MAT Mathematical Skills Sample Paper-4

Duration: 24 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions from the **Mathematical Skills** section of MAT.
- Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- There is **no** negative marking for unattempted questions.
- Suggested time for this section in the full MAT is **24 minutes**.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. A dishonest merchant sells goods at a 12.5% loss on the cost price but uses a weight of 28 grams instead of 36 grams. What is his overall profit or loss percentage?

(A) 12.5% loss

(B) 12.5% profit

(C) 14.28% profit

(D) No profit, no loss

Q2. If $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, find the total number of digits in the expansion of 6^{20}.

(A) 15

(B) 16

(C) 17

(D) 20

Q3. A cylindrical tank of radius 7 m is being filled with water through a pipe of diameter 14 cm at a rate of 5 m/s. How much will the water level rise in the tank after exactly one hour?

(A) 1.8 meters

(B) 3.6 meters

(C) 0.9 meters

(D) 7.2 meters

Q4. In how many distinct ways can the letters of the word 'CRICKET' be arranged such that the vowels always sit together?

(A) 240

(B) 120

(C) 720

(D) 360

Q5. What is the remainder when 2^{100} is divided by 101?

(A) 1

(B) 2

(C) 100

(D) 50

Q6. A sum of money compounded annually doubles itself in 6 years. In how many years will it become 8 times its initial value at the same interest rate?

(A) 18 years

(B) 24 years

(C) 12 years

(D) 16 years

Q7. Find the range of real values of x for which the inequality $x^2 - 5x + 6 < 0$ holds true.

(A) $x < 2$ or $x > 3$

(B) $2 < x < 3$

(C) $-3 < x < -2$

(D) $2 \leq x \leq 3$

Q8. The average mark of 40 students in a class was calculated as 68. Later, it was discovered that a score of 82 was incorrectly entered as 42. What is the corrected average score of the class?

(A) 67.0

(B) 69.0

(C) 68.5

(D) 70.0

Q9. A chord of length 16 cm is drawn inside a circle at a perpendicular distance of 6 cm from the central point. Compute the total diameter metric of this circle.

(A) 10 cm

(B) 20 cm

(C) 12 cm

(D) 24 cm

Q10. In a group of 60 professionals, 35 read business magazines, 25 read tech journals, and 15 read both types of publications. How many individuals in this group do not read either?

(A) 15

(B) 20

(C) 10

(D) 0

Q11. What is the total number of trailing zeros at the end of the evaluated product value of 80! (80 factorial)?

(A) 16

(B) 19

(C) 20

(D) 18

Q12. Two distinct trains running in opposite directions pass a post in 12 seconds and 18 seconds respectively. If they cross each other completely in 15 seconds, calculate the ratio of their speeds.

(A) 1 : 1

(B) 2 : 3

(C) 3 : 2

(D) 4 : 5

Q13. Find the sum of all two-digit numbers that yield a remainder of 2 when divided by 4.

(A) 1188

(B) 1210

(C) 1242

(D) 1300

Q14. A container contains a mixture of milk and water in the ratio 7 : 5. When 9 liters of this mixture is drawn off and replaced with pure water, the ratio shifts to 7 : 9. How many liters of milk were initially present in the container?

(A) 21 liters

(B) 15 liters

(C) 24 liters

(D) 18 liters

Q15. The total surface area of a solid hemisphere is equal to 462 cm^2. Determine its radius parameter. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(A) 10.5 cm

(B) 7 cm

(C) 14 cm

(D) 3.5 cm

Q16. In a simultaneous throw of two fair dice, what is the probability that the sum of the numbers showing on top is a prime number?

(A) 5/12

(B) 7/18

(C) 1/2

(D) 11/36

Q17. If the price of petroleum drops by 20%, by what percentage must a driver increase their consumption to keep their overall fuel expenditure unchanged?

(A) 20%

(B) 25%

(C) 16.67%

(D) 30%

Q18. Find the value of k for which the system of equations $3x + 4y = 12$ and $6x + ky = 24$ has an infinite number of solutions.

(A) $k = 4$

(B) $k = 8$

(C) $k = -8$

(D) $k = 12$

Q19. A path of uniform width 2 meters runs around the outside of a rectangular plot measuring 15 meters by 10 meters. Calculate the total surface area of this path.

(A) 116 sq m

(B) 100 sq m

(C) 58 sq m

(D) 150 sq m

Q20. The HCF and LCM of two positive numbers are 12 and 72 respectively. If one of the numbers is 24, find the other number.

(A) 36

(B) 48

(C) 18

(D) 12

Q21. A work crew of 15 laborers can complete a specific construction project in 8 days. How many extra workers are needed to complete the exact same project in just 5 days?

(A) 24

(B) 9

(C) 10

(D) 12

Q22. Find the sum of the infinite geometric series given by $12 + 4 + \frac{4}{3} + \frac{4}{9} + \ldots$

(A) 16

(B) 18

(C) 24

(D) 20

Q23. The total area of an equilateral triangle is measured to be $16\sqrt{3}$ cm^2. Find the total perimeter length of this triangle.

(A) 24 cm

(B) 12 cm

(C) 16 cm

(D) 32 cm

Q24. A bag contains 5 red balls and 4 green balls. If two balls are drawn at random simultaneously, what is the probability that both balls are of the same color?

(A) 4/9

(B) 5/9

(C) 1/2

(D) 2/9

Q25. An investor splits an amount between two schemes. Scheme A offers 8% simple interest and Scheme B offers 10% simple interest. The total annual interest earned across both schemes is $190. If the investment amounts were swapped, the total interest earned would be $170. How much money was originally invested in Scheme A?

(A) $1000

(B) $500

(C) $1500

(D) $1200

Q26. A boat can travel at a speed of 11 km/h in still water. If the speed of the river current is 3 km/h, how long will it take the boat to travel a distance of 56 km downstream?

(A) 7 hours

(B) 4 hours

(C) 8 hours

(D) 5 hours

Q27. A merchant buys an item for $400. She wants to mark up the price so that even after offering a 10% discount on the marked price, she still makes a profit of 20%. What should the marked price of the item be?

(A) $520

(B) $533.33

(C) \$500

(D) \$550

Q28. The lengths of the two parallel sides of a trapezoid are 12 cm and 18 cm. If the total area of the trapezoid is 150 cm^2, find the perpendicular distance between the parallel sides.

(A) 5 cm

(B) 10 cm

(C) 15 cm

(D) 20 cm

Q29. A data analysis report shows that a company's revenue increased by 10% in the first quarter, increased by 20% in the second quarter, and then dropped by 10% in the third quarter. What is the net percentage change in revenue across all three quarters combined?

(A) 20% increase

(B) 18.8% increase

(C) 15.5% increase

(D) 22.2% increase

Q30. A production manager tracks a factory line where the ratio of functional items to defective items is 19 : 1. If a random quality check sample contains 140 items, how many functional items are expected to be in that sample?

