius than a cylinder of the same mass to have the same inertia.

8. In which thermodynamic process does the internal energy of an ideal gas remain unchanged?

Correct Answer: Isothermal Process

Solution:

Step 1: Understanding the Concept:

Internal energy (U) of an ideal gas is a function solely of its absolute temperature (T). If the temperature of the system does not change, the internal energy must remain constant.

Step 2: Key Formula or Approach:

1. $\Delta U = nC_v\Delta T$.

2. If $\Delta T = 0$, then $\Delta U = 0$.

Step 3: Detailed Explanation:

1. In an Isothermal Process, the system is in contact with a heat reservoir, and the process happens slowly enough that the temperature remains constant ($T =$ constant). 2. Since internal energy for an ideal gas is defined as $U = \frac{f}{2}nRT$, no change in T means $\Delta U = 0$. 3. According to the First Law of Thermodynamics ($\Delta Q = \Delta U + \Delta W$), in this process, all heat added to the system is converted entirely into work.

Step 4: Final Answer:

The internal energy remains unchanged in an Isothermal Process.

> **Quick Tip:** Don't confuse "Internal Energy" with "Heat." In an Adiabatic process, no heat enters or leaves, but the internal energy can change because work is done, causing the temperature to rise or fall.

---

9. A gun fires 25 bullets per second. Each bullet has a mass of 10g and is fired with a velocity of 20m/s. Find the recoil force on the gun.

Correct Answer: 5 N

Solution:

Step 1: Understanding the Concept:

Recoil force is an application of Newton's Second Law and the Conservation of Momentum. The force exerted by the gun on the bullets (to accelerate them) is equal and opposite to the force exerted by the bullets on the gun (recoil force).

Step 2: Key Formula or Approach:

1. Force $(F) = \frac{\Delta p}{\Delta t}$.
2. For multiple bullets: $F = n \cdot m \cdot v$, where n is the number of bullets per second.

Step 3: Detailed Explanation:

1. Identify the given values: - Rate $(n) = 25$ bullets/s. - Mass $(m) = 10\,\text{g} = 0.01\,\text{kg}$. - Velocity $(v) = 20\,\text{m/s}$. 2. Calculate the momentum of a single bullet:

$$p = m \times v = 0.01 \times 20 = 0.2\,\text{kg} \cdot \text{m/s}$$

3. Calculate the total change in momentum per second (which is the Force):

$$F = n \times p = 25 \times 0.2$$

$$F = 5\,\text{N}$$

Step 4: Final Answer:

The recoil force on the gun is $5\,\text{N}$.

Quick Tip: Force is the same as the "Momentum Flow Rate." To find it quickly, just multiply (mass) × (velocity) × (frequency).

---

10. A satellite moves in an elliptical orbit around a planet such that its maximum distance and minimum distance from the planet are in the ratio 3:1. If its speed at the nearest point (perigee) is V, find its speed at the farthest point (Apogee).

Correct Answer: $\frac{V}{3}$

Solution:

Step 1: Understanding the Concept:

According to Kepler's Second Law (Law of Conservation of Angular Momentum), a satellite moves faster when it is closer to the planet and slower when it is farther away. The angular momentum $L = mvr\sin\theta$ remains constant. At the perigee and apogee, the velocity is perpendicular to the radius ($\theta = 90°$).

Step 2: Key Formula or Approach:

Conservation of Angular Momentum:

$$mv_p r_p = mv_a r_a \implies v_p r_p = v_a r_a$$

Step 3: Detailed Explanation:

1. Let the minimum distance (perigee) be r_p and the maximum distance (apogee) be r_a. 2. Given the ratio $r_a : r_p = 3 : 1$. This means $r_a = 3r_p$. 3. Let speed at perigee be $v_p = V$ and speed at apogee be v_a. 4. Applying the formula:

$$V \cdot r_p = v_a \cdot (3r_p)$$

5. Cancel r_p from both sides:

$$V = 3v_a \implies v_a = \frac{V}{3}$$

Step 4: Final Answer:

The speed at the farthest point (Apogee) is $\frac{V}{3}$.

Quick Tip: In elliptical orbits, velocity and distance are inversely proportional: $v \propto \frac{1}{r}$. If the distance triples, the speed must become one-third.

---

11. A particle moves such that its position is given by $y = t^2 + 2t + 3$ (m). Find the average acceleration of the particle between $t = 3s$ and $t = 6s$.

Correct Answer: $2\,\text{m/s}^2$

Solution:

Step 1: Understanding the Concept:

Average acceleration is the change in velocity divided by the time interval. To find this, we must first determine the velocity function by differentiating the position function.

Step 2: Key Formula or Approach:

1. Velocity $v = \frac{dy}{dt}$.

2. Average Acceleration $a_{avg} = \frac{v(t_2)-v(t_1)}{t_2-t_1}$.

Step 3: Detailed Explanation:

1. Differentiate $y = t^2 + 2t + 3$ to find velocity:

$$v(t) = \frac{d}{dt}(t^2 + 2t + 3) = 2t + 2$$

2. Find velocity at $t = 3s$:

$$v(3) = 2(3) + 2 = 8\,\text{m/s}$$

3. Find velocity at $t = 6s$:

$$v(6) = 2(6) + 2 = 14\,\text{m/s}$$

4. Calculate average acceleration:

$$a_{avg} = \frac{14-8}{6-3} = \frac{6}{3} = 2\,\text{m/s}^2$$

Step 4: Final Answer:

The average acceleration is $2\,\text{m/s}^2$.

> **Quick Tip:** Notice that the second derivative of position is $a = 2$. Since the acceleration is a constant, the instantaneous acceleration and average acceleration will be exactly the same regardless of the time interval.

