

# MHT-CET Mathematics Sample Paper-15

Duration: 90 Minutes

Maximum Marks: 100

## Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

**Q1.** The value of  $\lim_{x \rightarrow 0} \frac{e^{5x} - e^{2x}}{\sin(3x)}$  is:

- (A) 1
- (B) 3
- (C) 0
- (D) 5/3

**Q2.** If  $f(x) = \frac{\log(1+kx)}{\sin x}$  for  $x \neq 0$  and  $f(0) = 4$  is continuous at  $x = 0$ , then the value of  $k$  is:

- (A) 2
- (B) 4
- (C) 1
- (D) 8

**Q3.** If  $y = \tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$ , then  $\frac{dy}{dx}$  at  $x = 1$  is:

- (A) 1/4
- (B) 1/2
- (C) 1
- (D) 0



**Q4.** If  $x^y = e^{x-y}$ , then  $\frac{dy}{dx}$  is equal to:

- (A)  $\frac{\log x}{(1+\log x)^2}$
- (B)  $\frac{1}{(1+\log x)^2}$
- (C)  $\frac{\log x}{1+\log x}$
- (D)  $\frac{x-y}{1+\log x}$

**Q5.** If  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$ , then  $\frac{d^2y}{dx^2}$  at  $\theta = \pi/4$  is:

- (A)  $\frac{4\sqrt{2}}{3a}$
- (B)  $\frac{-4\sqrt{2}}{3a}$
- (C)  $\frac{3a}{4\sqrt{2}}$
- (D) 0

**Q6.** The equation of the tangent to the curve  $y = 2x^2 - x + 1$  at the point (1, 2) is:

- (A)  $3x - y - 1 = 0$
- (B)  $3x + y - 5 = 0$
- (C)  $x - 3y + 5 = 0$
- (D)  $2x - y = 0$

**Q7.** The maximum value of  $f(x) = xe^{-x}$  is:

- (A)  $e$
- (B)  $1/e$
- (C) 1
- (D)  $e^2$

**Q8.** The approximate value of  $\sqrt{25.2}$  using differentials is:

- (A) 5.01
- (B) 5.02
- (C) 5.05



(D) 5.002

**Q9.** The integral  $\int \frac{1}{x(x^5+1)} dx$  is equal to:

(A)  $\frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + C$

(B)  $\log \left| \frac{x^5}{x^5+1} \right| + C$

(C)  $5 \log \left| \frac{x^5+1}{x^5} \right| + C$

(D)  $\frac{1}{5} \log \left| \frac{x^5+1}{x^5} \right| + C$

**Q10.** The value of  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$  is:

(A)  $\pi$

(B)  $\pi/2$

(C)  $\pi/4$

(D) 0

**Q11.** The area bounded by the curve  $y^2 = 8x$  and the line  $x = 2$  is:

(A)  $16/3$  sq. units

(B)  $32/3$  sq. units

(C)  $64/3$  sq. units

(D) 8 sq. units

**Q12.** The order and degree of the differential equation  $\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2} = \frac{d^2y}{dx^2}$  are respectively:

(A) 2, 2

(B) 2, 3

(C) 1, 2

(D) 2, 1

**Q13.** The general solution of the differential equation  $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$  is:



- (A)  $e^y = e^x + \frac{x^3}{3} + C$
- (B)  $e^x = e^y + \frac{x^3}{3} + C$
- (C)  $e^{-y} = e^x + \frac{x^3}{3} + C$
- (D)  $e^y = e^x + x^3 + C$

**Q14.** If  $z = \frac{1+i}{1-i}$ , then  $z^{100}$  is equal to:

- (A)  $i$
- (B)  $-i$
- (C)  $1$
- (D)  $-1$

**Q15.** The square root of  $3 + 4i$  is:

- (A)  $\pm(2 + i)$
- (B)  $\pm(1 + 2i)$
- (C)  $\pm(2 - i)$
- (D)  $\pm(1 - 2i)$

**Q16.** If  $\alpha$  and  $\beta$  are roots of  $x^2 - 6x + 8 = 0$ , then the value of  $\alpha^3 + \beta^3$  is:

- (A) 72
- (B) 216
- (C) 144
- (D) 96

**Q17.** If the roots of  $ax^2 + bx + c = 0$  are in the ratio  $2 : 3$ , then:

- (A)  $6b^2 = 25ac$
- (B)  $25b^2 = 6ac$
- (C)  $b^2 = 6ac$
- (D)  $6b^2 = ac$



**Q18.** The  $n^{\text{th}}$  term of the series  $5 + 11 + 19 + 29 + \dots$  is:

- (A)  $n^2 + 3n + 1$
- (B)  $n^2 + n + 3$
- (C)  $2n^2 + 3$
- (D)  $n^2 + 4n$

**Q19.** If  $a, b, c$  are in G.P., then  $\log a, \log b, \log c$  are in:

- (A) A.P.
- (B) G.P.
- (C) H.P.
- (D) None of these

**Q20.** The coefficient of  $x^4$  in the expansion of  $(1 + 2x)^6$  is:

- (A) 240
- (B) 480
- (C) 15
- (D) 60

**Q21.** The number of ways in which 5 people can be seated around a circular table is:

- (A) 120
- (B) 24
- (C) 60
- (D) 12

**Q22.** If  $P(A) = 0.4, P(B) = 0.5$  and  $A, B$  are independent events, then  $P(A \cup B)$  is:

- (A) 0.9
- (B) 0.7
- (C) 0.2



(D) 0.1

**Q23.** The distance of the point  $(2, 3)$  from the line  $3x + 4y - 8 = 0$  is:

(A) 2 units

(B) 10 units

(C) 5 units

(D) 1 unit

**Q24.** The equation of the circle with center  $(1, -2)$  and passing through  $(4, 2)$  is:

(A)  $x^2 + y^2 - 2x + 4y - 20 = 0$

(B)  $x^2 + y^2 + 2x - 4y - 20 = 0$

(C)  $x^2 + y^2 - 2x + 4y + 5 = 0$

(D)  $x^2 + y^2 = 25$

**Q25.** The eccentricity of the ellipse  $9x^2 + 25y^2 = 225$  is:

(A)  $4/5$

(B)  $3/5$

(C)  $16/25$

(D)  $2/5$

**Q26.** The length of the latus rectum of the parabola  $y^2 = 12x$  is:

(A) 3

(B) 6

(C) 12

(D) 4

**Q27.** If the vectors  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} + \lambda\hat{k}$  are perpendicular, then  $\lambda$  is:

(A) 1

(B) -1



(C) 2

(D) 0

**Q28.** The direction cosines of the line joining  $(1, 0, 0)$  and  $(0, 1, 1)$  are:

(A)  $-1, 1, 1$

(B)  $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(C)  $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(D)  $\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

**Q29.** The value of  $\int e^x(\tan x + \sec^2 x)dx$  is:

(A)  $e^x \sec^2 x + C$

(B)  $e^x \tan x + C$

(C)  $e^x(\tan x + \sec x) + C$

(D)  $e^x/\tan x + C$

**Q30.** The probability of getting a sum of 9 when two dice are thrown is:

(A)  $1/9$

(B)  $1/6$

(C)  $1/4$

(D)  $1/12$

**Q31.** The equation of the plane passing through  $(1, 2, 3)$  and parallel to  $3x + 4y - 5z = 0$  is:

(A)  $3x + 4y - 5z + 4 = 0$

(B)  $3x + 4y - 5z - 4 = 0$

(C)  $3x + 4y - 5z = 0$

(D)  $x + y + z = 6$

**Q32.** If  $y = \log(\sec x + \tan x)$ , then  $\frac{dy}{dx}$  is:



- (A)  $\sec x$
- (B)  $\tan x$
- (C)  $\sec x \tan x$
- (D)  $\sec^2 x$

**Q33.** The area of the region bounded by  $y = \cos x$ ,  $x = 0$  and  $x = \pi/2$  is:

- (A) 1 sq. unit
- (B) 2 sq. units
- (C)  $1/2$  sq. unit
- (D)  $\pi$  sq. units