(A) 130 items

(B) 133 items

(C) 135 items

(D) 126 items

Detailed Solutions

Q1.

Solution

Concept:
In profit and loss problems involving faulty weights and nominal price variations, the net effect can be efficiently determined by analyzing the multiplier ratio of individual events. A nominal loss scales down the revenue factor, while using a lower weight for sales scales up the effective revenue factor because the merchant parts with less inventory than stated. The overall compounding factor is the product of these individual transaction ratios.

Solution:

(a) Consider the nominal pricing structure first. The merchant claims to sell the goods at a 12.5% loss on the cost price. Since 12.5% is equivalent to the fraction $\frac{1}{8}$, the effective selling price factor relative to the cost price becomes $1 - \frac{1}{8} = \frac{7}{8}$.

(b) Next, analyze the weight manipulation. The merchant provides a false weight of 28 grams while charging the customer for a full weight of 36 grams. This means the merchant recovers money equivalent to 36 grams from an actual inventory investment of only 28 grams.

(c) The weight multiplier factor helping the merchant earn a premium is calculated as $\frac{\text{Claimed Weight}}{\text{Actual Weight}} = \frac{36}{28}$. Simplifying this fraction by dividing both the numerator and the denominator by their greatest common divisor, 4, yields $\frac{9}{7}$.

(d) The overall compounding transactional multiplier is the product of the pricing factor and the weight manipulation factor, which is computed as Net Factor $= \frac{7}{8} \times \frac{9}{7}$. Canceling the common term 7 from the numerator and denominator simplifies the net factor to $\frac{9}{8}$.

(e) Since the net factor $\frac{9}{8}$ is strictly greater than 1, the merchant experiences an overall profit. The fractional gain is calculated as $\frac{9}{8} - 1 = \frac{1}{8}$. Converting this fractional gain into a percentage yields $\frac{1}{8} \times 100\% = 12.5\%$. Thus, despite the initial appearance of a loss, the weight cheating mechanism ensures a net profit.

Final Answer: The merchant achieves an overall profit of 12.5%.

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept:
To find the total number of digits in an exponential expression of the form A^B when expanded in base 10, common logarithms are used. The common logarithm of a number can be broken down into an integer part, known as the characteristic, and a fractional part, known as the mantissa. The total count of digits in the regular integer expansion is strictly equal to the value of the characteristic incremented by one.

Solution:

(a) Let the target value be represented by the variable Y, such that $Y = 6^{20}$. To determine the number of digits, take the common logarithm to the base 10 on both sides of the equation, yielding $\log_{10} Y = \log_{10}(6^{20})$.

(b) Apply the logarithmic power rule, which states that $\log(A^B) = B \cdot \log A$. This simplifies the expression to $\log_{10} Y = 20 \cdot \log_{10} 6$.

(c) Express the base number 6 as the product of its prime factors, 2×3. Using the logarithmic product identity $\log(M \cdot N) = \log M + \log N$, rewrite the equation as $\log_{10} Y = 20 \cdot (\log_{10} 2 + \log_{10} 3)$.

(d) Substitute the given logarithmic values into the simplified formula: $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$. This yields the expression $\log_{10} Y = 20 \cdot (0.3010 + 0.4771)$.

(e) Add the two values inside the parentheses together to get 0.7781. Next, carry out the scalar multiplication: $\log_{10} Y = 20 \cdot 0.7781 = 15.562$.

(f) Identify the characteristic of this resulting logarithm, which is the greatest integer less than or equal to the total value. The characteristic is 15. To find the total digit length, add 1 to this value, which gives $15 + 1 = 16$.

Final Answer: The total number of digits in the expansion is 16.

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept:
This problem is governed by the volumetric conservation principle of fluid dynamics. The total volume of water discharged by the inlet pipe over a specific time period must be exactly equal to the volume of water accumulated inside the cylindrical storage tank. Careful conversion of all metrics into standard uniform units is essential to avoid dimensional misalignment.

Solution:

(a) Analyze the inlet pipe data. The diameter of the pipe is given as 14 cm, which means its radius is 7 cm. Convert this dimensional metric into meters by dividing by 100, which gives $r = 0.07$ m.

(b) Calculate the cross-sectional area of the delivery pipe using the standard circle area formula, $A = \pi \cdot r^2$. Substituting the converted radius gives $A = \pi \cdot (0.07)^2 = 0.0049\pi \text{ m}^2$.

(c) The volume of water flowing out per second is the cross-sectional area multiplied by the flow speed: Rate $= 0.0049\pi \cdot 5 = 0.0245\pi \text{ m}^3/\text{s}$. Convert the duration of one hour into seconds: 1 hour = 3600 seconds.

(d) Calculate the total volume of water delivered during this time: $V = 0.0245\pi \cdot 3600 = 88.2\pi \text{ m}^3$.

(e) This volume causes the water level in the main tank to rise to a height of h. The main tank has a radius of 7 m, so its cross-sectional area is $\pi \cdot 7^2 = 49\pi \text{ m}^2$. Equate the two volumes: $49\pi \cdot h = 88.2\pi$.

(f) Cancel out the common factor π from both sides of the equation, leaving $49h = 88.2$. Isolating the height variable yields $h = \frac{88.2}{49} = 1.8$ meters.

Final Answer: The water level in the tank will rise by 1.8 meters.

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:
Permutation problems with grouping constraints can be solved using the string or block method. When specific elements must always stay together, they are treated as a single compound entity. The total number of permutations is then found by counting the arrangements of all items including this new compound entity, while factoring in identical elements and internal adjustments.
Solution:

(a) Analyze the letters of the word **'CRICKET'**. The word contains a total of 7 letters. Breaking down the components reveals the consonants are C, R, C, K, T and the vowels are I, E. Note that the consonant letter C appears twice.

(b) The problem requires that the vowels must always remain together. To satisfy this condition, bind the vowels I and E into a single compound block denoted as (IE).

(c) Now, treat this compound block as one single element alongside the remaining letters. The items to be arranged are now C, R, C, K, T, and the block (IE). This reduces the total number of items to arrange from 7 down to 6.

(d) Calculate the permutations of these 6 entities. Since the consonant letter C is repeated twice, divide the total factorial value by 2! to account for these identical elements. This gives $\frac{6!}{2!} = \frac{720}{2} = 360$ arrangements.

(e) Next, account for the internal arrangements within the vowel block itself. The two distinct vowels, I and E, can change positions with each other in $2! = 2$ different ways (either as IE or EI).

(f) Finally, apply the fundamental counting principle to find the total number of valid arrangements by multiplying the two values together: Total Arrangements $= 360 \times 2 = 720$.

Final Answer: The letters can be arranged in 720 distinct ways.

