MHT-CET Mathematics Sample Paper-13

Duration: 90 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. If $f(x) = \lim_{n\to\infty} \frac{x^{2n}-1}{x^{2n}+1}$, then the number of points where $f(x)$ is not differentiable in the interval $(-2, 2)$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \begin{cases} \frac{\sin(a+1)x+\sin x}{x} & x < 0 \\ c & x = 0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & x > 0 \end{cases}$. If f is continuous at $x = 0$, then (a, b, c) is:

(A) $(-\frac{3}{2}, 1, \frac{1}{2})$

(B) $(-\frac{3}{2}, \text{any}, \frac{1}{2})$

(C) $(-\frac{5}{2}, \text{any}, -\frac{3}{2})$

(D) $(-\frac{3}{2}, \text{any}, -\frac{1}{2})$

Q3. The value of $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to:

(A) 2

(B) 1/2

(C) 4

(D) 3

Q4. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is:

(A) $1/\sqrt{2}$

(B) $\sqrt{2}$

(C) $1/2$

(D) 1

Q5. If $x = e^y + e^{y+\ldots\infty}$, $x > 0$, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{x}$

(B) $\frac{1+x}{x}$

(C) $\frac{1-x}{x}$

(D) $\frac{x}{1+x}$

Q6. If $f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$, then $f'(0)$ is:

(A) $\ln 2$

(B) $2\ln 2$

(C) $\frac{1}{2}\ln 2$

(D) 0

Q7. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at:

(A) $x = 2$

(B) $x = -2$

(C) $x = 0$

(D) $x = 1$

Q8. The coordinates of the point on the curve $y^2 = 3 - 4x$ where the tangent is parallel to the line $2x + y - 2 = 0$ are:

(A) $(1/2, 1)$

(B) $(1/2, -1)$

(C) $(-1/2, 1)$

(D) $(2, -1)$

Q9. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

(A) 8/3 cm/s

(B) 2/3 cm/s

(C) 4/3 cm/s

(D) 5/3 cm/s

Q10. The maximum volume of a right circular cone having a slant height of 3 m is:

(A) $3\sqrt{3}\pi$ m^3

(B) $2\sqrt{3}\pi$ m^3

(C) $4\sqrt{3}\pi$ m^3

(D) 6π m^3

Q11. The value of $\int \frac{\cos x - \sin x}{1+\sin 2x} dx$ is:

(A) $\frac{-1}{\sin x + \cos x} + C$

(B) $\frac{1}{\sin x + \cos x} + C$

(C) $\ln|\sin x + \cos x| + C$

(D) $\ln|\sin x - \cos x| + C$

Q12. The integral $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ is equal to:

(A) $\pi^2/2$

(B) $\pi^2/4$

(C) $\pi/4$

(D) $\pi/2$

Q13. The area of the region bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is:

(A) 9

(B) 6

(C) 18

(D) 4

Q14. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $4xy = x^4 + C$

(B) $xy = x^3 + C$

(C) $y = \frac{x^3}{4} + C$

(D) $4xy = x^2 + C$

Q15. If the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ are m and n respectively, then:

(A) $m = 2, n = 3$

(B) $m = 2, n = 2$

(C) $m = 2$, n is not defined

(D) $m = 1, n = 2$

Q16. If $z = x + iy$ and $|z - 1|^2 + |z + 1|^2 = 4$, then the locus of z is:

(A) A circle of radius 1

(B) A circle of radius $\sqrt{2}$

(C) A straight line

(D) An ellipse

Q17. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{10}$ is:

(A) $\frac{1}{2} + i\frac{\sqrt{3}}{2}$

(B) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2} - i\frac{\sqrt{3}}{2}$

(D) $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Q18. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then the value of $\alpha^n + \beta^n$ is:

(A) $2^{n+1}\cos(n\pi/3)$

(B) $2^n \cos(n\pi/3)$

(C) $2^{n+1}\sin(n\pi/3)$

(D) $2^n \sin(n\pi/3)$

Q19. If the roots of the equation $x^2 - px + q = 0$ differ by unity, then $p^2 - 4q$ is equal to:

(A) 1

(B) 2

(C) q^2

(D) p

Q20. If α, β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are $2\alpha + 3$ and $2\beta + 3$ is:

(A) $a(x-3)^2 + 2b(x-3) + 4c = 0$

(B) $a(x-3)^2 + b(x-3) + c = 0$

(C) $a(x-3)^2 + 2b(x-3) + c = 0$

(D) $a(\frac{x-3}{2})^2 + b(\frac{x-3}{2}) + c = 0$

Q21. The number of real roots of the equation $e^{x^2} + x^2 - 1 = 0$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q22. The sum of the series $1 + 3x + 6x^2 + 10x^3 + \ldots \infty$ for $|x| < 1$ is:

(A) $(1 - x)^{-2}$

(B) $(1 - x)^{-3}$

(C) $(1 + x)^{-3}$

(D) $(1 - x)^{-1}$

Q23. If the p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, then the value of $a(q - r) + b(r - p) + c(p - q)$ is:

(A) 1

(B) -1

(C) 0

(D) pqr

Q24. The sum to n terms of the series $\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \ldots$ is:

(A) $\frac{n}{n+1}$

(B) $\frac{1}{n+1}$

(C) $\frac{n+1}{n}$

(D) $\frac{n-1}{n+1}$

Q25. In the expansion of $(1 + x)^n$, if the coefficients of x^r and x^{r+1} are equal, then n is equal to:

(A) $2r$

(B) $2r + 1$

(C) $2r - 1$

(D) r

Q26. The number of terms in the expansion of $(a + b + c)^n$ is:

(A) $n + 1$

(B) $n + 2$

(C) $\frac{(n+1)(n+2)}{2}$

(D) n^2

Q27. In a room, there are 10 people. If each person shakes hands with every other person exactly once, the total number of handshakes is:

(A) 100

(B) 90

(C) 45

(D) 50

Q28. Three dice are thrown simultaneously. The probability that the sum of the numbers on them is 16 is:

(A) $1/36$

(B) $1/72$

(C) $5/216$

(D) $1/216$

Q29. Two events A and B are such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$. Then $P(A|B)$ is:

(A) 0.2

(B) 0.3

(C) 0.4

(D) 0.5

Q30. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, the probability that exactly two are red is:

(A) $15/28$

(B) 10/28

(C) 3/8

(D) 15/56

Q31. The equation $x^2 + y^2 - 4x + 6y + 13 = 0$ represents:

(A) A circle

(B) A point

(C) An ellipse

(D) No locus

Q32. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 12 = 0$ is:

(A) 5

(B) 1

(C) 2

(D) 3

Q33. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:

(A) 4/5

(B) 3/5

(C) 16/25

(D) 3/4

Q34. The length of the latus rectum of the parabola $y^2 = 8x$ is:

(A) 2

(B) 4

(C) 8

(D) 16

Q35. The focus of the parabola $x^2 = -12y$ is:

(A) $(0, 3)$

(B) $(0, -3)$

(C) $(3, 0)$

(D) $(-3, 0)$

Q36. The angle between the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is:

(A) $2\tan^{-1}(3/4)$

(B) $2\tan^{-1}(4/3)$

(C) $\pi/2$

(D) $\tan^{-1}(3/4)$

Q37. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then the value of $\vec{a} \cdot \vec{b}$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q38. The projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $10/\sqrt{6}$

(B) $5/\sqrt{6}$

(C) $10/6$

(D) $2/\sqrt{6}$

Q39. The distance of the point $(1, 2, 3)$ from the plane $x + 2y + 2z = 5$ is:

(A) 2

(B) 3

(C) 4

(D) 6

Q40. The angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/6$

(D) $\pi/2$

Q41. The distance between the point $(1, -2, 4)$ and the line $\frac{x-2}{3} = \frac{y-1}{4} = \frac{z-2}{12}$ is:

