General Aptitude

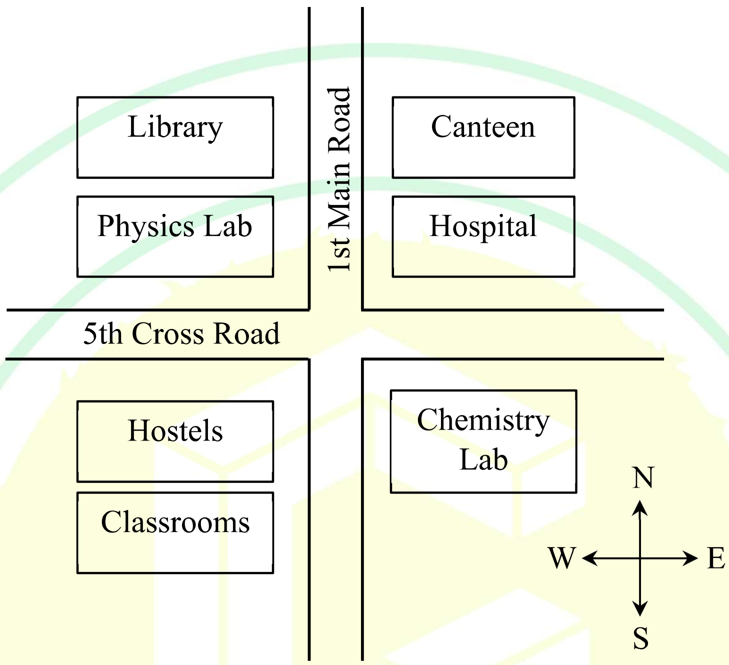
Q.1 – Q.5 Carry ONE mark Each

Q.1	Ravi had _______ younger brother who taught at _______ university. He was widely regarded as _______ honorable man. Select the option with the correct sequence of articles to fill in the blanks.
(A)	a; a; an
(B)	the; an; a
(C)	a; an; a
(D)	an; an; a

Q.2	The CEO's decision to downsize the workforce was considered <u>myopic</u> because it sacrificed long-term stability to accommodate short-term gains. Select the most appropriate option that can replace the word "myopic" without changing the meaning of the sentence.
(A)	visionary
(B)	shortsighted
(C)	progressive
(D)	innovative

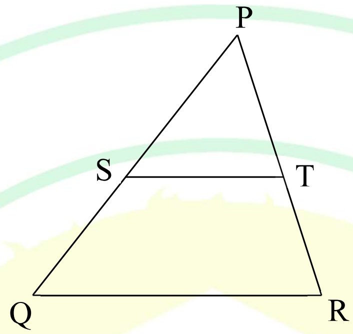
Q.3	The average marks obtained by a class in an examination were calculated as 30.8. However, while checking the marks entered, the teacher found that the marks of one student were entered incorrectly as 24 instead of 42. After correcting the marks, the average becomes 31.4. How many students does the class have?
(A)	25
(B)	28
(C)	30
(D)	32

Q.4	Consider the relationships among P, Q, R, S, and T: • P is the brother of Q. • S is the daughter of Q. • T is the sister of S. • R is the mother of Q. The following statements are made based on the relationships given above. (1) R is the grandmother of S. (2) P is the uncle of S and T. (3) R has only one son. (4) Q has only one daughter. Which one of the following options is correct?
(A)	Both (1) and (2) are true.
(B)	Both (1) and (3) are true.
(C)	Only (3) is true.
(D)	Only (4) is true.

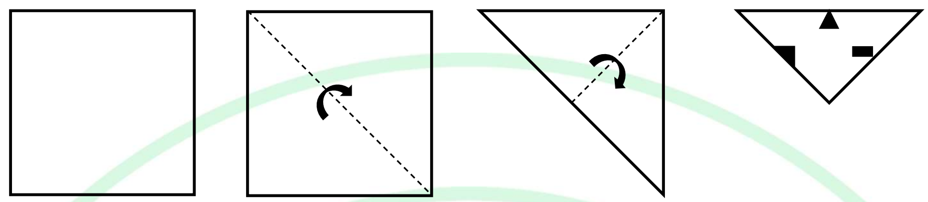
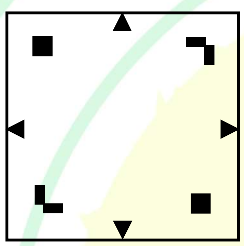
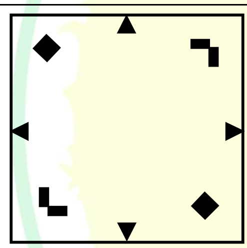
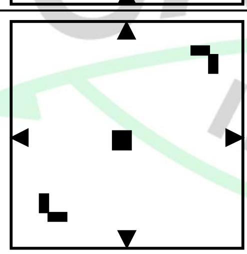
Q.5	According to the map shown in the figure, which one of the following statements is correct? Note: The figure shown is representative.
	
(A)	The library is located to the northwest of the canteen.
(B)	The hospital is located to the east of the chemistry lab.
(C)	The chemistry lab is to the southeast of physics lab.
(D)	The classrooms and canteen are next to each other.

Q.6 – Q.10 Carry TWO marks Each

Q.6	"I put the brown paper in my pocket along with the chalks, and possibly other things. I suppose every one must have reflected how primeval and how poetical are the things that one carries in one's pocket: the pocket-knife, for instance the type of all human tools, the infant of the sword. Once I planned to write a book of poems entirely about the things in my pocket. But I found it would be too long: and the age of the great epics is past." (From G.K. Chesterton's "A Piece of Chalk") Based only on the information provided in the above passage, which one of the following statements is true?
(A)	The author of the passage carries a mirror in his pocket to reflect upon things.
(B)	The author of the passage had decided to write a poem on epics.
(C)	The pocket-knife is described as the infant of the sword.
(D)	Epics are described as too inconvenient to write.