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept:
Number theory problems involving high modular powers can be solved using modular arithmetic theorems. When a number a is raised to a power and divided by a prime number p, Fermat's Little Theorem provides a direct solution. The theorem states that if p is a prime number and a is an integer not divisible by p, then $a^{p-1} \equiv 1 \pmod{p}$.

Solution:

(a) Identify the modular parameters from the given problem statement. The base value is $a = 2$, the exponential power is $n = 100$, and the divisor is $p = 101$.

(b) Examine the properties of the divisor, 101. Since its only integer divisors are 1 and itself, 101 is a prime number. Also, check that the base value 2 is not a multiple of 101, which satisfies the conditions for Fermat's Little Theorem.

(c) According to Fermat's Little Theorem, substituting the prime value $p = 101$ and base value $a = 2$ into the formula yields the congruence relation: $2^{101-1} \equiv 1 \pmod{101}$.

(d) Simplify the exponent term in the expression: $101 - 1 = 100$. This directly gives the simplified congruence expression: $2^{100} \equiv 1 \pmod{101}$.

(e) This modular relationship means that when the large number 2^{100} is divided by the prime number 101, the system leaves a positive remainder of 1. This theorem avoids the need for long cycles or manual expansions.

Final Answer: The remainder when the expression is divided by 101 is 1.

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:

Under a compound interest model, principal growth is exponential rather than linear. If a sum of money scales by a geometric factor K over a standard time interval T, it will scale by a factor of K^m over a total time period of $m \times T$. This geometric progression allows for rapid calculation without needing to find the specific compound interest rate.

Solution:

(a) Let the initial principal amount of money invested be represented by the variable P. The problem states that under compound interest conditions, this principal doubles itself over a time period of 6 years.

(b) This relationship can be written as $2P = P \cdot (1 + r)^6$, which simplifies to show that the growth multiplier factor is $(1 + r)^6 = 2$.

(c) The problem asks for the total time required for the principal to grow to 8 times its initial value, which can be written as $8P$. Express this target multiplier 8 as a power of the base growth factor 2, which gives $8 = 2^3$.

(d) Since the money doubles every 6 years, the compounding process can be viewed in successive blocks of time. In the first 6 years, the principal grows from P to $2P$. In the next 6 years, it doubles again from $2P$ to $4P$. In the third 6-year period, it doubles once more from $4P$ to $8P$.

(e) This sequence shows that three consecutive 6-year compounding cycles are required to reach the target amount. Calculate the total time by multiplying the number of cycles by the period length: Total Time $= 3 \times 6$ years $= 18$ years.

Final Answer: The sum of money will become 8 times its value in 18 years.

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:
Solving quadratic inequalities involves finding the roots of the corresponding quadratic equation to establish boundary checkpoints. These roots divide the real number line into distinct intervals. By checking the sign behavior of the quadratic expression within each interval, the region that satisfies the inequality can be determined.
Solution:

(a) Start with the given quadratic inequality: $x^2 - 5x + 6 < 0$. To analyze its sign behavior, factor the quadratic expression into a product of two linear binomial terms.

(b) Find two numbers that multiply to give the constant term $+6$ and add together to give the linear coefficient -5. These two numbers are -2 and -3. Rewrite the expression as $(x - 2)(x - 3) < 0$.

(c) Identify the critical transition roots by setting each linear factor to zero, which gives $x = 2$ and $x = 3$. These two values divide the real number line into three separate open intervals: $(-\infty, 2)$, $(2, 3)$, and $(3, \infty)$.

(d) Test the sign of the product within each interval. For the first interval $(-\infty, 2)$, pick $x = 0$, which gives $(-2)(-3) = +6 > 0$. For the third interval $(3, \infty)$, pick $x = 4$, which gives $(2)(1) = +2 > 0$. Both intervals yield positive values.

(e) For the middle interval $(2, 3)$, pick a test value such as $x = 2.5$. Substituting this value gives $(2.5 - 2)(2.5 - 3) = (0.5)(-0.5) = -0.25$, which is strictly less than zero.

(f) Since the inequality uses a strict less-than sign (< 0), the solution set is the open interval between the two roots where the expression is negative, which is $2 < x < 3$.

Final Answer: The inequality holds true for the range $2 < x < 3$.

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:
Statistical mean corrections can be calculated using net adjustments rather than recalculating the entire dataset sum from scratch. By finding the discrepancy between the correct value and the incorrect entry, the total error can be determined. Dividing this net difference by the total population size gives the correction factor for the average score.

Solution:

(a) Write down the initial values provided in the problem. The total student count is $N = 40$, and the calculated initial average score is $A_{\text{old}} = 68$.

(b) Calculate the total sum of the marks before correction by multiplying the student count by the initial average: Original Sum $= 40 \times 68 = 2720$.

(c) Identify the error details. An original score of 82 was mistyped as 42. This means the recorded value was lower than the actual value, creating a net deficit in the total sum.

(d) Calculate this net deficit by subtracting the incorrect entry from the correct score: Net Correction Value $= 82 - 42 = +40$. This positive value must be added back to restore the true sum.

(e) Find the corrected total sum by adding the adjustment value to the original sum: Corrected Sum $= 2720 + 40 = 2760$.

(f) Calculate the new corrected average score by dividing the corrected sum by the total student count: $A_{\text{new}} = \frac{2760}{40} = 69.0$. Alternatively, distributing the +40 deficit across the 40 students adds exactly $\frac{40}{40} = 1$ mark to the initial average, giving $68 + 1 = 69$.

Final Answer: The corrected average score of the class is 69.0.

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept:
This problem uses the geometric properties of a circle's chord. A fundamental theorem states that a line drawn perpendicularly from the center of a circle to a chord always bisects that chord into two equal halves. This perpendicular line, the radius, and half of the bisected chord form a right-angled triangle, allowing the use of the Pythagorean theorem.
Solution:

(a) Let the center of the circle be point O, and let the chord inside the circle be represented by the line segment AB. The problem states that the total length of the chord is $AB = 16$ cm.

(b) Draw a perpendicular line from the center O to meet the chord at point M. According to the chord bisector theorem, point M is the midpoint of AB, which means the segment length is $AM = \frac{16}{2} = 8$ cm.

(c) The perpendicular distance from the center to the chord is given as $OM = 6$ cm. Connect the center O to the chord endpoint A. The line segment OA represents the radius of the circle, r.

(d) This construction forms a right-angled triangle $\triangle OMA$, with the right angle located at vertex M. Apply the Pythagorean theorem to this triangle: $OA^2 = OM^2 + AM^2$.

(e) Substitute the known lengths into the equation: $r^2 = 6^2 + 8^2 = 36 + 64 = 100$. Taking the positive square root gives the radius value: $r = \sqrt{100} = 10$ cm.

(f) The problem asks for the total diameter of the circle. The diameter is twice the length of the radius, so calculate it as $D = 2 \times r = 2 \times 10 \text{ cm} = 20$ cm.

