KIITEE Mathematics Sample Paper – 4

Duration: 50 Minutes **Maximum Marks: 160**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **KIITEE** entrance.
- Each correct answer carries **+4 marks**. There is −1 **mark per wrong answer;** unattempted questions score **0**
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 (10+2) Mathematics — Calculus,Coordinate Geometry, Algebra , Probability & Statistics,Vector Algebra & 3D Geometry,Trigonometry,Permutation, Combination & Binomial Theorem**
- The test is computer based. Personal calculators, log tables, mobile phones, and other electronic gadgets are strictly prohibited.

Q1. If the line $y = mx + 1$ is tangent to the parabola $y^2 = 4x$, then the value of m is:

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 3

Q2. The value of $\lim_{x\to 0} \frac{1-\cos(2x)}{x \tan x}$ is:

(A) 0

(B) 1

(C) 2

(D) 4

Q3. Let A and B be two independent events such that $P(A) = 0.3$ and $P(B) = 0.4$. Then $P(A \cup B)$ is equal to:

(A) 0.7

(B) 0.12

(C) 0.58

(D) 0.82

Q4. If α and β are the roots of the equation $x^2 - 5x + 6 = 0$, then the equation whose roots are α^2 and β^2 is:

(A) $x^2 - 13x + 36 = 0$

(B) $x^2 + 13x + 36 = 0$

(C) $x^2 - 25x + 36 = 0$

(D) $x^2 - 13x - 36 = 0$

Q5. The distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q6. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is:

(A) $y = cx$

(B) $xy = c$

(C) $y = c \ln x$

(D) $y = x + c$

Q7. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is equal to:

(A) $\sqrt{2}\sin\theta$

(B) $-\sqrt{2}\sin\theta$

(C) $\frac{1}{\sqrt{2}}\sin\theta$

(D) $\sqrt{2}\cos\theta$

Q8. The number of ways in which 5 boys and 3 girls can be seated in a row such that no two girls are together is:

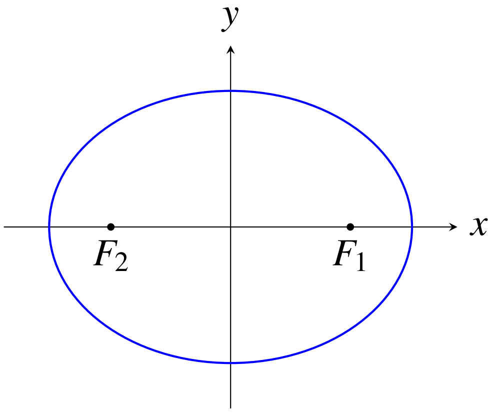
(A) 14400

(B) 2400

(C) 720

(D) 120

Q9. The eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is:



(A) $\frac{\sqrt{7}}{4}$

(B) $\frac{7}{16}$

(C) $\frac{3}{4}$

(D) $\frac{\sqrt{7}}{3}$

Q10. If the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A$ is equal to:

(A) $2I$

(B) $-2I$

(C) I

(D) O

Q11. The value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ is:

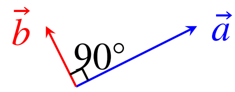
(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

Q12. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ are perpendicular, then the value of the scalar λ is:



(A) 2

(B) 1

(C) -1

(D) $-\frac{1}{2}$

Q13. The coefficient of x^4 in the expansion of $(1 + x)^{10}$ is:

(A) 210

(B) 120

(C) 45

(D) 252

Q14. The mean of 5 observations is 4 and their variance is 5.2. If three of the observations are 1, 2, and 6, then the other two observations are:

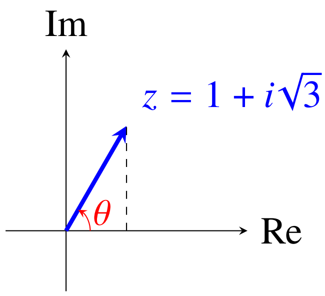
(A) 4, 7

(B) 3, 8

(C) 5, 6

(D) 2, 9

Q15. If $z = 1 + i\sqrt{3}$, then the amplitude (argument) of z is:



(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{4}$

Q16. The value of $\int e^x(\tan x + \ln(\sec x))\,dx$ is:

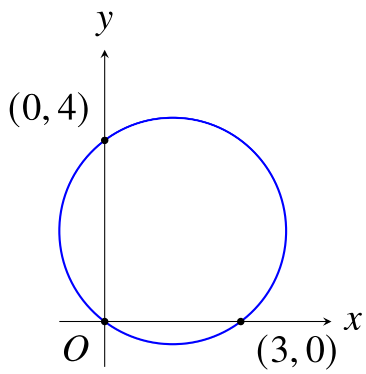
(A) $e^x \tan x + c$

(B) $e^x \ln(\sec x) + c$

(C) $e^x \sec x + c$

(D) $e^x \ln(\tan x) + c$

Q17. The equation of the circle passing through the origin and cutting intercepts 3 and 4 on the positive axes is:



(A) $x^2 + y^2 - 3x - 4y = 0$

(B) $x^2 + y^2 + 3x + 4y = 0$

(C) $x^2 + y^2 - 6x - 8y = 0$

(D) $x^2 + y^2 - 4x - 3y = 0$

Q18. The value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ is:

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{4}$

(D) 2π

Q19. If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx}$ is:

(A) $\sec x$

(B) $\tan x$

(C) $\sec x + \tan x$

(D) $\frac{1}{\sec x+\tan x}$

Q20. If A is a square matrix of order 3 and $|A| = 4$, then $|2A|$ is:

(A) 8

(B) 16

(C) 32

(D) 12

Q21. The value of $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ as $n \to \infty$ is:

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) ∞

Q22. The angle between the lines $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$ and $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z+2}{2}$ is:

(A) $\frac{\pi}{2}$

(B) $\cos^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{2}{9}\right)$

(D) 0

Q23. If the standard deviation of a data set is 4, then the variance is:

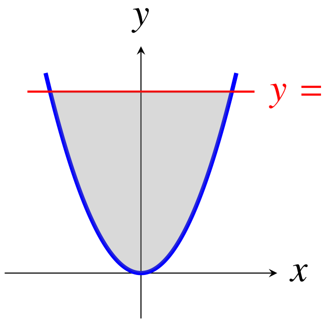
(A) 2

(B) 16

(C) 8

(D) 4

Q24. The area bounded by the curve $y = x^2$ and the line $y = 4$ is:



(A) $\frac{32}{3}$

(B) $\frac{16}{3}$

(C) $\frac{8}{3}$

(D) 16

Q25. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to:

(A) $\frac{\pi}{2}$

(B) π

(C) 0

(D) $\frac{\pi}{4}$

Q26. The number of terms in the expansion of $(x + y + z)^{10}$ is:

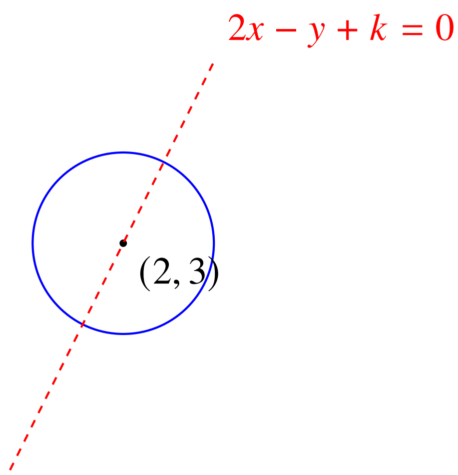
(A) 11

(B) 55

(C) 66

(D) 22

Q27. If the line $2x - y + k = 0$ is a normal to the circle $x^2 + y^2 - 4x - 6y + 11 = 0$, then k is:



(A) 1

(B) -1

(C) 2

(D) 0

Q28. The rate of change of the area of a circle with respect to its radius r when $r = 5$ cm is:

(A) 10π

(B) 5π

(C) 25π

(D) 20π

Q29. A bag contains 4 red and 6 black balls. A ball is drawn at random. The probability that it is red is:

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{1}{2}$

(D) $\frac{4}{5}$

Q30. If ω is an imaginary cube root of unity, then $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$ is:

(A) 32

(B) -32

(C) 64

(D) -64

Q31. The maximum value of $f(x) = \sin x + \cos x$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Q32. The projection of the vector $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$ is:

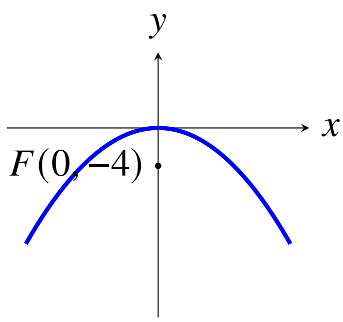
(A) $\frac{60}{\sqrt{114}}$

(B) $\frac{60}{\sqrt{59}}$

(C) $\sqrt{114}$

(D) $\frac{30}{\sqrt{114}}$

Q33. The focus of the parabola $x^2 = -16y$ is:



(A) $(4, 0)$

(B) $(0, 4)$

(C) $(0, -4)$

(D) $(-4, 0)$

Q34. The function $f(x) = x^3 - 3x^2 + 3x - 100$ is:

(A) Strictly increasing on $\mathbb{R}$

(B) Strictly decreasing on $\mathbb{R}$

(C) Increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$

(D) Decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$

Q35. The solution of the differential equation $\frac{dy}{dx} + y = e^{-x}$ with $y(0) = 0$ is:

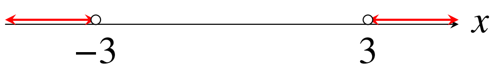
(A) $y = xe^{-x}$

(B) $y = xe^{x}$

(C) $y = e^{-x}$

(D) $y = (x + 1)e^{-x}$

Q36. The domain of the function $f(x) = \frac{1}{\sqrt{x^2-9}}$ is:



(A) $(-\infty, -3) \cup (3, \infty)$

(B) $[-3, 3]$

(C) $(-\infty, -3] \cup [3, \infty)$

(D) $\mathbb{R}$

Q37. The number of arrangements of the letters of the word 'BANANA' is:

(A) 720

(B) 60

(C) 120

(D) 24

Q38. If $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$, then the value of Δ is:

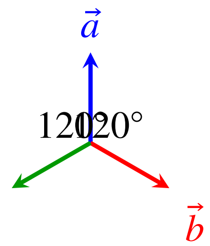
(A) 0

(B) $a + b + c$

(C) 1

(D) abc

Q39. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is:



(A) 0

(B) 1

(C) $-\frac{3}{2}$

(D) $\frac{3}{2}$

Q40. The value of $\cos 20° \cos 40° \cos 80°$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{1}{16}$

Detailed Solutions

Q1.

Solution

Concept:
For a parabola of the standard form $y^2 = 4ax$, the condition of tangency for a line given by the equation $y = mx + c$ is that the constant term must satisfy $c = \frac{a}{m}$. By identifying the parameters a and c from the given equations, we can solve directly for the unknown slope m.

Solution:
Step 1: Identify the parameters of the parabola. The given parabola is $y^2 = 4x$. Comparing this with the standard equation $y^2 = 4ax$, we get:

$$4a = 4 \implies a = 1$$

Step 2: Identify the parameters of the line. The given line is $y = mx + 1$. Comparing this with the slope-intercept form $y = mx + c$, we find the intercept:

$$c = 1$$

Step 3: Apply the standard condition for tangency to the parabola. The condition is:

$$c = \frac{a}{m}$$

Step 4: Substitute the values of $a = 1$ and $c = 1$ into the condition formula:

$$1 = \frac{1}{m}$$

Step 5: Solve the equation for the variable m:

$$m = 1$$

Thus, the value of the slope m for which the line is tangent to the parabola is equal to 1.

Final Answer: 1

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept:

To evaluate a limit of the form $\frac{0}{0}$ involving trigonometric functions, we can use the standard trigonometric identities to simplify the numerator and then utilize the fundamental limit theorem $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$.

Solution:

Step 1: Check the form of the limit by substituting $x = 0$ directly into the expression:

$$\frac{1-\cos(0)}{0\cdot\tan(0)} = \frac{1-1}{0} = \frac{0}{0}$$

This is an indeterminate form, so algebraic or trigonometric simplification is required.

Step 2: Use the double-angle trigonometric identity for the numerator:

$$1-\cos(2x) = 2\sin^2 x$$

Step 3: Substitute this identity back into the original limit expression:

$$\lim_{x\to 0} \frac{2\sin^2 x}{x\tan x}$$

Step 4: Rearrange the terms to form standard limits by dividing and multiplying by appropriate powers of x:

$$\lim_{x\to 0} 2\cdot\left(\frac{\sin x}{x}\right)^2 \cdot \frac{x}{\tan x}$$

Step 5: Apply the standard limit values as $x \to 0$:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x\to 0} \frac{\tan x}{x} = 1 \implies \lim_{x\to 0} \frac{x}{\tan x} = 1$$

Step 6: Substitute these evaluated standard limits into the expression:

$$2\cdot(1)^2\cdot 1 = 2$$

Thus, the evaluated limit value is equal to 2.

Final Answer: 2

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept:

For any two events A and B, the probability of their union is given by the addition theorem of probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If the two events are specified to be independent, the probability of their intersection simplifies to the product of their individual probabilities: $P(A \cap B) = P(A) \cdot P(B)$.

Solution:

Step 1: Note the given individual probabilities of the events from the problem statement:

$$P(A) = 0.3 \quad \text{and} \quad P(B) = 0.4$$

Step 2: Use the definition of independent events to calculate the probability of the intersection:

$$P(A \cap B) = P(A) \cdot P(B)$$

Step 3: Substitute the given numbers into the multiplication rule formula:

$$P(A \cap B) = 0.3 \cdot 0.4 = 0.12$$

Step 4: State the addition theorem of probability for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 5: Substitute all the calculated and given values into this equation:

$$P(A \cup B) = 0.3 + 0.4 - 0.12$$

Step 6: Perform the simple arithmetic addition and subtraction steps:

$$P(A \cup B) = 0.7 - 0.12 = 0.58$$

The probability that event A or event B occurs is equal to 0.58.

Final Answer: 0.58

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept:

For a quadratic equation $ax^2 + bx + c = 0$ with roots α and β, the sum of roots is given by $\alpha + \beta = -\frac{b}{a}$ and the product of roots is $\alpha\beta = \frac{c}{a}$. To construct a new quadratic equation with roots α^2 and β^2, we must find the new sum of roots $(\alpha^2 + \beta^2)$ and the new product of roots $(\alpha^2\beta^2)$, and write the equation as $x^2 - (\text{Sum})x + (\text{Product}) = 0$.