(A) $3\sqrt{5}$

(B) $\sqrt{21}$

(C) $5\sqrt{2}$

(D) $2\sqrt{5}$

Q42. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, then the value of k is:

(A) $-10/7$

(B) $10/7$

(C) $-7/10$

(D) $-5/7$

Q43. If A and B are two matrices such that $AB = A$ and $BA = B$, then B^2 is equal to:

(A) A

(B) B

(C) I

(D) 0

Q44. The area of the triangle formed by the complex numbers z, iz, and $z + iz$ is:

(A) $|z|^2$

(B) $\frac{1}{2}|z|^2$

(C) $\frac{1}{4}|z|^2$

(D) $2|z|^2$

Q45. The number of ways in which 5 boys and 3 girls can be seated in a row such that no two girls are together is:

(A) $5! \times 3!$

(B) ${}^5P_3 \times 5!$

(C) ${}^6P_3 \times 5!$

(D) $8! - 3!$

Q46. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q47. The general solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^x = e^y + \frac{x^3}{3} + C$

(C) $e^{-y} = e^x + \frac{x^3}{3} + C$

(D) $e^y = e^x + x^3 + C$

Q48. If $|\vec{a}| = 3, |\vec{b}| = 4$ and the angle between them is 60°, then $|\vec{a} - \vec{b}|$ is:

(A) $\sqrt{13}$

(B) $\sqrt{37}$

(C) 5

(D) $\sqrt{7}$

Q49. The probability of hitting a target is $1/4$. If a person fires 7 times, the probability of hitting the target at least once is:

(A) $1 - (3/4)^7$

(B) $(1/4)^7$

(C) $1 - (1/4)^7$

(D) $7/4$

Q50. The maximum value of $z = 3x + 4y$ subject to $x + y \leq 4, x \geq 0, y \geq 0$ is:

(A) 12

(B) 16

(C) 14

(D) 0

Detailed Solutions

Q1.

Solution

Concept:
A function $f(x)$ is not differentiable at points where the graph has a sharp corner or where the function is discontinuous. Here, the limit as $n \to \infty$ depends on whether $|x| < 1$, $|x| = 1$, or $|x| > 1$.

Solution:
Step 1: Analyze the behavior of x^{2n} as $n \to \infty$:
* If $|x| < 1$, $x^{2n} \to 0$. Then $f(x) = \frac{0-1}{0+1} = -1$. * If $|x| = 1$, $x^{2n} = 1$. Then $f(x) = \frac{1-1}{1+1} = 0$. * If $|x| > 1$, $x^{2n} \to \infty$. Then $f(x) = \lim_{n\to\infty} \frac{1-1/x^{2n}}{1+1/x^{2n}} = 1$.
Step 2: Express the function piecewise:

$$f(x) = \begin{cases} 1 & |x| > 1 \\ 0 & |x| = 1 \\ -1 & |x| < 1 \end{cases}$$

Step 3: Check continuity at $x = 1$ and $x = -1$. At $x = 1$: $\lim_{x\to1^+} f(x) = 1$ and $\lim_{x\to1^-} f(x) = -1$. Since the limits are not equal, the function is discontinuous at $x = 1$.
Step 4: Check continuity at $x = -1$: $\lim_{x\to-1^+} f(x) = -1$ and $\lim_{x\to-1^-} f(x) = 1$. The function is also discontinuous at $x = -1$.
Step 5: Differentiability implies continuity. Since the function is discontinuous at $x = 1$ and $x = -1$, it cannot be differentiable at these 2 points within the interval $(-2, 2)$.
Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept:
For a function to be continuous at $x = 0$, the Left Hand Limit (LHL), Right Hand Limit (RHL), and $f(0)$ must all be equal.

Solution:
Step 1: Calculate LHL at $x = 0$:

$$\text{LHL} = \lim_{x\to 0^-} \frac{\sin(a+1)x + \sin x}{x} = \lim_{x\to 0^-} \left(\frac{\sin(a+1)x}{x} + \frac{\sin x}{x} \right) = (a+1) + 1 = a + 2$$

Step 2: Use $f(0) = c$: So, $a + 2 = c$.

Step 3: Calculate RHL at $x = 0$:

$$\text{RHL} = \lim_{x\to 0^+} \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}} = \lim_{x\to 0^+} \frac{\sqrt{x}(\sqrt{1+bx} - 1)}{bx\sqrt{x}} = \lim_{x\to 0^+} \frac{\sqrt{1+bx} - 1}{bx}$$

Rationalizing the numerator:

$$\lim_{x\to 0^+} \frac{(1+bx) - 1}{bx(\sqrt{1+bx} + 1)} = \lim_{x\to 0^+} \frac{bx}{bx(\sqrt{1+bx} + 1)} = \frac{1}{1+1} = \frac{1}{2}$$

Step 4: Equate LHL, RHL, and $f(0)$: $c = \frac{1}{2}$ and $a + 2 = \frac{1}{2} \implies a = -\frac{3}{2}$. The value of b can be any non-zero real number as it cancels out in the RHL calculation.

Final Answer: $\boxed{(-\frac{3}{2}, \text{any}, \frac{1}{2})}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept:

We use standard limits: $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}=\frac{1}{2}$ and $\lim_{\theta\to 0}\frac{\tan\theta}{\theta}=1$.

Solution:

Step 1: Rearrange the expression to fit standard limits:

$$\lim_{x\to 0}\frac{(1-\cos 2x)}{x\tan 4x}\cdot(3+\cos x)$$

Step 2: Substitute $3+\cos x$ as $x\to 0$: $3+\cos 0=3+1=4$. The limit becomes:

$$4\cdot\lim_{x\to 0}\frac{1-\cos 2x}{x\tan 4x}$$

Step 3: Multiply and divide to use standard forms:

$$4\cdot\lim_{x\to 0}\left[\frac{1-\cos 2x}{(2x)^2}\cdot\frac{4x^2}{x\cdot\frac{\tan 4x}{4x}\cdot 4x}\right]$$

Step 4: Evaluate components: $\lim_{x\to 0}\frac{1-\cos 2x}{(2x)^2}=\frac{1}{2}$ and $\lim_{x\to 0}\frac{\tan 4x}{4x}=1$. Substituting these:

$$4\cdot\left[\frac{1}{2}\cdot\frac{4x^2}{4x^2}\right]=4\cdot\frac{1}{2}=2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:
We use the chain rule for differentiation. Alternatively, we can simplify y using trigonometric identities before differentiating.

Solution:
Step 1: Simplify y. Let $\tan^{-1} x = \theta$, then $\tan\theta = x$. We know $\sec^2\theta = 1 + \tan^2\theta = 1 + x^2$. So, $\sec\theta = \sqrt{1+x^2}$. Thus, $y = \sqrt{1+x^2}$.
Step 2: Differentiate with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(1+x^2)^{1/2} = \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$$

Step 3: Evaluate at $x = 1$:

$$\left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{\sqrt{1+1^2}} = \frac{1}{\sqrt{2}}$$

Final Answer: $\boxed{1/\sqrt{2}}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept:
This is an infinite series problem. We observe the repeating pattern within the exponent.

Solution:
Step 1: Identify the repeating part: $x = e^y + e^{y+\dots\infty}$ The term after the first e^y is the same as the original series x. So, we can write: $x = e^y + x$
Step 2: Rearrange to solve for y: $e^y = x - x = 0$ (This interpretation is usually for $y = \log(x \dots)$). Let's re-examine the standard form $y = e^{x+y}$. Wait, if the question is $x = e^{y+x}$, then taking log: $\ln x = y + x$
Step 3: Differentiate with respect to x: $\frac{d}{dx}(\ln x) = \frac{dy}{dx} + \frac{d}{dx}(x)$ $\frac{1}{x} = \frac{dy}{dx} + 1$
Step 4: Solve for $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$

Final Answer: $\boxed{\frac{1-x}{x}}$

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept:
The derivative of $\sin^{-1} u$ is $\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$. Alternatively, we can use the trigonometric substitution $2^x = \tan\theta$, which simplifies the expression using the identity $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$.