Final Answer: The total diameter of the circle is 20 cm.

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept:
Set theory problems involving overlapping groups can be solved using Venn diagrams or the principle of inclusion-exclusion. For two intersecting sets, the total number of unique items contained in their union is found by adding the sizes of the individual sets and then subtracting the size of their intersection to avoid double counting.

Solution:

(a) Define the sets based on the information provided. Let the universal set U represent the total number of professionals surveyed, so $n(U) = 60$.

(b) Let set B represent the professionals who read business magazines, so $n(B) = 35$. Let set T represent those who read tech journals, so $n(T) = 25$.

(c) The number of professionals who read both types of publications is represented by the intersection of the two sets, $n(B \cap T) = 15$.

(d) Apply the principle of inclusion-exclusion to calculate the total number of unique individuals who read at least one of the two magazines: $n(B \cup T) = n(B) + n(T) - n(B \cap T)$.

(e) Substitute the given values into the formula: $n(B \cup T) = 35 + 25 - 15 = 60 - 15 = 45$. This means 45 professionals read at least one magazine.

(f) The number of individuals who do not read either publication is the remainder of the total group outside this union. Calculate this by subtracting the union size from the universal set size: Non-readers $= n(U) - n(B \cup T) = 60 - 45 = 15$.

Final Answer: The number of individuals who do not read either magazine is 15.

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept:

The total number of trailing zeros at the end of a factorial expansion $N!$ is determined entirely by the highest power of 5 that divides into that factorial. In any factorial sequence, pairs of prime factors 2 and 5 combine to form a product of 10, which generates a trailing zero. Because multiples of 2 occur much more frequently than multiples of 5, the total count of trailing zeros is limited by the availability of prime factor 5. This value can be calculated using Legendre's formula.

Solution:

(a) Identify the target parameter from the problem statement, which is the factorial value $N = 80$. We need to compute the total number of times the prime number 5 divides into 80!.

(b) Apply Legendre's formula, which states that the exponent of a prime p in the prime factorization of $N!$ is calculated using the infinite series sum of integer floor quotients: $E_p(N!) = \lfloor \frac{N}{p^1} \rfloor + \lfloor \frac{N}{p^2} \rfloor + \lfloor \frac{N}{p^3} \rfloor + \ldots$

(c) Substitute $N = 80$ and the prime value $p = 5$ into the series expansion. This gives the calculation structure: Total Zeros $= \lfloor \frac{80}{5} \rfloor + \lfloor \frac{80}{25} \rfloor + \lfloor \frac{80}{125} \rfloor + \ldots$

(d) Calculate the value of the first term in the series. Dividing 80 by 5 gives exactly 16. Since 16 is an integer, its floor value remains $\lfloor 16 \rfloor = 16$. This means there are 16 numbers between 1 and 80 that are multiples of 5.

(e) Calculate the value of the second term in the series. Dividing 80 by 25 gives 3.2. Taking the lower integer floor of this value yields $\lfloor 3.2 \rfloor = 3$. This accounts for the extra factor of 5 contributed by numbers that are multiples of 25 (namely 25, 50, and 75).

(f) Evaluate the remaining terms in the series. Since the next power of 5 is 125, which is strictly greater than 80, the quotient inside the floor function is less than 1, yielding $\lfloor \frac{80}{125} \rfloor = 0$. Summing the individual non-zero components together gives Total Trailing Zeros $= 16+3 = 19$.

Final Answer: The total number of trailing zeros in 80! is 19.

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:
Time, speed, and distance problems involving crossing objects can be solved using relative speed principles. When two objects move in opposite directions, their relative speed is calculated by adding their individual speeds together. When a train passes a stationary point object like a post, the total distance traveled is equal to the length of that train. When two trains cross each other completely, the total distance traveled is the sum of both their lengths.

Solution:

(a) Let the speed of the first train be represented by the variable u and the speed of the second train be represented by the variable v. The problem states that they pass a fixed post in 12 seconds and 18 seconds respectively.

(b) Using the formula Distance = Speed × Time, determine the lengths of both trains. The length of the first train is computed as $L_1 = 12u$, and the length of the second train is computed as $L_2 = 18v$.

(c) Next, analyze the scenario where the two trains cross each other. Since they are running in opposite directions, their relative speed of approach is equal to the sum of their individual speeds, which is written as $u + v$.

(d) The total distance that must be covered for the two trains to clear each other completely is equal to the sum of their individual physical lengths, which is expressed as Total Distance = $L_1 + L_2 = 12u + 18v$.

(e) The problem states that the trains cross each other in 15 seconds. Set up the time equation using the relationship Distance = Relative Speed × Time, which gives the formula: $12u + 18v = 15 \times (u + v)$.

(f) Expand the right side of the equation to get $12u + 18v = 15u + 15v$. Group the terms for each variable on opposite sides of the equation by subtracting $12u$ and $15v$ from both sides, which simplifies to $18v - 15v = 15u - 12u$, or $3v = 3u$. This isolates the relationship to $\frac{u}{v} = \frac{1}{1}$, showing that their speeds are identical.

Final Answer: The ratio of their individual speeds is 1 : 1.

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept:
Problems involving sequences of numbers that change by a constant amount can be solved using the properties of an arithmetic progression. An arithmetic progression is a sequence of numbers where the difference between any two consecutive terms is a constant value. The total sum of an arithmetic progression can be calculated if the first term, the common difference, and the total number of terms are known.

Solution:

(a) Identify the boundary conditions and rules for the number sequence. We need to find two-digit numbers, which means the values must lie strictly within the range from 10 to 99. The numbers must also leave a remainder of 2 when divided by 4.

(b) Find the first term of the sequence. Test the lowest two-digit numbers: 10 divided by 4 is 2 with a remainder of 2. Therefore, the first term of our arithmetic progression is $a = 10$.

(c) Find the common difference for the sequence. Since the numbers are multiples of 4 with a fixed remainder offset, the sequence values will increase by 4 each time. This gives a common difference of $d = 4$. The next terms in the sequence are 14, 18, 22, and so on.

(d) Find the last term of the sequence below the upper boundary of 99. Testing the highest values shows that 96 is a multiple of 4, so $96 + 2 = 98$ will leave a remainder of 2. Thus, the last term of the progression is $a_n = 98$.

(e) Find the total number of terms, n, using the standard arithmetic progression term formula $a_n = a + (n-1)d$. Substituting our known values gives $98 = 10 + (n-1) \times 4$. Subtracting 10 yields $88 = 4(n-1)$, and dividing by 4 gives $22 = n - 1$, which means $n = 23$.

(f) Calculate the total sum of the progression using the series sum formula: $S_n = \frac{n}{2} \times (a + a_n)$. Substituting the values yields $S_{23} = \frac{23}{2} \times (10 + 98) = \frac{23}{2} \times 108 = 23 \times 54 = 1242$.

Final Answer: The sum of all such two-digit numbers is 1242.