Solution:

Step 1: Given the quadratic equation $x^2 - 5x + 6 = 0$, determine the values of the coefficients:

$$a = 1, \quad b = -5, \quad c = 6$$

Step 2: Calculate the sum and product of the initial roots α and β:

$$\alpha + \beta = -\frac{-5}{1} = 5$$

$$\alpha\beta = \frac{6}{1} = 6$$

Step 3: Alternatively, determine the explicit roots by factoring the equation:

$$x^2 - 5x + 6 = 0 \implies (x-2)(x-3) = 0 \implies \alpha = 2, \quad \beta = 3$$

Step 4: Calculate the values of the new roots for the required equation:

$$\alpha^2 = 2^2 = 4$$

$$\beta^2 = 3^2 = 9$$

Step 5: Find the sum and product of these newly squared roots:

$$\text{New Sum} = \alpha^2 + \beta^2 = 4 + 9 = 13$$

$$\text{New Product} = \alpha^2 \cdot \beta^2 = 4 \cdot 9 = 36$$

Step 6: Substitute these symmetric values into the standard quadratic construction formula:

$$x^2 - (\text{New Sum})x + (\text{New Product}) = 0 \implies x^2 - 13x + 36 = 0$$

Thus, the required quadratic equation is $x^2 - 13x + 36 = 0$.

Final Answer: $\boxed{x^2 - 13x + 36 = 0}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept:

The perpendicular distance d from a given point $P(x_1, y_1, z_1)$ to a plane given by the general equation $Ax + By + Cz + D = 0$ is calculated using the standard three-dimensional geometric formula:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Solution:

Step 1: Identify the coordinates of the given point $P(x_1, y_1, z_1)$ from the problem details:

$$x_1 = 2, \quad y_1 = 3, \quad z_1 = 4$$

Step 2: Identify the coefficient values from the plane equation $3x - 6y + 2z + 11 = 0$:

$$A = 3, \quad B = -6, \quad C = 2, \quad D = 11$$

Step 3: Substitute these values into the numerator of the perpendicular distance formula:

$$\text{Numerator} = |3(2) + (-6)(3) + 2(4) + 11|$$

$$\text{Numerator} = |6 - 18 + 8 + 11| = |7| = 7$$

Step 4: Substitute the plane coefficients into the denominator of the distance formula:

$$\text{Denominator} = \sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{9 + 36 + 4}$$

$$\text{Denominator} = \sqrt{49} = 7$$

Step 5: Compute the final value of the perpendicular distance d by division:

$$d = \frac{7}{7} = 1$$

The perpendicular distance from the point to the plane is exactly 1 unit.

Final Answer: 1

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:

A first-order differential equation of the form $\frac{dy}{dx} = \frac{y}{x}$ can be solved by applying the method of separation of variables. We group all terms containing the variable y with dy on one side and all terms containing the variable x with dx on the opposite side, followed by integration.

Solution:

Step 1: Write down the given first-order differential equation:

$$\frac{dy}{dx} = \frac{y}{x}$$

Step 2: Rearrange the terms to separate the variables y and x onto opposite sides of the equation:

$$\frac{1}{y}\,dy = \frac{1}{x}\,dx$$

Step 3: Apply the indefinite integral operator to both sides of the separated equation:

$$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$$

Step 4: Evaluate the standard logarithmic integrals on both sides:

$$\ln|y| = \ln|x| + C_1$$

Step 5: Rewrite the arbitrary constant of integration C_1 in logarithmic form as $\ln|c|$ to simplify the expression:

$$\ln|y| = \ln|x| + \ln|c|$$

Step 6: Combine the logarithmic terms on the right side using the product property of logarithms:

$$\ln|y| = \ln|c \cdot x|$$

Step 7: Take the exponential of both sides to remove the logarithms and solve for y:

$$y = cx$$

Thus, the general solution represents a family of straight lines passing through the origin.

Final Answer: $y = cx$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:
To evaluate the expression $\cos\theta - \sin\theta$ given the equation $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, we can apply algebraic rearrangement to group like trigonometric terms together and then utilize basic identity adjustments.

Solution:
Step 1: Write down the given primary trigonometric equation:

$$\sin\theta + \cos\theta = \sqrt{2}\cos\theta$$

Step 2: Rearrange the terms by subtracting $\cos\theta$ from both sides to isolate the $\sin\theta$ term:

$$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

Step 3: Factor out the common term $\cos\theta$ on the right side:

$$\sin\theta = (\sqrt{2} - 1)\cos\theta$$

Step 4: Rationalize the expression by multiplying both sides by $(\sqrt{2} + 1)$:

$$(\sqrt{2} + 1)\sin\theta = (\sqrt{2} + 1)(\sqrt{2} - 1)\cos\theta$$

Step 5: Simplify the right-hand side using the difference of squares identity $(a+b)(a-b) = a^2 - b^2$:

$$(\sqrt{2} + 1)\sin\theta = (2 - 1)\cos\theta \implies \sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

Step 6: Rearrange this resulting equation to match the form of the required expression:

$$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Thus, the expression $\cos\theta - \sin\theta$ simplifies exactly to $\sqrt{2}\sin\theta$.

Final Answer: $\boxed{\sqrt{2}\sin\theta}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept:
To solve permutation problems where certain items (like girls) must not be placed next to each other, we use the gap method. First, arrange the items that have no constraints (the boys) in a row. Then, identify the empty spaces or gaps created between and around them, and place the restricted items into these gaps.

Solution:
Step 1: Identify the total number of individuals of each category:

$$\text{Number of boys} = 5, \quad \text{Number of girls} = 3$$

Step 2: Arrange the 5 boys in a straight line row. The number of permutations for 5 distinct items is:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$$

Step 3: Determine the number of available gaps created around the boys where girls can be seated safely without being adjacent. For 5 boys, the total number of available gaps is:

$$\text{Number of gaps} = 5 + 1 = 6 \text{ gaps}$$

Step 4: Select 3 specific gaps out of the 6 available for the 3 girls and arrange them in those chosen positions. This is computed using permutations:

$$^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = 6 \times 5 \times 4 = 120 \text{ ways}$$

Step 5: Compute the total number of valid seating arrangements by applying the fundamental multiplication principle of combinatorics:

$$\text{Total ways} = 5! \times {}^6P_3 = 120 \times 120 = 14400$$

Thus, there are 14400 distinct valid seating arrangements.

Final Answer: 14400

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept:
The standard equation of an ellipse centered at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If $a > b$, the major axis lies along the horizontal x-axis. The eccentricity e, which measures the degree of elongation of the ellipse, is related to the semi-major axis a and semi-minor axis b by the standard formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:
Step 1: Write down the given equation of the ellipse from the problem text:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Step 2: Compare this given equation with the standard form to find a^2 and b^2:

$$a^2 = 16 \implies a = 4$$

$$b^2 = 9 \implies b = 3$$

Step 3: Observe that since $a^2 > b^2$ $(16 > 9)$, this is a horizontal ellipse.
Step 4: State the formula for the eccentricity e of a horizontal ellipse:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 5: Substitute the values of $a^2 = 16$ and $b^2 = 9$ into this eccentricity formula:

$$e = \sqrt{1 - \frac{9}{16}}$$

Step 6: Simplify the fraction inside the square root:

$$e = \sqrt{\frac{16 - 9}{16}} = \sqrt{\frac{7}{16}}$$

Step 7: Take the square root of the numerator and the denominator separately:

$$e = \frac{\sqrt{7}}{4}$$

Thus, the eccentricity value of the given ellipse is $\frac{\sqrt{7}}{4}$.

Final Answer: $\boxed{\frac{\sqrt{7}}{4}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept:

To compute $A^2 - 5A$ for a square matrix A, we evaluate the matrix product $A^2 = A \cdot A$ using standard row-by-column multiplication, scale matrix A by 5, and perform entry-wise subtraction.