---

12. $A = \frac{B}{CD^2}$**. If B, C and D have dimensions of inductive reactance, capacitive reactance and**

angular frequency respectively, find the dimension of A.

Correct Answer: $[T^2]$

Solution:

Step 1: Understanding the Concept:

In dimensional analysis, physical quantities like reactance (resistance) have specific dimensions. We can simplify the problem by substituting the dimensions of each variable into the formula.

Step 2: Key Formula or Approach:

1. Dimension of Inductive Reactance (B): $[ML^2T^{-3}A^{-2}]$ (same as resistance Ω).
2. Dimension of Capacitive Reactance (C): $[ML^2T^{-3}A^{-2}]$ (same as resistance Ω).
3. Dimension of Angular Frequency (D): $[T^{-1}]$.

Step 3: Detailed Explanation:

1. Plug the dimensions into the formula $A = \frac{B}{CD^2}$:

$$[A] = \frac{[\Omega]}{[\Omega] \cdot [T^{-1}]^2}$$

2. The dimensions of reactance cancel out:

$$[A] = \frac{1}{[T^{-2}]}$$

3. Moving the term to the numerator:

$$[A] = [T^2]$$

Step 4: Final Answer:

The dimension of A is $[T^2]$ or $[M^0L^0T^2]$.

Quick Tip: You don't need the full MLTA breakdown for reactance here! Since both B and C represent reactance, their units (Ω) cancel out immediately, leaving only the frequency term to solve.

---

13. A beam of unpolarized light of intensity I_0 is incident on a polarizer. A second polaroid is placed in the path such that its transmission axis makes an angle of 45° with the first polaroid. What is the intensity of light after it passes through the second polaroid?

Correct Answer: $\frac{I_0}{4}$

Solution:

Step 1: Understanding the Concept:

When unpolarized light passes through the first polarizer, its intensity is halved. When this polarized light passes through a second polarizer (analyzer), its intensity is governed by Malus's Law, which depends on the angle between the transmission axes of the two polarizers.

Step 2: Key Formula or Approach:

1. Intensity after first polarizer: $I_1 = \frac{I_0}{2}$.
2. Malus's Law for the second polarizer: $I_2 = I_1 \cos^2 \theta$.

Step 3: Detailed Explanation:

1. After the first polarizer, the intensity becomes $I_1 = \frac{I_0}{2}$. 2. The second polarizer is at an angle $\theta = 45°$ relative to the first. 3. Apply Malus's Law:

$$I_2 = \left(\frac{I_0}{2}\right) \cos^2(45°)$$

4. Since $\cos 45° = \frac{1}{\sqrt{2}}$, then $\cos^2(45°) = \frac{1}{2}$:

$$I_2 = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

Step 4: Final Answer:

The intensity of light after the second polaroid is $\frac{I_0}{4}$ or $0.25 I_0$.

> **Quick Tip:** Always remember the "Half-Intensity Rule": the first polarizer always cuts unpolarized light by 50%, regardless of its orientation. Malus's Law only applies to the polarizers that come *after* the first one.

14. For an electron of mass m and charge e, find the ratio of angular momentum (L) to magnetic dipole moment (μ).

Correct Answer: $\frac{2m}{e}$

Solution:

Step 1: Understanding the Concept:

A circulating electron creates a current loop, which has a magnetic moment. It also has angular momentum due to its orbital motion. The ratio between these two quantities is a fundamental property related to the gyromagnetic ratio.

Step 2: Key Formula or Approach:

1. Magnetic Moment $(\mu) = IA = \frac{evr}{2}$.
2. Angular Momentum $(L) = mvr$.
3. Ratio $= \frac{L}{\mu}$.

Step 3: Detailed Explanation:

1. For an electron in a circular orbit of radius r with velocity v: - The current $I = \frac{e}{T} = \frac{ev}{2\pi r}$. - The area $A = \pi r^2$. - $\mu = I \times A = \left(\frac{ev}{2\pi r}\right)(\pi r^2) = \frac{evr}{2}$. 2. The angular momentum is $L = mvr$. 3. Finding the ratio $\frac{L}{\mu}$:

$$\frac{L}{\mu} = \frac{mvr}{\frac{evr}{2}} = \frac{2mvr}{evr} = \frac{2m}{e}$$

Step 4: Final Answer:

The ratio of angular momentum to magnetic moment is $\frac{2m}{e}$.

> **Quick Tip:** The inverse of this ratio $(\frac{e}{2m})$ is known as the Gyromagnetic Ratio. Note that for an electron, because the charge is negative, the magnetic moment vector and angular momentum vector point in opposite directions.

---

15. Which of the following has the highest modulus of elasticity?

(A) Steel

(B) Aluminium

(C) Brass

(D) Glass

Correct Answer: (A) Steel

Solution:

Step 1: Understanding the Concept:

Modulus of elasticity (specifically Young's Modulus) is a measure of a material's stiffness or its resistance to elastic deformation. A material with a higher modulus requires more stress to produce the same amount of strain.

Step 2: Key Formula or Approach:

Young's Modulus $(Y) = \frac{\text{Stress}}{\text{Strain}}$.

Step 3: Detailed Explanation:

1. Steel has a Young's Modulus of approximately 200×10^9 Pa. 2. Aluminium is much more flexible, with a modulus of about 70×10^9 Pa. 3. Brass is approximately 100–120 $\times 10^9$ Pa. 4. Glass, while brittle, has a modulus of roughly 50–90 $\times 10^9$ Pa. 5. Comparing these values, Steel is the stiffest and most elastic material in the list because it returns to its original shape most forcefully after a large stress is applied.

Step 4: Final Answer:

Steel has the highest modulus of elasticity among the given options.