**Q34.** The radius of the circle  $x^2 + y^2 - 4x + 6y - 12 = 0$  is:

- (A) 5
- (B)  $\sqrt{13}$
- (C) 25
- (D) 7

**Q35.** In a Binomial Distribution with  $n = 10$  and  $p = 0.4$ , the variance is:

- (A) 4
- (B) 2.4
- (C) 6
- (D) 1.6

**Q36.** The angle between the lines  $\vec{r} = \hat{i} + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$  and  $\vec{r} = \hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$  is:

- (A)  $\cos^{-1}(16/21)$
- (B)  $\cos^{-1}(4/21)$
- (C)  $\cos^{-1}(8/21)$
- (D)  $90^\circ$



- Q37.** The integrating factor of the differential equation  $\frac{dy}{dx} - y \cot x = \csc x$  is:
- (A)  $\sin x$
  - (B)  $\csc x$
  - (C)  $\log(\sin x)$
  - (D)  $e^{\sin x}$
- Q38.** If  ${}^n C_{12} = {}^n C_8$ , then the value of  $n$  is:
- (A) 20
  - (B) 12
  - (C) 8
  - (D) 4
- Q39.** The slope of the normal to the curve  $y = x^3 + 2x + 6$  at  $x = 0$  is:
- (A) 2
  - (B) -2
  - (C)  $1/2$
  - (D)  $-1/2$
- Q40.** If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = 0$ , then  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  is:
- (A)  $3/2$
  - (B)  $-3/2$
  - (C) 0
  - (D) 1
- Q41.** The sum of the series  $1 + 1/2 + 1/4 + 1/8 + \dots$  to infinity is:
- (A) 1
  - (B) 2
  - (C) 1.5



(D)  $\infty$

**Q42.** The length of the major axis of the ellipse  $\frac{x^2}{16} + \frac{y^2}{9} = 1$  is:

(A) 4

(B) 8

(C) 3

(D) 6

**Q43.** If  $A$  and  $B$  are matrices such that  $AB$  exists, then  $(AB)^T$  is:

(A)  $A^T B^T$

(B)  $B^T A^T$

(C)  $AB$

(D)  $BA$

**Q44.** The value of  $\int_1^2 \frac{1}{x} dx$  is:

(A)  $\log 2$

(B) 1

(C) 0

(D)  $e$

**Q45.** The distance between the parallel lines  $y = x + 4$  and  $y = x + 1$  is:

(A) 3

(B)  $3/\sqrt{2}$

(C) 5

(D)  $1/\sqrt{2}$

**Q46.** The vertex of the parabola  $(y - 2)^2 = 8(x - 1)$  is:

(A) (0, 0)

(B) (1, 2)



- (C) (2, 1)
- (D) (-1, -2)

**Q47.** The value of  $i^{49}$  is:

- (A) 1
- (B) -1
- (C)  $i$
- (D)  $-i$

**Q48.** The derivative of  $\sin(x^2)$  with respect to  $x$  is:

- (A)  $\cos(x^2)$
- (B)  $2x \cos(x^2)$
- (C)  $-2x \cos(x^2)$
- (D)  $2 \cos x$

**Q49.** If the sum of two numbers is 10, then the maximum value of their product is:

- (A) 20
- (B) 25
- (C) 10
- (D) 100

**Q50.** The value of  $\Delta$  for the equation  $x^2 - 5x + 6 = 0$  is:

- (A) 1
- (B) 0
- (C) 25
- (D) 6



## Detailed Solutions

Q1.

## Solution

**Concept:**

The evaluation of limits involving exponential functions often utilizes the standard formula  $\lim_{x \rightarrow 0} \frac{e^{kx} - 1}{x} = k$ . For expressions involving the difference of two exponentials, we can subtract and add 1 in the numerator to apply this rule. Additionally, the trigonometric limit  $\lim_{x \rightarrow 0} \frac{\sin(ax)}{ax} = 1$  is frequently used. Alternatively, L'Hôpital's Rule can be applied when the limit results in an indeterminate form like  $0/0$ .

**Solution:**

1. Given limit:

$$L = \lim_{x \rightarrow 0} \frac{e^{5x} - e^{2x}}{\sin(3x)}$$

2. Observe that as  $x \rightarrow 0$ , the expression takes the indeterminate form  $\frac{e^0 - e^0}{\sin 0} = \frac{1-1}{0} = \frac{0}{0}$ .3. Applying L'Hôpital's Rule by differentiating the numerator and denominator with respect to  $x$ :

$$\text{Numerator derivative: } \frac{d}{dx}(e^{5x} - e^{2x}) = 5e^{5x} - 2e^{2x}$$

$$\text{Denominator derivative: } \frac{d}{dx}(\sin 3x) = 3 \cos 3x$$

4. Substitute the derivatives back into the limit:

$$L = \lim_{x \rightarrow 0} \frac{5e^{5x} - 2e^{2x}}{3 \cos 3x}$$

5. Evaluate the limit as  $x \rightarrow 0$ :

$$L = \frac{5e^0 - 2e^0}{3 \cos 0} = \frac{5(1) - 2(1)}{3(1)} = \frac{3}{3} = 1$$

**Final Answer:**

1

**Answer: (A)**[Go Back to Question 1](#)

Q2.

**Solution****Concept:**

A function  $f(x)$  is continuous at a point  $x = a$  if the limit of the function as  $x$  approaches  $a$  is equal to the value of the function at  $a$ . That is,  $\lim_{x \rightarrow a} f(x) = f(a)$ . For logarithmic and trigonometric limits, we use  $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ . These standard limits help in simplifying expressions involving parameters to find their values based on continuity.

**Solution:**

1. Given that  $f(x)$  is continuous at  $x = 0$ , we have:

$$\lim_{x \rightarrow 0} f(x) = f(0)$$

Given  $f(0) = 4$ .

2. Set up the limit equation:

$$\lim_{x \rightarrow 0} \frac{\log(1+kx)}{\sin x} = 4$$

3. To use standard limits, divide the numerator and denominator by  $x$ :

$$\lim_{x \rightarrow 0} \frac{\frac{\log(1+kx)}{x}}{\frac{\sin x}{x}} = 4$$

4. Adjust the numerator to match the standard form  $\frac{\log(1+kx)}{kx} \cdot k$ :

$$\frac{\lim_{x \rightarrow 0} \frac{\log(1+kx)}{kx} \cdot k}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = 4$$

5. Since  $\lim_{u \rightarrow 0} \frac{\log(1+u)}{u} = 1$  and  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ :

$$\frac{1 \cdot k}{1} = 4 \implies k = 4$$

**Final Answer:**

4

**Answer: (B)**

[Go Back to Question 2](#)



Q3.

**Solution****Concept:**

For inverse trigonometric functions involving square roots of the form  $\sqrt{1+x^2}$ , a common substitution is  $x = \tan \theta$ . This transforms the algebraic expression into a trigonometric one, often simplifying significantly using identities like  $1 + \tan^2 \theta = \sec^2 \theta$ . Half-angle formulas such as  $1 - \cos \theta = 2 \sin^2(\theta/2)$  and  $\sin \theta = 2 \sin(\theta/2) \cos(\theta/2)$  are then applied to reduce the inverse function.