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept:

Mixture and replacement problems can be solved by tracking the absolute quantities of individual components or by focusing on the component whose quantity is not added during the replacement step. When a portion of a mixture is removed, the relative ratio of the ingredients in the remaining solution stays exactly the same. The ratio only changes when a pure component is added back into the container.

Solution:

(a) Let the initial volume of milk and water in the container be represented in terms of a variable x. Based on the initial ratio of $7 : 5$, the initial volume of milk is $7x$ and the volume of water is $5x$, making the total initial mixture volume $12x$.

(b) A portion of 9 liters of the mixture is drawn off. Because the mixture is uniform, the liquid removed contains milk and water in the same $7 : 5$ ratio. The amount of milk removed is $9 \times \frac{7}{12} = \frac{21}{4}$ liters, and the amount of water removed is $9 \times \frac{5}{12} = \frac{15}{4}$ liters.

(c) Express the remaining quantities of both components after this removal step. The remaining volume of milk is $7x - \frac{21}{4}$, and the remaining volume of water is $5x - \frac{15}{4}$.

(d) Next, 9 liters of pure water is added to the container. This increases the water volume while the milk volume stays the same. The new volume of water is $(5x - \frac{15}{4}) + 9 = 5x + \frac{21}{4}$ liters.

(e) The problem states that the final ratio of milk to water becomes $7 : 9$. Set up the ratio equation: $\frac{7x-21/4}{5x+21/4} = \frac{7}{9}$. Divide both sides of the equation by 7 to simplify it, which gives $\frac{x-3/4}{5x+21/4} = \frac{1}{9}$.

(f) Cross-multiply to solve for x: $9 \times (x - \frac{3}{4}) = 5x + \frac{21}{4}$, which expands to $9x - \frac{27}{4} = 5x + \frac{21}{4}$. Group the terms to get $4x = \frac{21}{4} + \frac{27}{4} = \frac{48}{4} = 12$. Solving this gives $x = 3$. The initial volume of milk is $7x = 7 \times 3 = 21$ liters.

Final Answer: The initial quantity of milk in the container was 21 liters.

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:
Mensuration problems involving curved three-dimensional shapes require using the correct geometric surface area formulas. A solid hemisphere has two distinct surface regions: a curved upper dome and a flat circular base. The total surface area is the sum of these two individual regions. Calculating the total area requires adding the circular base area to the curved dome area.
Solution:

(a) Write down the surface area formulas for a sphere and hemisphere. A full sphere has a surface area of $4\pi r^2$. When cut in half, the curved dome area of the hemisphere is exactly half of that value, which is $2\pi r^2$.

(b) Because the problem specifies that the shape is a solid hemisphere, we must also include the boundary surface created by the flat cut. This flat base is a circle with an area given by the standard formula πr^2.

(c) Combine these two surface regions to find the total surface area (TSA) formula for a solid hemisphere: $\text{TSA} = 2\pi r^2 + \pi r^2 = 3\pi r^2$.

(d) The problem states that the total surface area of this solid hemisphere is equal to 462 cm^2. Set up the algebraic equation using the combined formula: $3\pi r^2 = 462$.

(e) Substitute the given value of $\pi = \frac{22}{7}$ into the equation to isolate the radius variable: $3 \times \frac{22}{7} \times r^2 = 462$. Multiply the constants together to simplify the expression to $\frac{66}{7} \times r^2 = 462$.

(f) Isolate the r^2 term by multiplying both sides by the reciprocal fraction $\frac{7}{66}$, which gives $r^2 = 462 \times \frac{7}{66}$. Dividing 462 by 66 simplifies to exactly 7, leaving $r^2 = 7 \times 7 = 49$. Take the positive square root to find the radius: $r = \sqrt{49} = 7$ cm.

Final Answer: The radius parameter of the solid hemisphere is 7 cm.

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept:
Probability problems with discrete outcomes can be solved by explicitly defining the total sample space and counting the favorable event outcomes. In a simultaneous roll of two independent, fair six-sided dice, each die has 6 possible outcomes, creating a total sample space of $6 \times 6 = 36$ equally likely coordinate pairs. The probability is the number of favorable outcomes divided by the total sample space size.

Solution:

(a) Determine the total number of possible outcomes for the experiment. Rolling two separate dice yields $6 \times 6 = 36$ unique combinations, ranging from a minimum possible sum of $1 + 1 = 2$ to a maximum possible sum of $6 + 6 = 12$.

(b) Identify the target condition for the event. The sum of the two numbers showing on top must be a prime number. List all prime numbers that fall within the achievable sum range between 2 and 12, which are 2, 3, 5, 7, and 11.

(c) Count the favorable combinations that sum to each prime number. For a sum of 2, there is only 1 valid pair: $(1, 1)$. For a sum of 3, there are 2 valid pairs: $(1, 2)$ and $(2, 1)$.

(d) Count the combinations for the next prime sums. For a sum of 5, there are 4 valid pairs: $(1, 4)$, $(2, 3)$, $(3, 2)$, and $(4, 1)$. For a sum of 7, there are 6 valid pairs: $(1, 6)$, $(2, 5)$, $(3, 4)$, $(4, 3)$, $(5, 2)$, and $(6, 1)$.

(e) Count the combinations for the final prime sum. For a sum of 11, there are 2 valid pairs: $(5, 6)$ and $(6, 5)$.

(f) Calculate the total number of favorable outcomes by summing the individual counts: $\text{Favorable Outcomes} = 1 + 2 + 4 + 6 + 2 = 15$. Finally, calculate the probability by dividing the favorable outcomes by the total sample space: $\text{Probability} = \frac{15}{36}$. Divide both numbers by 3 to simplify the fraction to $\frac{5}{12}$.

Final Answer: The probability that the sum of the numbers is a prime number is $5/12$.

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:

Percentage product stability problems are based on the inverse proportional relationship between two related variables. The total financial expenditure on a product is equal to the price per unit multiplied by the total consumption volume. If the price decreases, consumption must increase by a corresponding fractional amount to keep the final product value unchanged.

Solution:

(a) Let the initial price of petroleum per unit be represented by P and the initial consumption volume be represented by C. The initial total expenditure is the product of these two values, which can be written as $E = P \times C$.

(b) The problem states that the price of petroleum drops by 20%. Calculate the new price factor by subtracting 20% from the original price: $P_{\text{new}} = P - 0.20P = 0.80P$, which can also be written as the fraction $\frac{4}{5}P$.

(c) Let the new consumption volume required to keep expenditure constant be represented by C_{new}. The new expenditure equation is written as $E_{\text{new}} = P_{\text{new}} \times C_{\text{new}} = 0.80P \times C_{\text{new}}$.

(d) Since the driver wants to keep total expenditure unchanged, equate the new expenditure to the original value: $0.80P \times C_{\text{new}} = P \times C$. Divide both sides by the price variable P to simplify the equation to $0.80 \times C_{\text{new}} = C$.

