MHT-CET Mathematics Sample Paper- 11

Duration: 90 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. If $f(x) = \lim_{n\to\infty} \frac{\log(2+x)-x^{2n}\sin x}{1+x^{2n}}$, then the value of $f(1)$ so that $f(x)$ is continuous at $x = 1$ is:

(A) $\log 3$

(B) $\frac{1}{2}(\log 3 - \sin 1)$

(C) $\frac{1}{2}(\sin 1 - \log 3)$

(D) 0

Q2. The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3}$ is:

(A) $2/3$

(B) 0

(C) $1/3$

(D) 1

Q3. Let $f(x) = [x^2]\sin(\pi x)$ where [.] denotes the greatest integer function. The number of points in the interval $(0, 2)$ where $f(x)$ is discontinuous is:

(A) 3

(B) 2

(C) 1

(D) 0

Q4. If $x = e^y + e^{y+\ldots\infty}$, then $\frac{dy}{dx}$ is equal to:

(A) $\frac{1}{x}$

(B) $\frac{1}{1+x}$

(C) $\frac{1-x}{x}$

(D) $\frac{x}{1+x}$

Q5. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$, then $\frac{dy}{dx}$ is:

(A) $\frac{-x}{\sqrt{1-x^4}}$

(B) $\frac{x}{\sqrt{1-x^4}}$

(C) $\frac{-1}{2\sqrt{1-x^4}}$

(D) $\frac{1}{2\sqrt{1-x^4}}$

Q6. If $2^x + 2^y = 2^{x+y}$, then the value of $\frac{dy}{dx}$ at $x = y = 1$ is:

(A) 1

(B) 2

(C) -1

(D) 0

Q7. If $f(x) = \log_x(\ln x)$, then $f'(e)$ is:

(A) $1/e$

(B) e

(C) 1

(D) 0

Q8. If $y = (\sin x)^{\cos x}$, then $\frac{dy}{dx}$ at $x = \pi/2$ is:

(A) 0

(B) 1

(C) -1

(D) e

Q9. The function $f(x) = x^x$ has a stationary point at:

(A) $x = e$

(B) $x = 1/e$

(C) $x = 1$

(D) $x = \sqrt{e}$

Q10. The angle of intersection between the curves $y^2 = 4ax$ and $ay^2 = 4x^3$ at the origin is:

(A) $\pi/4$

(B) $\pi/3$

(C) $\pi/2$

(D) 0

Q11. A spherical balloon is being inflated at the rate of 35 cc/min. The rate of increase of its surface area when its radius is 14 cm is:

(A) 5 sq.cm/min

(B) 10 sq.cm/min

(C) 20 sq.cm/min

(D) 2.5 sq.cm/min

Q12. The maximum value of $f(x) = \frac{\log x}{x}$ in the interval $(0, \infty)$ is:

(A) $1/e$

(B) e

(C) 1

(D) e^2

Q13. The equation of the normal to the curve $x^2 = 4y$ which passes through the point $(1, 2)$ is:

(A) $x + y = 3$

(B) $x - y = 3$

(C) $x + y = 1$

(D) $2x + y = 4$

Q14. The value of $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ is:

(A) $-\frac{(x^4+1)^{1/4}}{x} + C$

(B) $\frac{(x^4+1)^{1/4}}{x} + C$

(C) $-\frac{(x^4+1)^{1/4}}{x^2} + C$

(D) $\frac{(x^4+1)^{1/4}}{x^4} + C$

Q15. The value of $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x+\cos^n x} \, dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q16. $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ is equal to:

(A) $e^x \tan(x/2) + C$

(B) $e^x \cot(x/2) + C$

(C) $e^x \sin(x/2) + C$

(D) $e^x \cos(x/2) + C$

Q17. The value of $\int_0^1 \frac{\log(1+x)}{1+x^2} \, dx$ is:

(A) $\frac{\pi}{8} \log 2$

(B) $\frac{\pi}{4} \log 2$

(C) $\frac{\pi}{2}\log 2$

(D) $\log 2$

Q18. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ is:

(A) $2[\sin x + x\cos\alpha] + C$

(B) $2[\sin x - x\cos\alpha] + C$

(C) $2[\cos x + x\sin\alpha] + C$

(D) None

Q19. The area bounded by the parabola $y^2 = 8x$ and its latus rectum is:

(A) $32/3$

(B) $16/3$

(C) $8/3$

(D) $64/3$

Q20. The area bounded by the curves $y = \sqrt{x}$ and $y = x^2$ is:

(A) $1/3$

(B) $2/3$

(C) 1

(D) $1/6$

Q21. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x = \sec x$ is:

(A) $\sec x$

(B) $\cos x$

(C) $\sin x$

(D) $\tan x$

Q22. The general solution of $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ is:

(A) $\sin(y/x) = cx$

(B) $\cos(y/x) = cx$

(C) $\tan(y/x) = cx$

(D) $\sin(y/x) = c/x$

Q23. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$ are:

(A) 2, 2

(B) 2, 1

(C) 3, 2

(D) 1, 2

Q24. If $|z - \frac{4}{z}| = 2$, then the maximum value of $|z|$ is:

(A) $\sqrt{5} + 1$

(B) $\sqrt{5} - 1$

(C) $\sqrt{5}$

(D) 2

Q25. If α, β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{19} + \beta^{19}$ is:

(A) 1

(B) -1

(C) 2

(D) 0

Q26. The area of the triangle formed by complex numbers z, iz and $z + iz$ in the Argand plane is:

(A) $|z|^2$

(B) $\frac{1}{2}|z|^2$

(C) $\frac{1}{4}|z|^2$

(D) 0

Q27. If α, β are the roots of $x^2 - px + q = 0$, then the value of $\alpha^4 + \beta^4$ is:

(A) $p^4 - 4p^2q + 2q^2$

(B) $p^4 - 4p^2q + q^2$

(C) $p^4 - 4p^2q - 2q^2$

(D) $p^4 + 4p^2q + 2q^2$

Q28. If the roots of $ax^2 + bx + c = 0$ are in the ratio $m : n$, then:

(A) $mna^2 = (m+n)^2 b$

(B) $mnb^2 = (m+n)^2 ac$

(C) $mnc^2 = (m+n)^2 ab$

(D) $abc = (m+n)^2$

Q29. If x is real, the minimum value of $\frac{x^2-x+1}{x^2+x+1}$ is:

(A) $1/3$

(B) 3

(C) $1/2$

(D) 1

Q30. If a, b, c are in H.P., then $\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in:

(A) A.P.

(B) G.P.

(C) H.P.

(D) None

Q31. The sum to n terms of the series $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5} + \ldots$ is:

(A) $\frac{n(n+3)}{4(n+1)(n+2)}$

(B) $\frac{n(n+1)}{4(n+2)(n+3)}$

(C) $\frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

(D) None

Q32. If $S_n = n^2P$ and $S_m = m^2P$ are sums of an A.P., then S_P is:

(A) P^3

(B) P^2

(C) $2P^2$

(D) P^4

Q33. The coefficient of x^n in the expansion of $(1+x)(1-x)^n$ is:

(A) $(n-1)(-1)^n$

(B) $(1-n)(-1)^n$

(C) $n(-1)^{n-1}$

(D) $n(-1)^n$

Q34. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in $(1+x)^n$ are in A.P., then:

(A) $n^2 - n(4r+1) + 4r^2 - 2 = 0$

(B) $n^2 - n(4r-1) + 4r^2 - 2 = 0$

(C) $n^2 - n(4r+1) + 4r^2 + 2 = 0$

(D) None

Q35. The number of ways in which 7 persons can be seated at a round table if two particular persons are not to sit together is:

(A) 120

(B) 480

(C) 720

(D) 960

Q36. A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that they are of the same color is:

(A) 5/108

(B) 1/6

(C) 4/9

(D) 5/18

Q37. If $nC_{12} = nC_8$, then the value of $22C_n$ is:

(A) 231

(B) 210

(C) 252

(D) 300

Q38. A problem in mathematics is given to three students whose chances of solving it are 1/2, 1/3, 1/4. The probability that the problem is solved is:

(A) 3/4

(B) 1/2

(C) 1/4

(D) 1/24

Q39. The distance between the parallel lines $y = mx + c$ and $y = mx + d$ is:

(A) $\frac{|c-d|}{\sqrt{1+m^2}}$

(B) $\frac{|c-d|}{\sqrt{1-m^2}}$

(C) $\frac{|c+d|}{\sqrt{1+m^2}}$

(D) $|c - d|$

Q40. If the line $y = mx + 1$ is a tangent to the circle $x^2 + y^2 = 1$, then m is:

(A) 1

(B) 0

(C) −1

(D) ∞

Q41. The equation of the circle passing through $(0,0)$, $(a,0)$ and $(0,b)$ is:

(A) $x^2 + y^2 + ax + by = 0$

(B) $x^2 + y^2 - ax - by = 0$

(C) $x^2 + y^2 - ax + by = 0$

(D) $x^2 + y^2 + ax - by = 0$

Q42. The length of the intercept made by the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ on the y-axis is:

(A) $2\sqrt{21}$

(B) 12

(C) $4\sqrt{3}$

(D) 10

Q43. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is:

(A) $4/3$

(B) $4/\sqrt{3}$

(C) $2/\sqrt{3}$

(D) $3/2$

Q44. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum is:

(A) 12

(B) 18

(C) 24

(D) 36

Q45. If P is a point on the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ and S, S' are foci, then $PS + PS'$ is:

(A) 4

(B) 8

(C) 16

(D) 7

Q46. The equation of the director circle of the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$ is:

(A) $x^2 + y^2 = 16$

(B) $x^2 + y^2 = 34$

(C) $x^2 + y^2 = 25$

(D) $x^2 + y^2 = 4$

Q47. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is:

(A) $3/2$

(B) $-3/2$

(C) 1

(D) 0

Q48. The scalar triple product $[\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{c} - \vec{a}]$ is:

(A) 0

(B) $2[\vec{a}\vec{b}\vec{c}]$

(C) $[\vec{a}\vec{b}\vec{c}]$

(D) None

Q49. The direction cosines of a line which is equally inclined to the coordinate axes are:

(A) $(1, 1, 1)$

(B) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(C) $(0, 0, 1)$

(D) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$

Q50. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ is:

(A) $\pi/3$

(B) $\cos^{-1}(1/2)$

(C) $\pi/2$

(D) $\pi/4$

Detailed Solutions

Q1.

Solution

Concept: Continuity using limits for different ranges of x.

Solution:

Step 1: Given

$$f(x) = \lim_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{1+x^{2n}}$$

For $|x| < 1$:

$$x^{2n} \to 0$$

Hence,

$$f(x) = \log(2+x)$$

For $|x| > 1$:

$$x^{2n} \to \infty$$

Dividing numerator and denominator by x^{2n}:

$$f(x) = \lim_{n\to\infty} \frac{\frac{\log(2+x)}{x^{2n}} - \sin x}{\frac{1}{x^{2n}} + 1}$$

$$f(x) = -\sin x$$

Step 2: Continuity at $x = 1$

$$f(1) = \lim_{x\to 1} f(x)$$

Left hand limit:

$$\lim_{x\to 1^-} f(x) = \log 3$$

Right hand limit:

$$\lim_{x\to 1^+} f(x) = -\sin 1$$

At $x = 1$,

$$x^{2n} = 1$$

Thus,

$$f(1) = \frac{\log 3 - \sin 1}{2}$$

Final Answer: $\boxed{\frac{1}{2}(\log 3 - \sin 1)}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: Expansion of $\sin\sqrt{t}$ near $t = 0$.

Solution:

Step 1: Given

$$\lim_{x\to 0} \frac{\int_0^{x^2} \sin\sqrt{t}\,dt}{x^3}$$

Using

$$\sin\sqrt{t} \sim \sqrt{t}$$

Therefore,

$$\int_0^{x^2} \sin\sqrt{t}\,dt \sim \int_0^{x^2} \sqrt{t}\,dt$$

$$= \left[\frac{2}{3}t^{3/2}\right]_0^{x^2}$$

$$= \frac{2}{3}(x^2)^{3/2}$$

$$= \frac{2}{3}x^3$$

Step 2: Evaluate limit

$$\lim_{x\to 0} \frac{\frac{2}{3}x^3}{x^3} = \frac{2}{3}$$

Final Answer: $\boxed{\frac{2}{3}}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: Greatest integer function is discontinuous when its argument crosses an integer.

Solution:

Step 1: Given

$$f(x) = [x^2]\sin(\pi x)$$

The function $[x^2]$ changes value when:

$$x^2 = 1, 2, 3, \ldots$$

Inside interval $(0, 2)$:

$$x = 1, \sqrt{2}, \sqrt{3}$$

Step 2: Check continuity

At $x = 1$:

$$\sin(\pi) = 0$$

Hence,

$$f(x) = 0$$

from both sides.

So function is continuous at $x = 1$.

At $x = \sqrt{2}$:

$$\sin(\pi\sqrt{2}) \neq 0$$

Jump occurs due to $[x^2]$.

Hence discontinuous.

At $x = \sqrt{3}$:

$$\sin(\pi\sqrt{3}) \neq 0$$

Hence discontinuous.

Step 3: Number of discontinuities

$$2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: Infinite exponential expression and implicit differentiation.

Solution:

Step 1: Given

$$x = e^y + e^{y+\ldots\infty}$$

The repeating part equals x itself.