Q.7	In the diagram, the lines QR and ST are parallel to each other. The shortest distance between these two lines is half the shortest distance between the point P and line QR. What is the ratio of the area of the triangle PST to the area of the trapezium SQRT? Note: The figure shown is representative.
	
(A)	$\frac{1}{3}$
(B)	$\frac{1}{4}$
(C)	$\frac{2}{5}$
(D)	$\frac{1}{2}$

Q.8	A fair six-faced dice, with the faces labelled '1', '2', '3', '4', '5', and '6', is rolled thrice. What is the probability of rolling '6' exactly once?
(A)	$\frac{75}{216}$
(B)	$\frac{1}{6}$
(C)	$\frac{1}{18}$
(D)	$\frac{25}{216}$

Q.9	A square paper, shown in figure (I), is folded along the dotted lines as shown in the figures (II) and (III). Then a few cuts are made as shown in figure (IV). Which one of the following patterns will be obtained when the paper is unfolded? Note: The figures shown are representative.
	 (I) (II) (III) (IV)
(A)	
(B)	
(C)	
(D)	

Q.10	A shop has 4 distinct flavors of ice-cream. One can purchase any number of scoops of any flavor. **The order in which the scoops are purchased is inconsequential.** If one wants to purchase 3 scoops of ice-cream, in how many ways can one make that purchase?
(A)	4
(B)	20
(C)	24
(D)	48

Q.11 – Q.35 Carry ONE mark Each

Q.11	Let $$S = \left\{ w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \in \mathbb{R}^3 : \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} [w_1 \quad w_2 \quad w_3] \text{ is diagonalizable and } \|w\| = 1 \right\},$$ where $\|w\| = (w_1^2 + w_2^2 + w_3^2)^{\frac{1}{2}}$. Then, which one of the following is TRUE?
(A)	S is compact and connected
(B)	S is neither compact nor connected
(C)	S is compact but not connected
(D)	S is connected but not compact
Q.12	Given that the Laplace transforms of $J_0(x)$, $J_0'(x)$ and $J_0''(x)$ exist, where $J_0(x)$ is the Bessel function. Let $Y = Y(s)$ be the Laplace transform of the Bessel function $J_0(x)$. Then, which one of the following is TRUE?
(A)	$$\frac{dY}{ds} + \frac{2sY}{s^2+1} = 0, \qquad s > 0$$
(B)	$$\frac{dY}{ds} - \frac{2sY}{s^2+1} = 0, \qquad s > 0$$
(C)	$$\frac{dY}{ds} - \frac{sY}{s^2+1} = 0, \qquad s > 0$$
(D)	$$\frac{dY}{ds} + \frac{sY}{s^2+1} = 0, \qquad s > 0$$

Q.13	To find a real root of the equation $x^3 + 4x^2 - 10 = 0$ in the interval $\left(1, \frac{3}{2}\right)$ by using the fixed-point iteration scheme, consider the following two statements: S1: The iteration scheme $x_{k+1} = \sqrt{\frac{10}{4+x_k}}$, $k = 0, 1, 2, \ldots$, converges for any initial guess $x_0 \in \left(1, \frac{3}{2}\right)$. S2: The iteration scheme $x_{k+1} = \frac{1}{2}\sqrt{10 - x_k^3}$, $k = 0, 1, 2, \ldots$, diverges for some initial guess $x_0 \in \left(1, \frac{3}{2}\right)$. Then, which one of the following is correct?
(A)	S1 is TRUE and S2 is FALSE
(B)	S2 is TRUE and S1 is FALSE
(C)	both S1 and S2 are TRUE
(D)	neither S1 nor S2 is TRUE

Q.14	For the linear programming problem: Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$ Subject to $\alpha x_1 + x_2 + x_3 = 4,$ $x_1 + \beta x_2 + x_4 = 8,$ $x_1, x_2, x_3, x_4 \geq 0,$ consider the following two statements: S1: If $\alpha = 2$ and $\beta = 1$, then $(x_1, x_2)^T$ forms an optimal basis. S2: If $\alpha = 1$ and $\beta = 4$, then $(x_3, x_2)^T$ forms an optimal basis. Then, which one of the following is correct?
(A)	S1 is TRUE and S2 is FALSE
(B)	S2 is TRUE and S1 is FALSE
(C)	both S1 and S2 are TRUE
(D)	neither S1 nor S2 is TRUE

Q.15	Consider the following subsets of the Euclidean space $\mathbb{R}^4$: $$S = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1^2 + x_2^2 + x_3^2 - x_4^2 = 0\},$$ $$T = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1^2 + x_2^2 + x_3^2 - x_4^2 = 1\}, \quad \text{and}$$ $$U = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1^2 + x_2^2 + x_3^2 - x_4^2 = -1\}.$$ Then, which one of the following is TRUE?
(A)	S is connected, but T and U are not connected
(B)	T and U are connected, but S is not connected
(C)	S and U are connected, but T is not connected
(D)	S and T are connected, but U is not connected
Q.16	Consider the system of ordinary differential equations $$\frac{dX}{dt} = MX,$$ where M is a 6×6 skew-symmetric matrix with entries in $\mathbb{R}$. Then, for this system, the origin is a stable critical point for
(A)	any such matrix M
(B)	only such matrices M whose rank is 2
(C)	only such matrices M whose rank is 4
(D)	only such matrices M whose rank is 6