> **Quick Tip:** In common language, we say rubber is "more elastic," but in physics, Steel is more elastic than rubber. This is because steel resists deformation much more strongly and requires much higher stress to stretch.

---

16. At which condition do the experimental P-V curve and predicted P-V curve (for an ideal gas) closely match?

(a) Low temperature and high pressure

(b) High temperature and low pressure

Correct Answer: (b) High temperature and low pressure

Solution:

Step 1: Understanding the Concept:

Real gases deviate from ideal behavior because the kinetic theory of gases assumes that gas molecules have no volume and no intermolecular forces. In reality, these assumptions only hold true under specific physical conditions.

Step 2: Key Formula or Approach:

An ideal gas follows $PV = nRT$. A real gas follows the Van der Waals equation:

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

Step 3: Detailed Explanation:

1. Low Pressure: When pressure is low, the volume of the gas is very large. This makes the actual volume of the molecules (nb) negligible compared to the total volume. 2. High Temperature: When temperature is high, molecules move with high kinetic energy, overcoming the attractive intermolecular forces ($\frac{an^2}{V^2}$). 3. Under these two conditions, real gases behave almost exactly like ideal gases, meaning the experimental and predicted curves will coincide.

Step 4: Final Answer:

The curves match closely at high temperature and low pressure.

> **Quick Tip:** To remember this, think of molecules as "social" vs "independent." At high temp and low pressure, they are far apart and moving too fast to "talk" to each other, behaving independently like an ideal gas.

---

17. Which law is the symmetrical counterpart of Faraday's law in electromagnetic induction?

(A) Ampere-Maxwell law

(B) Ampere's Circuital law

(C) Gauss's law

(D) Coulomb's law

Correct Answer: (A) Ampere-Maxwell law

Solution:

Step 1: Understanding the Concept:

Electromagnetism is built on mathematical symmetry. Faraday discovered that a changing magnetic field creates an electric field. Scientists looked for a law that described the inverse effect.

Step 2: Key Formula or Approach:

1. Faraday's Law: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ (Changing B creates E). 2. Ampere-Maxwell Law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_c + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ (Changing E creates B).

Step 3: Detailed Explanation:

1. Faraday's law shows that a time-varying magnetic flux induces an electromotive force (electric field). 2. Maxwell realized that Ampere's original law was incomplete. He added the "displacement current" term ($\frac{d\Phi_E}{dt}$), which shows that a time-varying electric flux induces a magnetic field. 3. This creates a perfect symmetry between electricity and magnetism, which is why the Ampere-Maxwell law is considered the counterpart to Faraday's law.

Step 4: Final Answer:

The symmetrical counterpart is the Ampere-Maxwell law.

> **Quick Tip:** This symmetry is what allows electromagnetic waves (like light) to travel through a vacuum. The changing E-field creates a B-field, which then creates an E-field, and so on.

---

18. A wire of fixed length is bent into a single circular turn, producing a magnetic field B at its center. If the same wire is bent into 3 circular turns (carrying the same current), what will be the magnetic field at the center?

Correct Answer: $9B$

Solution:

Step 1: Understanding the Concept:

The magnetic field at the center of a circular coil depends on both the number of turns (n) and the radius (r). When you use a fixed length of wire to make more turns, the radius of each turn must decrease.

Step 2: Key Formula or Approach:

1. Magnetic Field at center: $B = \frac{\mu_0 n I}{2r}$. 2. Length of wire is constant: $L = n(2\pi r) \implies r \propto \frac{1}{n}$.

Step 3: Detailed Explanation:

1. Case 1 ($n = 1$): Let radius be r_1. Field $B_1 = \frac{\mu_0 (1) I}{2r_1} = B$. 2. Case 2 ($n = 3$): Since the wire length is the same, $1(2\pi r_1) = 3(2\pi r_2)$. Therefore, $r_2 = \frac{r_1}{3}$. 3. Calculate the new field B_2:

$$B_2 = \frac{\mu_0 (3) I}{2r_2} = \frac{\mu_0 (3) I}{2(r_1/3)}$$

$$B_2 = \frac{\mu_0 (3 \times 3) I}{2r_1} = 9\left(\frac{\mu_0 I}{2r_1}\right)$$

4. Since the term in the parentheses is the original field B:

$$B_2 = 9B$$

Step 4: Final Answer:

The new magnetic field at the center will be $9B$.

Quick Tip: For a fixed length of wire, the magnetic field at the center is proportional to the square of the number of turns ($B \propto n^2$). If turns become n times, field becomes n^2 times.

---

19. Find the RMS current for the given expression: $i = 4\sqrt{2}\sin\omega t + 3\sqrt{2}\cos\omega t$

Correct Answer: 5 A

Solution:

Step 1: Understanding the Concept:

The root mean square (RMS) current for a sinusoidal wave is related to its peak amplitude (I_0) by the formula $I_{rms} = I_0/\sqrt{2}$. When we have a sum of a sine and cosine function of the same frequency, we must first find the resultant peak amplitude.

Step 2: Key Formula or Approach:

1. For $i = A\sin\omega t + B\cos\omega t$, the resultant peak amplitude is $I_0 = \sqrt{A^2 + B^2}$.
2. $I_{rms} = \frac{I_0}{\sqrt{2}}$.

Step 3: Detailed Explanation:

1. Identify the coefficients: $A = 4\sqrt{2}$ and $B = 3\sqrt{2}$. 2. Calculate the peak amplitude I_0:

$$I_0 = \sqrt{(4\sqrt{2})^2 + (3\sqrt{2})^2}$$

$$I_0 = \sqrt{(16 \times 2) + (9 \times 2)} = \sqrt{32 + 18} = \sqrt{50}$$

3. Simplify the square root: $I_0 = 5\sqrt{2}$ A. 4. Calculate the RMS value:

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5\sqrt{2}}{\sqrt{2}} = 5\,\text{A}$$

Step 4: Final Answer:

The RMS current is 5 A.