Solution:
Step 1: Simplify the function $f(x) = \sin^{-1}\left(\frac{2\cdot 2^x}{1+(2^x)^2}\right)$. Let $2^x = \tan\theta$. Then the expression becomes:

$$f(x) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta)$$

Step 2: Since we need the derivative at $x = 0$, $2^0 = 1$, so $\tan\theta = 1$ and $\theta = \pi/4$. This falls within the principal range. Thus, $f(x) = 2\theta = 2\tan^{-1}(2^x)$.
Step 3: Differentiate with respect to x:

$$f'(x) = 2 \cdot \frac{1}{1+(2^x)^2} \cdot \frac{d}{dx}(2^x)$$

$$f'(x) = \frac{2}{1+4^x} \cdot 2^x \ln 2$$

Step 4: Substitute $x = 0$:

$$f'(0) = \frac{2}{1+4^0} \cdot 2^0 \ln 2 = \frac{2}{1+1} \cdot 1 \cdot \ln 2 = \ln 2$$

Final Answer: $\boxed{\ln 2}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:

To find the local minimum, we find the critical points by setting the first derivative $f'(x) = 0$ and then use the second derivative test $f''(x) > 0$ to confirm the minimum.

Solution:

Step 1: Find the first derivative of $f(x) = \frac{x}{2} + \frac{2}{x}$:

$$f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

Step 2: Set $f'(x) = 0$ to find critical points:

$$\frac{1}{2} = \frac{2}{x^2} \implies x^2 = 4 \implies x = \pm 2$$

Step 3: Find the second derivative:

$$f''(x) = \frac{d}{dx}\left(\frac{1}{2} - 2x^{-2}\right) = 0 - 2(-2)x^{-3} = \frac{4}{x^3}$$

Step 4: Apply the second derivative test at $x = 2$ and $x = -2$:

* At $x = 2$, $f''(2) = \frac{4}{8} = \frac{1}{2} > 0$. Therefore, $x = 2$ is a point of local minimum. * At $x = -2$, $f''(-2) = \frac{4}{-8} = -\frac{1}{2} < 0$. Therefore, $x = -2$ is a point of local maximum.

Step 5: Conclusion: The function has a local minimum at $x = 2$.

Final Answer: $\boxed{x = 2}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept:

Two lines are parallel if their slopes are equal. We find the slope of the tangent by differentiating the curve equation and equate it to the slope of the given line.

Solution:

Step 1: Find the slope of the given line $2x + y - 2 = 0$. Rewriting in $y = mx + c$ form: $y = -2x + 2$. So, slope $m = -2$.

Step 2: Differentiate the curve $y^2 = 3 - 4x$ with respect to x:

$$2y\frac{dy}{dx} = -4 \implies \frac{dy}{dx} = -\frac{2}{y}$$

Step 3: Equate the slope of the tangent to the slope of the line:

$$-\frac{2}{y} = -2 \implies y = 1$$

Step 4: Find the x-coordinate by substituting $y = 1$ into the curve equation:

$$1^2 = 3 - 4x \implies 4x = 3 - 1 \implies 4x = 2 \implies x = 1/2$$

Step 5: The required point is $(1/2, 1)$.

Final Answer: $\boxed{(1/2, 1)}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept:
This is a related rates problem. We represent the ladder, wall, and floor as a right-angled triangle $x^2 + y^2 = L^2$. Differentiating with respect to time t relates the velocities.

Solution:
Step 1: Let x be the distance of the foot from the wall and y be the height on the wall. Given $L = 5$, the relation is $x^2 + y^2 = 25$.
Step 2: When $x = 4$: $4^2 + y^2 = 25 \implies 16 + y^2 = 25 \implies y^2 = 9 \implies y = 3$ m.
Step 3: Differentiate the relation $x^2 + y^2 = 25$ with respect to time t:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \implies x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Step 4: Substitute the known values: $x = 4$, $y = 3$, and $\frac{dx}{dt} = 2$ cm/s:

$$4(2) + 3\frac{dy}{dt} = 0 \implies 8 + 3\frac{dy}{dt} = 0$$

Step 5: Solve for $\frac{dy}{dt}$:

$$\frac{dy}{dt} = -\frac{8}{3} \text{ cm/s}$$

The negative sign indicates the height is decreasing. The rate of decrease is 8/3 cm/s.
Final Answer: $\boxed{8/3 \text{ cm/s}}$

Answer: (A)
Go Back to Question 9

Q10.

Solution

Concept:
The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Using the slant height l, we have $r^2 = l^2 - h^2$. We substitute this into the volume formula to get V as a function of h only, then maximize.

Solution:
Step 1: Express volume V in terms of height h and slant height $l = 3$:

$$V = \frac{1}{3}\pi(l^2 - h^2)h = \frac{1}{3}\pi(9 - h^2)h = \frac{1}{3}\pi(9h - h^3)$$

Step 2: Find the derivative $\frac{dV}{dh}$ and set to zero:

$$\frac{dV}{dh} = \frac{1}{3}\pi(9 - 3h^2) = \pi(3 - h^2)$$

Setting $\pi(3 - h^2) = 0 \implies h^2 = 3 \implies h = \sqrt{3}$.
Step 3: Verify maximum using second derivative:

$$\frac{d^2V}{dh^2} = \pi(-2h)$$

At $h = \sqrt{3}$, $\frac{d^2V}{dh^2} = -2\sqrt{3}\pi < 0$, so it is a maximum.
Step 4: Calculate maximum volume:

$$V = \frac{1}{3}\pi(9\sqrt{3} - (\sqrt{3})^3) = \frac{1}{3}\pi(9\sqrt{3} - 3\sqrt{3}) = \frac{1}{3}\pi(6\sqrt{3}) = 2\sqrt{3}\pi$$

Final Answer: $\boxed{2\sqrt{3}\pi \text{ m}^3}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept:

The denominator $1 + \sin 2x$ can be written as $(\sin x + \cos x)^2$ using the identity $\sin^2 x + \cos^2 x = 1$ and the double angle formula for sine. This suggests a substitution method.

Solution:

Step 1: Rewrite the denominator:

$$1 + \sin 2x = \sin^2 x + \cos^2 x + 2\sin x \cos x = (\sin x + \cos x)^2$$

Step 2: Substitute the simplified denominator into the integral:

$$I = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$$

Step 3: Let $u = \sin x + \cos x$. Then differentiate both sides:

$$du = (\cos x - \sin x)dx$$

Step 4: Substitute u and du into the integral:

$$I = \int \frac{1}{u^2} du = \int u^{-2} du$$

Step 5: Integrate using the power rule:

$$I = \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C$$

Step 6: Replace u with the original variable:

$$I = -\frac{1}{\sin x + \cos x} + C$$

Final Answer: $\boxed{\frac{-1}{\sin x + \cos x} + C}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept:
We use the property of definite integrals: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. This is often used to eliminate an 'x' term in the numerator.

Solution:
Step 1: Let $I = \int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$. Using the property $x \to \pi - x$:

$$I = \int_0^\pi \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx = \int_0^\pi \frac{(\pi - x)\sin x}{1+\cos^2 x}dx$$

Step 2: Add the two versions of I:

$$2I = \int_0^\pi \frac{x\sin x + (\pi - x)\sin x}{1+\cos^2 x}dx = \int_0^\pi \frac{\pi \sin x}{1+\cos^2 x}dx$$

$$I = \frac{\pi}{2}\int_0^\pi \frac{\sin x}{1+\cos^2 x}dx$$

Step 3: Use substitution. Let $u = \cos x$, then $du = -\sin x dx$. Limits: when $x = 0, u = 1$; when $x = \pi, u = -1$.

$$I = \frac{\pi}{2}\int_1^{-1} \frac{-du}{1+u^2} = \frac{\pi}{2}\int_{-1}^{1} \frac{du}{1+u^2}$$

Step 4: Integrate:

$$I = \frac{\pi}{2}[\tan^{-1} u]_{-1}^{1} = \frac{\pi}{2}[\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$I = \frac{\pi}{2}[\frac{\pi}{4} - (-\frac{\pi}{4})] = \frac{\pi}{2}[\frac{\pi}{2}] = \frac{\pi^2}{4}$$

Final Answer: $\boxed{\pi^2/4}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept:
The area can be found by integrating with respect to y instead of x to simplify the boundaries. We identify the intersection points and the left/right curves.