Q.17	Let $X = \{f \in C[0,1] : f(0) = 0 = f(1)\}$ with the norm $\|f\|_\infty = \sup_{0 \le t \le 1} |f(t)|$, where $C[0,1]$ is the space of all real-valued continuous functions on $[0,1]$. Let $Y = C[0,1]$ with the norm $\|f\|_2 = \left(\int_0^1 |f(t)|^2 dt\right)^{\frac{1}{2}}$. Let U_X and U_Y be the closed unit balls in X and Y centred at the origin, respectively. Consider $T: X \to \mathbb{R}$ and $S: Y \to \mathbb{R}$ given by $$Tf = \int_0^1 f(t)\,dt \quad \text{and} \quad Sf = \int_0^1 f(t)\,dt.$$ Consider the following statements: S1: $\sup_{f \in U_X} |Tf|$ is attained at a point of U_X. S2: $\sup_{f \in U_Y} |Sf|$ is attained at a point of U_Y. Then, which one of the following is correct?
(A)	S1 is TRUE and S2 is FALSE
(B)	S2 is TRUE and S1 is FALSE
(C)	both S1 and S2 are TRUE
(D)	neither S1 nor S2 is TRUE

Q.18	Let $g(x,y) = f(x,y)e^{2x+3y}$ be defined in $\mathbb{R}^2$, where $f(x,y)$ is a continuously differentiable non-zero homogeneous function of degree 4. Then, $x\frac{\partial g}{\partial x} + y\frac{\partial g}{\partial y} = 0$ holds for
(A)	all points (x,y) in $\mathbb{R}^2$
(B)	all points (x,y) on the line given by $2x + 3y + 4 = 0$
(C)	all points (x,y) in the region of $\mathbb{R}^2$ except on the line given by $2x + 3y + 4 = 0$
(D)	all points (x,y) on the line given by $2x + 3y = 0$
Q.19	The partial differential equation $$(1+x^2)\frac{\partial^2 u}{\partial x^2} + 2x(1-y^2)\frac{\partial^2 u}{\partial y \partial x} + (1-y^2)\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + (1-y^2)\frac{\partial u}{\partial y} = 0$$ is
(A)	elliptic in the region $\{(x,y) \in \mathbb{R}^2 : \lvert y \rvert \leq 1\}$
(B)	hyperbolic in the region $\{(x,y) \in \mathbb{R}^2 : \lvert y \rvert > 1\}$
(C)	elliptic in the region $\{(x,y) \in \mathbb{R}^2 : \lvert y \rvert > 1\}$
(D)	hyperbolic in the region $\{(x,y) \in \mathbb{R}^2 : \lvert y \rvert < 1\}$

Q.20	Let $u(x,t)$ be the solution of the following initial-boundary value problem $$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \qquad x \in (0,\pi), \qquad t > 0,$$ $$u(0,t) = u(\pi,t) = 0, \ \ u(x,0) = \sin 4x \cos 3x\,.$$ Then, for each $t > 0$, the value of $u\left(\frac{\pi}{4}, t\right)$ is
(A)	$$\frac{e^{-49t}}{2\sqrt{2}}(e^{48t} - 1)$$
(B)	$$\frac{e^{-49t}}{2\sqrt{2}}(1 - e^{48t})$$
(C)	$$\frac{e^{-49t}}{2\sqrt{2}}(1 + e^{48t})$$
(D)	$$\frac{e^{-49t}}{4\sqrt{2}}(1 - e^{48t})$$

Q.21	Consider the function $F: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by $$F(x,y) = (x^3 - 3xy^2 - 3x\,, 3x^2y - y^3 - 3y).$$ Then, for the function F, the inverse function theorem is
(A)	applicable at all points of $\mathbb{R}^2$
(B)	not applicable at exactly one point of $\mathbb{R}^2$
(C)	not applicable at exactly two points of $\mathbb{R}^2$
(D)	not applicable at exactly three points of $\mathbb{R}^2$

Q.22	Let the functions $f: \mathbb{R}^2 \to \mathbb{R}$ and $g: \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x_1, x_2) = x_1^2 + x_2^2 - 2x_1x_2,$ and $g(x_1, x_2) = 2x_1^2 + 2x_2^2 - x_1x_2.$ Consider the following statements: S1: For every compact subset K of $\mathbb{R}$, $f^{-1}(K)$ is compact. S2: For every compact subset K of $\mathbb{R}$, $g^{-1}(K)$ is compact. Then, which one of the following is correct?
(A)	S1 is TRUE and S2 is FALSE
(B)	S2 is TRUE and S1 is FALSE
(C)	both S1 and S2 are TRUE
(D)	neither S1 nor S2 is TRUE

Q.23	Let $p_A(x)$ denote the characteristic polynomial of a square matrix A. Then, for which of the following invertible matrices M, the polynomial $p_M(x) - p_{M^{-1}}(x)$ is constant?
(A)	$M = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$
(B)	$M = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$
(C)	$M = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$
(D)	$M = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

Q.24 Consider the balanced transportation problem with three sources S_1, S_2, S_3, and four destinations D_1, D_2, D_3, D_4, for minimizing the total transportation cost whose cost matrix is as follows:

	D_1	D_2	D_3	D_4	Supply
S_1	2	6	20	11	$\alpha + 10$
S_2	12	7	4	10	$\alpha + \lambda + 10$
S_3	8	14	16	11	5
Demand	$\alpha + 5$	10	$\lambda + 5$	$\alpha + \lambda$	

where $\alpha, \lambda > 0$. If the associated cost to the starting basic feasible solution obtained by using the North-West corner rule is 290, then which of the following is/are correct?