> **Quick Tip:** A common mistake is calculating the RMS of each part and adding them. Always find the combined peak amplitude first, then divide by $\sqrt{2}$.

---

20. What is the ratio of distance travelled by a freely falling body in successive equal intervals of time (starting from rest)?

Correct Answer: $1:3:5:7:\ldots$

Solution:

Step 1: Understanding the Concept:

This is a classic problem in kinematics known as Galileo's Law of Odd Numbers. For an object under constant acceleration starting from rest, the distances covered in consecutive equal time intervals follow a specific arithmetic progression.

Step 2: Key Formula or Approach:

Distance in the n^{th} second: $S_n = u + \frac{a}{2}(2n-1)$.

Step 3: Detailed Explanation:

1. For a free fall from rest, initial velocity $u = 0$ and acceleration $a = g$. 2. The distance covered in the n^{th} interval is:

$$S_n = 0 + \frac{g}{2}(2n-1) \implies S_n \propto (2n-1)$$

3. For successive intervals ($n = 1, 2, 3, \ldots$): - For $n = 1$: $S_1 = \frac{g}{2}(2(1)-1) = \frac{g}{2}(1)$ - For $n = 2$: $S_2 = \frac{g}{2}(2(2)-1) = \frac{g}{2}(3)$ - For $n = 3$: $S_3 = \frac{g}{2}(2(3)-1) = \frac{g}{2}(5)$ 4. The ratio is $1 : 3 : 5 : 7 : \ldots$

Step 4: Final Answer:

The ratio is the sequence of odd numbers: $1 : 3 : 5 : 7 : \ldots$

Quick Tip: Be careful with the wording! The ratio of **total** distance covered after time $t, 2t, 3t$ is $1^2 : 2^2 : 3^2$ (i.e., $1 : 4 : 9$), but the distance in **successive intervals** is the ratio of odd numbers.

---

21. A block of 10kg mass moving on a frictionless surface with 5m/s compresses a spring by 5cm and comes to rest. What is the force constant of the spring?

Correct Answer: 10^5 N/m

Solution:

Step 1: Understanding the Concept:

According to the Law of Conservation of Energy, the kinetic energy of the moving block is converted entirely into the elastic potential energy of the spring when the block comes to a momentary rest.

Step 2: Key Formula or Approach:

1. Kinetic Energy $(K.E.) = \frac{1}{2}mv^2$.

2. Elastic Potential Energy ($P.E.$) $= \frac{1}{2}kx^2$.
3. $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \implies k = \frac{mv^2}{x^2}$.

Step 3: Detailed Explanation:

1. Convert units to SI: - Mass (m) $= 10\,\text{kg}$. - Velocity (v) $= 5\,\text{m/s}$. - Compression (x) $= 5\,\text{cm} = 0.05\,\text{m} = 5 \times 10^{-2}\,\text{m}$. 2. Set up the energy balance:

$$10 \times (5)^2 = k \times (0.05)^2$$

$$10 \times 25 = k \times (0.0025)$$

3. Solve for k:

$$250 = k \times \frac{25}{10000}$$

$$k = \frac{250 \times 10000}{25} = 10 \times 10000 = 100,000\,\text{N/m}$$

Step 4: Final Answer:

The force constant (k) of the spring is $10^5\,\text{N/m}$.

Quick Tip: Always convert cm to m before squaring. In spring problems, x^2 is a very small number (0.0025), which often results in a very large value for the spring constant k.

---

22. A uniform rod of mass m and length l is rotating in a horizontal circle about a vertical axis passing through one of its ends with angular velocity ω. What is the angular momentum of the rod?

Correct Answer: $\frac{1}{3}ml^2\omega$

Solution:

Step 1: Understanding the Concept:

Angular momentum (L) of a rigid body is the product of its moment of inertia (I) about the axis of rotation and its angular velocity (ω). The moment of inertia depends on how the mass of the rod is distributed relative to the axis.

Step 2: Key Formula or Approach:

1. Angular Momentum $L = I\omega$.

2. Moment of Inertia of a uniform rod about an axis through one end: $I = \frac{1}{3}ml^2$.

Step 3: Detailed Explanation:

1. Identify the moment of inertia for the specific axis: For a rod of mass m and length l rotating about its end, $I = \frac{1}{3}ml^2$. 2. Given the angular velocity is ω. 3. Substitute I into the angular momentum formula:

$$L = \left(\frac{1}{3}ml^2\right) \times \omega = \frac{1}{3}ml^2\omega$$

Step 4: Final Answer:

The angular momentum of the rod is $\frac{1}{3}ml^2\omega$.

Quick Tip: If the rod were rotating about its **center** (center of mass), the moment of inertia would be $\frac{1}{12}ml^2$, and the angular momentum would be four times smaller.

---

23. If the threshold wavelengths of two metals are in the ratio 3:1, what is the ratio of their work functions?

Correct Answer: 1 : 3

Solution:

Step 1: Understanding the Concept:

The work function (ϕ) of a metal is the minimum energy required to eject an electron from its surface. It is inversely proportional to the threshold wavelength (λ_0).

