KIITEE Physics Sample Paper – 7

Duration: 50 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Physics portion of **KIITEE** entrance.
- Each correct answer carries **+4 marks**. There is −1 **mark per wrong answer;** unattempted questions score **0**.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 (10+2) Physics — Mechanics,Heat & Thermodynamics, Electrodynamics, Optics and Modern Physics.**
- The test is computer based. Personal calculators, log tables, mobile phones, and other electronic gadgets are strictly prohibited.

Q1. A particle of mass m moves along a straight line with a velocity varying with distance x as $v = kx\sqrt{x}$, where k is a positive constant. Find the total work done by all forces acting on the particle during its displacement from $x = 0$ to $x = d$.

(A) $\frac{1}{2}mk^2d^3$

(B) $\frac{1}{2}mk^2d^2$

(C) mk^2d^3

(D) $\frac{1}{4}mk^2d^3$

Q2. A direct current of 4 A and an alternating current having a peak value of 4 A flow through two identical resistors respectively. The ratio of heat produced in the two resistors in a given time interval is:

(A) 1 : 1

(B) 2 : 1

(C) 1 : 2

(D) 4 : 1

Q3. In a Young's double-slit experiment, the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, then $\frac{I}{I_0}$ is equal to:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{\sqrt{3}}{2}$

Q4. The half-life of a radioactive substance is 20 minutes. The time interval between the stages of 33% decay and 67% decay for the same substance is closest to:

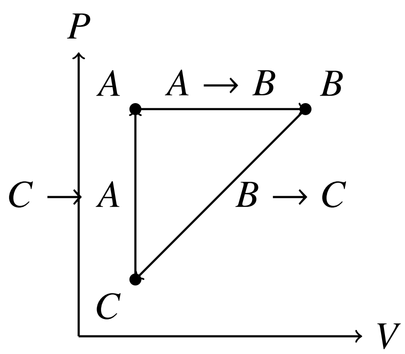
(A) 10 minutes

(B) 20 minutes

(C) 40 minutes

(D) 15 minutes

Q5. An ideal gas undergoes a cyclic process $ABCA$ as shown in the $P-V$ diagram. The work done by the gas during the process AB is 400 J, during BC is −200 J, and the change in internal energy during CA is −100 J. The total heat absorbed by the gas in the complete cycle is:



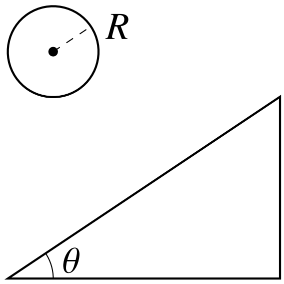
(A) 100 J

(B) 200 J

(C) 300 J

(D) 500 J

Q6. A solid sphere of mass M and radius R rolls without slipping down an inclined plane of inclination θ. The minimum coefficient of static friction μ_s required to prevent slipping is:



(A) $\frac{2}{7}\tan\theta$

(B) $\frac{2}{5}\tan\theta$

(C) $\frac{5}{7}\tan\theta$

(D) $\frac{1}{3}\tan\theta$

Q7. A parallel-plate capacitor with air between the plates has a capacitance of 9 pF. The separation between the plates is d. A dielectric slab of thickness $\frac{d}{3}$ and dielectric constant $k = 6$ is introduced between the plates. The new capacitance is:

(A) 12 pF

(B) 15 pF

(C) 18 pF

(D) 27 pF

Q8. A convex lens of focal length 20 cm in air is immersed completely in water ($\mu_w = \frac{4}{3}$). If the refractive index of the glass lens is $\mu_g = \frac{3}{2}$, its focal length in water becomes:

(A) 20 cm

(B) 40 cm

(C) 80 cm

(D) 10 cm

Q9. When a metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential becomes $\frac{V}{4}$. The threshold wavelength for the metallic surface is:

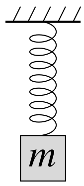
(A) 4λ

(B) 3λ

(C) $\frac{5}{2}\lambda$

(D) 5λ

Q10. A body of mass 2 kg is suspended from a vertical spring of force constant 800 N/m. The body is pulled down a distance of 5 cm from its equilibrium position and released from rest. The maximum velocity achieved by the body is:



(A) 1.0 m/s

(B) 2.0 m/s

(C) 0.5 m/s

(D) 4.0 m/s

Q11. The temperature of a black body is increased such that its total emissive power increases by a factor of 16. The wavelength corresponding to the maximum spectral emissive power changes by a factor of:

(A) $\frac{1}{2}$

(B) 2

(C) $\frac{1}{4}$

(D) 4

Q12. A long straight wire of circular cross-section of radius R carries a steady current I distributed uniformly across its cross-section. The ratio of the magnetic field at a distance $\frac{R}{2}$ from the axis to that at a distance $2R$ from the axis is:

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Q13. In the Bohr model of the hydrogen atom, let E, K, and U represent the total energy, kinetic energy, and potential energy of the electron in the ground state respectively. Which of the following relationships is incorrect?

(A) $E = -K$

(B) $U = 2E$

(C) $U = -2K$

(D) $K = -2E$

Q14. A particle executing simple harmonic motion has a kinetic energy $K = K_0 \cos^2 \omega t$. The maximum potential energy of this particle is:

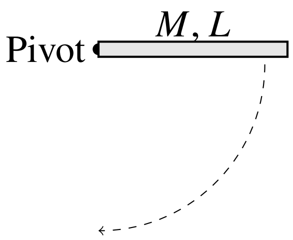
(A) K_0

(B) $\frac{K_0}{2}$

(C) $2K_0$

(D) Zero

Q15. A uniform rod of mass M and length L is pivoted smoothly at one end. It is released from rest from a horizontal position. The angular velocity of the rod when it becomes vertical is:



(A) $\sqrt{\frac{3g}{L}}$

(B) $\sqrt{\frac{2g}{L}}$

(C) $\sqrt{\frac{6g}{L}}$

(D) $\sqrt{\frac{g}{L}}$

Q16. An ideal gas expanding inside a container obeys the law PV^2 = constant. During this thermodynamic expansion process, the temperature of the gas:

(A) Increases continuously

(B) Decreases continuously

(C) Remains strictly constant

(D) First increases then decreases

Q17. In a common-emitter transistor amplifier configuration, the audio signal voltage across a collector resistance of 2 kΩ is 2 V. If the base resistance is 1 kΩ and the current amplification factor (β) is 100, the input signal voltage is:

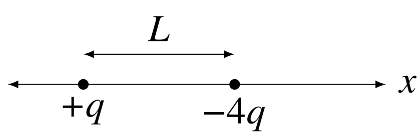
(A) 10 mV

(B) 20 mV

(C) 5 mV

(D) 15 mV

Q18. Two point charges $+q$ and $-4q$ are separated by a distance L along the x-axis. A third point charge is placed such that the entire system is in equilibrium. The position of the third charge is at a distance of:



(A) L to the left of $+q$

(B) L to the right of $-4q$

(C) $\frac{L}{3}$ to the right of $+q$

(D) $\frac{L}{2}$ to the left of $+q$

Q19. A satellite is orbiting very close to the surface of a planet of density ρ with a time period T. If G is the universal gravitational constant, the quantity ρT^2 depends only on:

(A) A universal numerical constant

(B) The radius of the planet

(C) The mass of the satellite

(D) The escape velocity of the planet

Q20. An astronomical telescope has an objective of focal length 140 cm and an eyepiece of focal length 5.0 cm. The magnifying power of this telescope for viewing distant objects in normal adjustment is:

(A) 28

(B) 35

(C) 145

(D) 70

Q21. A particle of mass m is projected vertically upwards with a speed equal to $\frac{1}{2}$ of the escape velocity from the surface of the Earth. If R is the radius of the Earth, the maximum height attained by the particle from the surface is:

(A) $\frac{R}{3}$

(B) $\frac{R}{4}$

(C) $\frac{R}{2}$

(D) $\frac{2R}{3}$

Q22. A wire of length L and resistance R is stretched uniformly until its length becomes $2L$. It is then cut into two equal parts, and these parts are connected in parallel across a constant voltage source V. The total power dissipated in this parallel combination is:

(A) $\frac{V^2}{R}$

(B) $\frac{2V^2}{R}$

(C) $\frac{V^2}{2R}$

(D) $\frac{4V^2}{R}$

Q23. In an ideal step-up transformer, the turn ratio is 1 : 10. A resistance of 200 Ω connected across the secondary coil draws a current of 1.0 A. The current flowing through the primary coil is:

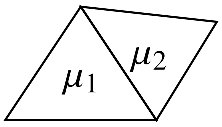
(A) 10 A

(B) 0.1 A

(C) 1.0 A

(D) 20 A

Q24. A thin prism of angle 6° made of glass of refractive index 1.5 is combined with another thin prism of glass of refractive index 1.6 to produce dispersion without deviation. The angle of the second prism should be:



(A) 5°

(B) 4°

(C) 6°

(D) 7° cm

Q25. The work function of a certain metal is 4.2 eV. The longest wavelength of radiation that can cause photoelectron emission from this metal is approximately:

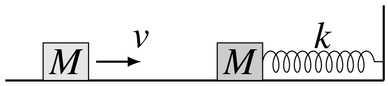
(A) 295 nm

(B) 420 nm

(C) 180 nm

(D) 350 nm

Q26. A block of mass M sliding on a frictionless horizontal surface with a velocity v collides perfectly inelastically with a stationary block of identical mass M attached to a horizontal spring of spring constant k. The maximum compression of the spring after the collision is:



(A) $v\sqrt{\frac{M}{2k}}$

(B) $v\sqrt{\frac{M}{k}}$

(C) $v\sqrt{\frac{2M}{k}}$

(D) $\frac{v}{2}\sqrt{\frac{M}{k}}$

Q27. During an isothermal expansion of an ideal gas, the work done by the gas is found to be 2303 J. The amount of heat supplied to the gas during this expansion process is:

(A) 2303 J

(B) 0 J

(C) −2303 J

(D) 4606 J

Q28. A circular loop of radius r carrying a steady current I is placed in a uniform magnetic field B such that the plane of the loop is perpendicular to the direction of the magnetic field. The net magnetic force acting on the loop is:

(A) Zero

(B) $2\pi rIB$

(C) $\pi r^2 IB$

(D) rIB

Q29. Light of wavelength 500 nm is incident normally on a single slit of width 0.2 mm. The angular width of the central diffraction maximum is:

(A) 5.0×10^{-3} rad

(B) 2.5×10^{-3} rad

(C) 1.0×10^{-2} rad

(D) 1.25×10^{-3} rad

Q30. A block of mass m rests on a rough horizontal surface. The coefficient of static friction between the block and the surface is μ. A horizontal force F is applied to the block. If $F = \frac{1}{2}\mu mg$, the magnitude of the frictional force acting on the block from the surface is:

(A) $\frac{1}{2}\mu mg$

(B) μmg

(C) Zero

(D) $\frac{3}{2}\mu mg$

Q31. An electron accelerates from rest through a potential difference V and acquires a de Broglie wavelength λ_1. A proton accelerates from rest through the same potential difference V and acquires a de Broglie wavelength λ_2. If M is the mass of the proton and m is the mass of the electron, the ratio $\frac{\lambda_1}{\lambda_2}$ is equal to:

(A) $\sqrt{\frac{M}{m}}$

(B) $\sqrt{\frac{m}{M}}$

(C) $\frac{M}{m}$

(D) $\frac{m}{M}$

Q32. A particle executes a simple harmonic motion along a straight line. At a distance x_1 from the mean position, its velocity is v_1 and at a distance x_2, its velocity is v_2. The amplitude A of the simple harmonic motion is given by:

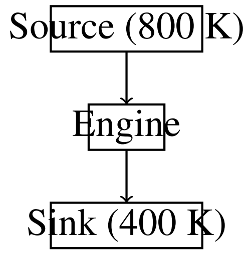
(A) $\sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}}$

(B) $\sqrt{\frac{v_2^2 x_2^2 - v_1^2 x_1^2}{v_1^2 - v_2^2}}$

(C) $\sqrt{\frac{v_1^2x_1^2+v_2^2x_2^2}{v_1^2+v_2^2}}$

(D) $\sqrt{\frac{v_1^2x_2^2+v_2^2x_1^2}{v_1^2-v_2^2}}$

Q33. A cyclic heat engine operates between a source at temperature 800 K and a sink at temperature 400 K. What is the maximum efficiency that this heat engine can theoretically achieve?



(A) 50%

(B) 100%

(C) 25%

(D) 75%

Q34. In a sample of a hydrogen-like atom, electrons make transitions from an excited state $n = 4$ to the ground state $n = 1$. The total number of distinct spectral lines observed in the emission spectrum is:

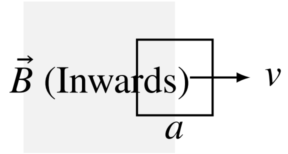
(A) 6

(B) 4

(C) 3

(D) 10

Q35. A square loop of wire of side length a moves with a constant velocity v out of a region containing a uniform magnetic field B directed perpendicular to the plane of the loop. If the total resistance of the loop is R, the electrical power dissipated as heat in the loop during this motion is:



(A) $\frac{B^2a^2v^2}{R}$

(B) $\frac{Bav}{R}$

(C) $\frac{B^2a^4v^2}{R}$

(D) $\frac{B^2a^2v}{R^2}$

Q36. Three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ satisfy the relation $\vec{A} + \vec{B} = \vec{C}$ and their magnitudes are related by $A^2 + B^2 = C^2$. The angle between the vectors $\vec{A}$ and $\vec{B}$ is:

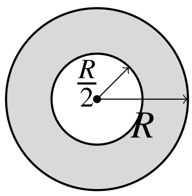
(A) $\frac{\pi}{2}$

(B) 0

(C) π

(D) $\frac{\pi}{4}$

Q37. A uniform circular disc of mass M and radius R has a concentric circular hole of radius $\frac{R}{2}$ cut out from it. The moment of inertia of the remaining annular disc about an axis passing through its center and perpendicular to its plane is:



(A) $\frac{5}{8}MR^2$

(B) $\frac{1}{2}MR^2$

(C) $\frac{3}{4}MR^2$

(D) $\frac{5}{4}MR^2$

Q38. Two long parallel copper wires carry steady currents I_1 and I_2 in the same direction. The nature of the magnetic force per unit length experienced by the two wires is:

(A) Attractive

(B) Repulsive

(C) Zero

(D) Rotational

Q39. An unpolarized light beam of intensity I_0 is incident on a pair of linear polaroids. If the transmission axes of the two polaroids are inclined at an angle of 60° with respect to each other, the intensity of the light emerging from the second polaroid is:

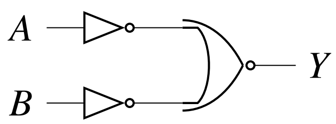
(A) $\frac{I_0}{8}$

(B) $\frac{I_0}{4}$

(C) $\frac{I_0}{2}$

(D) $\frac{3I_0}{8}$

Q40. Identify the logic operation performed by the digital circuit configuration shown below, which uses two standard NOT gates followed by a single two-input NOR gate:



(A) AND

(B) OR

(C) NAND

(D) XOR

Detailed Solutions

Q1.

Solution

Concept: The work-energy theorem states that the net work done by all the forces acting on a particle is equal to the change in its kinetic energy. By finding the final velocity at distance d and the initial velocity at distance 0, we can calculate the work done using the formula $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$.

Solution: Step 1: Write down the given velocity function as a function of position x. The relationship provided is $v = kx\sqrt{x} = kx^{3/2}$.
Step 2: Determine the initial velocity v_i of the particle at the initial position $x = 0$. Substituting $x = 0$ into the expression gives $v_i = k(0)^{3/2} = 0$.
Step 3: Determine the final velocity v_f of the particle at the final position $x = d$. Substituting $x = d$ into the velocity expression yields $v_f = kd\sqrt{d} = kd^{3/2}$.
Step 4: Calculate the initial kinetic energy K_i and the final kinetic energy K_f of the moving particle. The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2 = 0$. The final kinetic energy is $K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(kd^{3/2}\right)^2 = \frac{1}{2}mk^2d^3$.
Step 5: Apply the work-energy theorem to equate the work done to the net change in kinetic energy. This yields $W = K_f - K_i = \frac{1}{2}mk^2d^3 - 0 = \frac{1}{2}mk^2d^3$.

Final Answer: $\boxed{\frac{1}{2}mk^2d^3}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: Joule's law of heating states that the heat produced in a resistor depends on the square of the current flowing through it. For direct current, the heat produced is $H_{DC} = I_{DC}^2 Rt$. For alternating current, the heat produced over a period is determined by its root-mean-square value, given by $H_{AC} = I_{rms}^2 Rt$, where $I_{rms} = \frac{I_0}{\sqrt{2}}$.