(A) $\alpha^2 + \lambda^2 = 100$

(B) $\alpha^2 + \alpha\lambda = 150$

(C) The optimal cost of the transportation problem is 260

(D) The optimal cost of the transportation problem is 290

Q.25	Consider the following regions: $S_1 = \{(x_1, x_2) \in \mathbb{R}^2 :\ 2x_1 + x_2 \leq 4, \quad x_1 + 2x_2 \leq 5, \quad x_1, x_2 \geq 0\},$ $S_2 = \{(x_1, x_2) \in \mathbb{R}^2 :\ 2x_1 - x_2 \leq 5, \quad x_1 + 2x_2 \leq 5, \quad x_1, x_2 \geq 0\}.$ Then, which of the following is/are TRUE?
(A)	The maximum value of $x_1 + x_2$ is 3 on the region S_2
(B)	The maximum value of $x_1 + x_2$ is 5 on the region $S_2 - S_1$
(C)	The maximum value of $x_1 + x_2$ is 3 on the region $S_1 \cap S_2$
(D)	The maximum value of $x_1 + x_2$ is 4 on the region $S_1 \cup S_2$

Q.26	Let $f: \mathbb{R}^2 - \{(0,0)\} \longrightarrow \mathbb{R}$ be a function defined by $$f(x,y) = \left(\frac{x^2 - y^2}{x^2 + y^2}\right) + x \sin\left(\frac{1}{x^2 + y^2}\right).$$ Consider the following three statements: S1: $\lim_{x\to 0} \lim_{y\to 0} f(x,y)$ exists. S2: $\lim_{y\to 0} \lim_{x\to 0} f(x,y)$ exists. S3: $\lim_{(x,y)\to(0,0)} f(x,y)$ exists. Then, which of the following is/are correct?
(A)	S2 and S3 are TRUE and S1 is FALSE
(B)	S1 and S2 are TRUE and S3 is FALSE
(C)	S1 and S3 are TRUE and S2 is FALSE
(D)	S1, S2 and S3 all are TRUE

Q.27	Let M be a 7×7 matrix with entries in $\mathbb{R}$ and having the characteristic polynomial $$c_M(x) = (x-1)^\alpha (x-2)^\beta (x-3)^2, \quad \text{where } \alpha > \beta.$$ Let $\text{rank}(M - I_7) = \text{rank}(M - 2I_7) = \text{rank}(M - 3I_7) = 5$, where I_7 is the 7×7 identity matrix. If $m_M(x)$ is the minimal polynomial of M, then $m_M(5)$ is equal to _____ (in integer)

Q.28	Let $y = P_n(x)$ be the unique polynomial of degree n satisfying the Legendre differential equation $$(1-x^2)y'' - 2xy' + n(n+1)y = 0 \quad \text{and} \quad y(1) = 1.$$ Then, the value of $P'_{11}(1)$ is equal to ______ (in integer)
Q.29	Let $\hat{\boldsymbol{a}}$ be a unit vector parallel to the tangent at the point $P(1, 1, \sqrt{2})$ to the curve of intersection of the surfaces $2x^2 + 3y^2 - z^2 = 3$ and $x^2 + y^2 = z^2$. Then, the absolute value of the directional derivative of $$f(x,y,z) = x^2 + 2y^2 - 2\sqrt{11}\, z$$ at P in the direction of $\hat{\boldsymbol{a}}$ is ______ (in integer)
Q.30	The volume of the region bounded by the cylinders $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$ is ______ (rounded off to TWO decimal places)
Q.31	Let W be the vector space (over $\mathbb{R}$) consisting of all bounded real-valued solutions of the differential equation $$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0\,.$$ Then, the dimension of W is ______ (in integer)

Q.32	Let $\vec{F} = (y - z)\hat{\boldsymbol{\imath}} + (z - x)\hat{\boldsymbol{\jmath}} + (x - y)\hat{\boldsymbol{k}}$ be a vector field, and let S be the surface $x^2 + y^2 + (z-1)^2 = 9,\ 1 \le z \le 4.$ If $\hat{\boldsymbol{n}}$ denotes the unit outward normal vector to S, then the value of $$\frac{1}{\pi}\left\|\iint_S (\vec{\nabla} \times \vec{F}) \cdot \hat{\boldsymbol{n}}\, dS\right\|$$ is equal to _____ (in integer)
Q.33	Consider $$I = \frac{1}{2\pi i}\oint_C \frac{\sin z}{1 - \cos(z^3)}\, dz,$$ where $C = \{z \in \mathbb{C} : z = x + iy,\ \ \|x\| + \|y\| = 1,\ \ x, y \in \mathbb{R}\}$ is oriented positively as a simple closed curve. Then, the value of $120I$ is equal to _____ (in integer)
Q.34	Let $\alpha, \beta, \gamma, \delta \in \mathbb{R}$ be such that the quadrature formula $$\int_{-1}^{1} f(x)\, dx = \alpha f(-1) + \beta f(1) + \gamma f'(-1) + \delta f'(1)$$ is exact for all polynomials of degree less than or equal to 3. Then, $9(\alpha^2 + \beta^2 + \gamma^2 + \delta^2)$ is equal to _____ (in integer)

Q.35	Let $y(x)$ be the solution of the initial value problem $$\frac{dy}{dx} = \sin\big(\pi(x+y)\big), \qquad y(0) = 0.$$ Using Euler's method, with the step-size $h = 0.5$, the approximate value of $y(1.5) + 2y(1)$ is equal to ______ (in integer)