Step 2: Key Formula or Approach:

1. Work Function $\phi = \frac{hc}{\lambda_0}$.

2. Therefore, $\phi \propto \frac{1}{\lambda_0}$.

Step 3: Detailed Explanation:

1. Let the threshold wavelengths be λ_1 and λ_2. Given $\frac{\lambda_1}{\lambda_2} = \frac{3}{1}$. 2. Let the work functions be ϕ_1 and ϕ_2. 3. Using the inverse relationship:

$$\frac{\phi_1}{\phi_2} = \frac{\lambda_2}{\lambda_1}$$

4. Substitute the given ratio:

$$\frac{\phi_1}{\phi_2} = \frac{1}{3}$$

Step 4: Final Answer:

The ratio of their work functions is 1 : 3.

> **Quick Tip:** In the photoelectric effect, "threshold" values always have an inverse relationship. If threshold frequency is higher, work function is higher. If threshold **wavelength** is higher, work function is **lower**.

---

24. If the electric potential is $V = 3x^2 + 4x$, then the magnitude of the Electric field at $x = 1$m is?

Correct Answer: $10\,\text{V/m}$

Solution:

Step 1: Understanding the Concept:

The electric field (E) is the negative gradient of the electric potential (V). In a one-dimensional case, the electric field is the negative derivative of the potential with respect to distance.

Step 2: Key Formula or Approach:

1. $E = -\frac{dV}{dx}$.
2. Magnitude of $E = |-\frac{dV}{dx}|$.

Step 3: Detailed Explanation:

1. Differentiate the potential function $V = 3x^2 + 4x$ with respect to x:

$$\frac{dV}{dx} = \frac{d}{dt}(3x^2 + 4x) = 6x + 4$$

2. The electric field expression is:

$$E = -(6x + 4)$$

3. Calculate the value of E at $x = 1\,\text{m}$:

$$E = -(6(1) + 4) = -10\,\text{V/m}$$

4. The magnitude is the absolute value: $|-10| = 10\,\text{V/m}$.

Step 4: Final Answer:

The magnitude of the electric field at $x = 1\,\text{m}$ is $10\,\text{V/m}$.

> **Quick Tip:** The negative sign in $E = -dV/dx$ indicates that the electric field always points in the direction of decreasing potential (from "high voltage" to "low voltage").

---

25. Which of the following is not an extensive variable?

(a) Total mass

(b) Internal energy

(c) Volume

(d) Density

(e) Work done

Correct Answer: (d) Density

Solution:

Step 1: Understanding the Concept:

Thermodynamic variables are classified into two types: **Extensive** variables depend on the size or amount of matter in the system (e.g., mass, volume). **Intensive** variables are independent of the amount of matter (e.g., temperature, pressure, density).

Step 2: Key Formula or Approach:

If you divide a system into two equal parts, a variable is intensive if its value remains the same in each part, and extensive if its value is halved.

Step 3: Detailed Explanation:

1. **Mass, Internal Energy, and Volume** are all halved if you cut the system in half; thus, they are extensive. 2. **Work done** is a path function and technically depends on the scale of the system, often treated as extensive in context. 3. **Density** ($\rho = m/V$) is the ratio of two extensive variables. If you cut a block of lead in half, the density of each piece remains exactly the same. Therefore, density is an intensive variable.

Step 4: Final Answer:

Density is not an extensive variable.

> **Quick Tip:** A helpful rule of thumb: The ratio of two extensive properties (like Mass/Volume or Energy/Mass) always results in an intensive property.

---

26. What is the mass of one molecule of water in kg?

Correct Answer: 2.99×10^{-26} kg

Solution:

Step 1: Understanding the Concept:

To find the mass of a single molecule, we use the molar mass of the substance and divide it by Avogadro's number (N_A), which represents the number of molecules in one mole.

Step 2: Key Formula or Approach:

1. Molar mass of $H_2O = 18\,\text{g/mol} = 0.018\,\text{kg/mol}$.

2. Mass of one molecule $= \frac{\text{Molar Mass}}{N_A}$.

Step 3: Detailed Explanation:

1. Avogadro's number $N_A \approx 6.022 \times 10^{23}\,\text{mol}^{-1}$. 2. Calculate the mass:

$$\text{Mass} = \frac{0.018\,\text{kg}}{6.022 \times 10^{23}}$$

$$\text{Mass} = 0.002989 \times 10^{-23}\,\text{kg}$$

3. Express in scientific notation:

$$\text{Mass} \approx 2.99 \times 10^{-26}\,\text{kg}$$

Step 4: Final Answer:

The mass of one water molecule is 2.99×10^{-26} kg.

> **Quick Tip:** You can also calculate this in atomic mass units (amu). One water molecule is 18 amu. Since $1\,\text{amu} = 1.66 \times 10^{-27}$ kg, multiplying $18 \times 1.66 \times 10^{-27}$ gives the same result.

---

27. A capacitor has capacitance $C = 4\,\mu$F. If the energy stored in it is 18×10^{-8} J, find the charge stored in the capacitor.

Correct Answer: 1.2×10^{-6} C (or $1.2\,\mu$C)

Solution:

Step 1: Understanding the Concept:

Energy is stored in the electric field between the plates of a capacitor. This energy (U) can be expressed in terms of capacitance (C), voltage (V), or charge (Q).

Step 2: Key Formula or Approach:

1. Energy $U = \frac{Q^2}{2C}$.
2. Rearranging for charge: $Q = \sqrt{2CU}$.

Step 3: Detailed Explanation:

1. Identify given values in SI units: - $C = 4\,\mu\text{F} = 4 \times 10^{-6}$ F. - $U = 18 \times 10^{-8}$ J. 2. Substitute into the formula:

$$Q^2 = 2 \times (4 \times 10^{-6}) \times (18 \times 10^{-8})$$

$$Q^2 = 8 \times 18 \times 10^{-14}$$

$$Q^2 = 144 \times 10^{-14}$$

3. Take the square root:

$$Q = \sqrt{144 \times 10^{-14}} = 12 \times 10^{-7}\,\text{C}$$

4. Convert to standard scientific notation:

$$Q = 1.2 \times 10^{-6}\,\text{C}$$

Step 4: Final Answer:

The charge stored in the capacitor is $1.2 \times 10^{-6}\,\text{C}$.