Thus,

$$x = e^y + x$$

Interpreting correctly:

$$x = e^{y+x}$$

Taking logarithm:

$$\log x = y + x$$

$$y = \log x - x$$

Step 2: Differentiate

$$\frac{dy}{dx} = \frac{1}{x} - 1$$

$$= \frac{1-x}{x}$$

Final Answer: $\boxed{\frac{1-x}{x}}$

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept: Simplification using trigonometric identities.

Solution:

Step 1: Given

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$$

Let

$$a = \sqrt{1+x^2}, \quad b = \sqrt{1-x^2}$$

Then,

$$\tan y = \frac{a+b}{a-b}$$

Using identity:

$$\tan\left(\frac{\pi}{4}+\theta\right) = \frac{1+\tan\theta}{1-\tan\theta}$$

We get

$$\tan y = \tan\left(\frac{\pi}{4}+\tan^{-1}\frac{b}{a}\right)$$

Hence,

$$y = \frac{\pi}{4}+\tan^{-1}\frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}$$

Step 2: Differentiate

Let

$$u = \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}$$

Then,

$$y = \frac{\pi}{4}+\tan^{-1}u$$

Differentiating,

$$\frac{dy}{dx} = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

After simplification,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^4}}$$

Final Answer: $\boxed{-\frac{1}{2\sqrt{1-x^4}}}$

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: Implicit differentiation.

Solution:

Step 1: Given

$$2^x + 2^y = 2^{x+y}$$

Differentiate implicitly:

$$2^x \ln 2 + 2^y \ln 2 \frac{dy}{dx} = 2^{x+y} \ln 2 \left(1 + \frac{dy}{dx}\right)$$

Cancel $\ln 2$:

$$2^x + 2^y \frac{dy}{dx} = 2^{x+y} \left(1 + \frac{dy}{dx}\right)$$

Step 2: Put $x = y = 1$

$$2 + 2\frac{dy}{dx} = 4\left(1 + \frac{dy}{dx}\right)$$

$$2 + 2\frac{dy}{dx} = 4 + 4\frac{dy}{dx}$$

$$-2 = 2\frac{dy}{dx}$$

$$\frac{dy}{dx} = -1$$

Final Answer: $\boxed{-1}$

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: Logarithm differentiation using change of base formula.

Solution:

Step 1: Given

$$f(x) = \log_x(\ln x)$$

Using change of base:

$$f(x) = \frac{\ln(\ln x)}{\ln x}$$

Step 2: Differentiate

Using quotient rule:

$$f'(x) = \frac{\left(\frac{1}{\ln x} \cdot \frac{1}{x}\right)(\ln x) - \ln(\ln x)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

$$= \frac{1 - \ln(\ln x)}{x(\ln x)^2}$$

Step 3: Put $x = e$

$$\ln e = 1, \quad \ln 1 = 0$$

$$f'(e) = \frac{1}{e}$$

Final Answer: $\boxed{\frac{1}{e}}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: Logarithmic differentiation.

Solution:

Step 1: Given

$$y = (\sin x)^{\cos x}$$

Taking logarithm:

$$\ln y = \cos x \ln(\sin x)$$

Step 2: Differentiate

$$\frac{1}{y}\frac{dy}{dx} = -\sin x \ln(\sin x) + \cos x \cdot \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[-\sin x \ln(\sin x) + \frac{\cos^2 x}{\sin x}\right]$$

Step 3: Put $x = \frac{\pi}{2}$

$$\sin\frac{\pi}{2} = 1, \quad \cos\frac{\pi}{2} = 0$$

$$\ln 1 = 0$$

Hence,

$$\frac{dy}{dx} = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: Stationary point using first derivative.

Solution:

Step 1: Given

$$f(x) = x^x$$

Taking logarithm:

$$\ln f = x \ln x$$

Differentiate:

$$\frac{f'}{f} = \ln x + 1$$

$$f' = x^x(\ln x + 1)$$

Step 2: Stationary point

For stationary point:

$$f' = 0$$

Since $x^x \neq 0$,

$$\ln x + 1 = 0$$

$$\ln x = -1$$

$$x = e^{-1} = \frac{1}{e}$$

Final Answer: $\boxed{\frac{1}{e}}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: Angle of intersection using tangents at point of intersection.

Solution:

Step 1: First curve

$$y^2 = 4ax$$

At origin:

$$y^2 = 0$$

The tangent is:

$$y = 0$$

Step 2: Second curve

$$ay^2 = 4x^3$$

Lowest degree terms:

$$ay^2 = 0$$

Hence tangent is also:

$$y = 0$$

Step 3: Angle between tangents

Both tangents are same.

Therefore angle between curves:

$$0$$

Final Answer: $\boxed{0}$

Answer: (D)

Go Back to Question 10

Q11.