**Solution:**

1. Substitute  $x = \tan \theta$ , which implies  $\theta = \tan^{-1} x$ :

$$y = \tan^{-1} \left( \frac{\sqrt{1 + \tan^2 \theta} - 1}{\tan \theta} \right)$$

2. Use the identity  $\sqrt{1 + \tan^2 \theta} = \sec \theta$ :

$$y = \tan^{-1} \left( \frac{\sec \theta - 1}{\tan \theta} \right)$$

3. Convert to sine and cosine:

$$y = \tan^{-1} \left( \frac{\frac{1}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta}} \right) = \tan^{-1} \left( \frac{1 - \cos \theta}{\sin \theta} \right)$$

4. Apply half-angle identities:

$$y = \tan^{-1} \left( \frac{2 \sin^2(\theta/2)}{2 \sin(\theta/2) \cos(\theta/2)} \right) = \tan^{-1}(\tan(\theta/2))$$

$$y = \frac{\theta}{2} = \frac{1}{2} \tan^{-1} x$$

5. Differentiate with respect to  $x$ :

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1+x^2}$$

$$\text{At } x = 1: \frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1+1^2} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

**Final Answer:**

$$\boxed{1/4}$$

**Answer: (A)**

[Go Back to Question 3](#)



Q4.

**Solution****Concept:**

When an equation involves a variable in the exponent, such as  $x^y$ , logarithmic differentiation is the most effective approach. Taking the natural logarithm on both sides allows us to use the property  $\log(a^b) = b \log a$ , converting the exponential relationship into an implicit algebraic equation. After taking logs, we differentiate both sides with respect to  $x$ , applying the product rule and isolating  $\frac{dy}{dx}$ .

**Solution:**

1. Take natural logarithm on both sides:

$$\log(x^y) = \log(e^{x-y})$$

$$y \log x = (x - y) \log e$$

Since  $\log e = 1$ , we have  $y \log x = x - y$ .

2. Rearrange to solve for  $y$ :

$$y \log x + y = x$$

$$y(1 + \log x) = x$$

$$y = \frac{x}{1 + \log x}$$

3. Differentiate  $y$  with respect to  $x$  using the quotient rule  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ : Let  $u = x$  and  $v = 1 + \log x$ .  $\frac{du}{dx} = 1$  and  $\frac{dv}{dx} = \frac{1}{x}$ .

4. Substitute into the quotient rule:

$$\frac{dy}{dx} = \frac{(1 + \log x)(1) - x\left(\frac{1}{x}\right)}{(1 + \log x)^2}$$

$$\frac{dy}{dx} = \frac{1 + \log x - 1}{(1 + \log x)^2} = \frac{\log x}{(1 + \log x)^2}$$

**Final Answer:**

$$\frac{\log x}{(1 + \log x)^2}$$

**Answer: (1)**

[Go Back to Question 4](#)



Q5.

**Solution****Concept:**

Parametric differentiation is used when  $x$  and  $y$  are expressed in terms of a third variable, like  $\theta$ . The first derivative is found by  $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ . To find the second derivative  $\frac{d^2y}{dx^2}$ , we must differentiate  $\frac{dy}{dx}$  with respect to  $\theta$  and then multiply by  $\frac{d\theta}{dx}$  (the reciprocal of  $\frac{dx}{d\theta}$ ), following the chain rule. This is a common area where students forget the final  $\frac{d\theta}{dx}$  step.

**Solution:**

$$1. \text{ Find derivatives with respect to } \theta: x = a \cos^3 \theta \implies \frac{dx}{d\theta} = -3a \cos^2 \theta \sin \theta$$

$$y = a \sin^3 \theta \implies \frac{dy}{d\theta} = 3a \sin^2 \theta \cos \theta$$

$$2. \text{ Find the first derivative: } \frac{dy}{dx} = \frac{3a \sin^2 \theta \cos \theta}{-3a \cos^2 \theta \sin \theta} = -\frac{\sin \theta}{\cos \theta} = -\tan \theta$$

$$3. \text{ Find the second derivative: } \frac{d^2y}{dx^2} = \frac{d}{dx}(-\tan \theta) = \frac{d}{d\theta}(-\tan \theta) \cdot \frac{d\theta}{dx} \frac{d^2y}{d\theta^2} = (-\sec^2 \theta) \cdot \frac{1}{-3a \cos^2 \theta \sin \theta} = \frac{1}{3a \cos^4 \theta \sin \theta}$$

$$4. \text{ Evaluate at } \theta = \pi/4: \cos(\pi/4) = 1/\sqrt{2}, \sin(\pi/4) = 1/\sqrt{2} \frac{d^2y}{dx^2} = \frac{1}{3a(1/\sqrt{2})^4(1/\sqrt{2})} = \frac{1}{3a(1/4)(1/\sqrt{2})} = \frac{4\sqrt{2}}{3a}$$

**Final Answer:**

$$\boxed{\frac{4\sqrt{2}}{3a}}$$

**Answer: (A)**[Go Back to Question 5](#)

Q6.

**Solution****Concept:**

The slope of the tangent to a curve  $y = f(x)$  at a given point  $(x_1, y_1)$  is found by calculating the derivative  $\frac{dy}{dx}$  and evaluating it at that specific point. Once the slope  $m$  is determined, the equation of the tangent line can be written using the point-slope form:  $y - y_1 = m(x - x_1)$ . In the context of MHT-CET, this is a fundamental application of derivatives used to find linear approximations of curves.

**Solution:**

1. Given the equation of the curve:

$$y = 2x^2 - x + 1$$

2. Differentiate with respect to  $x$  to find the slope function:

$$\frac{dy}{dx} = 4x - 1$$

3. Calculate the slope  $m$  at the given point  $(1, 2)$ :

$$m = \left. \frac{dy}{dx} \right|_{x=1} = 4(1) - 1 = 3$$

4. Use the point-slope form for the equation of the tangent:

$$y - 2 = 3(x - 1)$$

5. Simplify the equation into the standard form  $Ax + By + C = 0$ :

$$y - 2 = 3x - 3$$

$$3x - y - 1 = 0$$

**Final Answer:**

$$3x - y - 1 = 0$$

**Answer: (A)**[Go Back to Question 6](#)

Q7.

**Solution****Concept:**

To find the maximum or minimum value of a function  $f(x)$ , we identify the critical points where the first derivative  $f'(x) = 0$ . After finding these critical values, we apply the Second Derivative Test. If  $f''(x) < 0$  at a critical point, the function has a local maximum there. For the function  $xe^{-x}$ , the product rule  $\frac{d}{dx}(uv) = u'v + uv'$  and the chain rule are essential for differentiation.

**Solution:**

1. Given the function:

$$f(x) = xe^{-x}$$

2. Find the first derivative using the product rule:

$$f'(x) = (1) \cdot e^{-x} + x \cdot (-e^{-x}) = e^{-x}(1 - x)$$

3. Set  $f'(x) = 0$  to find critical points:

$$e^{-x}(1 - x) = 0$$

Since  $e^{-x}$  is never zero for any real value of  $x$ , we must have  $1 - x = 0 \implies x = 1$ .

4. Find the second derivative to check for maxima:

$$f''(x) = \frac{d}{dx}[e^{-x}(1 - x)] = -e^{-x}(1 - x) + e^{-x}(-1) = e^{-x}(x - 2)$$

At  $x = 1$ ,  $f''(1) = e^{-1}(1 - 2) = -1/e$ . Since  $f''(1) < 0$ ,  $x = 1$  is a point of maximum.

5. Calculate the maximum value by substituting  $x = 1$  into  $f(x)$ :

$$f(1) = 1 \cdot e^{-1} = 1/e$$

**Final Answer:**

$$\boxed{1/e}$$

**Answer: (B)**

[Go Back to Question 7](#)



Q8.

**Solution****Concept:**

Differentials provide a method to approximate the value of a function  $f(x + \Delta x)$  using the formula  $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$ . This is based on the linear approximation of the curve at a known point  $x$ . For square roots, we choose  $x$  as the nearest perfect square to minimize the error  $\Delta x$  and ensure the derivative is easy to compute without a calculator.