> **Quick Tip:** There are three forms of the energy formula: $\frac{1}{2}CV^2$, $\frac{1}{2}QV$, and $\frac{Q^2}{2C}$. Choosing the one that uses your two known variables (C and U) directly avoids the need to calculate Voltage first.

---

28. The radius of the innermost orbit of a Hydrogen atom is 0.53Å. What is the radius of the 3rd orbit?

Correct Answer: 4.77Å

Solution:

Step 1: Understanding the Concept

According to Bohr's model of the atom, the radius of the permitted orbits for an electron is quantized. The radius depends on the principal quantum number (n), which represents the orbit level.

Step 2: Key Formula or Approach

The radius of the n^{th} orbit is given by:

$$r_n = r_0 \times n^2$$

where r_0 is the Bohr radius (radius of the 1st orbit).

Step 3: Detailed Explanation

1. Given: $r_0 = 0.53\text{Å}$ and we need to find the radius for the 3rd orbit ($n = 3$). 2. Apply the formula:

$$r_3 = 0.53 \times (3)^2$$

3. Calculate the square:

$$r_3 = 0.53 \times 9$$

4. Perform the multiplication:

$$r_3 = 4.77\,\text{Å}$$

Step 4: Final Answer

The radius of the 3rd orbit is 4.77Å.

Quick Tip: The radius grows with the square of the orbit number. This means the orbits get significantly further apart as you move away from the nucleus ($1 : 4 : 9 : 16 \ldots$).

---

29. The order of the electric field required to pull out electrons from a metal by field emission is

Correct Answer: $10^8\,\text{V/m}$

Solution:

Step 1: Understanding the Concept

Field emission (also known as Fowler-Nordheim emission) is the process where electrons are emitted from a metal surface due to the application of a very strong external electric field. This field lowers the potential barrier, allowing electrons to "tunnel" out.

Step 2: Key Formula or Approach

The electric field must be strong enough to overcome the work function of the metal by narrowing the surface potential barrier to a few nanometers.

Step 3: Detailed Explanation

1. For an electron to escape a metal without heat (thermionic emission) or light (photoelectric

emission), a massive external force is needed. 2. Experimental data and theoretical calculations show that the electric field strength required is typically in the range of 10^8 V/m. 3. If the field is weaker than this, the probability of electron tunneling is negligible.

Step 4: Final Answer

The order of the electric field required is 10^8 V/m.

> **Quick Tip:** To achieve such high fields in a lab, scientists often use very sharp needles. The electric field is much stronger at sharp points (lightning rod effect), allowing field emission at lower voltages.

---

30. The relative viscosity of blood ($\frac{\eta_{blood}}{\eta_{water}}$) is constant in which temperature range?

Correct Answer: 0°C to 37°C

Solution:

Step 1: Understanding the Concept

Viscosity is the internal friction of a fluid. Relative viscosity compares the viscosity of a fluid (like blood) to a reference fluid (usually water). While the absolute viscosity of both blood and water changes significantly with temperature, their ratio remains relatively stable within a specific biological range.

Step 2: Key Formula or Approach

Relative Viscosity $\eta_{rel} = \frac{\eta_{solution}}{\eta_{solvent}}$.

Step 3: Detailed Explanation

1. Both blood and water become "thinner" (less viscous) as they get warmer. 2. However, they change at approximately the same rate within the range of standard physiological and environmental temperatures. 3. Studies show that the ratio $\frac{\eta_{blood}}{\eta_{water}}$ remains nearly constant between 0°C and 37°C (human body temperature). 4. Beyond this range, especially as blood approaches higher temperatures, proteins may begin to denature, changing the ratio.

Step 4: Final Answer

The relative viscosity is constant in the range of 0°C to 37°C.

Quick Tip: Blood is a non-Newtonian fluid, meaning its viscosity also changes depending on how fast it is flowing (shear rate), not just the temperature!

---

31. The magnifying power of a simple microscope can be increased by using a lens of ____.

Correct Answer: Shorter focal length

Solution:

Step 1: Understanding the Concept

A simple microscope is essentially a single convex lens. The magnifying power is the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the unaided eye.

Step 2: Key Formula or Approach

The magnifying power (M) for an image formed at the least distance of distinct vision ($D = 25$ cm) is:

$$M = 1 + \frac{D}{f}$$

where f is the focal length of the lens.

Step 3: Detailed Explanation

1. From the formula $M = 1 + \frac{D}{f}$, we see that M is inversely proportional to the focal length f. 2. If the focal length is decreased, the value of $\frac{D}{f}$ increases, thereby increasing the magnifying power. 3. Therefore, a lens with higher optical power (shorter focal length) provides better magnification.

Step 4: Final Answer

Magnifying power is increased by using a lens with a **shorter focal length**.

Quick Tip: There is a limit to how much you can decrease the focal length. As the lens becomes more "curved" to shorten f, spherical and chromatic aberrations increase, which can blur the image.

---

32. In a Silicon (Si) crystal containing N atoms at absolute zero, the number of energy states in the valence band is ____.

Correct Answer: $4N$

Solution:

Step 1: Understanding the Concept

In an isolated Silicon atom, the outer shell ($n = 3$) has 4 valence electrons occupying the $3s$ and $3p$ orbitals. When N atoms come together to form a crystal, these energy levels split into bands.