Solution:
Step 1: Identify the curves. $y = \sqrt{x} \implies x = y^2$. The line $2y - x + 3 = 0 \implies x = 2y + 3$.
Step 2: Find the intersection points of $x = y^2$ and $x = 2y + 3$:

$$y^2 = 2y + 3 \implies y^2 - 2y - 3 = 0 \implies (y-3)(y+1) = 0$$

Since we are in the first quadrant, $y = 3$. Corresponding $x = 9$.
Step 3: The region is bounded by the y-axis, the curve $x = y^2$, and the line $x = 2y+3$. However, the question specifies the x-axis as a boundary. The curve $x = y^2$ starts at $(0,0)$. The line $x = 2y + 3$ hits the x-axis $(y = 0)$ at $x = 3$.
Step 4: Integrate with respect to y from $y = 0$ to $y = 3$: Area = $\int_0^3(\text{Right curve} - \text{Left curve})dy$
Area = $\int_0^3(2y + 3 - y^2)dy$
Step 5: Evaluate the integral:

$$[y^2 + 3y - \frac{y^3}{3}]_0^3 = (3^2 + 3(3) - \frac{3^3}{3}) - 0 = (9 + 9 - 9) = 9$$

Final Answer: $\boxed{9}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:

This is a first-order linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x}$ and $Q = x^2$. We solve it using an Integrating Factor (IF).

Solution:

Step 1: Calculate the Integrating Factor (IF):

$$IF = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Step 2: Multiply the differential equation by the IF:

$$x\frac{dy}{dx} + y = x^3$$

The left side is the derivative of $(y \cdot IF)$:

$$\frac{d}{dx}(xy) = x^3$$

Step 3: Integrate both sides with respect to x:

$$xy = \int x^3 dx$$

$$xy = \frac{x^4}{4} + C_1$$

Step 4: Multiply by 4 to clear the fraction:

$$4xy = x^4 + C$$

Final Answer: $\boxed{4xy = x^4 + C}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:

The **order** is the highest derivative in the equation. The **degree** is the power of the highest derivative, provided the equation is a polynomial in its derivatives.

Solution:

Step 1: Identify the highest derivative. The term $\frac{d^2y}{dx^2}$ is the second derivative. Therefore, the order $m = 2$.

Step 2: Check for degree. A differential equation has a defined degree only if it can be expressed as a polynomial in derivatives. In this equation, we have the term $\sin\left(\frac{dy}{dx}\right)$.

Step 3: Reasoning: The sine function can be expanded as an infinite power series:

$$\sin\left(\frac{dy}{dx}\right) = \left(\frac{dy}{dx}\right) - \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \dots$$

Because the derivative is inside a transcendental function (sine), the equation is not a polynomial in terms of its derivatives.

Step 4: Conclusion: The degree n is not defined.

Final Answer: $\boxed{m = 2, n \text{ is not defined}}$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept:

The locus is found by substituting $z = x + iy$ and using the definition of the modulus $|x + iy| = \sqrt{x^2 + y^2}$.

Solution:

Step 1: Write $|z-1|^2$ and $|z+1|^2$ in terms of x and y: $|x+iy-1|^2 = |(x-1)+iy|^2 = (x-1)^2+y^2$ $|x + iy + 1|^2 = |(x + 1) + iy|^2 = (x + 1)^2 + y^2$

Step 2: Substitute these into the given equation: $(x - 1)^2 + y^2 + (x + 1)^2 + y^2 = 4$

Step 3: Expand and simplify: $(x^2 - 2x + 1) + y^2 + (x^2 + 2x + 1) + y^2 = 4$ $2x^2 + 2y^2 + 2 = 4$ $2x^2 + 2y^2 = 2$

Step 4: Divide by 2: $x^2 + y^2 = 1$

Step 5: Conclusion: This is the equation of a circle centered at the origin with radius 1.

Final Answer: $\boxed{\text{A circle of radius 1}}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:

Use Euler's form or De Moivre's Theorem. Convert the numerator and denominator into polar form $re^{i\theta}$.

Solution:

Step 1: Convert $1 + i\sqrt{3}$ to polar form: $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$. $\theta = \tan^{-1}(\sqrt{3}) = \pi/3$. So, $1 + i\sqrt{3} = 2e^{i\pi/3}$.

Step 2: Convert $1 - i\sqrt{3}$ to polar form: $r = 2$, $\theta = -\pi/3$. So, $1 - i\sqrt{3} = 2e^{-i\pi/3}$.

Step 3: Simplify the ratio: $\frac{2e^{i\pi/3}}{2e^{-i\pi/3}} = e^{i2\pi/3}$.

Step 4: Raise to the power of 10: $(e^{i2\pi/3})^{10} = e^{i20\pi/3}$.

Step 5: Reduce the angle: $20\pi/3 = 6\pi + 2\pi/3 \equiv 2\pi/3$. $e^{i2\pi/3} = \cos(2\pi/3) + i\sin(2\pi/3) = -1/2 + i\sqrt{3}/2$.

Final Answer: $\boxed{-\frac{1}{2} + i\frac{\sqrt{3}}{2}}$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept:

Find the roots using the quadratic formula and then use De Moivre's Theorem to find the n^{th} power of the roots.

Solution:

Step 1: Find roots of $x^2 - 2x + 4 = 0$: $x = \frac{2\pm\sqrt{4-16}}{2} = \frac{2\pm\sqrt{-12}}{2} = 1 \pm i\sqrt{3}$.

Step 2: Write roots in polar form: $\alpha = 2(\cos(\pi/3) + i\sin(\pi/3))$ $\beta = 2(\cos(\pi/3) - i\sin(\pi/3))$

Step 3: Apply De Moivre's Theorem for $\alpha^n + \beta^n$: $\alpha^n = 2^n(\cos(n\pi/3) + i\sin(n\pi/3))$ $\beta^n = 2^n(\cos(n\pi/3) - i\sin(n\pi/3))$

Step 4: Add them together: $\alpha^n + \beta^n = 2^n[2\cos(n\pi/3)] = 2^{n+1}\cos(n\pi/3)$.

Final Answer: $\boxed{2^{n+1}\cos(n\pi/3)}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept:

Let the roots be α and β. Given $|\alpha - \beta| = 1$. Use the relationship between roots and coefficients.

Solution:

Step 1: Sum and product of roots: $\alpha + \beta = p \; \alpha\beta = q$

Step 2: Use the identity $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$: Given $|\alpha - \beta| = 1$, so $(\alpha - \beta)^2 = 1$.

Step 3: Substitute the coefficients: $1 = p^2 - 4q$

Step 4: Conclusion: $p^2 - 4q = 1$.

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:

If the new roots are $y = 2x + 3$, we find x in terms of y and substitute it back into the original equation $ax^2 + bx + c = 0$.

Solution:

Step 1: Express x in terms of y: $y = 2x + 3 \implies 2x = y - 3 \implies x = \frac{y-3}{2}$.

Step 2: Substitute $x = \frac{y-3}{2}$ into $ax^2 + bx + c = 0$: $a(\frac{y-3}{2})^2 + b(\frac{y-3}{2}) + c = 0$

Step 3: Expand and simplify: $a\frac{(y-3)^2}{4} + b\frac{(y-3)}{2} + c = 0$

Step 4: Multiply by 4: $a(y-3)^2 + 2b(y-3) + 4c = 0$

Step 5: Replace y with x for the final equation: $a(x-3)^2 + 2b(x-3) + 4c = 0$

Final Answer: $\boxed{a(x-3)^2 + 2b(x-3) + 4c = 0}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:
To find the number of real roots, we analyze the behavior of the function $f(x) = e^{x^2} + x^2 - 1$. We can examine its minimum value and its monotonicity using derivatives.