**Solution:**

1. Let the function be  $f(x) = \sqrt{x}$ . Then the derivative is  $f'(x) = \frac{1}{2\sqrt{x}}$ .
2. Identify  $x$  and  $\Delta x$  for the expression  $\sqrt{25.2}$ : Let  $x = 25$  (the nearest perfect square) and  $\Delta x = 0.2$ .
3. Calculate the values of the function and its derivative at  $x = 25$ :

$$f(25) = \sqrt{25} = 5$$

$$f'(25) = \frac{1}{2\sqrt{25}} = \frac{1}{10} = 0.1$$

4. Apply the approximation formula:

$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x$$

$$\sqrt{25.2} \approx 5 + (0.1)(0.2)$$

5. Calculate the final approximate value:

$$\sqrt{25.2} \approx 5 + 0.02 = 5.02$$

**Final Answer:**

5.02
------

Answer: (B)
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Q9.

**Solution****Concept:**

Integrals of the form  $\int \frac{1}{x(x^n+1)} dx$  are solved by a specific algebraic manipulation: multiplying the numerator and denominator by  $x^{n-1}$ . This allows for a substitution  $t = x^n$ , which simplifies the integrand into a form suitable for partial fractions or the standard logarithmic integration rule  $\int \frac{1}{t} dt = \log |t|$ . This is a high-yield pattern in MHT-CET calculus.

**Solution:**

1. Given the integral:

$$I = \int \frac{1}{x(x^5+1)} dx$$

2. Multiply the numerator and the denominator by  $x^4$ :

$$I = \int \frac{x^4}{x^5(x^5+1)} dx$$

3. Use substitution  $t = x^5$ , then  $dt = 5x^4 dx \implies x^4 dx = \frac{1}{5} dt$ :

$$I = \frac{1}{5} \int \frac{1}{t(t+1)} dt$$

4. Use the partial fraction decomposition  $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$ :

$$I = \frac{1}{5} \left( \int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right)$$

$$I = \frac{1}{5} (\log |t| - \log |t+1|) + C = \frac{1}{5} \log \left| \frac{t}{t+1} \right| + C$$

5. Resubstitute  $t = x^5$  to get the final answer:

$$I = \frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + C$$

**Final Answer:**

$$\boxed{\frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + C}$$

**Answer: (A)**

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## Q10.

## Solution

**Concept:**

A highly useful property of definite integrals is  $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ , often referred to as the "King's Property." In the case of trigonometric integrals with limits from 0 to  $\pi/2$ , applying this property converts  $\sin x$  to  $\cos x$  and vice-versa. Adding the original integral  $I$  to the transformed integral usually simplifies the integrand to 1, making the solution straightforward.

**Solution:**

1. Let the given integral be  $I$ :

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

— (Eq. 1)

2. Apply the property  $\int_0^a f(x)dx = \int_0^a f(a-x)dx$  by replacing  $x$  with  $\pi/2 - x$ : Since  $\sin(\pi/2 - x) = \cos x$  and  $\cos(\pi/2 - x) = \sin x$ , the integral becomes:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

— (Eq. 2)

3. Add Equation 1 and Equation 2:

$$2I = \int_0^{\pi/2} \left( \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \right) dx$$

4. Simplify the integrand:

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \int_0^{\pi/2} 1 dx$$

$$2I = [x]_0^{\pi/2} = \pi/2$$

5. Solve for the value of  $I$ :

$$I = \frac{\pi}{4}$$

**Final Answer:**

$$\boxed{\pi/4}$$

**Answer: (C)**

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Q11.

**Solution****Concept:**

The area under a curve  $y^2 = 4ax$  bounded by a vertical line  $x = h$  is calculated using the definite integral  $A = \int_a^b |y| dx$ . Since the parabola  $y^2 = 8x$  is symmetric about the x-axis, the total area consists of two equal parts: one above the x-axis and one below. We find the area of the upper half by integrating  $y = \sqrt{8x}$  from the vertex (0,0) to the line  $x = 2$ , and then double the result.

**Solution:**

1. Given the parabola  $y^2 = 8x$ , solving for  $y$  gives  $y = \pm\sqrt{8x} = \pm 2\sqrt{2}x^{1/2}$ .
2. The required area is bounded by  $x = 0$  and  $x = 2$ . Due to symmetry:

$$\text{Area} = 2 \int_0^2 2\sqrt{2}x^{1/2} dx$$

3. Take the constants outside the integral:

$$\text{Area} = 4\sqrt{2} \int_0^2 x^{1/2} dx$$

4. Integrate using the power rule  $\int x^n dx = \frac{x^{n+1}}{n+1}$ :

$$\text{Area} = 4\sqrt{2} \left[ \frac{x^{3/2}}{3/2} \right]_0^2 = 4\sqrt{2} \cdot \frac{2}{3} [x\sqrt{x}]_0^2$$

5. Substitute the limits:

$$\text{Area} = \frac{8\sqrt{2}}{3} [2\sqrt{2} - 0] = \frac{8\sqrt{2} \cdot 2\sqrt{2}}{3} = \frac{16 \cdot 2}{3} = \frac{32}{3}$$

**Final Answer:**

$$\boxed{32/3 \text{ sq. units}}$$

**Answer: (B)**

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Q12.

**Solution****Concept:**

The **Order** of a differential equation is the highest order derivative present in the equation. The **Degree** is the power of the highest order derivative, provided the differential equation can be expressed as a polynomial in terms of its derivatives. To determine the degree, all fractional powers or radicals involving the derivatives must be eliminated by algebraic manipulation.

**Solution:**

1. The given differential equation is:

$$[1 + (y')^2]^{3/2} = y''$$

2. To remove the fractional power  $3/2$ , we square both sides of the equation:

$$([1 + (y')^2]^{3/2})^2 = (y'')^2$$

$$[1 + (y')^2]^3 = (y'')^2$$

3. Now the equation is in polynomial form regarding the derivatives.

4. Identify the highest derivative: The highest derivative present is  $y''$  (the second derivative). Therefore, **Order = 2**.

5. Identify the power of the highest derivative: The power of  $y''$  is 2. Therefore, **Degree = 2**.

**Final Answer:**

2, 2
------

<b>Answer: (A)</b>
--------------------

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Q13.

**Solution****Concept:**

This is a first-order differential equation that can be solved using the **\*\*Variable Separable Method\*\***. We first simplify the right-hand side using exponent laws  $e^{a-b} = e^a \cdot e^{-b}$  to factor out common terms. Once the equation is in the form  $f(y)dy = g(x)dx$ , we integrate both sides to find the general solution, adding a constant of integration  $C$ .

**Solution:**

1. Write the given equation:

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

2. Use the law of exponents to factor the right side:

$$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y} = e^{-y}(e^x + x^2)$$

3. Separate the variables  $x$  and  $y$ :

$$\frac{dy}{e^{-y}} = (e^x + x^2)dx \implies e^y dy = (e^x + x^2)dx$$

4. Integrate both sides:

$$\int e^y dy = \int (e^x + x^2) dx$$

5. Perform the integration:

$$e^y = e^x + \frac{x^3}{3} + C$$

**Final Answer:**

$$e^y = e^x + \frac{x^3}{3} + C$$

**Answer: (A)**[Go Back to Question 13](#)

Q14.

**Solution****Concept:**

To simplify a complex number fraction, we multiply the numerator and denominator by the conjugate of the denominator. This process, known as rationalizing the denominator, helps us express the complex number in the standard  $a + bi$  form. Once simplified, we utilize the cyclic property of the powers of  $i$ , where  $i^1 = i$ ,  $i^2 = -1$ ,  $i^3 = -i$ , and  $i^4 = 1$ . Any power  $i^n$  is 1 if  $n$  is a multiple of 4.

**Solution:**

1. Simplify the expression inside the parenthesis:

$$z = \frac{1+i}{1-i}$$

2. Multiply by the conjugate of the denominator ( $1+i$ ):

$$z = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1^2-i^2}$$

3. Since  $i^2 = -1$ :

$$z = \frac{1+2i-1}{1-(-1)} = \frac{2i}{2} = i$$

4. Now, find  $z^{100}$ :

$$z^{100} = i^{100}$$

5. Check the power of  $i$ : Since 100 is a multiple of 4 ( $100 = 4 \times 25$ ), then  $i^{100} = (i^4)^{25} = 1^{25} = 1$ .