Solution:

Step 1: Write down the given 2×2 matrix A:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Step 2: Compute A^2 by multiplying matrix A by itself:

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(3) & 1(2) + 2(4) \\ 3(1) + 4(3) & 3(2) + 4(4) \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

Step 3: Multiply matrix A by the scalar 5:

$$5A = 5 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

Step 4: Substitute A^2 and $5A$ into the target expression $A^2 - 5A$:

$$A^2 - 5A = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

Step 5: Perform component-wise matrix subtraction:

$$A^2 - 5A = \begin{bmatrix} 7 - 5 & 10 - 10 \\ 15 - 15 & 22 - 20 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Step 6: Factor out the scalar 2 to write the result in terms of the identity matrix I:

$$A^2 - 5A = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2I$$

Final Answer: $\boxed{2I}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept:
Definite integrals involving trigonometric expressions can be simplified using King's Property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. This transformation allows us to add the original and modified integrals to eliminate complex terms.

Solution:
Step 1: Define the original definite integral as equation I:

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx \quad \text{— (1)}$$

Step 2: Apply King's Property by substituting x with $\left(0 + \frac{\pi}{2} - x\right) = \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\,dx$$

Step 3: Simplify using complementary angle identities $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx \quad \text{— (2)}$$

Step 4: Add equations (1) and (2) together:

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$$

Step 5: Simplify the integrand and evaluate the resulting definite integral:

$$2I = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

Step 6: Solve for the single integral value I:

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:

Two non-zero vector quantities $\vec{a}$ and $\vec{b}$ are mathematically defined as perpendicular (orthogonal) if and only if their scalar dot product is exactly equal to zero, i.e., $\vec{a} \cdot \vec{b} = 0$. The dot product of vectors expressed in orthogonal component form is computed by adding the products of their corresponding directional components.

Solution:

Step 1: Write down the component representations of the given vectors $\vec{a}$ and $\vec{b}$:

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$$

Step 2: State the formal condition for orthogonality of these vectors:

$$\vec{a} \cdot \vec{b} = 0$$

Step 3: Write out the algebraic calculation for the dot product using components:

$$(2)(1) + (-1)(\lambda) + (1)(-3) = 0$$

Step 4: Simplify the individual product terms in the equation:

$$2 - \lambda - 3 = 0$$

Step 5: Combine the constants to solve for the parameter λ:

$$-1 - \lambda = 0$$

$$\lambda = -1$$

Thus, the scalar parameter λ must be equal to -1 for the vectors to be perpendicular.

Final Answer: -1

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept:

According to the Binomial Theorem, the general term T_{r+1} in the algebraic expansion of a binomial expression $(1+x)^n$ is given by the formula $T_{r+1} = \binom{n}{r}x^r$. To find the specific coefficient of a particular power x^k, we set the exponent variable $r = k$ and compute the corresponding combination coefficient.

Solution:

Step 1: Identify the parameters from the given expression $(1+x)^{10}$:

$$n = 10, \quad \text{target power } k = 4$$

Step 2: Write out the general term formula for this binomial expansion:

$$T_{r+1} = \binom{10}{r}x^r$$

Step 3: Match the exponent of x to the target power of 4 by choosing:

$$r = 4$$

Step 4: Identify the combination value that represents the coefficient of x^4:

$$\text{Coefficient} = \binom{10}{4}$$

Step 5: Expand the combinatorial formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ with our chosen numbers:

$$\binom{10}{4} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$$

Step 6: Simplify the arithmetic expression safely:

$$\binom{10}{4} = \frac{5040}{24} = 210$$

Thus, the numerical coefficient of x^4 in the given binomial expansion is 210.

Final Answer: 210

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:

The mean $\bar{x}$ of n observations is given by $\bar{x} = \frac{\sum x_i}{n}$ and variance by $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$. Using these definitions, we set up a system of two algebraic equations to determine the two missing observations.

Solution:

Step 1: Let the two missing observations be a and b. The complete dataset is $\{1, 2, 6, a, b\}$ with $n = 5$.

Step 2: Use the given mean ($\bar{x} = 4$) to find the sum of the observations:

$$\frac{1+2+6+a+b}{5} = 4 \implies 9+a+b = 20 \implies a+b = 11 \quad \text{— (1)}$$

Step 3: Use the given variance ($\sigma^2 = 5.2$) to set up the sum of squares equation:

$$\sigma^2 = \frac{1^2+2^2+6^2+a^2+b^2}{5} - (4)^2 = 5.2$$

$$\frac{41+a^2+b^2}{5} - 16 = 5.2 \implies \frac{41+a^2+b^2}{5} = 21.2$$

$$41+a^2+b^2 = 106 \implies a^2+b^2 = 65 \quad \text{— (2)}$$

Step 4: Substitute $b = 11 - a$ from equation (1) into equation (2):

$$a^2 + (11-a)^2 = 65 \implies a^2 + 121 - 22a + a^2 = 65$$

$$2a^2 - 22a + 56 = 0 \implies a^2 - 11a + 28 = 0$$

Step 5: Factor the quadratic equation to find the possible values for a:

$$(a-4)(a-7) = 0 \implies a = 4 \quad \text{or} \quad a = 7$$

Step 6: Determine the corresponding values for b using equation (1):

$$\text{If } a = 4 \implies b = 7; \quad \text{If } a = 7 \implies b = 4$$

Thus, the two missing observations are 4 and 7.

Final Answer: $\boxed{4, 7}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:

For a complex number $z = x + iy$ located in the first quadrant of the Argand plane (where $x > 0$ and $y > 0$), the amplitude or principal argument θ is calculated directly using the basic inverse tangent geometric formula:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Solution:

Step 1: Identify the real component x and the imaginary component y from the complex number $z = 1 + i\sqrt{3}$:

$$x = 1, \quad y = \sqrt{3}$$

Step 2: Determine the quadrant location. Since both $x = 1 > 0$ and $y = \sqrt{3} > 0$, the point lies entirely within the first quadrant. Thus, $\theta = \alpha$.

Step 3: Apply the principal argument formula:

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$$

Step 4: Evaluate the inverse tangent expression for the standard angle value:

$$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

Thus, the amplitude (principal argument) of the complex number is exactly $\frac{\pi}{3}$ radians (or $60°$).

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept:

Integrals of the special structural form $\int e^x(f(x) + f'(x))\,dx$ can be evaluated immediately using the standard integration by parts identity theorem, which states that the result is always equal to $e^x f(x) + c$. We need to correctly identify which term represents the base function $f(x)$ and which term represents its derivative $f'(x)$.

Solution:

Step 1: Write down the given indefinite integral expression:

$$\int e^x(\tan x + \ln(\sec x))\,dx$$

Step 2: Let us test the functions to identify $f(x)$. Define the function $f(x)$ as:

$$f(x) = \ln(\sec x)$$

Step 3: Differentiate $f(x)$ with respect to x using the chain rule to verify its derivative:

$$f'(x) = \frac{1}{\sec x} \cdot \frac{d}{dx}(\sec x)$$

$$f'(x) = \frac{1}{\sec x} \cdot (\sec x \tan x) = \tan x$$

Step 4: Notice that the integrand matches the standard theorem form perfectly:

$$\text{Integrand} = e^x(f'(x) + f(x)) = e^x(\tan x + \ln(\sec x))$$

Step 5: Apply the integral theorem rule $\int e^x(f(x) + f'(x))\,dx = e^x f(x) + c$:

$$\int e^x(\tan x + \ln(\sec x))\,dx = e^x \ln(\sec x) + c$$

Thus, the evaluated indefinite integral is $e^x \ln(\sec x) + c$.