Solution

Concept: Related rates using sphere formulas.

Solution:

Step 1: Given

Rate of increase of volume:

$$\frac{dV}{dt} = 35 \text{ cc/min}$$

Volume of sphere:

$$V = \frac{4}{3}\pi r^3$$

Differentiate:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$35 = 4\pi(14)^2 \frac{dr}{dt}$$

Using

$$\pi = \frac{22}{7}$$

$$35 = 2464 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{5}{352}$$

Step 2: Surface area

$$S = 4\pi r^2$$

Differentiate:

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8 \times \frac{22}{7} \times 14 \times \frac{5}{352}$$

$$= 10$$

Final Answer: $\boxed{10 \text{ sq.cm/min}}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: Maximum value using derivatives.

Solution:

Step 1: Given

$$f(x) = \frac{\log x}{x}$$

Using natural logarithm:

$$f(x) = \frac{\ln x}{x}$$

Differentiate:

$$f'(x) = \frac{1 - \ln x}{x^2}$$

Step 2: Critical point

$$f'(x) = 0$$

$$1 - \ln x = 0$$

$$\ln x = 1$$

$$x = e$$

Step 3: Maximum value

$$f(e) = \frac{\ln e}{e} = \frac{1}{e}$$

Final Answer: $\boxed{\frac{1}{e}}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: Equation of normal to parabola.

Solution:

Given:

$$x^2 = 4y$$

For $x^2 = 4ay$,

$$a = 1$$

Normal at parameter t:

$$y = -tx + 2t + t^3$$

Passing through $(1, 2)$:

$$2 = -t + 2t + t^3$$

$$t^3 + t - 2 = 0$$

$$(t - 1)(t^2 + t + 2) = 0$$

$$t = 1$$

Hence normal:

$$y = -x + 3$$

$$x + y = 3$$

Final Answer: $\boxed{x + y = 3}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: Reverse differentiation method.

Solution:

Step 1: Consider

$$F(x) = -\frac{(x^4+1)^{1/4}}{x}$$

Differentiate:

$$F'(x) = -\left[\frac{x^3}{(x^4+1)^{3/4}} \cdot \frac{1}{x} - \frac{(x^4+1)^{1/4}}{x^2}\right]$$

$$= -\left[\frac{x^2}{(x^4+1)^{3/4}} - \frac{x^4+1}{x^2(x^4+1)^{3/4}}\right]$$

$$= \frac{1}{x^2(x^4+1)^{3/4}}$$

Thus,

$$\int \frac{dx}{x^2(x^4+1)^{3/4}} = -\frac{(x^4+1)^{1/4}}{x} + C$$

Final Answer: $\boxed{-\frac{(x^4+1)^{1/4}}{x} + C}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: Property of definite integrals.

Solution:

Step 1: Let

$$I = \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}\, dx$$

Using property:

$$\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$$

$$I = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x}\, dx$$

Step 2: Add both

$$2I = \int_0^{\pi/2} \frac{\sin^n x + \cos^n x}{\sin^n x + \cos^n x}\, dx$$

$$2I = \int_0^{\pi/2} 1\, dx$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: Trigonometric identity and reverse differentiation.

Solution:

Step 1: Simplify integrand

Using

$$\frac{1+\sin x}{1+\cos x} = \frac{(1+\sin x)^2}{1-\sin^2 x} = \frac{1+\sin x}{\cos x}$$

$$= \sec x + \tan x$$

Also,

$$\sec x + \tan x = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Hence,

$$I = \int e^x (\sec x + \tan x)\, dx$$

Step 2: Observe derivative

$$\frac{d}{dx}\left(e^x \tan\frac{x}{2}\right) = e^x \tan\frac{x}{2} + e^x \cdot \frac{1}{2}\sec^2\frac{x}{2}$$

Using identities,

$$= e^x \left(\frac{1+\sin x}{\cos x}\right)$$

Thus,

$$\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = e^x \tan\frac{x}{2} + C$$

Final Answer: $\boxed{e^x \tan\frac{x}{2} + C}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: Substitution in definite integral.

Solution:

Step 1: Let

$$x = \tan\theta$$

Then,

$$dx = \sec^2\theta\, d\theta$$

and

$$1 + x^2 = \sec^2\theta$$

Limits:

$$x = 0 \Rightarrow \theta = 0$$

$$x = 1 \Rightarrow \theta = \frac{\pi}{4}$$

Thus,

$$I = \int_0^{\pi/4} \log(1 + \tan\theta)\, d\theta$$

Using identity:

$$1 + \tan\theta = \frac{\sin\theta + \cos\theta}{\cos\theta}$$

$$= \frac{\sqrt{2}\sin(\theta + \pi/4)}{\cos\theta}$$

Hence,

$$I = \int_0^{\pi/4} \log\sqrt{2}\, d\theta$$

$$= \frac{1}{2}\log 2 \cdot \frac{\pi}{4}$$

$$= \frac{\pi}{8}\log 2$$

Final Answer: $\boxed{\frac{\pi}{8}\log 2}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: Trigonometric identities.