Q.36 – Q.65 Carry TWO marks Each

Q.36	Consider the linear system $A\boldsymbol{x} = \boldsymbol{b}$, where $A = [a_{ij}]$, $i, j = 1, 2, 3$, and $a_{ii} \neq 0$ for $i = 1, 2, 3$, is a matrix with entries in $\mathbb{R}$. For $D = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$, let $$D^{-1}A = \begin{bmatrix} 1 & 1 & -2 \\ 3 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad D^{-1}\boldsymbol{b} = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}.$$ Consider the following two statements: S1: The approximation of $\boldsymbol{x}$ after one iteration of the Jacobi scheme with initial vector $\boldsymbol{x_0} = [1 \quad 1 \quad 1]^T$ is $\boldsymbol{x_1} = [5 \quad -1 \quad -1]^T$. S2: There exists an initial vector $\boldsymbol{x_0}$ for which Jacobi iterative scheme diverges. Then, which one of the following is correct?
(A)	S1 is TRUE and S2 is FALSE
(B)	S2 is TRUE and S1 is FALSE
(C)	both S1 and S2 are TRUE
(D)	neither S1 nor S2 is TRUE

Q.37	Let $y(x)$ be the solution of the differential equation $$x^2y'' + 7xy' + 9y = x^{-3}\log_e x, \quad x > 0,$$ satisfying $y(1) = 0$ and $y'(1) = 0$. Then, the value of $y(e)$ is equal to
(A)	$\frac{1}{3}e^{-3}$
(B)	$\frac{1}{6}e^{-3}$
(C)	$\frac{2}{3}e^{-3}$
(D)	$\frac{1}{2}e^{-3}$

Q.38	Let $y_1(x)$ and $y_2(x)$ be the two linearly independent solutions of the differential equation $$(1+x^2)y'' - xy' + (\cos^2 x)y = 0,$$ satisfying the initial conditions $$y_1(0) = 3, \quad y_1'(0) = -1 \text{ and } y_2(0) = -5, \quad y_2'(0) = 2.$$ Define $W(x) = \begin{vmatrix} y_1(x) & y_2(x) \\ y_1'(x) & y_2'(x) \end{vmatrix}$. Then, the value of $W\left(\frac{1}{2}\right)$ is
(A)	$\frac{\sqrt{5}}{4}$
(B)	$\frac{\sqrt{5}}{2}$
(C)	$\frac{2}{\sqrt{5}}$
(D)	$\frac{4}{\sqrt{5}}$

Q.39	Let C be the curve of intersection of the surfaces $z^2 = x^2 + y^2$ and $4x + z = 7$. If P is a point on C at a minimum distance from the xy-plane, then the distance of P from the origin is
(A)	$\frac{7}{5}$
(B)	$\frac{7\sqrt{2}}{5}$
(C)	$\frac{14}{5}$
(D)	$\frac{14\sqrt{2}}{5}$

Q.40	Let $u(x,t)$ be the solution of the initial-value problem $$\frac{\partial^2 u}{\partial t^2} - 9\frac{\partial^2 u}{\partial x^2} = 0, \quad x \in \mathbb{R}, \quad t > 0, \; u(x,0) = e^x, \; \frac{\partial u}{\partial t}(x,0) = \sin x.$$ Then, the value of $u\left(\frac{\pi}{2}, \frac{\pi}{6}\right)$ is
(A)	$\frac{1}{2}\left(e^{\pi} - \frac{1}{3}\right)$
(B)	$\frac{1}{2}\left(e^{\pi} + \frac{1}{3}\right)$
(C)	$\frac{1}{2}\left(e^{\pi} + \frac{5}{3}\right)$
(D)	$\frac{1}{2}\left(e^{\pi} - \frac{5}{3}\right)$

Q.41	Let T be the Möbius transformation that maps the points 0, $\frac{1}{2}$ and 1 conformally onto the points -3, ∞ and 2, respectively, in the extended complex plane. If T maps the circle centred at 1 with radius k onto a straight line given by the equation $\alpha x + \beta y + \gamma = 0$, then the value of $\frac{2k(\alpha+\beta)+\gamma}{\alpha+\beta-2k\gamma}$ is equal to
(A)	$\frac{1}{7}$
(B)	$\frac{2}{7}$
(C)	$\frac{1}{3}$
(D)	$\frac{2}{3}$

Q.42	Let $U = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$ and $D = \{z \in \mathbb{C} : \|z\| < 1\}$, where $\text{Im}(z)$ denotes the imaginary part of z. Let S be the set of all bijective analytic functions $f: U \to D$ such that $f(i) = 0$. Then, the value of $\sup_{f \in S} \|f(4i)\|$ is
(A)	0
(B)	$\frac{1}{4}$
(C)	$\frac{1}{2}$
(D)	$\frac{3}{5}$

Q.43	Let Ω be a non-empty open connected subset of $\mathbb{C}$ and $f:\Omega\to\mathbb{C}$ be a non-constant function. Let the functions $f^2:\Omega\to\mathbb{C}$ and $f^3:\Omega\to\mathbb{C}$ be defined by $$f^2(z)=\big(f(z)\big)^2 \text{ and } f^3(z)=\big(f(z)\big)^3,\quad z\in\Omega.$$ Consider the following two statements: S1: If f is continuous in Ω and f^2 is analytic in Ω, then f is analytic in Ω. S2: If f^2 and f^3 are analytic in Ω, then f is analytic in Ω. Then, which one of the following is correct?
(A)	S1 is TRUE and S2 is FALSE
(B)	S2 is TRUE and S1 is FALSE
(C)	both S1 and S2 are TRUE
(D)	neither S1 nor S2 is TRUE