Final Answer: $e^x \ln(\sec x) + c$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept:
The general equation of a circle is given by $x^2 + y^2 + 2gx + 2fy + c = 0$. If the circle passes through the origin $(0, 0)$, then the constant term is $c = 0$. If it cuts intercepts of lengths a and b on the positive x and y axes respectively, it passes through the points $(a, 0)$ and $(0, b)$, which allows us to find the parameter values g and f.

Solution:
Step 1: Since the circle passes through the origin $(0, 0)$, substituting these coordinates into the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$ yields:

$$c = 0$$

Step 2: The circle cuts a positive intercept of length 3 on the x-axis, meaning it passes through the coordinate point:

$$(3, 0)$$

Step 3: Substitute the point $(3, 0)$ into the circle equation with $c = 0$:

$$3^2 + 0^2 + 2g(3) + 2f(0) = 0 \implies 9 + 6g = 0 \implies 2g = -3$$

Step 4: The circle cuts a positive intercept of length 4 on the y-axis, meaning it passes through the coordinate point:

$$(0, 4)$$

Step 5: Substitute the point $(0, 4)$ into the circle equation:

$$0^2 + 4^2 + 2g(0) + 2f(4) = 0 \implies 16 + 8f = 0 \implies 2f = -4$$

Step 6: Substitute the found coefficients $2g = -3$, $2f = -4$, and $c = 0$ back into the general equation:

$$x^2 + y^2 - 3x - 4y = 0$$

Thus, the required equation of the circle is $x^2 + y^2 - 3x - 4y = 0$.

Final Answer: $x^2 + y^2 - 3x - 4y = 0$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:
To evaluate the sum of multiple inverse tangent terms, we can use the fundamental addition formula for standard principal values: $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when the product $xy > 1$.

Solution:
Step 1: Identify the three terms in the given expression:

$$S = \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$$

Step 2: Evaluate the first term, which corresponds to a standard angle value:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Step 3: Isolate the remaining two terms to apply the inverse trigonometric addition identity:

$$\text{Part 2} = \tan^{-1}(2) + \tan^{-1}(3)$$

Step 4: Check the product of the arguments: $x = 2$ and $y = 3 \implies xy = 2 \cdot 3 = 6$. Since $6 > 1$, apply the formula containing the π adjustment factor:

$$\tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}\left(\frac{2+3}{1-2\cdot 3}\right)$$

Step 5: Simplify the fractional argument inside the function:

$$\text{Part 2} = \pi + \tan^{-1}\left(\frac{5}{1-6}\right) = \pi + \tan^{-1}\left(\frac{5}{-5}\right) = \pi + \tan^{-1}(-1)$$

Step 6: Use the odd function property $\tan^{-1}(-x) = -\tan^{-1}(x)$:

$$\text{Part 2} = \pi - \tan^{-1}(1) = \pi - \frac{\pi}{4}$$

Step 7: Combine all evaluated parts together to find the full sum S:

$$S = \frac{\pi}{4} + \left(\pi - \frac{\pi}{4}\right) = \pi$$

The final simplified value of the total expression is exactly π.

Final Answer: π

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept:
The derivative of a composite logarithmic function $y = \ln(f(x))$ is found by applying the standard chain rule of calculus, which states that $\frac{dy}{dx} = \frac{1}{f(x)} \cdot f'(x)$. Here, the inner function is a sum of standard basic trigonometric functions.

Solution:
Step 1: State the given logarithmic function:

$$y = \ln(\sec x + \tan x)$$

Step 2: Apply the chain rule of differentiation. Differentiate the outer logarithm function first:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$

Step 3: Compute the derivatives of the standard inner trigonometric terms:

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

Step 4: Substitute these inner derivatives back into the chain rule expression:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot \left(\sec x \tan x + \sec^2 x\right)$$

Step 5: Factor out the common term $\sec x$ from the numerator terms:

$$\frac{dy}{dx} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

Step 6: Cancel the identical factor $(\sec x + \tan x)$ from both the numerator and denominator:

$$\frac{dy}{dx} = \sec x$$

The derivative of the function simplifies cleanly to $\sec x$.

Final Answer: $\sec x$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:

For any square matrix A of order n, multiplying the entire matrix by a scalar value k scales every single row by that factor. Therefore, when evaluating the determinant, the scalar pulls out raised to the power of the matrix order, following the rule: $|kA| = k^n|A|$.

Solution:

Step 1: Identify the given properties of the matrix from the question statement:

$$\text{Order of matrix } A \implies n = 3$$

$$\text{Determinant value } |A| = 4$$

Step 2: State the standard determinant scaling theorem formula:

$$|kA| = k^n|A|$$

Step 3: Identify the required scalar multiplier value from the problem:

$$k = 2$$

Step 4: Substitute the specific values $k = 2$, $n = 3$, and $|A| = 4$ into the theorem formula:

$$|2A| = 2^3 \cdot |A|$$

Step 5: Calculate the value of the exponential term:

$$2^3 = 2 \times 2 \times 2 = 8$$

Step 6: Perform the final scalar multiplication step:

$$|2A| = 8 \cdot 4 = 32$$

The determinant value of the scaled matrix $2A$ is equal to 32.

Final Answer: 32

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept:
To evaluate an infinite series whose terms are rational fractions, we can expand the general term using the method of partial fractions into a telescoping series form, where consecutive internal terms cancel each other out out to infinity.

Solution:
Step 1: Write down the general term of the summation:

$$T_r = \frac{1}{r(r+1)}$$

Step 2: Resolve the expression into partial fractions by rewriting the numerator:

$$T_r = \frac{(r+1)-r}{r(r+1)} = \frac{r+1}{r(r+1)} - \frac{r}{r(r+1)}$$

$$T_r = \frac{1}{r} - \frac{1}{r+1}$$

Step 3: Write out the partial sum S_n by expanding the terms from $r = 1$ to n:

$$S_n = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

Step 4: Observe the telescoping cancellation of all intermediate adjacent fractional terms:

$$S_n = 1 - \frac{1}{n+1}$$

Step 5: Apply the limit as n approaches infinity to find the sum of the infinite series:

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)$$

Step 6: Since $\lim_{n\to\infty} \frac{1}{n+1} = 0$, evaluate the final result:

$$S = 1 - 0 = 1$$

The total sum of the infinite series is exactly 1.

Final Answer: 1

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:

The angle θ between two straight lines in three-dimensional space is determined entirely by the direction ratios of the lines. If the direction vectors of the lines are $\vec{b}_1 = (a_1, b_1, c_1)$ and $\vec{b}_2 = (a_2, b_2, c_2)$, the angle satisfies the standard vector formula:

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Solution:

Step 1: Extract the direction ratios of the first line from the denominators of its symmetric equation:

$$a_1 = 2, \quad b_1 = 1, \quad c_1 = -2 \implies \vec{b}_1 = 2\hat{i} + \hat{j} - 2\hat{k}$$

Step 2: Extract the direction ratios of the second line in a similar manner:

$$a_2 = 1, \quad b_2 = 2, \quad c_2 = 2 \implies \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

Step 3: Compute the scalar dot product of the two direction vectors:

$$\vec{b}_1 \cdot \vec{b}_2 = (2)(1) + (1)(2) + (-2)(2)$$

$$\vec{b}_1 \cdot \vec{b}_2 = 2 + 2 - 4 = 0$$

Step 4: Since the numerator of the cosine formula represents the dot product and evaluates to 0, substitute it into the full angle equation:

$$\cos\theta = 0$$

Step 5: Determine the principal angle value whose cosine is zero:

$$\theta = \frac{\pi}{2}$$

The two lines intersect each other perpendicularly at an angle of $\frac{\pi}{2}$ radians.