Solution:

Step 1: Simplify numerator

$$\cos 2x - \cos 2\alpha = -2\sin(x+\alpha)\sin(x-\alpha)$$

Also,

$$\cos x - \cos\alpha = -2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}$$

Therefore,

$$\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} = 2\cos\frac{x+\alpha}{2}\cos\frac{x-\alpha}{2}$$

Using product formula:

$$= 2(\cos x + \cos\alpha)$$

Step 2: Integrate

$$I = \int 2(\cos x + \cos\alpha)\,dx$$

$$= 2(\sin x + x\cos\alpha) + C$$

Final Answer: $\boxed{2[\sin x + x\cos\alpha] + C}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: Area bounded by parabola and latus rectum.

Solution:

Step 1: Given parabola

$$y^2 = 8x$$

Comparing with

$$y^2 = 4ax$$

$$4a = 8 \Rightarrow a = 2$$

Latus rectum:

$$x = a = 2$$

Endpoints:

$$y = \pm 4$$

Step 2: Area

Area between parabola and latus rectum:

$$A = \int_{-4}^{4} \left(2 - \frac{y^2}{8}\right) dy$$

Using symmetry:

$$A = 2\int_{0}^{4} \left(2 - \frac{y^2}{8}\right) dy$$

$$= 2\left[2y - \frac{y^3}{24}\right]_0^4$$

$$= 2\left(8 - \frac{64}{24}\right)$$

$$= 2\left(8 - \frac{8}{3}\right)$$

$$= \frac{32}{3}$$

Final Answer: $\boxed{\frac{32}{3}}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: Area between curves using definite integration.

Solution:

Step 1: Given curves

$$y = \sqrt{x}, \quad y = x^2$$

Points of intersection:

$$\sqrt{x} = x^2$$

$$x = x^4$$

$$x(x^3 - 1) = 0$$

$$x = 0, 1$$

Step 2: Area between curves

For $0 < x < 1$:

$$\sqrt{x} > x^2$$

Hence,

$$A = \int_0^1 (\sqrt{x} - x^2)\, dx$$

$$= \int_0^1 \left(x^{1/2} - x^2\right)\, dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{3}$$

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: Integrating factor.

Solution:

$$\frac{dy}{dx} + y\tan x = \sec x$$

Here,

$$P = \tan x$$

$$IF = e^{\int \tan x\,dx}$$

$$= e^{\log(\sec x)} = \sec x$$

Final Answer: $\boxed{\sec x}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: Homogeneous differential equation.

Solution:

Put

$$v = \frac{y}{x} \Rightarrow y = vx$$

Then,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substitute:

$$x\frac{dv}{dx} = \tan v$$

$$\cot v\,dv = \frac{dx}{x}$$

Integrating:

$$\log(\sin v) = \log(cx)$$

$$\sin\left(\frac{y}{x}\right) = cx$$

Final Answer: $\boxed{\sin(y/x) = cx}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: Order and degree.

Solution:

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$$

Squaring:

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

Highest order derivative:

$$\frac{d^2y}{dx^2}$$

Hence,

$$\text{Order} = 2, \quad \text{Degree} = 2$$

Final Answer: $\boxed{2, 2}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: Triangle inequality.

Solution:

Given:

$$\left|z - \frac{4}{z}\right| = 2$$

Let

$$|z| = r$$

Then,

$$\left|r - \frac{4}{r}\right| \leq 2$$

Using upper bound:

$$r - \frac{4}{r} = 2$$

$$r^2 - 2r - 4 = 0$$

$$r = 1 + \sqrt{5}$$

Final Answer: $\boxed{1 + \sqrt{5}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: Cube roots of unity.

Solution:

$$x^2 + x + 1 = 0$$

Roots:

$$\alpha = \omega, \quad \beta = \omega^2$$

$$\alpha^{19} + \beta^{19} = \omega^{19} + \omega^{38}$$

Using modulo 3:

$$19 \equiv 1, \quad 38 \equiv 2$$

$$= \omega + \omega^2$$

Since

$$1 + \omega + \omega^2 = 0$$

$$\omega + \omega^2 = -1$$

Final Answer: $\boxed{-1}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: Area in Argand plane.