Q.44	In the following, all subsets of Euclidean spaces are considered with the respective subspace topologies. Define an equivalence relation $\sim$ on the sphere $$S = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\}$$ by $(x_1, x_2, x_3) \sim (y_1, y_2, y_3)$ if $x_3 = y_3$, for $(x_1, x_2, x_3), (y_1, y_2, y_3) \in S$. Let $[x_1, x_2, x_3]$ denote the equivalence class of (x_1, x_2, x_3), and let X denote the set of all such equivalence classes. Let $\mathcal{L} : S \to X$ be given by $\mathcal{L}\big((x_1, x_2, x_3)\big) = [x_1, x_2, x_3]$. If X is provided with the quotient topology induced by the map $\mathcal{L}$, then which one of the following is TRUE?
(A)	X is homeomorphic to $\{x \in \mathbb{R} : -1 \le x \le 1\}$
(B)	X is homeomorphic to $\{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1\}$
(C)	X is homeomorphic to $\{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 \le 1\}$
(D)	X is homeomorphic to $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1^2 + x_2^2 = 1 \text{ and } -1 \le x_3 \le 1\}$

Q.45	Consider the following two spaces: $X = (C[-1,1],\ \|\cdot\|_\infty)$, the space of all real-valued continuous functions defined on $[-1,1]$ equipped with the norm $\| f \|_\infty = \sup_{t\in[-1,1]} \|f(t)\|$. $Y = (C[-1,1],\ \|\cdot\|_2)$, the space of all real-valued continuous functions defined on $[-1,1]$ equipped with the norm $\| f \|_2 = \left(\int_{-1}^{1} \|f(t)\|^2 dt\right)^{1/2}$. Let W be the linear span over $\mathbb{R}$ of all the Legendre polynomials. Then, which one of the following is correct?
(A)	W is dense in X but not in Y
(B)	W is dense in Y but not in X
(C)	W is dense in both X and Y
(D)	W is dense neither in X nor in Y

Q.46	Consider the metric spaces $X = (\mathbb{R}, d_1)$ and $Y = ([0,1], d_2)$ with the metrics defined by $d_1(x,y) = \lvert x - y\rvert$, $x, y \in \mathbb{R}$ and $d_2(x,y) = \lvert x - y\rvert$, $x, y \in [0,1]$, respectively. Then, which one of the following is TRUE?
(A)	$\left[0, \frac{1}{4}\right)$ is open in X but not in Y
(B)	$\left[0, \frac{1}{4}\right)$ is open in Y but not in X
(C)	$\left[0, \frac{1}{4}\right)$ is open in both X and Y
(D)	$\left[0, \frac{1}{4}\right)$ is open neither in X nor in Y
Q.47	Let K be an algebraically closed field containing a finite field F. Let L be the subfield of K consisting of elements of K that are algebraic over F. Consider the following statements: S1: L is algebraically closed. S2: L is infinite. Then, which one of the following is correct?
(A)	S1 is TRUE and S2 is FALSE
(B)	S2 is TRUE and S1 is FALSE
(C)	both S1 and S2 are TRUE
(D)	neither S1 nor S2 is TRUE

Q.48	Let $M_2(\mathbb{R})$ be the vector space (over $\mathbb{R}$) of all 2×2 matrices with entries in $\mathbb{R}$. Consider the linear transformation $T: M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by $T(X) = AXB$, where $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 5 \\ -2 & -1 \end{bmatrix}$. If P is the matrix representation of T with respect to the standard basis of $M_2(\mathbb{R})$, then which of the following is/are TRUE?
(A)	P is an invertible matrix
(B)	The trace of P is 25
(C)	The rank of $(P^2 - 4I_4)$ is 4, where I_4 is the 4×4 identity matrix
(D)	The nullity of $(P - 2I_4)$ is 0, where I_4 is the 4×4 identity matrix

Q.49	Consider the linear programming problem (LPP): Maximize $Z = 3x_1 + 5x_2$ Subject to $x_1 + x_3 = 4$, $2x_2 + x_4 = 12$, $3x_1 + 2x_2 + x_5 = 18$, $x_1, x_2, x_3, x_4, x_5 \geq 0$. Given that $x_B = (x_3, x_2, x_1)^T$ forms the optimal basis of the LPP with basis matrix B and respective $B^{-1} = \begin{bmatrix} \alpha & \beta & -\beta \\ 0 & \gamma & 0 \\ 0 & -\beta & \beta \end{bmatrix}$. If (p, q, r) is the optimal solution of the dual of the LPP, then which of the following is/are TRUE?
(A)	$\alpha + 3\beta + 2\gamma = 3$
(B)	$\alpha - 3\beta + 4\gamma = 1$
(C)	$p + q + r = \frac{5}{2}$
(D)	$p^2 + q^2 + r^2 = \frac{17}{4}$