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept:
In statistics, the variance of a given population or sample dataset measures the dispersion of the data points and is mathematically defined as the square of the standard deviation (σ). Therefore, the relationship can be written simply as: Variance $= \sigma^2$.

Solution:
Step 1: Identify the standard deviation value provided in the problem statement:

$$\text{Standard Deviation } (\sigma) = 4$$

Step 2: Recall the foundational statistical definition that relates variance directly to standard deviation:

$$\text{Variance} = (\text{Standard Deviation})^2$$

Step 3: Substitute the numerical value into this definition formula:

$$\text{Variance} = 4^2$$

Step 4: Perform the simple squaring computation:

$$\text{Variance} = 4 \times 4 = 16$$

Thus, the variance of the data set is exactly 16.

Final Answer: 16

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:

The area bounded by a curve and a line can be computed using definite integration. For a vertical parabola $y = x^2$ bounded by a horizontal upper line $y = c$, the area is symmetric about the y-axis. It can be found by evaluating the integral $\int_{-a}^{a} (c - x^2)\, dx$, or equivalently $2 \int_0^a (c - x^2)\, dx$, where a is the positive x-intercept boundary point.

Solution:

Step 1: Find the intersection points of the curve $y = x^2$ and the line $y = 4$:

$$x^2 = 4 \implies x = -2 \quad \text{and} \quad x = 2$$

Step 2: Set up the area definite integral using the upper function minus the lower function:

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\, dx$$

Step 3: Use the even function symmetry property to rewrite the bounds from 0 to 2:

$$\text{Area} = 2 \int_0^2 (4 - x^2)\, dx$$

Step 4: Apply standard power integration rules to evaluate the terms:

$$\text{Area} = 2 \left[4x - \frac{x^3}{3} \right]_0^2$$

Step 5: Substitute the upper limit value 2 into the evaluated expression:

$$\text{Area} = 2 \left(4(2) - \frac{2^3}{3} \right) = 2 \left(8 - \frac{8}{3} \right)$$

Step 6: Simplify the fractional arithmetic expressions:

$$\text{Area} = 2 \left(\frac{24 - 8}{3} \right) = 2 \left(\frac{16}{3} \right) = \frac{32}{3}$$

The total enclosed bounded area is $\frac{32}{3}$ square units.

Final Answer: $\frac{32}{3}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept:

For any valid input value $k \in [-1, 1]$, the inverse trigonometric co-function identity states that the sum of the sine and cosine inverse functions is a constant value: $\sin^{-1} k + \cos^{-1} k = \frac{\pi}{2}$. By writing out this identity for two variables x and y, we can solve for the required sum.

Solution:

Step 1: Write down the two individual standard co-function identity equations for variables x and y:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \quad \text{— (Equation 1)}$$

$$\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2} \quad \text{— (Equation 2)}$$

Step 2: Add Equation 1 and Equation 2 together:

$$\left(\sin^{-1} x + \cos^{-1} x\right) + \left(\sin^{-1} y + \cos^{-1} y\right) = \frac{\pi}{2} + \frac{\pi}{2}$$

Step 3: Rearrange and regroup the terms to isolate the known sum and the unknown sum:

$$\left(\sin^{-1} x + \sin^{-1} y\right) + \left(\cos^{-1} x + \cos^{-1} y\right) = \pi$$

Step 4: Substitute the given condition $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$ into the grouped equation:

$$\frac{\pi}{2} + \left(\cos^{-1} x + \cos^{-1} y\right) = \pi$$

Step 5: Isolate the target expression by subtracting $\frac{\pi}{2}$ from both sides:

$$\cos^{-1} x + \cos^{-1} y = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

The value of the expression $\cos^{-1} x + \cos^{-1} y$ is exactly equal to $\frac{\pi}{2}$.

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:

The total number of distinct terms in the expansion of a multinomial expression containing m terms raised to a positive integer power n, of the form $(x_1 + x_2 + \cdots + x_m)^n$, is given by the standard combination formula:

$$\text{Total Terms} = \binom{n+m-1}{m-1}$$

Solution:

Step 1: Identify the parameters from the given expression $(x + y + z)^{10}$:

$$\text{Power exponent } n = 10$$

$$\text{Number of base variables } m = 3 \quad (\text{since there are } x, y, z)$$

Step 2: Substitute these values into the multinomial terms formula:

$$\text{Total Terms} = \binom{10+3-1}{3-1}$$

Step 3: Simplify the upper and lower terms of the combination:

$$\text{Total Terms} = \binom{12}{2}$$

Step 4: Compute the value of the combination $\binom{12}{2}$ using factorials:

$$\binom{12}{2} = \frac{12 \times 11}{2 \times 1}$$

Step 5: Perform the simple division and multiplication steps:

$$\text{Total Terms} = 6 \times 11 = 66$$

Thus, the number of terms in the algebraic expansion is exactly 66.

Final Answer: 66

Answer: (C)

Go Back to Question 26

Q27.

Solution

Concept:
A fundamental geometric property of any circle is that any line acting as a normal to the circle must pass directly through its geometric center. By finding the coordinates of the center from the circle equation and substituting them into the line equation, we can solve for the unknown parameter k.

Solution:
Step 1: Write down the given general equation of the circle:

$$x^2 + y^2 - 4x - 6y + 11 = 0$$

Step 2: Find the coordinates of the circle's center (h, k_{center}) by comparing it to the standard general form $x^2 + y^2 + 2gx + 2fy + c = 0$:

$$2g = -4 \implies g = -2 \implies h = -g = 2$$

$$2f = -6 \implies f = -3 \implies k_{center} = -f = 3$$

Thus, the center of the circle is located at the point $(2, 3)$.
Step 3: State the condition that the normal line $2x - y + k = 0$ must pass through this center point $(2, 3)$.
Step 4: Substitute the coordinates $x = 2$ and $y = 3$ directly into the line equation:

$$2(2) - 3 + k = 0$$

Step 5: Simplify the arithmetic expression to solve for k:

$$4 - 3 + k = 0$$

$$1 + k = 0 \implies k = -1$$

The value of the unknown parameter k is equal to -1.

Final Answer: -1

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept:

The rate of change of a geometric quantity with respect to a specific variable is found by taking its first derivative with respect to that variable. For the area A of a circle as a function of its radius r, the rate of change with respect to r is given by $\frac{dA}{dr}$.

Solution:

Step 1: State the standard formula for the area A of a circle of radius r:

$$A = \pi r^2$$

Step 2: Differentiate the area function A with respect to the radius variable r:

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2)$$

Step 3: Apply the standard power rule of differentiation ($\frac{d}{dr}[r^2] = 2r$):

$$\frac{dA}{dr} = 2\pi r$$

Step 4: Identify the specific value of the radius given in the problem statement:

$$r = 5 \text{ cm}$$

Step 5: Substitute $r = 5$ into the derived rate of change formula:

$$\frac{dA}{dr} = 2\pi(5) = 10\pi$$

The rate of change of the area with respect to its radius at that instant is 10π cm^2/cm.