(e) Isolate the new consumption variable: $C_{\text{new}} = \frac{C}{0.80} = \frac{5}{4}C = 1.25C$. This shows that the new consumption must be 1.25 times the original volume.

(f) Calculate the percentage increase in consumption by finding the fractional change: Increase $= 1.25C - C = 0.25C$. Converting this fraction into a percentage yields $0.25 \times 100\% = 25\%$. Alternatively, use the standard shortcut formula: $\frac{R}{100-R} \times 100\% = \frac{20}{80} \times 100\% = 25\%$.

Final Answer: The driver must increase consumption by 25% to keep expenditure the same.

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept:
The conditions for solutions in a system of simultaneous linear equations can be determined by analyzing the ratios of their coefficients. A system of two linear equations in the form $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ represents two lines in a plane. For the system to have an infinite number of solutions, the two lines must lie exactly on top of each other, meaning they are identical. This requires the ratios of their corresponding coefficients to be equal.

Solution:

(a) Write down the two linear equations given in the problem statement: the first equation is $3x + 4y = 12$ and the second equation is $6x + ky = 24$.

(b) Identify the individual coefficients for both equations. For the first equation, the coefficients are $a_1 = 3$, $b_1 = 4$, and $c_1 = 12$. For the second equation, the coefficients are $a_2 = 6$, $b_2 = k$, and $c_2 = 24$.

(c) Set up the mathematical condition required for the system to have an infinite number of solutions. The ratio of the coefficients must satisfy the extended equality: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

(d) Substitute the known values into this ratio structure to get the equation: $\frac{3}{6} = \frac{4}{k} = \frac{12}{24}$.

(e) Simplify the known numeric ratios. Notice that $\frac{3}{6} = \frac{1}{2}$ and $\frac{12}{24} = \frac{1}{2}$. Both ratios reduce to exactly $\frac{1}{2}$, confirming that the constant terms and x-coefficients scale by the same factor.

(f) Set the remaining variable ratio equal to this constant scaling factor to solve for k: $\frac{4}{k} = \frac{1}{2}$. Cross-multiplying to isolate the variable gives $k = 4 \times 2 = 8$.

Final Answer: The system has infinite solutions when $k = 8$.

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept:
The area of a uniform border path running around the outside of a central rectangular plot can be calculated using nested area subtraction. The total surface area of the path is equal to the area of the larger outer rectangle (which includes both the plot and the path) minus the area of the smaller inner rectangular plot. Accurate outer dimensions must be found by adding the path width to both ends of each dimension.

Solution:

(a) Identify the dimensions of the inner rectangular plot from the problem details. The inner length is $L_{\text{in}} = 15$ meters and the inner width is $W_{\text{in}} = 10$ meters.

(b) Calculate the surface area of this inner plot using the standard area formula Area = Length × Width: Inner Area = $15 \times 10 = 150$ square meters.

(c) Determine the dimensions of the larger outer boundary. The path has a uniform width of 2 meters and runs completely around the outside of the plot. This means the path adds 2 meters to both sides of the length and both sides of the width.

(d) Calculate the new outer length by adding the path width twice: $L_{\text{out}} = 15+2+2 = 19$ meters. Similarly, calculate the new outer width: $W_{\text{out}} = 10 + 2 + 2 = 14$ meters.

(e) Calculate the total surface area of this larger outer rectangle using the new dimensions: Outer Area = 19×14. Breaking down this multiplication gives $19 \times 14 = 266$ square meters.

(f) Find the net surface area of the path by subtracting the inner plot area from the total outer area: Path Area = Outer Area − Inner Area = $266 - 150 = 116$ square meters.

Final Answer: The total surface area of the path is 116 sq m.

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:
A fundamental theorem in number theory states that for any two positive integers, the product of their Highest Common Factor (HCF) and their Lowest Common Multiple (LCM) is always exactly equal to the product of the two original numbers themselves. This mathematical relationship can be written as $\text{HCF}(A, B) \times \text{LCM}(A, B) = A \times B$. If any three of these values are known, the fourth can be found using basic algebra.

Solution:

(a) Write down the values provided in the problem statement. The Highest Common Factor is $\text{HCF} = 12$, the Lowest Common Multiple is $\text{LCM} = 72$, and one of the two numbers is given as $n_1 = 24$.

(b) Let the second, unknown positive number be represented by the variable n_2.

(c) Set up the algebraic equation based on the product property theorem: $\text{HCF} \times \text{LCM} = n_1 \times n_2$.

(d) Substitute the known values into the product formula to get the equation: $12 \times 72 = 24 \times n_2$.

(e) Calculate the product on the left side of the equation: $12 \times 72 = 864$. This simplifies the equation to $864 = 24 \times n_2$.

(f) Isolate the unknown variable n_2 by dividing both sides of the equation by 24: $n_2 = \frac{864}{24}$. To simplify this calculation, notice that $\frac{12}{24} = \frac{1}{2}$, so the equation can be rewritten as $n_2 = \frac{72}{2} = 36$.

Final Answer: The value of the other positive number is 36.

Answer: (A)

Go Back to Question 20

Here is the detailed solution for the final batch of questions (Q21 to Q30) formatted precisely according to your LaTeX template structure, commands, and formatting rules.

Q21.

Solution

Concept:
Work allocation problems operate under an inverse relationship metric between available manpower and time required, assuming the operational efficiency of each unit remains uniform. The total volume of work can be measured in compound terms of man-days, which is calculated as the product of the total number of workers and the number of days they work. This total man-days value must remain constant across different staffing setups for the same project.

Solution:

(a) Identify the initial parameters from the problem statement. The initial size of the workforce is $M_1 = 15$ laborers, and the time required to finish the construction project is $D_1 = 8$ days.

(b) Calculate the total volume of work required for this project by multiplying the workforce size by the number of days: Total Work $= M_1 \times D_1 = 15 \times 8 = 120$ man-days.

(c) Now, consider the target scenario where the project duration must be shortened to a new time frame of $D_2 = 5$ days. Let the total number of workers needed for this faster timeline be represented by the variable M_2.

(d) Because the total work volume remains the same, set up the work conservation equation: $M_2 \times D_2 =$ Total Work. This gives the equation $M_2 \times 5 = 120$.

(e) Isolate the workforce variable M_2 by dividing the total man-days by the new number of days: $M_2 = \frac{120}{5} = 24$ workers. This means a total workforce of 24 laborers is needed to complete the project in 5 days.

(f) The problem asks for the number of extra workers that must be added to the original team. Calculate this by subtracting the initial workforce size from the required total: Extra Workers $= M_2 - M_1 = 24 - 15 = 9$.

Final Answer: The contractor must hire 9 extra workers to meet the deadline.