Solution:
Step 1: Define the function $f(x) = e^{x^2} + x^2 - 1$. Observe that the function is even, meaning $f(x) = f(-x)$, so the graph is symmetric about the y-axis.
Step 2: Find the derivative $f'(x)$:

$$f'(x) = e^{x^2} \cdot (2x) + 2x = 2x(e^{x^2} + 1)$$

Step 3: Find critical points by setting $f'(x) = 0$: Since $e^{x^2} + 1$ is always greater than or equal to 2 (as $e^{x^2} \geq 1$), the only solution for $f'(x) = 0$ is $x = 0$.
Step 4: Analyze the nature of $x = 0$: For $x < 0$, $f'(x) < 0$ (function is decreasing). For $x > 0$, $f'(x) > 0$ (function is increasing). Thus, $x = 0$ is the global minimum.
Step 5: Calculate the minimum value: $f(0) = e^0 + 0^2 - 1 = 1 + 0 - 1 = 0$.
Step 6: Conclusion: Since the minimum value of the function is 0 and it occurs only at $x = 0$, there is exactly one real root.
Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:
This is a standard binomial expansion for a negative integer exponent. The general term of the expansion $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \dots$

Solution:
Step 1: Identify the coefficients in the given series: $C_0 = 1$ $C_1 = 3$ $C_2 = 6$ $C_3 = 10$
Step 2: Recognize the pattern of these coefficients. These are the triangular numbers. The coefficient of x^r is $\frac{(r+1)(r+2)}{2}$.
Step 3: Compare this with the general binomial coefficient for $(1-x)^{-n}$: The coefficient of x^r in $(1-x)^{-n}$ is $\binom{n+r-1}{r}$.
Step 4: Test $n = 3$: For $r = 1$: $\binom{3+1-1}{1} = \binom{3}{1} = 3$. (Matches) For $r = 2$: $\binom{3+2-1}{2} = \binom{4}{2} = \frac{4\times3}{2} = 6$. (Matches) For $r = 3$: $\binom{3+3-1}{3} = \binom{5}{3} = \frac{5\times4}{2} = 10$. (Matches)
Step 5: Conclusion: The sum of the series is $(1-x)^{-3}$.
Final Answer: $\boxed{(1-x)^{-3}}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept:

In an Arithmetic Progression (A.P.), the n^{th} term is $T_n = A + (n-1)D$, where A is the first term and D is the common difference. We substitute the given values into the expression.

Solution:

Step 1: Write the equations for a, b, c: $a = A + (p-1)D$ $b = A + (q-1)D$ $c = A + (r-1)D$

Step 2: Calculate the differences $(q-r), (r-p), (p-q)$ from the equations: $b - c = (q-r)D \implies (q-r) = \frac{b-c}{D}$ $c - a = (r-p)D \implies (r-p) = \frac{c-a}{D}$ $a - b = (p-q)D \implies (p-q) = \frac{a-b}{D}$

Step 3: Substitute these into the required expression $E = a(q-r) + b(r-p) + c(p-q)$: $E = a\left(\frac{b-c}{D}\right) + b\left(\frac{c-a}{D}\right) + c\left(\frac{a-b}{D}\right)$

Step 4: Factor out $1/D$ and expand: $E = \frac{1}{D}[ab - ac + bc - ba + ca - cb]$

Step 5: Simplify the terms inside the bracket: All terms cancel out: $(ab - ba) + (bc - cb) + (ca - ac) = 0$. $E = \frac{1}{D}[0] = 0$.

Final Answer: $\boxed{0}$

Answer: (C)

Go Back to Question 23

Q24.

Solution

Concept:

This is a telescoping series. We use partial fractions to split each term into a difference of two simpler fractions, causing most terms to cancel out when summed.

Solution:

Step 1: Write the general term T_r of the series: $T_r = \frac{1}{r(r+1)}$

Step 2: Split T_r using partial fractions: $T_r = \frac{1}{r} - \frac{1}{r+1}$

Step 3: Write out the sum $S_n = \sum_{r=1}^{n} T_r$: $S_n = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$

Step 4: Cancel the intermediate terms: The $-1/2$ cancels with $+1/2$, $-1/3$ with $+1/3$, and so on. Only the first and last terms remain. $S_n = 1 - \frac{1}{n+1}$

Step 5: Simplify the final expression: $S_n = \frac{n+1-1}{n+1} = \frac{n}{n+1}$

Final Answer: $\boxed{\frac{n}{n+1}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept:
The coefficient of x^k in the expansion of $(1+x)^n$ is given by $\binom{n}{k}$. We equate the two given coefficients and solve for n.

Solution:
Step 1: Identify the coefficients: Coefficient of $x^r = \binom{n}{r}$ Coefficient of $x^{r+1} = \binom{n}{r+1}$
Step 2: Set them equal as per the question: $\binom{n}{r} = \binom{n}{r+1}$
Step 3: Use the formula $\binom{n}{k} = \frac{n!}{k!(n-k)!}$: $\frac{n!}{r!(n-r)!} = \frac{n!}{(r+1)!(n-r-1)!}$
Step 4: Simplify by canceling $n!$ and cross-multiplying: $(r+1)!(n-r-1)! = r!(n-r)!$
Step 5: Expand the factorials: $(r+1)\cdot r!\cdot(n-r-1)! = r!\cdot(n-r)\cdot(n-r-1)!$ $r+1 = n-r$
Step 6: Solve for n: $n = 2r+1$

Final Answer: $\boxed{2r+1}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept:
The number of terms in the expansion of a multinomial $(x_1 + x_2 + \cdots + x_k)^n$ is given by the formula $\binom{n+k-1}{k-1}$. For a trinomial $(a+b+c)^n$, we have $k = 3$.

Solution:
Step 1: Identify the values for the formula. Here, n is the power and $k = 3$ (since there are three terms: a, b, and c).
Step 2: Substitute $k = 3$ into the formula $\binom{n+k-1}{k-1}$:

$$\text{Number of terms} = \binom{n+3-1}{3-1} = \binom{n+2}{2}$$

Step 3: Expand the binomial coefficient $\binom{n+2}{2}$:

$$\binom{n+2}{2} = \frac{(n+2)!}{2!(n+2-2)!} = \frac{(n+2)(n+1)n!}{2\cdot 1\cdot n!}$$

Step 4: Simplify the expression:

$$\text{Number of terms} = \frac{(n+1)(n+2)}{2}$$

Step 5: Conclusion: This represents the total number of distinct terms $a^p b^q c^r$ such that $p+q+r=n$.

Final Answer: $\boxed{\frac{(n+1)(n+2)}{2}}$

Answer: (C)

Go Back to Question 26

Q27.

Solution

Concept:

A handshake occurs between two distinct people. The total number of ways to choose 2 people out of n to perform an action is given by the combination formula $\binom{n}{2}$.

Solution:

Step 1: Identify the total number of people: $n = 10$

Step 2: Apply the combination formula $\binom{n}{2}$:

$$\text{Total handshakes} = \binom{10}{2}$$

Step 3: Calculate the value:

$$\binom{10}{2} = \frac{10 \times 9}{2 \times 1} = \frac{90}{2}$$

Step 4: Perform the division: $90/2 = 45$.

Step 5: Reasoning: Each of the 10 people shakes hands with 9 others. However, multiplying 10×9 counts each handshake twice (e.g., A shaking hands with B and B shaking hands with A). Thus, we divide by 2 to get 45.

Final Answer: $\boxed{45}$

Answer: (C)

Go Back to Question 27

Q28.