Final Answer: $\boxed{10\pi}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept:

According to the classical definition of probability, the probability $P(E)$ of an event occurring is given by the ratio of the number of favorable outcomes $n(E)$ to the total number of possible outcomes in the sample space $n(S)$, expressed as $P(E) = \frac{n(E)}{n(S)}$.

Solution:

Step 1: Count the individual components inside the bag from the problem description:

$$\text{Number of red balls} = 4$$

$$\text{Number of black balls} = 6$$

Step 2: Calculate the total number of possible outcomes $n(S)$ by summing all the balls together:

$$n(S) = 4 + 6 = 10$$

Step 3: Identify the number of favorable outcomes $n(E)$ for the target event (drawing a red ball):

$$n(E) = 4$$

Step 4: Apply the classical probability definition formula:

$$P(\text{Red}) = \frac{n(E)}{n(S)} = \frac{4}{10}$$

Step 5: Simplify the fraction by dividing both the numerator and denominator by their greatest common divisor, which is 2:

$$P(\text{Red}) = \frac{2}{5}$$

The probability of choosing a red ball from the bag is exactly $\frac{2}{5}$ (or 0.4).

Final Answer: $\boxed{\frac{2}{5}}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept:
For the complex cube roots of unity, the properties $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ apply. We rewrite internal terms using $1 + \omega^2 = -\omega$ and $1 + \omega = -\omega^2$ to simplify powers before evaluation.

Solution:
Step 1: Write down the given expression to evaluate:

$$E = (1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$$

Step 2: Rearrange and substitute $1 + \omega^2 = -\omega$ into the first block, and $1 + \omega = -\omega^2$ into the second:

$$E = (1 + \omega^2 - \omega)^5 + (1 + \omega - \omega^2)^5$$

$$E = (-\omega - \omega)^5 + (-\omega^2 - \omega^2)^5 = (-2\omega)^5 + (-2\omega^2)^5$$

Step 3: Expand the powers using scalar exponential rules ($(-2)^5 = -32$):

$$E = -32\omega^5 - 32\omega^{10}$$

Step 4: Reduce higher powers of ω using the identity $\omega^3 = 1$:

$$\omega^5 = \omega^3 \cdot \omega^2 = \omega^2$$

$$\omega^{10} = (\omega^3)^3 \cdot \omega = \omega$$

Step 5: Substitute these simplified powers back into the expression:

$$E = -32\omega^2 - 32\omega = -32(\omega^2 + \omega)$$

Step 6: Apply the property $\omega^2 + \omega = -1$ derived from the core identity to find the final value:

$$E = -32(-1) = 32$$

Final Answer: 32

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept:
An algebraic expression of the form $f(x) = a\sin x + b\cos x$ can be simplified using trigonometric identities into a single sine or cosine wave. The mathematical absolute maximum and minimum values of such functions are bounded by the formula:

$$-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$$

Solution:
Step 1: Identify the coefficients from the given function $f(x) = \sin x + \cos x$:

$$a = 1, \quad b = 1$$

Step 2: State the standard bounding range formula for the maximum value:

$$\text{Maximum Value} = \sqrt{a^2+b^2}$$

Step 3: Substitute the specific coefficients $a = 1$ and $b = 1$ into the formula:

$$\text{Maximum Value} = \sqrt{1^2+1^2}$$

Step 4: Simplify the arithmetic expression inside the square root:

$$\text{Maximum Value} = \sqrt{1+1} = \sqrt{2}$$

Thus, the absolute maximum value that the function can attain for any real input x is equal to $\sqrt{2}$.

Final Answer: $\sqrt{2}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept:

The scalar projection of a vector quantity $\vec{a}$ onto another vector quantity $\vec{b}$ represents the length of the orthogonal projection segment of $\vec{a}$ along the directional line of $\vec{b}$. It is calculated using the standard vector formula:

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Solution:

Step 1: Write down the component forms of the vectors $\vec{a}$ and $\vec{b}$:

$$\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$$

$$\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$$

Step 2: Compute the scalar dot product $\vec{a} \cdot \vec{b}$ in the numerator:

$$\vec{a} \cdot \vec{b} = (1)(7) + (3)(-1) + (7)(8)$$

$$\vec{a} \cdot \vec{b} = 7 - 3 + 56 = 60$$

Step 3: Compute the magnitude of the target base vector $\vec{b}$ in the denominator:

$$|\vec{b}| = \sqrt{7^2 + (-1)^2 + 8^2}$$

$$|\vec{b}| = \sqrt{49 + 1 + 64} = \sqrt{114}$$

Step 4: Substitute the calculated dot product and magnitude values back into the projection formula:

$$\text{Projection} = \frac{60}{\sqrt{114}}$$

Thus, the scalar projection value of vector $\vec{a}$ on vector $\vec{b}$ is exactly $\frac{60}{\sqrt{114}}$.

Final Answer: $\boxed{\frac{60}{\sqrt{114}}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept:

A parabola of the form $x^2 = -4ay$ is symmetric about the vertical y-axis and opens directly downwards. The vertex is located at the origin $(0, 0)$, and the focus point lies on the axis of symmetry inside the curve at a distance a below the vertex, giving it the coordinates $(0, -a)$.

Solution:

Step 1: Write down the given equation of the parabola from the problem description:

$$x^2 = -16y$$

Step 2: Compare this equation with the downward standard parabola form to find a:

$$-4a = -16 \implies 4a = 16 \implies a = 4$$

Step 3: State the standard formula for the focus coordinates of a vertical downward-opening parabola:

$$\text{Focus} = (0, -a)$$

Step 4: Substitute the computed value of the parameter $a = 4$ into the coordinates formula:

$$\text{Focus} = (0, -4)$$

Thus, the focal point coordinates of the given downward parabola are $(0, -4)$.

Final Answer: $(0, -4)$

Answer: (C)

Go Back to Question 33

Q34.

Solution

Concept:

To determine the monotonicity of a continuous differentiable function $f(x)$ on the set of real numbers, we evaluate its first derivative $f'(x)$. If $f'(x) > 0$ for all real numbers x (except at isolated points where it can equal zero), the function is classified as strictly increasing on $\mathbb{R}$.

Solution:

Step 1: State the given cubic polynomial function:

$$f(x) = x^3 - 3x^2 + 3x - 100$$

Step 2: Compute the first derivative of the function with respect to x:

$$f'(x) = \frac{d}{dx}(x^3 - 3x^2 + 3x - 100)$$

$$f'(x) = 3x^2 - 6x + 3$$

Step 3: Factor out the common scalar constant 3 from the derivative terms:

$$f'(x) = 3(x^2 - 2x + 1)$$

Step 4: Identify the expression inside the parentheses as a perfect square trinomial:

$$x^2 - 2x + 1 = (x - 1)^2$$

$$f'(x) = 3(x - 1)^2$$

Step 5: Analyze the sign behavior of the factored derivative expression. Since $(x - 1)^2$ is a squared term, it is strictly positive for all $x \neq 1$, and exactly zero at $x = 1$.

$$f'(x) \geq 0 \quad \forall x \in \mathbb{R}$$

Step 6: Since the derivative is never negative and only vanishes at a single isolated point ($x = 1$), the function is strictly increasing throughout its entire domain.

Final Answer: Strictly increasing on $\mathbb{R}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept:

A first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ is solved by calculating an integrating factor I.F. $= e^{\int P(x)\,dx}$. The general solution is then found using the standard formula $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.})\,dx + C$.