**Final Answer:**

1
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<b>Answer: (C)</b>
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## Q15.

**Solution****Concept:**

The square root of a complex number  $a + bi$  can be found by assuming  $\sqrt{a + bi} = x + iy$ . Squaring both sides leads to the equations  $x^2 - y^2 = a$  and  $2xy = b$ . Alternatively, use the shortcut formula:

$$\sqrt{a + bi} = \pm \left( \sqrt{\frac{|z| + a}{2}} + i \cdot \operatorname{sgn}(b) \sqrt{\frac{|z| - a}{2}} \right)$$

where  $|z| = \sqrt{a^2 + b^2}$ .

**Solution:**

1. For  $z = 3 + 4i$ , we have  $a = 3, b = 4$ . Calculate  $|z| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$ .
2. Find the real part  $x$ :

$$x = \sqrt{\frac{5 + 3}{2}} = \sqrt{\frac{8}{2}} = 2$$

3. Find the imaginary part  $y$ :

$$y = \sqrt{\frac{5 - 3}{2}} = \sqrt{\frac{2}{2}} = 1$$

4. Since the imaginary part  $b = 4$  is positive, the signs of  $x$  and  $y$  in the final answer must be the same:

$$\sqrt{3 + 4i} = \pm(2 + i)$$

5. Verification:  $(2 + i)^2 = 4 + 4i + i^2 = 4 + 4i - 1 = 3 + 4i$ .

**Final Answer:**

$$\boxed{\pm(2 + i)}$$

**Answer: (A)**

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Q16.

**Solution****Concept:**

For a quadratic equation  $ax^2 + bx + c = 0$  with roots  $\alpha$  and  $\beta$ , the sum of roots is given by  $\alpha + \beta = -b/a$  and the product of roots is  $\alpha\beta = c/a$ . To find the value of symmetric functions of roots like  $\alpha^3 + \beta^3$ , we use the algebraic identity  $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ . This method is much faster than solving for the individual roots using the quadratic formula, which is a key strategy for time management in the MHT-CET.

**Solution:**

1. Given the quadratic equation:

$$x^2 - 6x + 8 = 0$$

Comparing this with  $ax^2 + bx + c = 0$ , we get  $a = 1, b = -6, c = 8$ .

2. Calculate the sum and product of the roots: Sum  $(\alpha + \beta) = -(-6)/1 = 6$

Product  $(\alpha\beta) = 8/1 = 8$

3. Apply the identity for the sum of cubes:

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

4. Substitute the calculated values into the identity:

$$\alpha^3 + \beta^3 = (6)^3 - 3(8)(6)$$

$$\alpha^3 + \beta^3 = 216 - 144$$

5. Final calculation results in:

$$\alpha^3 + \beta^3 = 72$$

**Final Answer:**

72
----

Answer: (A)
-------------

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Q17.

**Solution****Concept:**

When the roots of a quadratic equation  $ax^2 + bx + c = 0$  are in a specific ratio  $m : n$ , we can represent them as  $m\alpha$  and  $n\alpha$ . By applying the standard sum and product rules, we can establish a relationship between the coefficients. The specific condition for the ratio  $m : n$  is often expressed as  $(m + n)^2 ac = mn b^2$ . This allows us to eliminate the unknown variable  $\alpha$  and find the required algebraic condition directly.

**Solution:**

1. Let the roots be  $2\alpha$  and  $3\alpha$  (since the ratio is 2 : 3).
2. Use the Sum of Roots formula:

$$2\alpha + 3\alpha = -b/a \implies 5\alpha = -b/a \implies \alpha = -b/(5a)$$

3. Use the Product of Roots formula:

$$(2\alpha)(3\alpha) = c/a \implies 6\alpha^2 = c/a$$

4. Substitute the value of  $\alpha$  from step 2 into the product equation:

$$6 \left( \frac{-b}{5a} \right)^2 = \frac{c}{a}$$

5. Simplify the expression:

$$6 \left( \frac{b^2}{25a^2} \right) = \frac{c}{a}$$

Multiply by  $25a^2$ :

$$6b^2 = 25ac$$

**Final Answer:**

$$\boxed{6b^2 = 25ac}$$

**Answer: (A)**

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Q18.

**Solution****Concept:**

In a sequence where the differences between consecutive terms form an Arithmetic Progression (A.P.), the general term  $T_n$  is a quadratic expression of the form  $an^2 + bn + c$ . This is known as the "Method of Successive Differences." For MHT-CET, if you are unsure of the formula, a quick way to verify is to substitute  $n = 1, 2, 3$  into the options and check which one generates the sequence terms 5, 11, 19, 29.

**Solution:**

1. Observe the given series: 5, 11, 19, 29, ...

2. Find the first set of differences:  $11 - 5 = 6$

$$19 - 11 = 8$$

$29 - 19 = 10$  The first differences (6, 8, 10) are in A.P. with common difference  $d = 2$ .

3. Since the first differences are in A.P., the general term is  $T_n = an^2 + bn + c$ . For  $n = 1$ :  
 $a + b + c = 5$

$$\text{For } n = 2: 4a + 2b + c = 11$$

$$\text{For } n = 3: 9a + 3b + c = 19$$

4. Solving these equations: Subtracting eq. 1 from eq. 2:  $3a + b = 6$

Subtracting eq. 2 from eq. 3:  $5a + b = 8$

Subtracting these results:  $2a = 2 \implies a = 1$ . Then  $3(1) + b = 6 \implies b = 3$ . Then  
 $1 + 3 + c = 5 \implies c = 1$ .

5. The  $n^{\text{th}}$  term is:

$$T_n = n^2 + 3n + 1$$

**Final Answer:**

$$n^2 + 3n + 1$$

**Answer: (A)**

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Q19.

**Solution****Concept:**

The properties of Geometric Progression (G.P.) and logarithms are closely linked. If  $a, b, c$  are in G.P., then  $b^2 = ac$ . Taking the natural logarithm of both sides of this property allows us to use the rule  $\log(x^y) = y \log x$  and  $\log(xy) = \log x + \log y$ . This transforms the geometric relationship into an arithmetic relationship, which is a standard conceptual question in the sequences and series topic.

**Solution:**

1. Given that  $a, b, c$  are in Geometric Progression (G.P.).
2. The condition for three numbers to be in G.P. is:

$$b^2 = ac$$

3. Take natural logarithms on both sides:

$$\log(b^2) = \log(ac)$$

4. Apply logarithmic properties:  $\log(b^2) = 2 \log b$   
 $\log(ac) = \log a + \log c$

5. Thus, we get:

$$2 \log b = \log a + \log c$$

6. This is the condition for  $x, y, z$  to be in Arithmetic Progression (A.P.), where  $2y = x + z$ . Therefore,  $\log a, \log b, \log c$  are in A.P.

**Final Answer:**

A.P.

Answer: (A)

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Q20.

**Solution****Concept:**

The general term  $T_{r+1}$  in the binomial expansion of  $(a + x)^n$  is given by  $T_{r+1} = \binom{n}{r} a^{n-r} x^r$ . To find the coefficient of a specific power of  $x$ , say  $x^k$ , we identify the value of  $r$  that makes the power of  $x$  equal to  $k$ . For  $(1 + 2x)^6$ , the power of  $x$  is directly determined by  $r$ . Remember that the coefficient includes both the binomial coefficient  $\binom{n}{r}$  and any numerical factors attached to  $x$ .