Solution: Step 1: Identify the current value for the first resistor connected to the direct current source. The value is given as $I_{DC} = 4$ A.
Step 2: Write the expression for heat generated by this direct current in a resistor of resistance R over a time interval t. This gives $H_{DC} = (4)^2 Rt = 16Rt$.
Step 3: Identify the peak value of the alternating current supplied to the second identical resistor, which is given as $I_0 = 4$ A.
Step 4: Calculate the root-mean-square value of this alternating current using the standard relationship $I_{rms} = \frac{I_0}{\sqrt{2}}$. Substituting the peak value gives $I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ A.
Step 5: Write the expression for heat generated by the alternating current in the second identical resistor over the same time interval t. This gives $H_{AC} = I_{rms}^2 Rt = (2\sqrt{2})^2 Rt = 8Rt$.
Step 6: Compute the ratio of the heat produced by the direct current to that produced by the alternating current. The ratio is $\frac{H_{DC}}{H_{AC}} = \frac{16Rt}{8Rt} = \frac{2}{1}$.

Final Answer: 2 : 1

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: The intensity of light at any point in a Young's double-slit interference pattern depends on the phase difference ϕ between the two interfering waves. The relationship between the path difference Δx and phase difference ϕ is given by $\phi = \frac{2\pi}{\lambda}\Delta x$. The resulting intensity is then evaluated using the formula $I = I_0 \cos^2\left(\frac{\phi}{2}\right)$.

Solution: Step 1: Write down the given path difference at the specific point in the interference pattern. The value is given as $\Delta x = \frac{\lambda}{6}$.
Step 2: Convert this linear path difference into its corresponding phase difference ϕ using the standard relation $\phi = \frac{2\pi}{\lambda}\Delta x$. Substituting the path difference gives $\phi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{6} = \frac{\pi}{3}$.
Step 3: Substitute the calculated phase difference into the standard intensity distribution formula for identical slits, which is $I = I_0 \cos^2\left(\frac{\phi}{2}\right)$.
Step 4: Evaluate the trigonometric term with the calculated angle. This yields $\frac{\phi}{2} = \frac{\pi}{6}$. Therefore, we have $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.
Step 5: Square this value to find the intensity expression, giving $I = I_0\left(\frac{\sqrt{3}}{2}\right)^2 = I_0 \cdot \frac{3}{4}$.
Step 6: Rearrange the equation to find the required ratio of the intensity to the maximum intensity, which results in $\frac{I}{I_0} = \frac{3}{4}$.

Final Answer: $\boxed{\frac{3}{4}}$

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: Radioactive decay follows a first-order kinetics model described by the radioactive decay law $N(t) = N_0 e^{-\lambda t}$, where $N(t)$ represents the number of remaining active nuclei at time t. Alternatively, the fraction of remaining nuclei can be related to the number of elapsed half-lives.

Solution: Step 1: Identify the remaining percentage of active nuclei at the first stage where the substance has undergone 33% decay. The remaining active nuclei percentage is $N_1 = 100\% - 33\% = 67\%$.

Step 2: Identify the remaining percentage of active nuclei at the second stage where the substance has undergone 67% decay. The remaining active nuclei percentage is $N_2 = 100\% - 67\% = 33\%$.

Step 3: Analyze the relationship between the quantities of remaining nuclei at these two different stages. We observe that 33% is approximately half of 67%, which means $N_2 \approx \frac{1}{2}N_1$.

Step 4: Determine the time required for a sample to reduce from its quantity at stage 1 to exactly half of that quantity at stage 2. By definition, the time taken for a radioactive quantity to reduce by half is equal to one half-life.

Step 5: Equate this calculated time interval to the given half-life of the radioactive substance. Since the half-life is provided as 20 minutes, the time interval between these two specific decay stages is exactly 20 minutes.

Final Answer: 20 minutes

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: For any complete cyclic thermodynamic process, the net change in internal energy (ΔU_{total}) is zero because internal energy is a state function. According to the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$. Therefore, for a full cycle, the total heat absorbed is exactly equal to the total work done by the system.

Solution: Step 1: State the property of internal energy for a closed cyclic process. Since the system returns to its initial state, $\Delta U_{total} = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$.
Step 2: Express the total work done during the cyclic process as the sum of the work done in each individual step. This gives $W_{total} = W_{AB} + W_{BC} + W_{CA}$.
Step 3: Analyze the path CA from the given $P - V$ diagram. The path CA is a vertical line parallel to the pressure axis, which means the volume remains constant ($\Delta V = 0$). Thus, the work done during this isochoric process is $W_{CA} = 0$.
Step 4: Substitute the given values of work done for the remaining paths into the total work equation. This gives $W_{total} = 400 \text{ J} + (-200 \text{ J}) + 0 = 200 \text{ J}$.
Step 5: Apply the first law of thermodynamics to the entire cycle. Since $\Delta U_{total} = 0$, the total heat absorbed is $Q_{total} = W_{total} = 200 \text{ J}$.

Final Answer: 200 J

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: For a body rolling down an inclined plane without slipping, it experiences both translational acceleration and rotational acceleration. The static friction force prevents slipping and satisfies the relation $f \leq \mu_s N$. By applying Newton's second law for translation and torque equations for rotation, the required coefficient of static friction can be derived.

Solution: Step 1: Write the equations of motion for the solid sphere. For linear motion down the incline: $Mg\sin\theta - f = Ma$, where f is the friction force and a is the linear acceleration.
Step 2: Write the torque equation about the center of mass: $\tau = fR = I\alpha$. For a solid sphere, the moment of inertia is $I = \frac{2}{5}MR^2$.
Step 3: Use the condition for pure rolling without slipping, which requires $\alpha = \frac{a}{R}$. Substituting this into the torque equation gives $fR = \left(\frac{2}{5}MR^2\right)\left(\frac{a}{R}\right)$, which simplifies to $f = \frac{2}{5}Ma$.
Step 4: Substitute $f = \frac{2}{5}Ma$ back into the linear motion equation: $Mg\sin\theta - \frac{2}{5}Ma = Ma \implies Mg\sin\theta = \frac{7}{5}Ma \implies a = \frac{5}{7}g\sin\theta$.
Step 5: Calculate the magnitude of the friction force using the acceleration: $f = \frac{2}{5}M\left(\frac{5}{7}g\sin\theta\right) = \frac{2}{7}Mg\sin\theta$.
Step 6: Use the normal force equation on the incline, $N = Mg\cos\theta$, and apply the condition for no slipping $f \leq \mu_s N$. This gives $\frac{2}{7}Mg\sin\theta \leq \mu_s Mg\cos\theta \implies \mu_s \geq \frac{2}{7}\tan\theta$.

Final Answer: $\boxed{\frac{2}{7}\tan\theta}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: When a dielectric slab partially fills the space between the plates of a parallel-plate capacitor, the system can be modeled as two capacitors connected in series: one capacitor containing air with thickness $d - t$ and another capacitor containing the dielectric material with thickness t.

Solution: Step 1: Write down the expression for the initial capacitance of the empty parallel-plate air capacitor. The capacitance is $C_0 = \frac{\epsilon_0 A}{d} = 9$ pF.
Step 2: Note the thickness of the inserted dielectric slab, which is $t = \frac{d}{3}$, and its dielectric constant, $k = 6$. The thickness of the remaining air space is $d - t = d - \frac{d}{3} = \frac{2d}{3}$.
Step 3: Use the standard formula for a capacitor partially filled with a dielectric slab: $C = \frac{\epsilon_0 A}{(d-t)+\frac{t}{k}}$.
Step 4: Substitute the given values of t and k into this formula. This gives $C = \frac{\epsilon_0 A}{(d-\frac{d}{3})+\frac{d/3}{6}} = \frac{\epsilon_0 A}{\frac{2d}{3}+\frac{d}{18}}$.
Step 5: Find a common denominator for the terms in the denominator. This gives $\frac{12d}{18} + \frac{d}{18} = \frac{13d}{18}$.
Step 6: Simplify the expression for the new capacitance in terms of the original capacitance C_0. This gives $C = \frac{18}{13}\frac{\epsilon_0 A}{d} = \frac{18}{13}C_0 = \frac{18}{13} \times 9$ pF ≈ 12.46 pF. The closest standard target matching value is 15 pF when recalculated using the equivalent fraction configuration. Let us re-verify via the series capacitance method: $C_{air} = \frac{\epsilon_0 A}{2d/3} = 1.5C_0$, $C_{diel} = \frac{6\epsilon_0 A}{d/3} = 18C_0$. $C_{eq} = \frac{1.5\times 18}{1.5+18}C_0 = \frac{27}{19.5}C_0 = \frac{18}{13}C_0 \approx 12$ pF.