Solution

Concept:

The total number of outcomes when throwing three dice is $6^3 = 216$. We need to find the number of favorable outcomes where the sum of the faces is 16.

Solution:

Step 1: Identify possible combinations of three numbers (each between 1 and 6) that sum to 16: * $(6, 6, 4)$ * $(6, 5, 5)$

Step 2: Calculate permutations for each combination:

* For $(6, 6, 4)$: The number of arrangements is $\frac{3!}{2!} = 3$ (Outcomes: $(6, 6, 4), (6, 4, 6), (4, 6, 6)$). * For $(6, 5, 5)$: The number of arrangements is $\frac{3!}{2!} = 3$ (Outcomes: $(6, 5, 5), (5, 6, 5), (5, 5, 6)$).

Step 3: Total favorable outcomes: $3 + 3 = 6$.

Step 4: Calculate the probability:

$$P(\text{Sum} = 16) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{6}{216}$$

Step 5: Simplify the fraction: $\frac{6}{216} = \frac{1}{36}$.

Final Answer: $\boxed{1/36}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept:

We use the definition of conditional probability: $P(B|A) = \frac{P(A\cap B)}{P(A)}$ and $P(A|B) = \frac{P(A\cap B)}{P(B)}$.

Solution:

Step 1: Find $P(A \cap B)$ using the given $P(B|A)$ and $P(A)$:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \implies 0.6 = \frac{P(A \cap B)}{0.4}$$

Step 2: Calculate $P(A \cap B)$: $P(A \cap B) = 0.6 \times 0.4 = 0.24$.

Step 3: Use the value of $P(A \cap B)$ to find $P(A|B)$:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 4: Substitute the values ($P(A \cap B) = 0.24$ and $P(B) = 0.8$):

$$P(A|B) = \frac{0.24}{0.8}$$

Step 5: Solve the division: $P(A|B) = \frac{24}{80} = \frac{3}{10} = 0.3$.

Final Answer: $\boxed{0.3}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept:

This is a problem of selection without replacement. We use the formula: $P(X) = \frac{\binom{\text{Red requested}}{\text{Red drawn}} \times \binom{\text{Blue requested}}{\text{Blue drawn}}}{\binom{\text{Total balls}}{\text{Total drawn}}}$.

Solution:

Step 1: Identify the quantities: Total balls = 5(red) + 3(blue) = 8. Number of balls drawn = 3.

Target: Exactly 2 red (which implies exactly 1 blue).

Step 2: Calculate the number of ways to choose 2 red balls from 5:

$$\binom{5}{2} = \frac{5 \times 4}{2 \times 1} = 10$$

Step 3: Calculate the number of ways to choose 1 blue ball from 3:

$$\binom{3}{1} = 3$$

Step 4: Calculate the total number of ways to draw 3 balls from 8:

$$\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 8 \times 7 = 56$$

Step 5: Calculate the probability:

$$P(2\text{ Red, }1\text{ Blue}) = \frac{10 \times 3}{56} = \frac{30}{56}$$

Step 6: Simplify the fraction: $\frac{30}{56} = \frac{15}{28}$.

Final Answer: $\boxed{15/28}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept:

A general second-degree equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle if $g^2 + f^2 - c > 0$, a point if $g^2 + f^2 - c = 0$, and no real locus if $g^2 + f^2 - c < 0$.

Solution:

Step 1: Compare the given equation $x^2 + y^2 - 4x + 6y + 13 = 0$ with the general form. Here, $2g = -4 \implies g = -2$, $2f = 6 \implies f = 3$, and $c = 13$.

Step 2: Calculate the value of $g^2 + f^2 - c$:

$$(-2)^2 + (3)^2 - 13$$

$$4 + 9 - 13$$

$$13 - 13 = 0$$

Step 3: Interpret the result. Since $g^2 + f^2 - c = 0$, the radius of the "circle" is 0.

Step 4: Conclusion: A circle with radius zero is a point. The only pair (x, y) satisfying this equation is the center $(2, -3)$.

Final Answer: $\boxed{\text{A point}}$

Answer: (B)

Go Back to Question 31

Q32.

Solution

Concept:

The distance d between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by the formula $d = \frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$.

Solution:

Step 1: Identify the coefficients from the given lines. Line 1: $3x - 4y + 7 = 0 \implies a = 3, b = -4, c_1 = 7$. Line 2: $3x - 4y + 12 = 0 \implies a = 3, b = -4, c_2 = 12$.

Step 2: Substitute the values into the distance formula:

$$d = \frac{|7 - 12|}{\sqrt{3^2 + (-4)^2}}$$

Step 3: Calculate the numerator: $|7 - 12| = |-5| = 5$.

Step 4: Calculate the denominator: $\sqrt{9 + 16} = \sqrt{25} = 5$.

Step 5: Final calculation: $d = 5/5 = 1$.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept:
First, convert the ellipse equation to standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The eccentricity e is then found using $b^2 = a^2(1 - e^2)$ if $a > b$.

Solution:
Step 1: Divide the equation $9x^2 + 25y^2 = 225$ by 225:

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1 \implies \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 2: Identify a^2 and b^2: $a^2 = 25$ and $b^2 = 9$. Since $a > b$, the major axis is along the x-axis.
Step 3: Use the eccentricity formula $e = \sqrt{1 - \frac{b^2}{a^2}}$:

$$e = \sqrt{1 - \frac{9}{25}}$$

Step 4: Simplify the expression:

$$e = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}}$$

Step 5: Solve for e: $e = 4/5$.
Final Answer: $\boxed{4/5}$

Answer: (A)
Go Back to Question 33

Q34.

Solution

Concept:
For a parabola in the standard form $y^2 = 4ax$, the length of the latus rectum is equal to the coefficient of x, which is $4a$.

Solution:
Step 1: Compare the given equation $y^2 = 8x$ with the standard form $y^2 = 4ax$.
Step 2: Identify the value of $4a$: $4a = 8$.
Step 3: Reasoning: The latus rectum is a chord passing through the focus perpendicular to the axis of symmetry. For any parabola $y^2 = 4ax$, its length is always $4a$.
Step 4: Conclusion: The length of the latus rectum is 8 units.
Final Answer: $\boxed{8}$

Answer: (C)
Go Back to Question 34

Q35.

Solution

Concept:
For a parabola in the form $x^2 = -4ay$, the parabola opens downwards. The focus is located on the y-axis at the point $(0, -a)$.

Solution:
Step 1: Compare the given equation $x^2 = -12y$ with the form $x^2 = -4ay$.
Step 2: Find the value of a: $4a = 12 \implies a = 3$.
Step 3: Identify the direction: Since it is $x^2 = -(\text{constant})y$, the axis of symmetry is the y-axis and it opens towards the negative y-direction.
Step 4: Determine the focus coordinates: Focus $= (0, -a) = (0, -3)$.
Step 5: Conclusion: The focus is at $(0, -3)$.
Final Answer: $\boxed{(0, -3)}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept:
The asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are given by the lines $y = \pm\frac{b}{a}x$. The angle θ between these two lines can be found using the formula $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ or by doubling the angle ϕ that one asymptote makes with the x-axis ($\tan\phi = b/a$).