Q.50	Let $0 < \alpha < 1$. Define $$C^{\alpha}[0,1] = \left\{ f:[0,1] \to \mathbb{R} : \sup_{\substack{s \neq t \\ s,t \in [0,1]}} \frac{\lvert f(t) - f(s) \rvert}{\lvert t - s \rvert^{\alpha}} < \infty \right\}.$$ It is given that $C^{\alpha}[0,1]$ is a Banach space with respect to the norm $\lVert \cdot \rVert_{\alpha}$ given by $$\lVert f \rVert_{\alpha} = \lvert f(0) \rvert + \sup_{\substack{s \neq t \\ s,t \in [0,1]}} \frac{\lvert f(t) - f(s) \rvert}{\lvert t - s \rvert^{\alpha}}.$$ Let $C[0,1]$ be the space of all real-valued continuous functions on $[0,1]$ with the norm $\lVert f \rVert_{\infty} = \sup_{0 \le t \le 1} \lvert f(t) \rvert$. If $T: C^{\alpha}[0,1] \to C[0,1]$ is the map $Tf = f$, $f \in C^{\alpha}[0,1]$, then which of the following is/are TRUE?
(A)	T is a compact linear map
(B)	Image of T is closed in $C[0,1]$
(C)	Image of T is dense in $C[0,1]$
(D)	T is not a bounded linear map

Q.51	Let $u(x,t)$ be the solution of the initial value problem $$\frac{\partial u}{\partial t}+3\frac{\partial u}{\partial x}=u,\quad x\in\mathbb{R},\qquad t>0,\qquad u(x,0)=\cos x,$$ and let $v(x,t)$ be the solution of the initial value problem $$\frac{\partial v}{\partial t}+3\frac{\partial v}{\partial x}=v^2,\quad x\in\mathbb{R},\qquad t>0,\qquad v(x,0)=\cos x.$$ Then, which of the following is/are TRUE?
(A)	$\lvert u(x,t)\rvert \le e^t$ for all $x\in\mathbb{R}$ and for all $t>0$
(B)	$v(x,1)$ is not defined for certain values of $x\in\mathbb{R}$
(C)	$v(x,1)$ is not defined for any $x\in\mathbb{R}$
(D)	$u(2\pi,\pi)=-e^{\pi}$

Q.52	Let $u(x,t)$ be the solution of the initial-boundary value problem $$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \qquad t > 0,$$ $$u(0,t) = u(1,t) = 0, u(x,0) = 2x(1-x).$$ Then, which of the following is/are TRUE?
(A)	$0 \le u(x,t) \le \frac{1}{4}$ for all $t \ge 0$ and $x \in [0,1]$
(B)	$u(x,t) = u(1-x,t)$ for all $t \ge 0$ and $x \in [0,1]$
(C)	$\int_0^1 (u(x,t))^2\, dx$ is a decreasing function of t
(D)	$\int_0^1 (u(x,t))^2\, dx$ is not a decreasing function of t
Q.53	Consider the function $f: \mathbb{R}^2 \to \mathbb{R}^2$ given by $$f(x,y) = (e^{2\pi x}\cos 2\pi y,\ \ e^{2\pi x}\sin 2\pi y).$$ Then, which of the following is/are TRUE?
(A)	If G is open in $\mathbb{R}^2$, then $f(G)$ is open in $\mathbb{R}^2$
(B)	If G is closed in $\mathbb{R}^2$, then $f(G)$ is closed in $\mathbb{R}^2$
(C)	If G is dense in $\mathbb{R}^2$, then $f(G)$ is dense in $\mathbb{R}^2$
(D)	f is surjective

Q.54	Let $\{x_k\}_{k=1}^{\infty}$ be an orthonormal set of vectors in a real Hilbert space X with inner product $<\cdot\,,\ \cdot>$. Let $n \in \mathbb{N}$, and let Y be the linear span of $\{x_k\}_{k=1}^{n}$ over $\mathbb{R}$. For $x \in X$, let $S_n(x) = \sum_{k=1}^{n} < x, x_k > x_k$. Then, which of the following is/are TRUE?
(A)	$S_n(x)$ is the orthogonal projection of x onto Y
(B)	$S_n(x)$ is the orthogonal projection of x onto $Y^{\perp}$
(C)	$(x - S_n(x))$ is orthogonal to $S_n(x)$ for all x in X
(D)	$\sum_{k=1}^{n} \lvert < x, x_k > \rvert^2 = \lVert x \rVert^2$ for all x in X
Q.55	Consider the sequence $\{f_n\}$ of continuous functions on $[0,1]$ defined by $$f_1(x) = \frac{x}{2}, \qquad f_{n+1}(x) = f_n(x) - \frac{1}{2}\left(\left(f_n(x)\right)^2 - x \right), \qquad n = 1, 2, 3, \ldots .$$ Then, which of the following is/are TRUE?
(A)	The sequence $\{f_n\}$ converges pointwise but not uniformly on $[0,1]$
(B)	The sequence $\{f_n\}$ converges uniformly on $[0,1]$
(C)	$\sqrt{x} - f_n(x) > \frac{2\sqrt{x}}{2+n\sqrt{x}}$ for all $x \in [0,1]$ and $n = 1, 2, 3, \ldots$
(D)	$0 \le f_n(x) \le \sqrt{x}$ for all $x \in [0,1]$ and $n = 1, 2, 3, \ldots$

Q.56	For $x \in (0,\pi)$, let $u_n(x) = \frac{\sin nx}{\sqrt{n}}$, $n = 1,2,3,\ldots$. Then, which of the following is/are TRUE?
(A)	$\sum_{n=1}^{\infty} u_n(x)$ converges uniformly on $(0,\pi)$
(B)	$\sum_{n=1}^{\infty} u_n'(x)$ converges uniformly on $(0,\pi)$
(C)	$\sum_{n=1}^{\infty} u_n(x)$ converges pointwise but not uniformly on $(0,\pi)$
(D)	$\sum_{n=1}^{\infty} u_n(x)$ converges uniformly on every compact subset of $(0,\pi)$
Q.57	Let $\mathbb{R}^1$ and $\mathbb{R}^2$ be provided with the respective Euclidean topologies, and let $$S^1 = \{(x_1,x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 = 1\}$$ be assigned the subspace topology induced from $\mathbb{R}^2$. If $f: S^1 \to \mathbb{R}^1$ is a non-constant continuous function, then which of the following is/are TRUE?
(A)	f maps closed sets to closed sets
(B)	f is injective
(C)	f is surjective
(D)	There exists $\lambda \in \mathbb{R}$ such that $f(\cos\lambda, \sin\lambda) = f(-\cos\lambda, -\sin\lambda)$