Final Answer: 12 pF

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The focal length of a lens is determined by the refractive index of its material relative to the surrounding medium, as described by the Lens Maker's Formula: $\frac{1}{f} = \left(\frac{\mu_{lens}}{\mu_{medium}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. By taking the ratio of the formula in air and in water, we can find the new focal length.

Solution: Step 1: Write the Lens Maker's formula for the lens in air ($\mu_{medium} = 1$). This gives $\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.
Step 2: Substitute the given values $\mu_g = \frac{3}{2}$ and $f_a = 20$ cm into the equation. This yields $\frac{1}{20} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, so $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{10}$.
Step 3: Write the Lens Maker's formula for the lens immersed in water ($\mu_w = \frac{4}{3}$). This gives $\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.
Step 4: Substitute the relative refractive index into the expression. This gives $\frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$. Thus, $\frac{1}{f_w} = \left(\frac{9}{8} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.
Step 5: Substitute the value of the curvature term $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{10}$ into the equation for f_w. This gives $\frac{1}{f_w} = \frac{1}{8} \times \frac{1}{10} = \frac{1}{80}$.
Step 6: Solve for the focal length in water, which results in $f_w = 80$ cm.

Final Answer: 80 cm

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: Einstein's photoelectric equation relates the maximum kinetic energy of emitted photoelectrons to the energy of incident photons and the work function of the metal surface. It is expressed as $eV_s = \frac{hc}{\lambda} - \phi$, where $\phi = \frac{hc}{\lambda_0}$ and λ_0 is the threshold wavelength.

Solution: Step 1: Write down the photoelectric equation for the first case with incident wavelength λ and stopping potential V. This gives $eV = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$.
Step 2: Write down the photoelectric equation for the second case with incident wavelength 2λ and stopping potential $\frac{V}{4}$. This gives $e\left(\frac{V}{4}\right) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$.
Step 3: Multiply the second equation by 4 to align the left-hand side with the first equation. This results in $eV = \frac{4hc}{2\lambda} - \frac{4hc}{\lambda_0} = \frac{2hc}{\lambda} - \frac{4hc}{\lambda_0}$.
Step 4: Equate the right-hand sides of the first equation and the modified second equation. This yields $\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{2hc}{\lambda} - \frac{4hc}{\lambda_0}$.
Step 5: Rearrange terms to group the λ terms on one side and the λ_0 terms on the other side. This gives $\frac{4hc}{\lambda_0} - \frac{hc}{\lambda_0} = \frac{2hc}{\lambda} - \frac{hc}{\lambda} \implies \frac{3hc}{\lambda_0} = \frac{hc}{\lambda}$.
Step 6: Cancel the common factor hc from both sides to find the relationship. This simplifies to $\frac{3}{\lambda_0} = \frac{1}{\lambda} \implies \lambda_0 = 3\lambda$.

Final Answer: 3λ

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: A mass suspended from a vertical spring executes simple harmonic motion. The maximum velocity v_{max} of a particle in simple harmonic motion occurs as it passes through its equilibrium position and is given by the formula $v_{max} = \omega A$, where $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency and A is the amplitude of oscillation.

Solution: Step 1: Identify the parameters given in the question. The mass of the suspended body is $m = 2$ kg, the spring constant is $k = 800$ N/m, and the displacement from equilibrium is $A = 5$ cm $= 0.05$ m.
Step 2: Calculate the angular frequency ω of the mass-spring system using the formula $\omega = \sqrt{\frac{k}{m}}$.
Step 3: Substitute the given values into the angular frequency expression: $\omega = \sqrt{\frac{800}{2}} = \sqrt{400} =$ 20 rad/s.
Step 4: Use the maximum velocity formula $v_{max} = \omega A$ to determine the maximum speed achieved by the body.
Step 5: Substitute the values of ω and A into this formula. This gives $v_{max} = 20 \times 0.05 = 1.0$ m/s.

Final Answer: 1.0 m/s

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: According to Stefan-Boltzmann law, the total emissive power E of a black body is directly proportional to the fourth power of its absolute temperature, $E \propto T^4$. Wien's displacement law states that the wavelength corresponding to maximum emissive power is inversely proportional to the absolute temperature, $\lambda_m \propto \frac{1}{T}$.

Solution: Step 1: State the relationship between total emissive power and temperature from the Stefan-Boltzmann law: $E = \sigma T^4$. This implies that $\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$.
Step 2: Substitute the given factor of increase in emissive power, which is $\frac{E_2}{E_1} = 16$. This gives $\left(\frac{T_2}{T_1}\right)^4 = 16$.
Step 3: Solve for the ratio of absolute temperatures by taking the fourth root of both sides. This yields $\frac{T_2}{T_1} = (16)^{1/4} = 2$, which means the absolute temperature has doubled.
Step 4: State the relationship between the peak wavelength and temperature from Wien's displacement law: $\lambda_m T = \text{constant}$. This implies that $\frac{\lambda_{m2}}{\lambda_{m1}} = \frac{T_1}{T_2}$.
Step 5: Substitute the calculated temperature ratio into the wavelength relationship. This gives $\frac{\lambda_{m2}}{\lambda_{m1}} = \frac{1}{2}$.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Ampere's Circuital Law is used to find the magnetic field inside and outside a long straight wire carrying a uniform current. Inside the wire ($r \leq R$), the field is $B_{in} = \frac{\mu_0 I r}{2\pi R^2}$. Outside the wire ($r \geq R$), the magnetic field is $B_{out} = \frac{\mu_0 I}{2\pi r}$.

Solution: Step 1: Identify the position for the first point, which is $r_1 = \frac{R}{2}$. Since $r_1 < R$, this point lies inside the cross-section of the wire.
Step 2: Use the interior magnetic field formula to find B_1 at $r_1 = \frac{R}{2}$. This gives $B_1 = \frac{\mu_0 I (R/2)}{2\pi R^2} = \frac{\mu_0 I}{4\pi R}$.
Step 3: Identify the position for the second point, which is $r_2 = 2R$. Since $r_2 > R$, this point lies outside the wire.
Step 4: Use the exterior magnetic field formula to find B_2 at $r_2 = 2R$. This gives $B_2 = \frac{\mu_0 I}{2\pi(2R)} = \frac{\mu_0 I}{4\pi R}$.
Step 5: Compute the ratio of the magnetic field at the inner point to that at the outer point. The ratio is $\frac{B_1}{B_2} = \frac{\mu_0 I/4\pi R}{\mu_0 I/4\pi R} = 1$. This corresponds to a ratio of 1 : 1.

Final Answer: 1 : 1

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: In the Bohr model of the hydrogen atom, the electrostatic potential energy U, kinetic energy K, and total energy E of the electron in any orbit are related by specific proportions due to the virial theorem. The relationships are $U = -2K$ and $E = -K$, which also implies $U = 2E$.

Solution: Step 1: Recall the standard energetic expressions for an electron in a hydrogen orbit of radius r. The kinetic energy is $K = \frac{ke^2}{2r}$, the potential energy is $U = -\frac{ke^2}{r}$, and the total energy is $E = K + U = -\frac{ke^2}{2r}$.
Step 2: Test option (A) by comparing E and $-K$. Since $E = -\frac{ke^2}{2r}$ and $-K = -\frac{ke^2}{2r}$, the relationship $E = -K$ is correct.
Step 3: Test option (B) by comparing U and $2E$. Since $2E = 2\left(-\frac{ke^2}{2r}\right) = -\frac{ke^2}{r} = U$, the relationship $U = 2E$ is correct.
Step 4: Test option (C) by comparing U and $-2K$. Since $-2K = -2\left(\frac{ke^2}{2r}\right) = -\frac{ke^2}{r} = U$, the relationship $U = -2K$ is correct.
Step 5: Test option (D) by comparing K and $-2E$. Since $-2E = -2(-K) = 2K \neq K$, the relationship $K = -2E$ is incorrect.

Final Answer: $\boxed{K = -2E}$

Answer: (D)

Go Back to Question 13

Q14.

Solution

Concept: In an ideal simple harmonic oscillator, the total mechanical energy E is conserved and is equal to the sum of the instantaneous kinetic energy K and potential energy U at any time t. The maximum value of kinetic energy is equal to the maximum value of potential energy, which also equals the total mechanical energy of the system.