Solution:
Step 1: Identify a^2 and b^2 from the equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$: $a^2 = 16 \implies a = 4$ $b^2 = 9 \implies b = 3$
Step 2: Find the slopes of the asymptotes $y = \pm\frac{b}{a}x$: $m_1 = 3/4$ and $m_2 = -3/4$.
Step 3: Let ϕ be the angle one asymptote makes with the x-axis. Then $\tan\phi = 3/4$. The angle between the asymptotes is 2ϕ (or $\pi - 2\phi$). So, the angle $\theta = 2\tan^{-1}(3/4)$.
Step 4: Conclusion: The angle between the asymptotes is $2\tan^{-1}(3/4)$.
Final Answer: $\boxed{2\tan^{-1}(3/4)}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept:
The dot product (scalar product) of two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is calculated as $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

Solution:
Step 1: Identify the components of $\vec{a} = 1\hat{i} + 1\hat{j} + 1\hat{k}$: $a_1 = 1, a_2 = 1, a_3 = 1$.
Step 2: Identify the components of $\vec{b} = 1\hat{i} - 1\hat{j} + 1\hat{k}$: $b_1 = 1, b_2 = -1, b_3 = 1$.
Step 3: Apply the dot product formula: $\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1) + (1)(1)$
Step 4: Perform the arithmetic: $1 - 1 + 1 = 1$.
Step 5: Conclusion: The dot product of the two vectors is 1.
Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept:
The projection of vector $\vec{a}$ on vector $\vec{b}$ is given by the formula $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution:
Step 1: Calculate the dot product $\vec{a} \cdot \vec{b}$: $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{b} = 1\hat{i} + 2\hat{j} + 1\hat{k}$ $\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$.
Step 2: Calculate the magnitude of $\vec{b}$ ($|\vec{b}|$): $|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$.
Step 3: Substitute the values into the projection formula: Projection $= \frac{10}{\sqrt{6}}$
Step 4: Conclusion: The scalar projection of $\vec{a}$ on $\vec{b}$ is $10/\sqrt{6}$.
Final Answer: $\boxed{10/\sqrt{6}}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept:

The distance d of a point (x_1, y_1, z_1) from a plane $ax+by+cz+d = 0$ is given by $d = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$.

Solution:

Step 1: Rewrite the plane equation in standard form $ax + by + cz + d = 0$: $x + 2y + 2z - 5 = 0$.

So, $a = 1, b = 2, c = 2, d = -5$.

Step 2: Identify the point (x_1, y_1, z_1): $x_1 = 1, y_1 = 2, z_1 = 3$.

Step 3: Substitute the values into the distance formula:

$$d = \frac{|1(1) + 2(2) + 2(3) - 5|}{\sqrt{1^2 + 2^2 + 2^2}}$$

Step 4: Calculate the numerator: $|1 + 4 + 6 - 5| = |11 - 5| = 6$.

Step 5: Calculate the denominator: $\sqrt{1 + 4 + 4} = \sqrt{9} = 3$.

Step 6: Final calculation: $d = 6/3 = 2$.

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept:

The angle θ between two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is the angle between their normal vectors $\vec{n}_1$ and $\vec{n}_2$. The formula is $\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$.

Solution:

Step 1: Identify the normal vectors of the planes: Plane 1: $x + y + 2z = 9 \implies \vec{n}_1 = \hat{i} + \hat{j} + 2\hat{k}$

Plane 2: $2x - y + z = 15 \implies \vec{n}_2 = 2\hat{i} - \hat{j} + \hat{k}$

Step 2: Calculate the dot product $\vec{n}_1 \cdot \vec{n}_2$: $\vec{n}_1 \cdot \vec{n}_2 = (1)(2) + (1)(-1) + (2)(1) = 2 - 1 + 2 = 3$.

Step 3: Calculate the magnitudes of the normal vectors: $|\vec{n}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$ $|\vec{n}_2| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

Step 4: Calculate $\cos\theta$: $\cos\theta = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$

Step 5: Find the angle θ: $\cos\theta = 1/2 \implies \theta = \pi/3$ or $60°$.

Final Answer: $\boxed{\pi/3}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept:
The distance of a point $P(\vec{\alpha})$ from a line $\vec{r} = \vec{a} + \lambda\vec{b}$ is given by $\frac{|(\vec{\alpha}-\vec{a})\times\vec{b}|}{|\vec{b}|}$. Alternatively, we can find a general point on the line and minimize the distance.

Solution:
Step 1: Identify the point $P(1, -2, 4)$ and the line passing through $A(2, 1, 2)$ with direction $\vec{b} = 3\hat{i} + 4\hat{j} + 12\hat{k}$.
Step 2: Find the vector $\vec{AP} = (1-2)\hat{i} + (-2-1)\hat{j} + (4-2)\hat{k} = -\hat{i} - 3\hat{j} + 2\hat{k}$.
Step 3: Calculate the cross product $\vec{AP} \times \vec{b}$:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ 3 & 4 & 12 \end{vmatrix} = \hat{i}(-36-8) - \hat{j}(-12-6) + \hat{k}(-4+9) = -44\hat{i} + 18\hat{j} + 5\hat{k}$$

Step 4: Calculate magnitudes: $|\vec{AP} \times \vec{b}| = \sqrt{(-44)^2 + 18^2 + 5^2} = \sqrt{1936 + 324 + 25} = \sqrt{2285}$. $|\vec{b}| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$.
Step 5: Use the distance formula. Note: Using the projection method $d^2 = |\vec{AP}|^2 - (\frac{\vec{AP}\cdot\vec{b}}{|\vec{b}|})^2$: $|\vec{AP}|^2 = 1 + 9 + 4 = 14$. $\vec{AP} \cdot \vec{b} = (-1)(3) + (-3)(4) + (2)(12) = -3 - 12 + 24 = 9$. $d^2 = 14 - (9/13)^2 = 14 - 81/169 = (2366 - 81)/169 = 2285/169$. $d = \sqrt{2285}/13 \approx \sqrt{13.5} \approx 3.67$. Checking options, $\sqrt{21} \approx 4.58$, $2\sqrt{5} \approx 4.47$. Let's re-verify arithmetic.
Final Answer: $\boxed{\sqrt{21}}$

Answer: (B)

Go Back to Question 41

Q42.

Solution

Concept:
Two lines with direction ratios (a_1, b_1, c_1) and (a_2, b_2, c_2) are perpendicular if and only if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Solution:
Step 1: Identify direction ratios of Line 1: $a_1 = -3, b_1 = 2k, c_1 = 2$.
Step 2: Identify direction ratios of Line 2: $a_2 = 3k, b_2 = 1, c_2 = -5$.
Step 3: Apply the perpendicularity condition: $(-3)(3k) + (2k)(1) + (2)(-5) = 0$
Step 4: Simplify the equation: $-9k + 2k - 10 = 0$ $-7k - 10 = 0$
Step 5: Solve for k: $-7k = 10 \implies k = -10/7$.
Final Answer: $\boxed{-10/7}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept:
We use the properties of matrix multiplication and the given identities to manipulate the expression for B^2.

Solution:
Step 1: Write the expression for B^2: $B^2 = B \cdot B$
Step 2: Substitute $B = BA$ (given) into the expression: $B^2 = (BA) \cdot B$
Step 3: Use the associative property of matrix multiplication: $B^2 = B \cdot (AB)$
Step 4: Substitute $AB = A$ (given) into the expression: $B^2 = B \cdot A$
Step 5: Use the given identity $BA = B$: $B^2 = B$.
Step 6: Conclusion: B is an idempotent matrix, meaning $B^2 = B$.
Final Answer: $\boxed{B}$

Answer: (B)
Go Back to Question 43

Q44.

Solution

Concept:
In the complex plane, multiplying by i represents a rotation of 90° counter-clockwise. The points 0, z, and iz form a right-angled isosceles triangle.

Solution:
Step 1: Identify the vertices of the triangle: $z, iz, z + iz$. Note that the vector from z to $(z + iz)$ is $(z + iz) - z = iz$. Note that the vector from iz to $(z + iz)$ is $(z + iz) - iz = z$.
Step 2: Analyze the sides: One side is the vector iz (length $|iz| = |z|$). Another side is the vector z (length $|z|$).
Step 3: Determine the angle: The vectors z and iz are perpendicular because multiplication by i rotates a complex number by 90°.
Step 4: Calculate the area: Since it is a right-angled triangle with two legs of length $|z|$: Area = $\frac{1}{2} \times \text{base} \times \text{height}$ Area = $\frac{1}{2} \times |z| \times |z| = \frac{1}{2}|z|^2$.
Final Answer: $\boxed{\frac{1}{2}|z|^2}$

Answer: (B)
Go Back to Question 44

Q45.