**Solution:**

1. Identify the general term in the expansion of  $(1 + 2x)^6$ :

$$T_{r+1} = \binom{6}{r} (1)^{6-r} (2x)^r$$

$$T_{r+1} = \binom{6}{r} \cdot 2^r \cdot x^r$$

2. We need the coefficient of  $x^4$ . Therefore, set  $r = 4$ .
3. Substitute  $r = 4$  into the expression: Coefficient =  $\binom{6}{4} \cdot 2^4$
4. Calculate the binomial coefficient  $\binom{6}{4}$ :  $\binom{6}{4} = \binom{6}{6-4} = \binom{6}{2} = \frac{6 \times 5}{2 \times 1} = 15$
5. Calculate the full coefficient: Coefficient =  $15 \cdot 2^4 = 15 \cdot 16 = 240$

**Final Answer:**

240

**Answer: (A)**[Go Back to Question 20](#)

Q21.

**Solution****Concept:**

Permutations in a circular arrangement differ from linear arrangements because there is no fixed starting or ending point. In a circle, shifting every person by one position results in the same relative arrangement. Therefore, to find the number of ways to arrange  $n$  distinct objects in a circle, we fix one object's position and arrange the remaining  $(n - 1)$  objects. The formula for circular permutations is  $(n - 1)!$ .

**Solution:**

1. Identify the number of people to be seated:  $n = 5$
2. Recall the formula for circular permutations of  $n$  distinct items: Number of ways =  $(n - 1)!$
3. Substitute  $n = 5$  into the formula: Number of ways =  $(5 - 1)! = 4!$
4. Calculate the factorial value:  $4! = 4 \times 3 \times 2 \times 1 = 24$
5. Therefore, there are 24 distinct ways to seat 5 people around a circular table.

**Final Answer:**

24
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Answer: (B)
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Q22.

**Solution****Concept:**

Two events  $A$  and  $B$  are said to be independent if the occurrence of one does not affect the probability of the other. Mathematically, for independent events, the probability of their intersection is the product of their individual probabilities:  $P(A \cap B) = P(A) \cdot P(B)$ . To find the probability of the union of two events, we use the Addition Theorem of Probability:  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .

**Solution:**

1. Given probabilities:  $P(A) = 0.4$   $P(B) = 0.5$
2. Since  $A$  and  $B$  are independent events, calculate  $P(A \cap B)$ :

$$P(A \cap B) = P(A) \cdot P(B) = 0.4 \times 0.5 = 0.20$$

3. Use the Addition Theorem to find  $P(A \cup B)$ :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

4. Substitute the known values:

$$P(A \cup B) = 0.4 + 0.5 - 0.2$$

5. Final calculation:

$$P(A \cup B) = 0.9 - 0.2 = 0.7$$

**Final Answer:**

0.7
-----

Answer: (B)
-------------

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Q23.

**Solution****Concept:**

The perpendicular distance  $d$  from a point  $(x_1, y_1)$  to a line defined by the equation  $Ax + By + C = 0$  is given by the formula:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

This formula calculates the shortest distance between the point and the line. It is a very common topic in MHT-CET coordinate geometry, often paired with finding the distance between parallel lines or the area of a triangle.

**Solution:**

1. Identify the point  $(x_1, y_1)$  and the line coefficients: Point:  $(2, 3) \implies x_1 = 2, y_1 = 3$  Line:  $3x + 4y - 8 = 0 \implies A = 3, B = 4, C = -8$
2. Substitute these values into the distance formula:

$$d = \frac{|3(2) + 4(3) - 8|}{\sqrt{3^2 + 4^2}}$$

3. Simplify the numerator: Numerator =  $|6 + 12 - 8| = |10| = 10$
4. Simplify the denominator: Denominator =  $\sqrt{9 + 16} = \sqrt{25} = 5$
5. Calculate the final distance:

$$d = \frac{10}{5} = 2 \text{ units}$$

**Final Answer:**

2 units
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Answer: (A)
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Q24.

**Solution****Concept:**

The standard equation of a circle with center  $(h, k)$  and radius  $r$  is  $(x - h)^2 + (y - k)^2 = r^2$ . If the circle passes through a specific point  $(x_1, y_1)$ , the radius  $r$  can be found using the distance formula between the center and that point. Expanding the squared terms leads to the general form of the circle equation:  $x^2 + y^2 + 2gx + 2fy + c = 0$ .

**Solution:**

1. Given the center  $(h, k) = (1, -2)$ . The circle passes through  $(4, 2)$ .
2. Calculate the radius squared ( $r^2$ ) using the distance formula:

$$r^2 = (4 - 1)^2 + (2 - (-2))^2$$

$$r^2 = 3^2 + 4^2 = 9 + 16 = 25$$

3. Write the equation in standard form:

$$(x - 1)^2 + (y - (-2))^2 = 25$$

$$(x - 1)^2 + (y + 2)^2 = 25$$

4. Expand the equation:

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 25$$

5. Rearrange into general form:

$$x^2 + y^2 - 2x + 4y + 5 - 25 = 0$$

$$x^2 + y^2 - 2x + 4y - 20 = 0$$

**Final Answer:**

$$x^2 + y^2 - 2x + 4y - 20 = 0$$

**Answer: (A)**[Go Back to Question 24](#)

Q25.

**Solution****Concept:**

The eccentricity ( $e$ ) of an ellipse measures how much the ellipse deviates from being a perfect circle. For an ellipse in standard form  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , where  $a > b$ , the eccentricity is calculated using the formula  $e = \sqrt{1 - \frac{b^2}{a^2}}$ . The first step in such problems is always to convert the given general equation into the standard form by dividing by the constant term.

**Solution:**

1. Given equation:  $9x^2 + 25y^2 = 225$ .
2. Divide the entire equation by 225 to bring it to standard form:

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

3. Identify  $a^2$  and  $b^2$ : Here,  $a^2 = 25$  and  $b^2 = 9$ . Since  $a^2 > b^2$ , the major axis is horizontal.
4. Apply the eccentricity formula:

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{25}}$$

5. Simplify the expression:

$$e = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

**Final Answer:**

$$\boxed{\frac{4}{5}}$$

**Answer: (A)****Go Back to Question 25**

Q26.

**Solution****Concept:**

A parabola of the form  $y^2 = 4ax$  has its vertex at the origin  $(0, 0)$  and is symmetric about the  $x$ -axis. The parameter  $4a$  represents the length of the latus rectum, which is a chord passing through the focus and perpendicular to the axis of symmetry. To find the length of the latus rectum, we simply compare the given equation with the standard form  $y^2 = 4ax$ .

**Solution:**

1. Given the equation of the parabola:

$$y^2 = 12x$$

2. Compare this with the standard equation of a parabola:

$$y^2 = 4ax$$

3. By comparison, we find the value of  $4a$ :

$$4a = 12$$

4. The length of the latus rectum of a parabola is defined as the absolute value of  $4a$ .
5. Therefore, the length of the latus rectum is 12.

**Final Answer:**

12
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Answer: (C)
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Q27.

**Solution****Concept:**

Two vectors  $\vec{a}$  and  $\vec{b}$  are perpendicular (orthogonal) if and only if their dot product (scalar product) is zero. For vectors given in component form  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ , the dot product is calculated as  $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ . This principle is a cornerstone of vector algebra used to find unknown parameters or angles.

**Solution:**

1. Given vectors:  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

$$\vec{b} = \hat{i} + 3\hat{j} + \lambda\hat{k}$$

2. Since the vectors are perpendicular, their dot product must be zero:

$$\vec{a} \cdot \vec{b} = 0$$

3. Calculate the dot product using components:

$$(2)(1) + (-1)(3) + (1)(\lambda) = 0$$

4. Simplify the algebraic equation:

$$2 - 3 + \lambda = 0$$

$$-1 + \lambda = 0$$

5. Solve for  $\lambda$ :

$$\lambda = 1$$

**Final Answer:**

$$\boxed{1}$$

**Answer: (A)**[Go Back to Question 27](#)

Q28.

**Solution****Concept:**

The direction ratios (D.R.s) of a line joining two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  are  $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$ . To find the direction cosines (D.C.s), denoted as  $l, m, n$ , we divide each direction ratio by the magnitude (distance) between the two points,  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ . Note that D.C.s satisfy the property  $l^2 + m^2 + n^2 = 1$ .