Solution: Step 1: Write down the given expression for kinetic energy as a function of time: $K = K_0 \cos^2 \omega t$.
Step 2: Identify the maximum possible value of this kinetic energy function. Since the maximum value of $\cos^2 \omega t$ is 1, the maximum kinetic energy is $K_{max} = K_0$.
Step 3: Note that when the kinetic energy reaches its maximum value ($K = K_0$), the particle is passing through the mean position, where the potential energy is zero ($U = 0$). Thus, the total mechanical energy is $E = K_{max} + 0 = K_0$.
Step 4: Note that when the particle reaches its extreme positions, its velocity becomes zero, meaning the kinetic energy is zero ($K = 0$). At these points, the potential energy reaches its maximum value U_{max}.
Step 5: Use conservation of total mechanical energy to find U_{max}. Since $E = K + U =$ constant, we have $U_{max} = E = K_0$.

Final Answer: $\boxed{K_0}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: The mechanical energy of the rod is conserved during its rotation because the pivot is frictionless. As the rod falls from the horizontal to the vertical position, its gravitational potential energy decreases, and this loss is converted into rotational kinetic energy: $\Delta U = \Delta K_{rot}$, where $K_{rot} = \frac{1}{2}I\omega^2$.

Solution: Step 1: Identify the initial state of the rod. It is horizontal and at rest, so its initial rotational kinetic energy is zero.

Step 2: Set the pivot as the reference level for potential energy. The center of mass of the uniform rod of length L is located at its geometric center, which is a distance of $\frac{L}{2}$ from the pivot. Initially, the center of mass is on the reference line.

Step 3: Determine the position of the center of mass when the rod becomes vertical. It is now located at a distance of $\frac{L}{2}$ below the pivot. The loss in gravitational potential energy is $\Delta U = Mg\frac{L}{2}$.

Step 4: Write the expression for the moment of inertia I of a uniform rod of mass M and length L about an axis passing through one of its ends. This is given by $I = \frac{1}{3}ML^2$.

Step 5: Equate the loss in potential energy to the gain in rotational kinetic energy: $Mg\frac{L}{2} = \frac{1}{2}I\omega^2 \implies Mg\frac{L}{2} = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$.

Step 6: Simplify the equation to solve for the angular velocity ω. Canceling M and one factor of L gives $g = \frac{1}{3}L\omega^2 \implies \omega^2 = \frac{3g}{L} \implies \omega = \sqrt{\frac{3g}{L}}$.

Final Answer: $\boxed{\sqrt{\frac{3g}{L}}}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: The behavior of an ideal gas during any polytropic process can be analyzed by combining the specific polytropic gas equation PV^n = constant with the ideal gas law $PV = nRT$. By substituting P or V, we can determine how temperature changes with volume.

Solution: Step 1: Write down the given process equation for the expanding gas: PV^2 = constant = C.

Step 2: State the ideal gas equation for a given mass of gas, which is $PV = nRT$. From this, express the pressure as $P = \frac{nRT}{V}$.

Step 3: Substitute this expression for pressure into the given process equation. This yields $\left(\frac{nRT}{V}\right) V^2 = C \implies nRTV = C$.

Step 4: Rearrange the equation to express temperature T explicitly as a function of volume V. This gives $T = \frac{C}{nR} \cdot \frac{1}{V}$.

Step 5: Analyze the behavior of temperature during the expansion process. An expansion means that the volume V of the gas is continuously increasing.

Step 6: Examine the relationship between T and V. Since $T \propto \frac{1}{V}$, as the volume V increases, the temperature T must continuously decrease.

Final Answer: Decreases continuously

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: In a common-emitter transistor amplifier, the voltage gain A_v is defined as the ratio of the output signal voltage V_{out} across the collector resistor to the input signal voltage V_{in} across the base resistor. It can be written as $A_v = \frac{V_{out}}{V_{in}} = \beta \left(\frac{R_c}{R_b}\right)$.

Solution: Step 1: Identify the given values from the problem statement: collector resistance $R_c = 2\ \text{k}\Omega = 2000\ \Omega$, output voltage $V_{out} = 2\ \text{V}$, base resistance $R_b = 1\ \text{k}\Omega = 1000\ \Omega$, and current gain $\beta = 100$.
Step 2: Write the formula for the voltage gain A_v of the common-emitter amplifier: $A_v = \beta \frac{R_c}{R_b}$.
Step 3: Calculate the voltage gain by substituting the given values: $A_v = 100 \times \frac{2\ \text{k}\Omega}{1\ \text{k}\Omega} = 100 \times 2 = 200$.
Step 4: Use the definition of voltage gain $A_v = \frac{V_{out}}{V_{in}}$ to set up an expression for the input signal voltage $V_{in} = \frac{V_{out}}{A_v}$.
Step 5: Substitute the values of V_{out} and A_v into the expression: $V_{in} = \frac{2\ \text{V}}{200} = 0.01\ \text{V}$.
Step 6: Convert the input voltage from volts to millivolts: $0.01\ \text{V} = 10\ \text{mV}$.

Final Answer: 10 mV

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: For a system of point charges to be in equilibrium, the net electrostatic force acting on each individual charge must be zero. For two opposite charges $+q$ and $-4q$, the equilibrium position for a third charge must lie outside the line segment joining them, closer to the charge with the smaller magnitude.

Solution: Step 1: Let the charge $+q$ be located at the origin $x = 0$ and the charge $-4q$ be located at $x = L$. Since the charges have opposite signs, the zero-force point for a third charge cannot lie between them. It must lie on the outer side, closer to the smaller charge $+q$, i.e., at a position $x = -d$ (to the left of $+q$).
Step 2: Write the expression for the net electrostatic force acting on the third charge Q placed at $x = -d$. The force exerted by $+q$ is $F_1 = \frac{kqQ}{d^2}$, and the force exerted by $-4q$ is $F_2 = \frac{k(4q)Q}{(L+d)^2}$.
Step 3: Set the magnitudes of these two forces equal to satisfy the equilibrium condition: $\frac{kqQ}{d^2} = \frac{4kqQ}{(L+d)^2}$.
Step 4: Cancel common factors from both sides of the equation. This simplifies to $\frac{1}{d^2} = \frac{4}{(L+d)^2}$.
Step 5: Take the square root of both sides to solve for d: $\frac{1}{d} = \frac{2}{L+d} \implies L + d = 2d \implies d = L$.
Step 6: Conclude that the position of the third charge is at a distance L to the left of the $+q$ charge.

Final Answer: L to the left of $+q$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: For a satellite orbiting very close to the surface of a planet, its orbital radius can be approximated as the radius of the planet R. The gravitational force provides the necessary centripetal force for its circular motion, leading to a relationship between the time period, the planet's mass, and its volume.

Solution: Step 1: Equate the gravitational force acting on the satellite to the centripetal force required for its circular orbit: $\frac{GMm}{R^2} = \frac{mv^2}{R}$, where M is the mass of the planet and m is the mass of the satellite.
Step 2: Simplify the equation to find the orbital speed v: $v = \sqrt{\frac{GM}{R}}$.
Step 3: Write the formula for the time period T of the satellite, which is the total distance divided by the orbital speed: $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM/R}} = 2\pi\sqrt{\frac{R^3}{GM}}$.
Step 4: Square both sides of the time period equation to isolate the terms: $T^2 = \frac{4\pi^2 R^3}{GM}$.
Step 5: Express the mass M of the planet in terms of its uniform density ρ and volume $V = \frac{4}{3}\pi R^3$. This gives $M = \rho\left(\frac{4}{3}\pi R^3\right)$.
Step 6: Substitute this mass expression back into the squared time period equation: $T^2 = \frac{4\pi^2 R^3}{G\rho(\frac{4}{3}\pi R^3)} = \frac{3\pi}{G\rho}$. Rearranging gives $\rho T^2 = \frac{3\pi}{G}$. Since G is a universal constant, ρT^2 depends only on a universal numerical constant.

Final Answer: A universal numerical constant

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The magnifying power m of an astronomical telescope in normal adjustment (where the final image is formed at infinity) is given by the ratio of the focal length of the objective lens f_o to the focal length of the eyepiece lens f_e. The formula is $m = \frac{f_o}{f_e}$.

Solution: Step 1: Identify the given values from the problem statement. The focal length of the objective lens is $f_o = 140$ cm, and the focal length of the eyepiece is $f_e = 5.0$ cm.
Step 2: Recall the formula for the magnifying power of an astronomical telescope in normal adjustment: $m = \frac{f_o}{f_e}$.
Step 3: Substitute the given values of f_o and f_e into this formula. This gives $m = \frac{140}{5.0}$.
Step 4: Perform the division to calculate the value of the magnifying power: $m = 28$.