Step 2: Key Formula or Approach

Each Si atom provides 4 valence electrons and has 8 available states (2 in s and 6 in p) in its outer shell.

Step 3: Detailed Explanation

1. For N atoms, there are a total of $8N$ available states in the valence shell. 2. These states split into two bands separated by an energy gap (E_g). 3. The lower band (Valence Band) contains $4N$ states and is completely filled by the $4N$ electrons at absolute zero temperature (0 K). 4. The upper band (Conduction Band) also contains $4N$ states but is completely empty at 0 K.

Step 4: Final Answer

The number of energy states in the valence band is $4N$.

> **Quick Tip:** At absolute zero, Silicon acts as a perfect insulator because the valence band is full and the conduction band is empty, meaning no electrons are free to move.

---

33. Two particles with charges $2q$ and q having equal momentum enter a uniform magnetic field in a direction perpendicular to the field. Find the ratio of the radii of their circular paths.

Correct Answer: 1 : 2

Solution:

Step 1: Understanding the Concept

When a charged particle enters a magnetic field perpendicularly, it experiences a Lorentz force that acts as a centripetal force, causing the particle to move in a circular path.

Step 2: Key Formula or Approach

The radius (r) of the circular path is given by:

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

where p is the momentum, q is the charge, and B is the magnetic field.

Step 3: Detailed Explanation

1. We are given that momentum (p) and magnetic field (B) are the same for both particles. 2. Therefore, r is inversely proportional to the charge: $r \propto \frac{1}{q}$. 3. Let r_1 be the radius for charge $q_1 = 2q$, and r_2 be the radius for charge $q_2 = q$. 4. The ratio is:

$$\frac{r_1}{r_2} = \frac{q_2}{q_1}$$

5. Substituting the values:

$$\frac{r_1}{r_2} = \frac{q}{2q} = \frac{1}{2}$$

Step 4: Final Answer

The ratio of the radii is 1 : 2.

> **Quick Tip:** A larger charge experiences a stronger magnetic force, which "pulls" it into a tighter (smaller) circle. This is why the particle with $2q$ has half the radius of the particle with q.

---

34. The instantaneous displacement of a wave is given by $y = 2\sin pt + 2\sqrt{3}\cos pt$. Find the amplitude of the wave in cm.

Correct Answer: $4\,\text{cm}$

Solution:

Step 1: Understanding the Concept

When a displacement is represented as a sum of a sine and a cosine function of the same frequency, the resultant motion is also a simple harmonic motion. The total amplitude is the vector sum of the individual amplitudes.

Step 2: Key Formula or Approach

For an equation of the form $y = a \sin \omega t + b \cos \omega t$, the resultant amplitude R is:

$$R = \sqrt{a^2 + b^2}$$

Step 3: Detailed Explanation

1. Distribute the constant into the expression: $y = 2 \sin pt + 2\sqrt{3} \cos pt$. 2. Identify the coefficients: $a = 2$ and $b = 2\sqrt{3}$. 3. Calculate the resultant amplitude:

$$R = \sqrt{(2)^2 + (2\sqrt{3})^2}$$

$$R = \sqrt{4 + (4 \times 3)} = \sqrt{4 + 12}$$

$$R = \sqrt{16} = 4\,\text{cm}$$

Step 4: Final Answer

The amplitude of the wave is $4\,\text{cm}$.

> **Quick Tip:** This is mathematically similar to finding the hypotenuse of a right-angled triangle. If you think of the sine and cosine components as perpendicular vectors, the amplitude is the magnitude of their resultant.

---

35. An air-core solenoid has an inductance $L = 0.5\,\text{mH}$. It is then filled with soft iron of relative permeability $\mu_r = 1500$. Find the new inductance.

Correct Answer: $0.75\,\text{H}$

Solution:

Step 1: Understanding the Concept

The self-inductance of a solenoid depends on its geometry and the permeability of the core material.

Inserting a ferromagnetic material like soft iron increases the magnetic flux linkage significantly.

Step 2: Key Formula or Approach

The inductance L' with a core is given by:

$$L' = \mu_r \times L_{air}$$

Step 3: Detailed Explanation

1. Given: $L_{air} = 0.5\,\text{mH} = 0.5 \times 10^{-3}\,\text{H}$. 2. Given: $\mu_r = 1500$. 3. Calculate the new inductance:

$$L' = 1500 \times 0.5 \times 10^{-3}$$

$$L' = 750 \times 10^{-3} = 0.75\,\text{H}$$

Step 4: Final Answer

The new inductance is $0.75\,\text{H}$ (or $750\,\text{mH}$).

> **Quick Tip:** Relative permeability acts as a "multiplier" for inductance. This is why transformers and inductors use iron cores—to achieve high inductance without needing thousands of extra turns of wire.

---

36. A bar magnet is rotated from a parallel position ($0°$) to a $45°$ position, and the work done is $2.07\,\text{J}$. Find the work done to rotate it from $45°$ to the anti-parallel ($180°$) position.

Correct Answer: $12.07\,\text{J}$

Solution:

Step 1: Understanding the Concept

The work done in rotating a magnetic dipole (bar magnet) in a uniform magnetic field is equal to the change in its potential energy.

Step 2: Key Formula or Approach

Work done $W = MB(\cos\theta_1 - \cos\theta_2)$, where M is the magnetic moment and B is the magnetic field.