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:
An infinite geometric series is an infinite sequence of numbers where each term after the first is found by multiplying the previous term by a fixed, non-zero number called the common ratio. If the absolute value of this common ratio is strictly less than 1, the series converges to a finite value. The sum of a converging infinite geometric series can be calculated using the standard formula $S_\infty = \frac{a}{1-r}$, where a is the first term and r is the common ratio.
Solution:

(a) Write down the terms of the infinite geometric series provided in the problem statement: $12 + 4 + \frac{4}{3} + \frac{4}{9} + \dots$

(b) Identify the first term of this sequence, which is represented by the variable a. The first term is $a = 12$.

(c) Find the common ratio, r, by dividing any term in the sequence by the term that immediately precedes it. Dividing the second term by the first term gives: $r = \frac{4}{12} = \frac{1}{3}$.

(d) Check the convergence condition for the series. Calculate the absolute value of the common ratio: $|r| = |\frac{1}{3}| = \frac{1}{3}$. Since $\frac{1}{3}$ is strictly less than 1, the series converges, meaning it sums to a fixed finite number.

(e) Substitute the first term $a = 12$ and the common ratio $r = \frac{1}{3}$ into the infinite sum formula: $S_\infty = \frac{12}{1-1/3}$.

(f) Simplify the denominator subtraction term: $1 - \frac{1}{3} = \frac{2}{3}$. This changes the sum expression to $S_\infty = \frac{12}{2/3}$. Simplify this fraction by multiplying by the reciprocal: $S_\infty = 12 \times \frac{3}{2} = 6 \times 3 = 18$.

Final Answer: The sum of the infinite geometric series is 18.

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept:
An equilateral triangle is a regular polygon where all three interior angles measure exactly 60 degrees and all three bounding sides have the same length. Because of this symmetry, the total surface area of an equilateral triangle can be calculated using only its side length, s, with the standard area formula Area $= \frac{\sqrt{3}}{4}s^2$. Once the side length is found, the total perimeter is simply three times that value.

Solution:

(a) Let the side length of the equilateral triangle be represented by the variable s. The problem states that the total measured area of the triangle is $16\sqrt{3}$ cm^2.

(b) Set up the algebraic equation by setting the standard area formula for an equilateral triangle equal to the given area value: $\frac{\sqrt{3}}{4}s^2 = 16\sqrt{3}$.

(c) Isolate the side variable s^2 by dividing both sides of the equation by $\sqrt{3}$. This cancels out the radical term from both sides, leaving the simplified equation: $\frac{1}{4}s^2 = 16$.

(d) Multiply both sides of the equation by 4 to clear the fraction from the left side: $s^2 = 16 \times 4 = 64$.

(e) Find the side length by taking the positive square root of both sides of the equation: $s = \sqrt{64} = 8$ cm. This means each of the three sides of the triangle measures exactly 8 cm.

(f) The problem asks for the total perimeter length of the triangle. Since an equilateral triangle has three identical sides, calculate the perimeter by multiplying the side length by 3: Perimeter $= 3 \times s = 3 \times 8$ cm $= 24$ cm.

Final Answer: The total perimeter length of the triangle is 24 cm.

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept:
Probability problems involving multiple item selections can be solved using combinations to count possibilities. When drawing objects simultaneously from a container without replacement, the order of selection does not matter. The probability of an event is the number of favorable selection combinations divided by the total number of possible combinations in the sample space.
Solution:

(a) Analyze the contents of the bag. The bag contains 5 red balls and 4 green balls, which means the total number of balls is $5 + 4 = 9$. The experiment involves drawing 2 balls at random simultaneously.

(b) Calculate the total number of ways to choose 2 balls from the 9 available balls. Use the combination formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$, which gives: Total Outcomes $= \binom{9}{2} = \frac{9\times8}{2\times1} = 36$.

(c) Identify the favorable condition: both drawn balls must be of the same color. This can happen in two mutually exclusive ways: either both balls are red, or both balls are green.

(d) Calculate the number of ways to choose 2 red balls from the 5 available red balls: Red Combinations $= \binom{5}{2} = \frac{5\times4}{2\times1} = 10$.

(e) Calculate the number of ways to choose 2 green balls from the 4 available green balls: Green Combinations $= \binom{4}{2} = \frac{4\times3}{2\times1} = 6$.

(f) Find the total number of favorable outcomes by adding the two values together: Favorable Outcomes $= 10 + 6 = 16$. Finally, calculate the probability by dividing the favorable outcomes by the total sample space: Probability $= \frac{16}{36}$. Divide the numerator and denominator by 4 to simplify the fraction to $\frac{4}{9}$.

Final Answer: The probability that both balls are the same color is $4/9$.

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept:
Problems involving multiple investment allocations can be solved using a system of simultaneous linear equations. Simple interest is calculated as the product of the principal amount, the annual interest rate, and the time period in years. When the investment amounts are swapped between the two schemes, it changes the coefficients of the variables, creating a solvable system of equations.
Solution:

(a) Let the principal amount originally invested in Scheme A be represented by the variable x, and the amount invested in Scheme B be represented by the variable y.

(b) Write the interest equation for the initial setup. Scheme A pays 8% interest and Scheme B pays 10% interest, producing a total annual return of \$190. This gives the equation: $0.08x + 0.10y = 190$. Multiply by 100 to clear the decimals: $8x + 10y = 19000$.

(c) Write the interest equation for the swapped setup. The investment amounts are reversed, so y is invested in Scheme A and x is invested in Scheme B, producing a total return of \$170. This gives the equation: $0.08y + 0.10x = 170$. Clear the decimals to get: $10x + 8y = 17000$.

(d) Add the two linear equations together to find a relationship for the combined investment: $(8x + 10y) + (10x + 8y) = 19000 + 17000$, which simplifies to $18x + 18y = 36000$. Divide by 18 to get: $x + y = 2000$.

(e) Subtract the second equation from the first to find the difference between the investments: $(8x + 10y) - (10x + 8y) = 19000 - 17000$, which simplifies to $-2x + 2y = 2000$. Divide by 2 to get: $-x + y = 1000$.

(f) Add this difference equation to the combined investment equation to isolate y: $(x + y) + (-x + y) = 2000 + 1000$, which simplifies to $2y = 3000$, or $y = 1500$. Substitute $y = 1500$ back into $x + y = 2000$ to find the principal for Scheme A: $x = 2000 - 1500 = 500$.

Final Answer: The money originally invested in Scheme A was \$500.

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept:
Relative velocity principles govern problems involving motion through a moving medium, such as a boat traveling on a river. When a boat travels downstream, it moves in the same direction as the river current, meaning the water helps push the boat along. To find the net downstream speed, add the speed of the boat in still water to the speed of the river current.

Solution:

(a) Write down the velocity components provided in the problem statement. The speed of the boat in still water is $v_b = 11$ km/h, and the speed of the river current is $v_c = 3$ km/h.