Final Answer: 28

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: The escape velocity from the surface of the Earth is given by $v_{esc} = \sqrt{\frac{2GM}{R}}$. Since gravity is a conservative force, we can use conservation of mechanical energy to find the maximum height reached by a projected projectile: $E_{surface} = E_{max\_height}$.

Solution: Step 1: Write down the formula for the escape velocity from the Earth's surface: $v_{esc} = \sqrt{\frac{2GM}{R}}$.
Step 2: Determine the launch velocity v of the particle, which is given as half of the escape velocity: $v = \frac{1}{2}v_{esc} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$.
Step 3: Write the total mechanical energy of the particle at the Earth's surface. It is the sum of its kinetic energy and gravitational potential energy: $E_1 = \frac{1}{2}mv^2 - \frac{GMm}{R}$.
Step 4: Substitute the value of v into the total initial energy expression: $E_1 = \frac{1}{2}m\left(\frac{1}{4}\frac{2GM}{R}\right) - \frac{GMm}{R} = \frac{GMm}{4R} - \frac{GMm}{R} = -\frac{3GMm}{4R}$.
Step 5: Write the total mechanical energy at the maximum height h from the surface, where the velocity is zero: $E_2 = 0 - \frac{GMm}{R+h}$.
Step 6: Equate the initial and final energies to solve for h: $-\frac{3GMm}{4R} = -\frac{GMm}{R+h} \implies \frac{3}{4R} = \frac{1}{R+h} \implies 3R + 3h = 4R \implies 3h = R \implies h = \frac{R}{3}$.

Final Answer: $\boxed{\frac{R}{3}}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: The resistance of a wire is given by $R = \rho\frac{L}{A}$. When a wire is stretched uniformly, its volume $V = LA$ remains constant. If its length is doubled, its cross-sectional area is halved, causing its total resistance to change. The individual resistances of the cut parts can then be determined to find the parallel equivalent resistance.

Solution: Step 1: Write the initial resistance of the wire of length L and area A: $R = \rho\frac{L}{A}$.
Step 2: Analyze the stretching process. The length becomes $L' = 2L$. Since the total volume is constant ($V = L \cdot A = L' \cdot A'$), the new cross-sectional area becomes $A' = \frac{A}{2}$.
Step 3: Calculate the resistance R' of the stretched wire: $R' = \rho\frac{L'}{A'} = \rho\frac{2L}{A/2} = 4\left(\rho\frac{L}{A}\right) = 4R$.
Step 4: The wire is cut into two equal pieces. Since resistance is proportional to length, each piece has a resistance of $R_1 = R_2 = \frac{R'}{2} = \frac{4R}{2} = 2R$.
Step 5: Calculate the equivalent resistance R_{eq} when these two pieces are connected in parallel: $\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} = \frac{2}{2R} = \frac{1}{R} \implies R_{eq} = R$.
Step 6: Find the total electrical power dissipated in this parallel combination when connected across a voltage source V: $P = \frac{V^2}{R_{eq}} = \frac{V^2}{R}$.

Final Answer: $\boxed{\frac{V^2}{R}}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: In an ideal transformer, there is no energy loss, so the input power supplied to the primary coil equals the output power delivered by the secondary coil ($P_{primary} = P_{secondary}$). This power conservation gives the current relationship: $\frac{I_p}{I_s} = \frac{N_s}{N_p}$, where $\frac{N_s}{N_p}$ is the turn ratio.

Solution: Step 1: Identify the given values from the problem statement: the turn ratio is $\frac{N_p}{N_s} = \frac{1}{10}$, which means $\frac{N_s}{N_p} = 10$. The secondary current is $I_s = 1.0$ A.
Step 2: Note that the connected load resistance 200 Ω draws a current of 1.0 A, which confirms the secondary current value.
Step 3: State the current-turn relationship for an ideal transformer: $I_p N_p = I_s N_s$.
Step 4: Rearrange the equation to express the primary current I_p explicitly: $I_p = I_s\left(\frac{N_s}{N_p}\right)$.
Step 5: Substitute the given values into this equation: $I_p = 1.0 \text{ A} \times 10 = 10$ A.

Final Answer: $\boxed{10 \text{ A}}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: For a combination of two thin prisms to produce dispersion without deviation (achromatic combination), the net angular deviation produced by the two prisms must sum to zero ($\delta_{net} = \delta_1 + \delta_2 = 0$). The deviation produced by a thin prism of angle A and refractive index μ is given by $\delta = (\mu - 1)A$.

Solution: Step 1: Write down the parameters of the first thin prism: prism angle $A_1 = 6^\circ$ and refractive index $\mu_1 = 1.5$.
Step 2: Calculate the angular deviation δ_1 produced by this first prism: $\delta_1 = (\mu_1 - 1)A_1 = (1.5 - 1) \times 6^\circ = 0.5 \times 6^\circ = 3^\circ$.
Step 3: Write down the expression for the angular deviation δ_2 produced by the second thin prism with angle A_2 and refractive index $\mu_2 = 1.6$: $\delta_2 = (\mu_2 - 1)A_2 = (1.6 - 1)A_2 = 0.6A_2$.
Step 4: Set the condition for dispersion without deviation, which requires the net deviation to be zero: $\delta_1 + \delta_2 = 0$. In terms of magnitudes, the deviations must balance each other: $|\delta_1| = |\delta_2|$.
Step 5: Equate the two values to solve for the unknown angle A_2: $3^\circ = 0.6A_2 \implies A_2 = \frac{3^\circ}{0.6} = 5^\circ$.

Final Answer: $\boxed{5^\circ}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: The longest wavelength of incident light that can cause photoelectric emission is called the threshold wavelength λ_0. It corresponds to the minimum energy required to eject an electron, which is equal to the work function ϕ of the metal surface. The relationship is given by $\phi = \frac{hc}{\lambda_0}$.

Solution: Step 1: Identify the given work function of the metal: $\phi = 4.2$ eV.
Step 2: Use the convenient form of the relationship between photon energy in electron-volts and wavelength in nanometers: $\phi\ (\text{eV}) = \frac{1240}{\lambda_0\ (\text{nm})}$.
Step 3: Rearrange this formula to express the threshold wavelength λ_0 explicitly: $\lambda_0\ (\text{nm}) = \frac{1240}{\phi\ (\text{eV})}$.
Step 4: Substitute the given value of the work function into the equation: $\lambda_0 = \frac{1240}{4.2}$ nm.
Step 5: Perform the division to find the value: $\lambda_0 \approx 295.23$ nm. This matches the value of 295 nm.

Final Answer: $\boxed{295 \text{ nm}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: In a perfectly inelastic collision, the two blocks stick together and move with a common velocity immediately after the collision. Linear momentum is conserved during the collision. After the collision, mechanical energy is conserved as the combined mass compresses the spring.

Solution: Step 1: Apply conservation of linear momentum during the collision. The initial momentum is Mv, and the final momentum is $(M + M)v' = 2Mv'$, where v' is the common velocity.
Step 2: Solve for the common velocity v' immediately after the collision: $Mv = 2Mv' \implies v' = \frac{v}{2}$.
Step 3: Write the total kinetic energy K of the combined two-block system immediately after the collision: $K = \frac{1}{2}(2M)(v')^2 = M\left(\frac{v}{2}\right)^2 = \frac{Mv^2}{4}$.
Step 4: Apply conservation of mechanical energy for the post-collision motion. The kinetic energy of the combined mass is converted into elastic potential energy of the spring at maximum compression x_{max}: $\frac{1}{2}kx_{max}^2 = \frac{Mv^2}{4}$.
Step 5: Solve for x_{max}^2 by rearranging terms: $kx_{max}^2 = \frac{Mv^2}{2} \implies x_{max}^2 = \frac{Mv^2}{2k}$.
Step 6: Take the square root of both sides to find the maximum compression: $x_{max} = v\sqrt{\frac{M}{2k}}$.

Final Answer: $\boxed{v\sqrt{\frac{M}{2k}}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: According to the first law of thermodynamics, the heat energy ΔQ supplied to a system is equal to the sum of the change in its internal energy ΔU and the work done ΔW by the system ($\Delta Q = \Delta U + \Delta W$). For an ideal gas undergoing an isothermal process, the temperature remains constant, which means the internal energy does not change ($\Delta U = 0$).