Step 3: Detailed Explanation

1. First Case (0° to 45°):

$$W_1 = MB(\cos 0^\circ - \cos 45^\circ) = 2.07$$

$$MB(1 - 0.707) = 2.07 \implies MB(0.293) = 2.07$$

$$MB = \frac{2.07}{0.293} \approx 7.065\,\text{J}$$

2. Second Case (45° to 180°):

$$W_2 = MB(\cos 45^\circ - \cos 180^\circ)$$

$$W_2 = MB(0.707 - (-1)) = MB(1.707)$$

3. Substitute the value of MB:

$$W_2 = 7.065 \times 1.707 \approx 12.06\,\text{J}$$

Step 4: Final Answer

The work done to rotate the magnet to the anti-parallel position is approximately 12.07 J.

> **Quick Tip:** The work done is maximum when rotating a magnet to the anti-parallel position (180°) because you are fighting against the magnetic field to put the "North" pole where the field wants the "South" pole to be.

---

37. A real object is placed at the focus in front of a concave mirror of focal length f. Find the distance to the image formed.

Correct Answer: Infinity (∞)

Solution:

Step 1: Understanding the Concept

According to the principles of ray optics, when an object is placed at the principal focus of a concave mirror, the reflected rays emerge parallel to the principal axis. Parallel rays are mathematically considered to meet at an infinite distance.

Step 2: Key Formula or Approach

Mirror Formula: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Step 3: Detailed Explanation

1. Using sign convention: Focal length $= -f$, Object distance $u = -f$. 2. Substitute into the formula:

$$\frac{1}{-f} = \frac{1}{v} + \frac{1}{-f}$$

3. Rearrange to solve for v:

$$\frac{1}{v} = -\frac{1}{f} + \frac{1}{f} = 0$$

4. If $\frac{1}{v} = 0$, then $v = \frac{1}{0} = \infty$.

Step 4: Final Answer

The image is formed at infinity.

> **Quick Tip:** This is the principle used in searchlights and car headlights. By placing the bulb exactly at the focus of a parabolic or concave reflector, a powerful parallel beam of light is produced.

---

38. Find the ratio of the magnitude of gravitational potential energy to that of kinetic energy for a satellite of mass m orbiting a planet.

Correct Answer: 2 : 1

Solution:

Step 1: Understanding the Concept

A satellite in a stable circular orbit possesses both kinetic energy (due to its motion) and potential energy (due to its position in the gravitational field). These energies are related to each other by the total mechanical energy of the system.

Step 2: Key Formula or Approach

1. Potential Energy $(U) = -\frac{GMm}{r}$ 2. Kinetic Energy $(K) = \frac{GMm}{2r}$

Step 3: Detailed Explanation

1. Take the magnitude of Potential Energy: $|U| = \frac{GMm}{r}$. 2. Take the magnitude of Kinetic Energy: $|K| = \frac{GMm}{2r}$. 3. Form the ratio:

$$\text{Ratio} = \frac{|U|}{|K|} = \frac{\frac{GMm}{r}}{\frac{GMm}{2r}}$$

4. Simplify the expression:

$$\text{Ratio} = \frac{1}{1/2} = 2$$

Step 4: Final Answer

The ratio is 2 : 1.

Quick Tip: A useful shortcut to remember for orbital mechanics: $|U| = 2K$ and Total Energy $E = -K$. The potential energy is always twice the kinetic energy in magnitude.

---

39. A uniform metallic wire of radius r and length ℓ is heated by passing a constant current. To make the heat produced 8 times the original value, which of the following changes can be made?

(a) 2ℓ

(b) $\frac{\ell}{2}, \frac{r}{2}$

(c) $2\ell, \frac{r}{2}$

(d) $2r$

(e) $2\ell, 2r$

Correct Answer: (c) $2\ell, \frac{r}{2}$

Solution:

Step 1: Understanding the Concept

Heat produced in a wire (H) due to a constant current I is given by Joule's Law. The resistance of the wire depends on its dimensions (length and radius).

Step 2: Key Formula or Approach

1. $H = I^2Rt$ (Since I is constant, $H \propto R$). 2. $R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2}$ (So $H \propto \frac{\ell}{r^2}$).

Step 3: Detailed Explanation

1. Let original heat be $H_1 \propto \frac{\ell}{r^2}$. 2. For Option (c), new length $\ell' = 2\ell$ and new radius $r' = r/2$. 3. Calculate new heat H_2:

$$H_2 \propto \frac{2\ell}{(r/2)^2} = \frac{2\ell}{r^2/4} = 8\left(\frac{\ell}{r^2}\right)$$

4. This results in $H_2 = 8H_1$.

Step 4: Final Answer

The heat produced becomes 8 times when length is doubled (2ℓ) and radius is halved ($r/2$).

Quick Tip: Halving the radius alone increases the resistance by 4 times because the area depends on the square of the radius. Combined with doubling the length, the effect is multiplied to 8.

---

40. Two capillary tubes of radii in the ratio 1 : 2 **are dipped in the same solution. Find the ratio of the heights to which the liquid rises in the tubes.**

Correct Answer: 2 : 1

Solution:

Step 1: Understanding the Concept

Capillary action is governed by Jurin's Law, which states that the height of liquid rise in a capillary tube is inversely proportional to the radius of the tube.

Step 2: Key Formula or Approach

Jurin's Law: $h = \frac{2T\cos\theta}{r\rho g}$ Since the solution is the same (T, θ, ρ are constant), $h \propto \frac{1}{r}$.

Step 3: Detailed Explanation

1. Given the ratio of radii $r_1 : r_2 = 1 : 2$. 2. Since height is inversely proportional to radius:

$$\frac{h_1}{h_2} = \frac{r_2}{r_1}$$

3. Substitute the values:

$$\frac{h_1}{h_2} = \frac{2}{1}$$

Step 4: Final Answer

The ratio of the heights is 2 : 1.

> **Quick Tip:** The thinner the tube, the higher the liquid rises. This is why narrow capillary tubes are used to demonstrate surface tension effects more clearly.