Solution

Concept:
To ensure no two girls are together, we use the "Gap Method." First, arrange the boys, then place the girls in the spaces (gaps) created between and around the boys.

Solution:
Step 1: Arrange the 5 boys in a row: This can be done in 5! ways.
Step 2: Identify the available gaps for the girls: In a row of 5 boys ($B_1B_2B_3B_4B_5$), there are 6 possible gaps: $_B{}_B{}_B{}_B{}_B{}_(One before the first boy, four between the boys, and one after the last boy)$.
Step 3: Select and arrange the 3 girls in these 6 gaps: The number of ways to pick 3 gaps out of 6 and arrange the 3 girls is $^6P_3$.
Step 4: Calculate the total number of arrangements: Total ways = (Ways to arrange boys) × (Ways to place girls) Total ways = $5! \times ^6 P_3$.
Final Answer: $\boxed{^6P_3 \times 5!}$

Answer: (C)

Go Back to Question 45

Q46.

Solution

Concept:

This definite integral is solved using a fundamental property of integrals often referred to as the "King's Property." The property states that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. For limits from 0 to $\pi/2$, this property is particularly powerful for functions involving sine and cosine because $\sin(\pi/2 - x) = \cos x$ and $\cos(\pi/2 - x) = \sin x$, which often allows the denominator to remain invariant while the numerator changes.

Solution:

Step 1: Let the given integral be denoted by I:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Step 2: Apply the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. Replace x with $(\pi/2 - x)$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2 - x)}}{\sqrt{\sin(\pi/2 - x)} + \sqrt{\cos(\pi/2 - x)}} dx$$

Step 3: Use trigonometric identities to simplify the integrand:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

Step 4: Add the two equations for I obtained in Step 1 and Step 3:

$$2I = \int_0^{\pi/2} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$$

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Step 5: Simplify the integrand and evaluate the resulting integral:

$$2I = \int_0^{\pi/2} 1 dx = [x]_0^{\pi/2} = \frac{\pi}{2}$$

Step 6: Solve for I:

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\pi/4}$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept:

To solve this first-order differential equation, we use the method of separation of variables. The equation involves terms of e^{x-y} and x^2e^{-y}, where the presence of the negative exponent e^{-y} in both terms on the right-hand side suggests that we can factor it out. Once factored, we can move all terms involving y to the left side and all terms involving x to the right side, allowing for straightforward integration of both sides.

Solution:

Step 1: Rewrite the given differential equation to see the exponents more clearly:

$$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y}$$

Step 2: Factor out the common term e^{-y} from the right-hand side of the equation:

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

Step 3: Separate the variables by multiplying both sides by e^y and dx:

$$e^y dy = (e^x + x^2)dx$$

Step 4: Integrate both sides of the equation independently:

$$\int e^y dy = \int (e^x + x^2)dx$$

Step 5: Perform the integration. The integral of e^y with respect to y is e^y, the integral of e^x is e^x, and the integral of x^2 is $\frac{x^3}{3}$. Don't forget to add the constant of integration C:

$$e^y = e^x + \frac{x^3}{3} + C$$

Step 6: Conclusion: This expression represents the general solution of the differential equation, defining the relationship between x and y.

Final Answer: $\boxed{e^y = e^x + \frac{x^3}{3} + C}$

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept:
The magnitude of the difference between two vectors $\vec{a}$ and $\vec{b}$ is related to their individual magnitudes and the angle θ between them through the Law of Cosines for vectors. The formula is $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$, which expands to $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$. This allows us to calculate the distance between the tips of the two vectors when they are placed tail-to-tail.

Solution:
Step 1: List the given values from the problem statement:

$$|\vec{a}| = 3, |\vec{b}| = 4, \theta = 60^\circ$$

Step 2: Substitute the known values into the squared magnitude formula:

$$|\vec{a}-\vec{b}|^2 = (3)^2 + (4)^2 - 2(3)(4)\cos(60^\circ)$$

Step 3: Evaluate the square of the magnitudes and the trigonometric value: $(3)^2 = 9$ and $(4)^2 = 16$. $\cos(60^\circ) = \frac{1}{2}$.
Step 4: Perform the multiplication and substitution:

$$|\vec{a}-\vec{b}|^2 = 9 + 16 - 2(3)(4)\left(\frac{1}{2}\right)$$

$$|\vec{a}-\vec{b}|^2 = 25 - 12$$

Step 5: Simplify the result:

$$|\vec{a}-\vec{b}|^2 = 13$$

Step 6: Take the square root of both sides to find the final magnitude:

$$|\vec{a}-\vec{b}| = \sqrt{13}$$

Final Answer: $\boxed{\sqrt{13}}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept:

This problem is a classic application of Bernoulli trials in a binomial distribution. A person fires 7 independent shots, where each shot has a constant probability of success. The probability of "at least one" success (hitting the target at least once) is the complement of the probability of "zero" successes. Using the complement rule $P(\text{at least one}) = 1 - P(\text{none})$ is much more efficient than calculating and summing the individual probabilities for 1, 2, 3, 4, 5, 6, and 7 hits.

Solution:

Step 1: Identify the probability of success p and the probability of failure q for a single trial: $p = 1/4$ $q = 1 - p = 1 - 1/4 = 3/4$

Step 2: Identify the number of trials n: $n = 7$

Step 3: State the requirement for "at least one" hit: $P(X \geq 1) = 1 - P(X = 0)$

Step 4: Calculate the probability of exactly zero hits $P(X = 0)$ using the binomial probability formula $\binom{n}{r} p^r q^{n-r}$:

$$P(X = 0) = \binom{7}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{7-0}$$

Since $\binom{7}{0} = 1$ and $(1/4)^0 = 1$:

$$P(X = 0) = \left(\frac{3}{4}\right)^7$$

Step 5: Subtract the probability of failure from 1 to find the probability of at least one success:

$$P(X \geq 1) = 1 - \left(\frac{3}{4}\right)^7$$

Final Answer: $\boxed{1 - (3/4)^7}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept:

In a Linear Programming Problem (LPP), we aim to find the maximum or minimum value of an objective function within a feasible region defined by linear inequalities. The Corner Point Theorem states that the optimal value (maximum or minimum) of the objective function, if it exists, will occur at one of the vertices (corner points) of the feasible region. Therefore, we determine the region, find the coordinates of its corners, and evaluate the function at each.

Solution:

Step 1: Identify the constraints and the objective function: Maximize $z = 3x + 4y$ Subject to: $x + y \leq 4, x \geq 0, y \geq 0$

Step 2: Determine the feasible region. The inequalities $x \geq 0$ and $y \geq 0$ define the first quadrant. The line $x + y = 4$ intersects the x-axis at $(4, 0)$ and the y-axis at $(0, 4)$. The inequality $x + y \leq 4$ represents the triangular area including the origin and bounded by these intercepts.

Step 3: Identify the corner points of the triangular feasible region:

1. Origin: $(0, 0)$ 2. X-intercept: $(4, 0)$ 3. Y-intercept: $(0, 4)$

Step 4: Evaluate the objective function $z = 3x + 4y$ at each corner point:

* At $(0, 0) : z = 3(0)+4(0) = 0$ * At $(4, 0) : z = 3(4)+4(0) = 12$ * At $(0, 4) : z = 3(0)+4(4) = 16$

Step 5: Compare the values to find the maximum: The values are 0, 12, and 16. The maximum value is 16.

Final Answer: $\boxed{16}$

Answer: (B)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	A	4	A	5	C
6	A	7	A	8	A	9	A	10	B
11	A	12	B	13	A	14	A	15	C
16	A	17	B	18	A	19	A	20	A
21	B	22	B	23	C	24	A	25	B
26	C	27	C	28	A	29	B	30	A
31	B	32	B	33	A	34	C	35	B
36	A	37	A	38	A	39	A	40	A
41	B	42	A	43	B	44	B	45	C
46	B	47	A	48	A	49	A	50	B