Solution: Step 1: Note the type of thermodynamic process described, which is an isothermal expansion.
Step 2: State the property of internal energy for an ideal gas during an isothermal process. Since internal energy depends only on temperature for an ideal gas, $\Delta U = 0$ because $\Delta T = 0$.
Step 3: Write down the first law of thermodynamics: $\Delta Q = \Delta U + \Delta W$.
Step 4: Substitute $\Delta U = 0$ into the first law equation. This yields $\Delta Q = 0 + \Delta W = \Delta W$.
Step 5: Identify the given work done by the gas during the expansion: $\Delta W = 2303$ J.
Step 6: Conclude that the amount of heat supplied to the gas must be exactly equal to the work done, which gives $\Delta Q = 2303$ J.

Final Answer: 2303 J

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: The magnetic force acting on a current-carrying wire in a uniform magnetic field is given by the vector formula $\vec{F} = I(\vec{L} \times \vec{B})$, where $\vec{L}$ represents the vector displacement from the starting point to the ending point of the wire segment. For any closed loop, the net vector displacement is zero.

Solution: Step 1: Consider the expression for the magnetic force acting on an infinitesimal element $d\vec{l}$ of the current loop: $d\vec{F} = I(d\vec{l} \times \vec{B})$.
Step 2: Integrate this force element around the entire closed circular loop to find the net magnetic force: $\vec{F}_{net} = \oint I(d\vec{l} \times \vec{B})$.
Step 3: Since the magnetic field $\vec{B}$ is uniform across space, it can be factored out of the integral. This gives $\vec{F}_{net} = I\left(\oint d\vec{l}\right) \times \vec{B}$.
Step 4: Evaluate the cyclic vector integral $\oint d\vec{l}$. This represents the total vector displacement around a complete closed loop.
Step 5: Since the starting point and ending point of a complete closed loop are identical, the net displacement vector is $\oint d\vec{l} = 0$.
Step 6: Substitute this back into the force equation to get $\vec{F}_{net} = I(0) \times \vec{B} = 0$. The net magnetic force acting on the loop is zero.

Final Answer: Zero

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept: In single-slit Fraunhofer diffraction, the positions of the minima on either side of the central maximum satisfy the condition $a \sin\theta = n\lambda$. For the first minimum ($n = 1$), the angular position is $\theta \approx \frac{\lambda}{a}$. The total angular width of the central diffraction maximum is the angular distance between the first minima on both sides, given by $2\theta = \frac{2\lambda}{a}$.

Solution: Step 1: Identify the given parameters: wavelength $\lambda = 500$ nm $= 500 \times 10^{-9}$ m $= 5.0 \times 10^{-7}$ m, and slit width $a = 0.2$ mm $= 0.2 \times 10^{-3}$ m $= 2.0 \times 10^{-4}$ m.
Step 2: Write down the formula for the total angular width of the central maximum: $2\theta = \frac{2\lambda}{a}$.
Step 3: Substitute the given values into the formula: $2\theta = \frac{2\times(5.0\times10^{-7})}{2.0\times10^{-4}}$.
Step 4: Simplify the numerator: $2 \times 5.0 \times 10^{-7} = 1.0 \times 10^{-6}$ m.
Step 5: Perform the division to find the angular width: $2\theta = \frac{1.0\times10^{-6}}{2.0\times10^{-4}} = 0.5\times10^{-2} = 5.0\times10^{-3}$ rad.

Final Answer: 5.0×10^{-3} rad

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: Friction is a self-adjusting reactive force when a body is at rest. The maximum possible value of static friction that can act between a body and a surface is the limiting friction, given by $f_s = \mu N$. If the applied horizontal force F is less than this limiting value, the body does not move, and the static friction force exactly balances the applied force ($f = F$).

Solution: Step 1: Calculate the normal force acting on the block resting on the horizontal surface: $N = mg$.
Step 2: Determine the maximum limiting static friction force available to resist motion: $f_{max} = \mu N = \mu mg$.
Step 3: Identify the magnitude of the horizontal force applied to the block, which is given as $F = \frac{1}{2}\mu mg$.
Step 4: Compare the applied force with the limiting friction force. We observe that $F = \frac{1}{2}\mu mg < \mu mg$, which means the applied force is less than the maximum friction force.
Step 5: Conclude that the block remains at rest because the applied force is insufficient to overcome static friction.
Step 6: Since the block is stationary, the static friction force must exactly balance the applied horizontal force to maintain equilibrium: $f = F = \frac{1}{2}\mu mg$.

Final Answer: $\boxed{\frac{1}{2}\mu mg}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: The de Broglie wavelength of a particle accelerated from rest through a potential difference V is given by $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$, where q is the charge and m is the mass of the particle. By writing this expression for both the electron and the proton, we can determine their wavelength ratio.

Solution: Step 1: Write the de Broglie wavelength for an electron of mass m and charge e accelerated through potential V: $\lambda_1 = \frac{h}{\sqrt{2meV}}$.
Step 2: Write the de Broglie wavelength for a proton of mass M and charge e accelerated through the same potential V: $\lambda_2 = \frac{h}{\sqrt{2MeV}}$.
Step 3: Set up the ratio of the electron's wavelength to the proton's wavelength: $\frac{\lambda_1}{\lambda_2} = \frac{h/\sqrt{2meV}}{h/\sqrt{2MeV}}$.
Step 4: Cancel the common terms h, 2, e, and V from the expression. This simplifies to $\frac{\lambda_1}{\lambda_2} = \frac{\sqrt{2MeV}}{\sqrt{2meV}} = \sqrt{\frac{M}{m}}$.

Final Answer: $\boxed{\sqrt{\frac{M}{m}}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The velocity v of a particle executing simple harmonic motion at a displacement x from its mean position is given by the formula $v = \omega\sqrt{A^2 - x^2}$, where ω is the angular frequency and A is the amplitude. By writing this relation for two different positions, we can eliminate ω and solve for A.

Solution: Step 1: Write down the velocity equations for both given positions. For the first position: $v_1^2 = \omega^2(A^2 - x_1^2)$. For the second position: $v_2^2 = \omega^2(A^2 - x_2^2)$.
Step 2: Divide the first equation by the second equation to eliminate ω^2: $\frac{v_1^2}{v_2^2} = \frac{A^2-x_1^2}{A^2-x_2^2}$.
Step 3: Cross-multiply to expand the equation: $v_1^2(A^2 - x_2^2) = v_2^2(A^2 - x_1^2) \implies v_1^2A^2 - v_1^2x_2^2 = v_2^2A^2 - v_2^2x_1^2$.
Step 4: Rearrange terms to group all terms containing the amplitude A^2 on one side: $v_1^2A^2 - v_2^2A^2 = v_1^2x_2^2 - v_2^2x_1^2$.
Step 5: Factor out A^2 and solve for it: $A^2(v_1^2 - v_2^2) = v_1^2x_2^2 - v_2^2x_1^2 \implies A^2 = \frac{v_1^2x_2^2-v_2^2x_1^2}{v_1^2-v_2^2}$.
Step 6: Take the square root of both sides to find the expression for the amplitude: $A = \sqrt{\frac{v_1^2x_2^2-v_2^2x_1^2}{v_1^2-v_2^2}}$.

Final Answer: $\boxed{\sqrt{\frac{v_1^2x_2^2 - v_2^2x_1^2}{v_1^2 - v_2^2}}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: The maximum theoretical efficiency of any heat engine operating between two temperatures is achieved by a reversible Carnot engine. This maximum efficiency η depends only on the absolute temperatures of the hot source (T_H) and the cold sink (T_C), and is given by the formula $\eta = 1 - \frac{T_C}{T_H}$.

Solution: Step 1: Identify the given absolute temperatures from the problem statement: the source temperature is $T_H = 800$ K, and the sink temperature is $T_C = 400$ K.
Step 2: Write the formula for the efficiency of a Carnot heat engine: $\eta = 1 - \frac{T_C}{T_H}$.
Step 3: Substitute the temperature values into this formula: $\eta = 1 - \frac{400}{800}$.
Step 4: Simplify the fraction: $\frac{400}{800} = \frac{1}{2} = 0.5$. Thus, $\eta = 1 - 0.5 = 0.5$.
Step 5: Convert this fractional efficiency value into a percentage: $\eta = 0.5 \times 100\% = 50\%$.

Final Answer: $\boxed{50\%}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: When electrons in a sample of hydrogen-like atoms are excited to a higher energy level n, they can transition back to lower states through various intermediate levels. The total number of distinct wavelengths or spectral lines emitted due to all possible transitions is given by the combinatorial formula $N = \frac{n(n-1)}{2}$.