Solution:

Step 1: Identify the coefficient functions $P(x)$ and $Q(x)$ from the given differential equation $\frac{dy}{dx} + y = e^{-x}$:

$$P(x) = 1, \quad Q(x) = e^{-x}$$

Step 2: Compute the integrating factor (I.F.):

$$\text{I.F.} = e^{\int 1\,dx} = e^x$$

Step 3: Set up the standard linear equation solution formula:

$$y \cdot e^x = \int e^{-x} \cdot e^x\,dx + C$$

Step 4: Simplify the integrand on the right side using exponent laws ($e^{-x} \cdot e^x = e^0 = 1$):

$$y \cdot e^x = \int 1\,dx + C \implies y \cdot e^x = x + C$$

Step 5: Use the given initial boundary condition $y(0) = 0$ to find the value of C:

$$0 \cdot e^0 = 0 + C \implies C = 0$$

Step 6: Substitute $C = 0$ back into the solution expression:

$$y \cdot e^x = x \implies y = xe^{-x}$$

The particular solution of the differential equation is $y = xe^{-x}$.

Final Answer: $\boxed{y = xe^{-x}}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept:
For a real-valued function involving a square root in the denominator, of the form $f(x) = \frac{1}{\sqrt{g(x)}}$, the domain consists of all values of x for which the expression inside the radical is strictly greater than zero, i.e., $g(x) > 0$.

Solution:
Step 1: Set up the domain inequality condition for the given function $f(x) = \frac{1}{\sqrt{x^2-9}}$:

$$x^2 - 9 > 0$$

Step 2: Factor the algebraic quadratic expression using the difference of squares rule:

$$(x-3)(x+3) > 0$$

Step 3: Determine the critical boundary points where the expression equals zero:

$$x = 3 \quad \text{and} \quad x = -3$$

Step 4: Apply the sign interval method (wavy curve method) to test the intervals:

$$\text{For } x > 3 \implies (+)(+) > 0 \quad \text{(Valid)}$$

$$\text{For } -3 < x < 3 \implies (-)(+) < 0 \quad \text{(Invalid)}$$

$$\text{For } x < -3 \implies (-)(-) > 0 \quad \text{(Valid)}$$

Step 5: Combine the valid intervals into interval notation format:

$$x \in (-\infty, -3) \cup (3, \infty)$$

Thus, the domain of the given function is $(-\infty, -3) \cup (3, \infty)$.

Final Answer: $\boxed{(-\infty, -3) \cup (3, \infty)}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept:
The total number of distinct linear permutations of a word containing a total of n letters, where some letters are repeated, is given by the multinomial permutation formula:

$$\text{Total Arrangements} = \frac{n!}{p_1! \cdot p_2! \cdot \dots \cdot p_k!}$$

where p_i represents the frequency of each repeating distinct letter.

Solution:
Step 1: Count the total number of letters (n) in the given word 'BANANA':

$$n = 6$$

Step 2: Analyze the repetition frequency of each unique letter within the word:

$$\text{Letter 'B'} \implies 1 \text{ time}$$

$$\text{Letter 'A'} \implies 3 \text{ times}$$

$$\text{Letter 'N'} \implies 2 \text{ times}$$

Step 3: Substitute these values into the permutation formula for repeated items:

$$\text{Total Arrangements} = \frac{6!}{1! \cdot 3! \cdot 2!}$$

Step 4: Expand the factorials to perform the simplification steps:

$$6! = 720, \quad 3! = 6, \quad 2! = 2$$

$$\text{Total Arrangements} = \frac{720}{1 \times 6 \times 2} = \frac{720}{12}$$

Step 5: Perform the final division operation:

$$\text{Total Arrangements} = 60$$

There are exactly 60 distinct ways to arrange the letters of the word 'BANANA'.

Final Answer: 60

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept:

The determinant of a matrix can be simplified using standard elementary row or column operations. If an operation causes two columns or two rows to become identical or proportional, the value of the determinant is immediately zero.

Solution:

Step 1: Write down the given 3×3 determinant equation:

$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

Step 2: Apply the elementary column operation $C_3 \to C_3 + C_2$ to add the elements of the second column to the third column:

$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

Step 3: Factor out the common algebraic scalar expression $(a + b + c)$ from the third column:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

Step 4: Observe the columns of the new determinant. Column 1 (C_1) and Column 3 (C_3) are completely identical:

$$C_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad C_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Step 5: Apply the property that a determinant with two identical columns is equal to zero:

$$\Delta = (a+b+c) \cdot 0 = 0$$

The value of the given determinant expression is exactly 0.

Final Answer: 0

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept:
To evaluate a symmetric sum of scalar dot products $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ given a zero vector sum equation, we can take the vector dot product of the sum equation with itself, or equivalently square the magnitude equation $|\vec{a} + \vec{b} + \vec{c}|^2 = 0$.

Solution:
Step 1: State the definition parameters for unit vectors given in the question:

$$|\vec{a}| = 1, \quad |\vec{b}| = 1, \quad |\vec{c}| = 1$$

Step 2: Write down the primary given vector equation:

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

Step 3: Take the square of the magnitude on both sides of the equation:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

Step 4: Expand the expression using vector dot product identities:

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

Step 5: Substitute the known unit magnitudes into the expanded equation:

$$1^2 + 1^2 + 1^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

Step 6: Isolate the target scalar product sum expression:

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$$

The value of the required sum of dot products is equal to $-\frac{3}{2}$.

Final Answer: $-\frac{3}{2}$

Answer: (C)

Go Back to Question 39

Q40.

Solution

Concept:

To evaluate a product of cosine terms with doubling angles, $\cos\theta\cos 2\theta\cos 4\theta\ldots\cos 2^{n-1}\theta$, we can repeatedly apply the sine double-angle product identity $2\sin\theta\cos\theta = \sin 2\theta$ by multiplying and dividing the entire expression by $2\sin\theta$.

Solution:

Step 1: Write down the given product expression:

$$P = \cos 20^\circ \cos 40^\circ \cos 80^\circ$$

Step 2: Multiply and divide the expression by $2\sin 20^\circ$:

$$P = \frac{2\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 3: Simplify the first two terms in the numerator using $2\sin 20^\circ \cos 20^\circ = \sin 40^\circ$:

$$P = \frac{\sin 40^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 4: Multiply the numerator and denominator by 2 to form another double angle identity:

$$P = \frac{2\sin 40^\circ \cos 40^\circ \cos 80^\circ}{4\sin 20^\circ}$$

$$P = \frac{\sin 80^\circ \cos 80^\circ}{4\sin 20^\circ}$$

Step 5: Multiply by 2 once more to simplify the final pair of terms:

$$P = \frac{2\sin 80^\circ \cos 80^\circ}{8\sin 20^\circ} = \frac{\sin 160^\circ}{8\sin 20^\circ}$$

Step 6: Apply the supplementary angle identity $\sin(180^\circ - \theta) = \sin\theta$ to simplify the numerator:

$$\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ$$

Step 7: Substitute this back into the expression fraction:

$$P = \frac{\sin 20^\circ}{8\sin 20^\circ} = \frac{1}{8}$$

The final numerical value of the trigonometric product expression is exactly $\frac{1}{8}$.

Final Answer: $\boxed{\frac{1}{8}}$

Answer: (C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	C	4	A	5	A
6	A	7	A	8	A	9	A	10	A
11	B	12	C	13	A	14	A	15	B
16	B	17	A	18	B	19	A	20	C
21	B	22	A	23	B	24	A	25	A
26	C	27	B	28	A	29	A	30	A
31	C	32	A	33	C	34	A	35	A
36	A	37	B	38	A	39	C	40	C