**Solution:**

- Given points  $A(1, 0, 0)$  and  $B(0, 1, 1)$ .
- Find the direction ratios (D.R.s) of the line  $AB$ :  $a = 0 - 1 = -1$   
 $b = 1 - 0 = 1$   
 $c = 1 - 0 = 1$
- Calculate the magnitude  $r$ :

$$r = \sqrt{a^2 + b^2 + c^2} = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$$

- The direction cosines (D.C.s) are given by  $l = a/r, m = b/r, n = c/r$ :

$$l = \frac{-1}{\sqrt{3}}, m = \frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$$

- The D.C.s are  $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ .

**Final Answer:**

$$\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

**Answer: (B)**[Go Back to Question 28](#)

Q29.

**Solution****Concept:**

There is a standard integral property in calculus involving exponential functions:  $\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$ . This rule states that if the integrand consists of  $e^x$  multiplied by the sum of a function and its derivative, the result is simply  $e^x$  times the function. Identifying  $f(x)$  and its derivative  $f'(x)$  is the primary task here.

**Solution:**

1. Given integral:

$$\int e^x (\tan x + \sec^2 x) dx$$

2. Identify the function  $f(x)$ : Let  $f(x) = \tan x$ .

3. Find the derivative of  $f(x)$ : We know that  $\frac{d}{dx}(\tan x) = \sec^2 x$ , so  $f'(x) = \sec^2 x$ .

4. The integral is now in the form  $\int e^x [f(x) + f'(x)] dx$ .

5. Apply the standard formula:

$$\int e^x (\tan x + \sec^2 x) dx = e^x \tan x + C$$

**Final Answer:**

$$e^x \tan x + C$$

**Answer: (B)**

[Go Back to Question 29](#)



Q30.

**Solution****Concept:**

When two fair dice are thrown, the total number of outcomes (sample space) is  $6 \times 6 = 36$ . Probability is defined as the number of favorable outcomes divided by the total number of outcomes. To find the probability of a specific sum, we list all possible pairs  $(d_1, d_2)$  where  $1 \leq d_1, d_2 \leq 6$  that add up to the required value.

**Solution:**

1. Total number of outcomes when two dice are thrown:

$$n(S) = 6 \times 6 = 36$$

2. Let  $E$  be the event of getting a sum of 9. List the favorable outcomes: Pairs could be  $(3, 6)$ ,  $(4, 5)$ ,  $(5, 4)$ , and  $(6, 3)$ .

3. Count the number of favorable outcomes:

$$n(E) = 4$$

4. Calculate the probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36}$$

5. Simplify the fraction:

$$P(E) = \frac{1}{9}$$

**Final Answer:**

$$\boxed{\frac{1}{9}}$$

**Answer: (A)**[Go Back to Question 30](#)

Q31.

**Solution****Concept:**

Two planes are parallel if they have the same direction ratios for their normal vectors. The general equation of a plane is  $Ax + By + Cz + D = 0$ , where  $(A, B, C)$  are the direction ratios of the normal. Therefore, any plane parallel to  $3x + 4y - 5z = 0$  must be of the form  $3x + 4y - 5z + k = 0$ . The constant  $k$  is determined by substituting the coordinates of a point through which the plane passes.

**Solution:**

1. The given plane is  $3x + 4y - 5z = 0$ . The equation of any plane parallel to it is:

$$3x + 4y - 5z + k = 0$$

2. The plane passes through the point  $(1, 2, 3)$ . Substitute  $x = 1, y = 2, z = 3$  into the equation to find  $k$ :

$$3(1) + 4(2) - 5(3) + k = 0$$

3. Calculate the sum:

$$3 + 8 - 15 + k = 0$$

$$11 - 15 + k = 0$$

$$-4 + k = 0 \implies k = 4$$

4. Substitute the value of  $k$  back into the parallel plane equation:

$$3x + 4y - 5z + 4 = 0$$

**Final Answer:**

$$3x + 4y - 5z + 4 = 0$$

**Answer: (A)**[Go Back to Question 31](#)

Q32.

**Solution****Concept:**

To differentiate a composite logarithmic function like  $\log(f(x))$ , we use the Chain Rule:  $\frac{d}{dx}[\log(f(x))] = \frac{1}{f(x)} \cdot f'(x)$ . For the specific function  $f(x) = \sec x + \tan x$ , the derivatives are  $\frac{d}{dx}(\sec x) = \sec x \tan x$  and  $\frac{d}{dx}(\tan x) = \sec^2 x$ . Simplifying the resulting expression usually reveals a fundamental trigonometric identity.

**Solution:**

1. Given function:

$$y = \log(\sec x + \tan x)$$

2. Apply the Chain Rule for differentiation:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$

3. Differentiate the terms inside the parenthesis:

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$$

4. Factor out  $\sec x$  from the numerator:

$$\frac{dy}{dx} = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

5. Cancel the common term  $(\sec x + \tan x)$ :

$$\frac{dy}{dx} = \sec x$$

**Final Answer:**

$$\boxed{\sec x}$$

**Answer: (A)**[Go Back to Question 32](#)

Q33.

**Solution****Concept:**

The area bounded by a curve  $y = f(x)$ , the x-axis, and vertical lines  $x = a$  and  $x = b$  is given by the definite integral  $\int_a^b |f(x)| dx$ . For the cosine function between 0 and  $\pi/2$ , the function is entirely non-negative, so we simply integrate  $\cos x$ . This is a basic application of definite integration in finding areas of standard trigonometric regions.

**Solution:**

1. Set up the integral for the area:

$$A = \int_0^{\pi/2} \cos x \, dx$$

2. Find the antiderivative of  $\cos x$ : The integral of  $\cos x$  is  $\sin x$ .

3. Apply the limits of integration:

$$A = [\sin x]_0^{\pi/2}$$

4. Calculate the difference:

$$A = \sin(\pi/2) - \sin(0)$$

5. Substitute the values:

$$A = 1 - 0 = 1 \text{ sq. unit}$$

**Final Answer:**

$$\boxed{1 \text{ sq. unit}}$$

**Answer: (A)**[Go Back to Question 33](#)

Q34.

**Solution****Concept:**

The general equation of a circle is  $x^2 + y^2 + 2gx + 2fy + c = 0$ . The center of this circle is given by  $(-g, -f)$  and the radius is calculated using the formula  $r = \sqrt{g^2 + f^2 - c}$ . When given an equation, we first divide by any coefficients of  $x^2$  and  $y^2$  (to make them 1) and then identify  $g$ ,  $f$ , and  $c$  by comparing coefficients.

**Solution:**

1. Given circle equation:

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

2. Compare with  $x^2 + y^2 + 2gx + 2fy + c = 0$ :  $2g = -4 \implies g = -2$

$$2f = 6 \implies f = 3$$

$$c = -12$$

3. Use the radius formula  $r = \sqrt{g^2 + f^2 - c}$ :

$$r = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

4. Simplify the terms inside the square root:

$$r = \sqrt{4 + 9 + 12}$$

$$r = \sqrt{25}$$

5. Final result:

$$r = 5$$

**Final Answer:**

$$\boxed{5}$$

**Answer: (A)**[Go Back to Question 34](#)

Q35.

**Solution****Concept:**

In a Binomial Distribution  $B(n, p)$ , where  $n$  is the number of independent trials and  $p$  is the probability of success in each trial, the variance ( $\sigma^2$ ) is given by the formula  $Var(X) = npq$ . Here,  $q = 1 - p$  represents the probability of failure. The variance measures the spread of the distribution around its mean ( $np$ ).