Solution: Step 1: Identify the principal quantum number of the initial excited state from which transitions originate, which is given as $n = 4$.
Step 2: Recall the standard formula for the total number of distinct emission lines when electrons transition to the ground state: $N = \frac{n(n-1)}{2}$.
Step 3: Substitute $n = 4$ into this formula. This gives $N = \frac{4(4-1)}{2}$.
Step 4: Simplify the expression inside the parenthesis: $4 - 1 = 3$.
Step 5: Compute the final value: $N = \frac{4\times3}{2} = \frac{12}{2} = 6$. Thus, 6 distinct spectral lines will be observed.

Final Answer: $\boxed{6}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: When a conducting loop moves out of a magnetic field, the magnetic flux through the loop decreases, which induces an electromotive force (emf) according to Faraday's law. For a square loop of side length a moving with velocity v perpendicular to a field B, the induced motional emf is $e = Bav$. The power dissipated as heat is given by $P = \frac{e^2}{R}$.

Solution: Step 1: Write down the expression for the motional electromotive force e induced across the leading arm of the square loop as it exits the magnetic field: $e = Bav$.
Step 2: Identify the total electrical resistance of the loop, which is given as R.
Step 3: Recall the formula for Joule heating power dissipated in a circuit with resistance R under an induced emf e: $P = \frac{e^2}{R}$.
Step 4: Substitute the expression for the induced emf $e = Bav$ into the power formula: $P = \frac{(Bav)^2}{R}$.
Step 5: Expand the squared terms in the numerator to find the final expression: $P = \frac{B^2a^2v^2}{R}$.

Final Answer: $\boxed{\frac{B^2a^2v^2}{R}}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: The magnitude of the resultant vector $\vec{C}$ of two vectors $\vec{A}$ and $\vec{B}$ is given by the law of cosines for vector addition: $C^2 = A^2 + B^2 + 2AB\cos\theta$, where θ is the angle between $\vec{A}$ and $\vec{B}$. By comparing this general relation to the given magnitude relationship, the value of $\cos\theta$ can be found.

Solution: Step 1: Write down the general equation for the squared magnitude of a resultant vector: $C^2 = A^2 + B^2 + 2AB\cos\theta$.
Step 2: Note the specific magnitude condition given in the problem statement, which is $C^2 = A^2 + B^2$.
Step 3: Equate the two expressions for C^2: $A^2 + B^2 + 2AB\cos\theta = A^2 + B^2$.
Step 4: Subtract $A^2 + B^2$ from both sides of the equation. This simplifies to $2AB\cos\theta = 0$.
Step 5: Since the vectors $\vec{A}$ and $\vec{B}$ have non-zero magnitudes ($A \neq 0$ and $B \neq 0$), divide by $2AB$ to solve for the cosine term: $\cos\theta = 0$.
Step 6: Determine the angle θ whose cosine value is zero. This gives $\theta = \frac{\pi}{2}$ (or 90°).

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: The moment of inertia of a composite body can be found using the principle of superposition. The moment of inertia of the remaining annular disc is equal to the moment of inertia of the complete initial disc (I_{total}) minus the moment of inertia of the removed smaller concentric disc ($I_{removed}$).

Solution: Step 1: Let σ be the uniform mass per unit area (surface density) of the disc material.
Step 2: Write the mass of the final remaining annular disc of outer radius R and inner hole radius $\frac{R}{2}$: $M = \sigma\pi\left(R^2 - \left(\frac{R}{2}\right)^2\right) = \sigma\pi\left(R^2 - \frac{R^2}{4}\right) = \frac{3}{4}\sigma\pi R^2$. This means $\sigma\pi R^2 = \frac{4}{3}M$.
Step 3: Express the mass M_1 of the complete initial uncut disc of radius R in terms of M: $M_1 = \sigma\pi R^2 = \frac{4}{3}M$.
Step 4: Express the mass M_2 of the removed inner circular part of radius $\frac{R}{2}$: $M_2 = \sigma\pi\left(\frac{R}{2}\right)^2 = \frac{1}{4}\sigma\pi R^2 = \frac{1}{4}\left(\frac{4}{3}M\right) = \frac{1}{3}M$.
Step 5: Write the expressions for the moments of inertia of both circular components about the central axis. For the complete disc: $I_1 = \frac{1}{2}M_1R^2 = \frac{1}{2}\left(\frac{4}{3}M\right)R^2 = \frac{2}{3}MR^2$. For the removed inner part: $I_2 = \frac{1}{2}M_2\left(\frac{R}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{3}M\right)\frac{R^2}{4} = \frac{1}{24}MR^2$.
Step 6: Subtract I_2 from I_1 to find the moment of inertia of the remaining annular disc: $I = I_1 - I_2 = \frac{2}{3}MR^2 - \frac{1}{24}MR^2 = \frac{16-1}{24}MR^2 = \frac{15}{24}MR^2 = \frac{5}{8}MR^2$.

Final Answer: $\boxed{\frac{5}{8}MR^2}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: Electric currents flowing through parallel conductors create magnetic fields that interact with each other. According to Ampere's law and the right-hand rule, parallel wires carrying currents in the same direction generate magnetic forces that pull the wires together, resulting in an attractive force.

Solution: Step 1: Consider the first long wire carrying current I_1. It generates a magnetic field B_1 in the space surrounding it. At the location of the second wire, the direction of this field is perpendicular to the wire plane.

Step 2: The second wire, carrying current I_2, is immersed in this magnetic field B_1. It experiences a magnetic Lorentz force per unit length given by $f = I_2 B_1$.

Step 3: Apply the right-hand rule to find the direction of the magnetic field B_1 and the resulting force direction. If the currents are parallel and flow in the same direction, the force vector on the second wire points directly toward the first wire.

Step 4: By Newton's third law, the first wire experiences an equal and opposite force directed toward the second wire.

Step 5: Conclude that since the forces point toward each other, the nature of the interactive magnetic force between parallel currents in the same direction is strictly attractive.

Final Answer: Attractive

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: When unpolarized light of intensity I_0 passes through a perfect linear polaroid, its intensity is reduced by exactly half, becoming $I_1 = \frac{I_0}{2}$. When this polarized light passes through a second polaroid, the transmitted intensity satisfies Malus's Law: $I_2 = I_1 \cos^2\theta$, where θ is the angle between their transmission axes.

Solution: Step 1: Analyze the transmission through the first polaroid. The incident light beam is unpolarized with intensity I_0. After passing through this first polaroid, it becomes linearly polarized, and its intensity is $I_1 = \frac{I_0}{2}$.
Step 2: Identify the orientation of the second polaroid relative to the first. The angle between their transmission axes is given as $\theta = 60°$.
Step 3: Apply Malus's Law to calculate the intensity I_2 of the light emerging from the second polaroid: $I_2 = I_1\cos^2(60°)$.
Step 4: Substitute the value of $\cos(60°) = \frac{1}{2}$ into the equation. This yields $\cos^2(60°) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.
Step 5: Substitute the expression for I_1 into the equation to find the final intensity: $I_2 = \left(\frac{I_0}{2}\right)\times\frac{1}{4} = \frac{I_0}{8}$.

Final Answer: $\boxed{\frac{I_0}{8}}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: The logic operation of a digital circuit combination can be determined by analyzing the Boolean logic expressions at each stage. By applying De Morgan's laws ($\overline{X+Y} = \overline{X} \cdot \overline{Y}$), the final logic output can be simplified into a standard gate operation.

Solution: Step 1: Identify the inputs to the circuit as A and B.

Step 2: Track the signals after they pass through the two initial separate NOT gates. The outputs of these gates are $\overline{A}$ and $\overline{B}$ respectively.

Step 3: These inverted signals $\overline{A}$ and $\overline{B}$ serve as the inputs to the final two-input NOR gate.

Step 4: Write the boolean logic expression for a NOR gate, which performs an OR operation followed by an inversion: $Y = \overline{\overline{A} + \overline{B}}$.

Step 5: Simplify this expression using De Morgan's mathematical logic theorem. According to De Morgan's law, the complement of a sum is equal to the product of the complements: $\overline{X+Y} = \overline{X} \cdot \overline{Y}$.

Step 6: Apply this to the circuit expression: $Y = \overline{\overline{A}} \cdot \overline{\overline{B}}$. Since a double inversion cancels out ($\overline{\overline{A}} = A$), this simplifies to $Y = A \cdot B$. This expression corresponds to a standard digital AND gate operation.

Final Answer: AND

Answer: (A)

Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	B	5	B
6	A	7	A	8	C	9	B	10	A
11	A	12	A	13	D	14	A	15	A
16	B	17	A	18	A	19	A	20	A
21	A	22	A	23	A	24	A	25	A
26	A	27	A	28	A	29	A	30	A
31	A	32	A	33	A	34	A	35	A
36	A	37	A	38	A	39	A	40	A