(b) Determine the total travel distance required, which is given as $d = 56$ km. The direction of travel is specified as downstream.

(c) Calculate the net downstream speed, s_{down}, by adding the speed of the river current to the still-water speed of the boat: $s_{\text{down}} = v_b + v_c = 11 \text{ km/h} + 3 \text{ km/h} = 14 \text{ km/h}$.

(d) Use the standard time-distance formula, Time $= \frac{\text{Distance}}{\text{Speed}}$, to find the duration of the trip.

(e) Substitute the travel distance and the calculated downstream speed into the formula: Time $= \frac{56 \text{ km}}{14 \text{ km/h}}$.

(f) Calculate the quotient. Dividing 56 by 14 gives exactly 4. Therefore, it will take the boat a total time duration of 4 hours to complete the downstream journey.

Final Answer: The total travel time required for the journey is 4 hours.

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept:
Retail pricing problems connect the cost price, marked price, and selling price through percentage profit and discount rates. Profit is always calculated as a percentage markup on the cost price, establishing the final selling price. A discount is a percentage reduction applied to the marked price to arrive at that same selling price, creating a link that allows the required marked price to be found.

Solution:

(a) Write down the pricing parameters provided in the problem. The cost price of the item is $CP = \$400$. The merchant wants to earn a target profit percentage of 20% on this cost price.

(b) Calculate the target selling price (SP) required to achieve this profit. A 20% profit means the selling price must be 120% of the cost price: $SP = 400 \times \left(1 + \frac{20}{100}\right) = 400 \times 1.20 = \$480$.

(c) Next, analyze the relationship between the selling price and the marked price (MP). The merchant plans to offer a 10% discount on the marked price, meaning the customer pays 90% of the marked price.

(d) Set up the algebraic equation linking these two pricing factors together: $0.90 \times MP =$ Selling Price. Substituting the required selling price gives the equation: $\frac{9}{10} \times MP = 480$.

(e) Isolate the marked price variable by multiplying both sides of the equation by the reciprocal fraction $\frac{10}{9}$: $MP = 480 \times \frac{10}{9} = \frac{4800}{9}$.

(f) Simplify the fraction by dividing both the numerator and the denominator by 3, which gives $\frac{1600}{3}$. Converting this improper fraction into a decimal value yields \$533.33.

Final Answer: The merchant should set the marked price of the item to \$533.33.

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept:
A trapezoid is a quadrilateral with at least one pair of parallel sides. The total area of a trapezoid depends on the lengths of these parallel sides and the perpendicular height distance separating them. The area is calculated by multiplying the average length of the two parallel sides by the perpendicular height, using the formula Area $= \frac{1}{2} \times (a + b) \times h$.

Solution:

(a) Identify the geometric parameters provided in the problem statement. The lengths of the two parallel sides are $a = 12$ cm and $b = 18$ cm. The total area of the trapezoid is given as Area $= 150\ \text{cm}^2$.

(b) Let the unknown perpendicular distance between these parallel sides be represented by the height variable h.

(c) Substitute the known side lengths and the total area into the standard trapezoid area formula: $150 = \frac{1}{2} \times (12 + 18) \times h$.

(d) Simplify the expression inside the parentheses by adding the two parallel side lengths together: $12 + 18 = 30$ cm. This updates the equation to: $150 = \frac{1}{2} \times 30 \times h$.

(e) Calculate the fractional product on the right side of the equation. Half of 30 is 15, which simplifies the linear equation to: $150 = 15 \times h$.

(f) Isolate the height variable h by dividing both sides of the equation by 15: $h = \frac{150}{15} = 10$ cm. This confirms that the perpendicular distance between the parallel sides is exactly 10 cm.

Final Answer: The perpendicular distance between the parallel sides is 10 cm.

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept:
When a baseline value experiences multiple percentage changes in sequence, the effects compound sequentially rather than adding together linearly. Each percentage shift alters the base value for the next calculation step. The final net change can be found by applying successive decimal multipliers to an initial baseline value of 100.

Solution:

(a) Let the initial revenue of the company before the first quarter be represented by a baseline value of 100.

(b) In the first quarter, the revenue increases by 10%. Calculate the new value by adding 10% to the base: $\text{Value}_1 = 100 \times \left(1 + \frac{10}{100}\right) = 100 \times 1.10 = 110$.

(c) In the second quarter, the revenue increases by 20%. This percentage increase applies to the new base value of 110, not the original 100. Calculate the updated value: $\text{Value}_2 = 110 \times \left(1 + \frac{20}{100}\right) = 110 \times 1.20 = 132$.

(d) In the third quarter, revenue drops by 10%. This decrease applies to the second-quarter value of 132. Calculate the final revenue value: $\text{Value}_3 = 132 \times \left(1 - \frac{10}{100}\right) = 132 \times 0.90 = 118.8$.

(e) Determine the net percentage change across all three quarters by comparing the final compiled value to the initial baseline value of 100: $\text{Net Change} = 118.8 - 100 = 18.8$.

(f) Since this net difference is positive, the revenue experienced an overall increase of 18.8% from its starting value. This demonstrates why sequential percentage changes must be calculated step-by-step.

Final Answer: The net percentage change across all quarters is an 18.8% increase.

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept:
Ratio problems involving sample distributions use the properties of proportional scaling. A ratio provides the relative proportions of different components within a whole system. By summing the individual parts of the ratio, the size of a single standard proportional unit can be found. Dividing the total sample size by this sum gives a scaling factor that determines the expected count for each component.

Solution:

(a) Identify the ratio components provided in the problem details. The ratio of functional items to defective items on the production line is given as $19 : 1$.

(b) Calculate the total number of parts in one full ratio cycle by adding the individual components together: Total Parts $= 19 + 1 = 20$ parts. This means that out of every 20 items produced, 19 are expected to be functional and 1 is expected to be defective.

(c) The quality check sample contains a total of $N = 140$ items. This sample size represents a multiple of the basic ratio structure.

(d) Calculate the scaling factor, k, by dividing the total sample size by the total number of parts in the ratio: $k = \frac{140}{20} = 7$. This means the sample contains exactly 7 complete ratio cycles.

(e) Find the expected number of functional items in the sample by multiplying the functional component of the ratio by this scaling factor: Functional Items $= 19 \times k = 19 \times 7$.

(f) Calculate the product: $19 \times 7 = 133$ items. To verify, calculate the expected defective items: $1 \times 7 = 7$ items. Adding the two groups together confirms the total sample size: $133 + 7 = 140$.

Final Answer: The expected number of functional items in the sample is 133.

Answer: (B)

Go Back to Question 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	A	4	C	5	A
6	A	7	B	8	B	9	B	10	A
11	B	12	A	13	C	14	A	15	B
16	A	17	B	18	B	19	A	20	A
21	B	22	B	23	A	24	A	25	B
26	B	27	B	28	B	29	B	30	B