Solution:

Let

$$z = x + iy$$

Then,

$$|z|^2 = x^2 + y^2$$

Area of triangle:

$$= \frac{1}{2}(x^2 + y^2)$$

$$= \frac{1}{2}|z|^2$$

Final Answer: $\boxed{\frac{1}{2}|z|^2}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: Relations between roots.

Solution:

Given:

$$\alpha + \beta = p, \quad \alpha\beta = q$$

$$\alpha^2 + \beta^2 = p^2 - 2q$$

Then,

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2q^2$$

$$= (p^2 - 2q)^2 - 2q^2$$

$$= p^4 - 4p^2q + 2q^2$$

Final Answer: $\boxed{p^4 - 4p^2q + 2q^2}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: Roots in ratio.

Solution:

Let roots be:

$$mk, \quad nk$$

Using relations:

$$k(m+n) = \frac{-b}{a}$$

and

$$mnk^2 = \frac{c}{a}$$

Substituting k:

$$mnb^2 = (m+n)^2 ac$$

Final Answer: $\boxed{mnb^2 = (m+n)^2 ac}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: Minimum value.

Solution:

Let

$$y = \frac{x^2 - x + 1}{x^2 + x + 1}$$

Then,

$$(y-1)x^2 + (y+1)x + (y-1) = 0$$

For real x:

$$(y+1)^2 - 4(y-1)^2 \geq 0$$

$$3y^2 - 10y + 3 \leq 0$$

$$y = \frac{1}{3}, \ 3$$

Hence,

$$\frac{1}{3} \leq y \leq 3$$

Minimum value: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: Harmonic progression relation.

Solution:

Since

$$a, b, c \text{ are in H.P.}$$

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$2ac = b(a + c)$$

Hence,

$$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$$

are in A.P.

Final Answer: $\boxed{\text{A.P.}}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: Telescoping series.

Solution:

$$\frac{1}{r(r+1)(r+2)} = \frac{1}{2}\left[\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right]$$

Thus,

$$S_n = \frac{1}{2}\left[\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)}\right]$$

$$= \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

Final Answer: $\boxed{\frac{1}{4} - \frac{1}{2(n+1)(n+2)}}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept: Sum of A.P.

Solution:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Given:

$$S_n = n^2 P$$

Comparing:

$$a = P, \quad d = 2P$$

Hence,

$$S_P = \frac{P}{2}[2P + (P-1)(2P)] = P^3$$

Final Answer: $\boxed{P^3}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: Binomial coefficient.

Solution:

Coefficient of x^n in

$$(1+x)(1-x)^n$$

is

$$(-1)^n + n(-1)^{n-1}$$

$$= (1-n)(-1)^n$$

Final Answer: $\boxed{(1-n)(-1)^n}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: Consecutive binomial coefficients in A.P.

Solution:

Given:

$$2^nC_r = {}^nC_{r-1} + {}^nC_{r+1}$$

Using identities:

$${}^nC_{r-1} = {}^nC_r \frac{r}{n-r+1}$$

$${}^nC_{r+1} = {}^nC_r \frac{n-r}{r+1}$$

Simplifying:

$$n^2 - n(4r+1) + 4r^2 - 2 = 0$$

Final Answer: $\boxed{n^2 - n(4r+1) + 4r^2 - 2 = 0}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: Circular permutation.

Solution:

Total arrangements:

$$(7-1)! = 720$$

When two particular persons sit together:

$$(6-1)! \times 2 = 240$$

Required arrangements:

$$720 - 240 = 480$$

Final Answer: $\boxed{480}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: Probability using combinations.

Solution:

Total ways:

$$^9C_2 = 36$$

Favorable ways:

$$^5C_2 + {}^4C_2 = 10 + 6 = 16$$

$$P = \frac{16}{36} = \frac{4}{9}$$

Final Answer: $\boxed{\frac{4}{9}}$

Answer: (C)

Go Back to Question 36

Q37.

Solution

Concept: Combination property.

Solution:

Given:

$$^nC_{12} = {}^nC_8$$

$$n = 12 + 8 = 20$$

Thus,

$$^{22}C_n = {}^{22}C_{20} = {}^{22}C_2$$

$$= \frac{22 \cdot 21}{2} = 231$$

Final Answer: $\boxed{231}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: Probability of at least one event.

Solution:

$$P(\text{none}) = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) = \frac{1}{4}$$

$$P(\text{at least one}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Final Answer: $\boxed{\frac{3}{4}}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: Distance between parallel lines.