Q.58	Let X be an uncountable set. Let the topology on X be defined by declaring a subset $U \subseteq X$ to be open if $X - U$ is either empty or finite or countable, and the empty set to be open. Then, which of the following is/are TRUE?
(A)	Every compact subset of X is closed
(B)	Every closed subset of X is compact
(C)	X is T_1 (singleton subsets are closed) but not T_2 (Hausdorff)
(D)	X is T_2 (Hausdorff)

Q.59	All rings considered below are assumed to be associative and commutative with $1 \neq 0$. Further, all ring homomorphisms map 1 to 1. Consider the following statements about such a ring R: P1: R is isomorphic to the product of two rings R_1 and R_2. P2: $\exists\, r_1, r_2 \in R$ such that $r_1^2 = r_1 \neq 0 \neq r_2 = r_2^2,\ r_1 r_2 = 0$ and $r_1 + r_2 = 1$. P3: $\exists$ ideals $I_1, I_2 \subseteq R$ with $R \neq I_1 \neq (0) \neq I_2 \neq R$ such that $R = I_1 + I_2$ and $I_1 \cap I_2 = (0)$. P4: $\exists\, a, b \in R$ with $a \neq 0 \neq b$ such that $ab = 0$. Then, which of the following is/are TRUE?
(A)	$P_1 \implies P_2$
(B)	$P_2 \implies P_3$
(C)	$P_3 \implies P_4$
(D)	$P_4 \implies P_1$

Q.60	Let $E \subset F$ and $F \subset K$ be field extensions which are not algebraic. Let $\alpha \in K$ be algebraic over F and $\alpha \notin F$. Let L be the subfield of K generated over E by the coefficients of the monic polynomial of minimal degree over F which has α as a zero. Then, which of the following is/are TRUE?
(A)	$F(\alpha) \supset L(\alpha)$ is a finite extension if and only if $F \supset L$ is a finite extension
(B)	The dimension of $L(\alpha)$ over L is greater than the dimension of $F(\alpha)$ over F
(C)	The dimension of $L(\alpha)$ over L is smaller than the dimension of $F(\alpha)$ over F
(D)	$F(\alpha) \supset L(\alpha)$ is an algebraic extension if and only if $F \supset L$ is an algebraic extension

Q.61	Consider the inner product space of all real-valued continuous functions defined on $[-1, 1]$ with the inner product $$\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)\ dx.$$ If $p(x) = \alpha + \beta x^2 - 30x^4$, $\alpha, \beta \in \mathbb{R}$ is orthogonal to all the polynomials having degree less than or equal to 3, with respect to this inner product, then $\alpha + 5\,\beta$ is equal to _____ (in integer)

Q.62	For $X = (x_1, x_2, x_3)^T \in \mathbb{R}^3$, consider the quadratic form: $$Q(X) = 2x_1^2 + 2x_2^2 + 3x_3^2 + 4x_1x_2 + 2x_1x_3 + 2x_2x_3.$$ Let M be the symmetric matrix associated with the quadratic form $Q(X)$ with respect to the standard basis of $\mathbb{R}^3$. Let $Y = (y_1, y_2, y_3)^T \in \mathbb{R}^3$ be a non-zero vector, and let $$a_n = \frac{Y^T(M + I_3)^{n+1}Y}{Y^T(M + I_3)^n Y}, \qquad n = 1, 2, 3, \dots,$$ where I_3 is the 3×3 identity matrix. Then, the value of $\lim_{n\to\infty} a_n$ is _____ (in integer)
Q.63	Let α, β be distinct non-zero real numbers, and let $Q(z)$ be a polynomial of degree less than 5. If the function $$f(z) = \frac{\alpha^6 \sin \beta z - \beta^6 (e^{2\alpha z} - Q(z))}{z^6}$$ satisfies Morera's theorem in $\mathbb{C}\setminus\{0\}$, then the value of $\frac{\alpha}{4\beta}$ is equal to _____ (in integer)
Q.64	Let G be a group with identity element e, and let $g, h \in G$ be such that the following hold: (i) $g \neq e,\ g^2 = e,$ (ii) $h \neq e,\ h^2 \neq e,$ and $ghg^{-1} = h^2.$ Then, the least positive integer n for which $h^n = e$ is _____ (in integer)

Q.65	Let $(\mathbb{R}^2, d_1)$ and $(\mathbb{R}^2, d_2)$ be two metric spaces with $$d_1\big((x_1,x_2),(y_1,y_2)\big) = \lvert x_1 - y_1\rvert + \lvert x_2 - y_2\rvert$$ and $$d_2\big((x_1,x_2),(y_1,y_2)\big) = \frac{d_1\big((x_1,x_2),(y_1,y_2)\big)}{1 + d_1\big((x_1,x_2),(y_1,y_2)\big)}\ .$$ If the open ball centred at $(0,0)$ with radius $\frac{1}{7}$ in $(\mathbb{R}^2, d_1)$ is equal to the open ball centred at $(0,0)$ with radius $\frac{1}{\alpha}$ in $(\mathbb{R}^2, d_2)$, then the value of α is ______ (in integer)