Solution:

For lines

$$y = mx + c, \qquad y = mx + d$$

Distance:

$$D = \frac{|c - d|}{\sqrt{1 + m^2}}$$

Final Answer: $\boxed{\frac{|c - d|}{\sqrt{1 + m^2}}}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: Tangent condition to circle.

Solution:

Circle:

$$x^2 + y^2 = 1$$

Line:

$$y = mx + 1$$

Tangency condition:

$$\frac{1}{\sqrt{m^2 + 1}} = 1$$

$$m = 0$$

Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 40

Q41.

Solution

Concept: Circle through three points.

Solution:

General equation:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Using points $(0, 0)$, $(a, 0)$, $(0, b)$:

$$c = 0, \qquad g = -\frac{a}{2}, \qquad f = -\frac{b}{2}$$

Hence,

$$x^2 + y^2 - ax - by = 0$$

Final Answer: $\boxed{x^2 + y^2 - ax - by = 0}$

Answer: (B)

Go Back to Question 41

Q42.

Solution

Concept: Length of intercept.

Solution:

Given:

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

Put $x = 0$:

$$y^2 - 6y - 12 = 0$$

$$y = 3 \pm \sqrt{21}$$

Intercept length:

$$(3 + \sqrt{21}) - (3 - \sqrt{21}) = 2\sqrt{21}$$

Final Answer: $\boxed{2\sqrt{21}}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: Hyperbola parameter relations.

Solution:

$$\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$$

Given:

$$2b = c \Rightarrow c^2 = 4b^2$$

Using

$$c^2 = a^2 + b^2$$

$$4b^2 = a^2 + b^2 \Rightarrow a^2 = 3b^2$$

$$e = \frac{c}{a} = \sqrt{\frac{4b^2}{3b^2}} = \frac{2}{\sqrt{3}}$$

Final Answer: $\boxed{\frac{2}{\sqrt{3}}}$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept: Area using latus rectum.

Solution:

$$x^2 = 12y \Rightarrow 4a = 12 \Rightarrow a = 3$$

Ends of latus rectum:

$$(\pm 2a, a) = (\pm 6, 3)$$

Base:

$$12$$

Height:

$$3$$

Area:

$$\frac{1}{2} \times 12 \times 3 = 18$$

Final Answer: $\boxed{18}$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept: Sum of focal distances.

Solution:

$$\frac{x^2}{16} + \frac{y^2}{7} = 1 \Rightarrow a = 4$$

For ellipse:

$$PS + PS' = 2a$$

$$= 2(4) = 8$$

Final Answer: $\boxed{8}$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: Director circle of hyperbola.

Solution:

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

$$a^2 = 25, \quad b^2 = 9$$

Director circle:

$$x^2 + y^2 = a^2 - b^2$$

$$x^2 + y^2 = 25 - 9 = 16$$

Final Answer: $\boxed{x^2 + y^2 = 16}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: Dot product identity.

Solution:

$$\vec{a} + \vec{b} + \vec{c} = 0$$

Squaring:

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2S = 0$$

where

$$S = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

Since vectors are unit vectors:

$$1 + 1 + 1 + 2S = 0$$

$$3 + 2S = 0 \Rightarrow S = -\frac{3}{2}$$

Final Answer: $\boxed{-\frac{3}{2}}$

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept: Scalar triple product.

Solution:

$$[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}]$$

Expanding:

$$= [\vec{a}\vec{b}\vec{c}] - [\vec{b}\vec{c}\vec{a}]$$

Using cyclic property:

$$[\vec{b}\vec{c}\vec{a}] = [\vec{a}\vec{b}\vec{c}]$$

Hence,

$$0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept: Direction cosines equally inclined.

Solution:

$$l = m = n = k$$

Using:

$$l^2 + m^2 + n^2 = 1$$

$$3k^2 = 1 \Rightarrow k = \frac{1}{\sqrt{3}}$$

Hence,

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Final Answer:: $\boxed{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$

Answer: (B)

Go Back to Question 49

Q50.

Solution

Concept: Angle between planes.

Solution:

Normals:

$$\vec{n}_1 = (2, -1, 1), \quad \vec{n}_2 = (1, 1, 2)$$

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$$

$$= \frac{3}{\sqrt{6}\cdot\sqrt{6}} = \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (A)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	B	4	C	5	C
6	C	7	A	8	A	9	B	10	D
11	B	12	A	13	A	14	A	15	B
16	A	17	A	18	A	19	A	20	A
21	A	22	A	23	A	24	A	25	B
26	B	27	A	28	B	29	A	30	A
31	C	32	A	33	B	34	A	35	B
36	C	37	A	38	A	39	A	40	B
41	B	42	A	43	C	44	B	45	B
46	A	47	B	48	A	49	B	50	A