**Solution:**

1. Given parameters:  $n = 10$

$$p = 0.4$$

2. Calculate the probability of failure  $q$ :

$$q = 1 - p = 1 - 0.4 = 0.6$$

3. Apply the formula for variance in a Binomial Distribution:

$$\text{Variance} = npq$$

4. Substitute the values:

$$\text{Variance} = 10 \times 0.4 \times 0.6$$

5. Calculate the final value:

$$\text{Variance} = 4 \times 0.6 = 2.4$$

**Final Answer:**

2.4
-----

Answer: (B)
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Q36.

**Solution****Concept:**

The angle  $\theta$  between two lines in 3D space, given by their vector equations  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ , is determined by the angle between their direction vectors  $\vec{b}_1$  and  $\vec{b}_2$ . The formula used is:

$$\cos \theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$$

This calculation ignores the passing points  $\vec{a}_1$  and  $\vec{a}_2$  and focuses entirely on the orientation of the lines.

**Solution:**

1. Identify the direction vectors from the given equations:  $\vec{b}_1 = 2\hat{i} + 2\hat{j} + \hat{k}$   $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$
2. Calculate the dot product  $\vec{b}_1 \cdot \vec{b}_2$ :

$$\vec{b}_1 \cdot \vec{b}_2 = (2 \times 3) + (2 \times 2) + (1 \times 6) = 6 + 4 + 6 = 16$$

3. Calculate the magnitudes of both vectors:  $|\vec{b}_1| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$   
 $|\vec{b}_2| = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7$
4. Substitute values into the cosine formula:

$$\cos \theta = \frac{16}{3 \times 7} = \frac{16}{21}$$

5. Solve for  $\theta$ :

$$\theta = \cos^{-1}\left(\frac{16}{21}\right)$$

**Final Answer:**

$$\cos^{-1}(16/21)$$

**Answer: (A)**[Go Back to Question 36](#)

Q37.

**Solution****Concept:**

A first-order linear differential equation is represented as  $\frac{dy}{dx} + Py = Q$ , where  $P$  and  $Q$  are functions of  $x$ . The **Integrating Factor (I.F.)** is a function used to solve such equations, calculated as  $\text{I.F.} = e^{\int P dx}$ . This factor, when multiplied throughout the equation, makes the left side a perfect derivative of the product ( $y \times \text{I.F.}$ ).

**Solution:**

1. Write the given differential equation:

$$\frac{dy}{dx} - y \cot x = \csc x$$

2. Identify  $P(x)$  by comparing with the standard form:  $P = -\cot x$ . (Note: The negative sign is crucial).
3. Calculate the integral of  $P$ :

$$\int P dx = \int -\cot x dx = -\log(\sin x)$$

4. Using logarithmic properties:  $-\log(\sin x) = \log((\sin x)^{-1}) = \log(\csc x)$
5. Calculate the Integrating Factor:

$$\text{I.F.} = e^{\int P dx} = e^{\log(\csc x)}$$

Since  $e^{\log f(x)} = f(x)$ , we get  $\text{I.F.} = \csc x$ .

**Final Answer:**

$$\boxed{\csc x}$$

**Answer: (B)**

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Q38.

**Solution****Concept:**

In combinatorics, the property of complementary combinations states that  ${}^n C_r = {}^n C_{n-r}$ . From this, we derive a very useful rule: if  ${}^n C_x = {}^n C_y$ , then either  $x = y$  or  $x + y = n$ . This property is a frequent guest in MHT-CET questions involving binomial coefficients and permutations, allowing for the quick determination of the value of  $n$ .

**Solution:**

1. Given the equation:

$${}^n C_{12} = {}^n C_8$$

2. Identify the values of  $x$  and  $y$ :  $x = 12$  and  $y = 8$ .

3. Check the conditions: Since  $12 \neq 8$ , we must apply the condition  $x + y = n$ .

4. Calculate  $n$ :

$$n = 12 + 8$$

$$n = 20$$

5. This property ensures that the number of ways to choose 12 items out of 20 is exactly the same as choosing 8 items (which is equivalent to leaving 12 behind).

**Final Answer:**

20
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Answer: (A)
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Q39.

**Solution****Concept:**

The slope of the tangent to a curve  $y = f(x)$  at  $x = a$  is  $m_t = f'(a)$ . The normal to the curve at the same point is perpendicular to the tangent. Therefore, the slope of the normal ( $m_n$ ) is the negative reciprocal of the slope of the tangent, provided  $m_t \neq 0$ . The relationship is:

$$m_n = -\frac{1}{m_t} = -\frac{1}{f'(a)}$$

**Solution:**

1. Given the curve equation:

$$y = x^3 + 2x + 6$$

2. Find the derivative to get the slope of the tangent:

$$\frac{dy}{dx} = 3x^2 + 2$$

3. Evaluate the slope of the tangent at  $x = 0$ :

$$m_t = 3(0)^2 + 2 = 2$$

4. Use the relationship between the slopes of tangent and normal:

$$m_n = -\frac{1}{m_t}$$

5. Substitute the value:

$$m_n = -\frac{1}{2}$$

**Final Answer:**

$$\boxed{-1/2}$$

**Answer: (D)**

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Q40.

**Solution****Concept:**

For any three vectors  $\vec{a}, \vec{b}, \vec{c}$ , the square of their sum is given by  $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ . If the vectors are unit vectors, their magnitudes are all equal to 1. This identity is extremely useful for finding the sum of dot products when the sum of the vectors themselves is known (especially when it is zero).

**Solution:**

1. Given that  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors:  $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ .
2. Given the sum of vectors is zero:  $\vec{a} + \vec{b} + \vec{c} = 0 \implies |\vec{a} + \vec{b} + \vec{c}|^2 = 0$ .
3. Expand the expression using the vector identity:

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

4. Substitute the magnitudes of unit vectors:

$$1^2 + 1^2 + 1^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

5. Solve for the required sum:

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -3/2$$

**Final Answer:**

$$\boxed{-3/2}$$

**Answer: (B)****Go Back to Question 40**

Q41.

**Solution****Concept:**

The angle  $\theta$  between two planes  $A_1x + B_1y + C_1z + D_1 = 0$  and  $A_2x + B_2y + C_2z + D_2 = 0$  is the angle between their normal vectors  $\vec{n}_1 = (A_1, B_1, C_1)$  and  $\vec{n}_2 = (A_2, B_2, C_2)$ . The cosine of this angle is given by:

$$\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$$

If the dot product is zero, the planes are perpendicular ( $90^\circ$ ). If the normal vectors are proportional, the planes are parallel ( $0^\circ$ ).

**Solution:**

1. Identify the normal vectors from the plane equations: Plane 1:  $2x - y + z = 11 \implies \vec{n}_1 = (2, -1, 1)$  Plane 2:  $x + y + 2z = 3 \implies \vec{n}_2 = (1, 1, 2)$
2. Calculate the dot product  $\vec{n}_1 \cdot \vec{n}_2$ :

$$\vec{n}_1 \cdot \vec{n}_2 = (2 \times 1) + (-1 \times 1) + (1 \times 2) = 2 - 1 + 2 = 3$$

3. Calculate the magnitudes:  $|\vec{n}_1| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$   $|\vec{n}_2| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$

4. Substitute into the cosine formula:

$$\cos \theta = \frac{3}{\sqrt{6} \cdot \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$$

5. Determine the angle: Since  $\cos \theta = 1/2$ , then  $\theta = 60^\circ$  or  $\pi/3$ .

**Final Answer:**

60°

Answer: (B)

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Q42.

**Solution****Concept:**

The shortest distance between two skewed lines  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$  is the projection of the vector joining two points on the lines onto the common perpendicular. The formula is:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

If the lines intersect, the shortest distance will be zero.

**Solution:**

1. Extract vectors from the given lines: Line 1:  $\vec{a}_1 = (\hat{i} + \hat{j})$ ,  $\vec{b}_1 = (2\hat{i} - \hat{j} + \hat{k})$  Line 2:  $\vec{a}_2 = (2\hat{i} + \hat{j} - \hat{k})$ ,  $\vec{b}_2 = (3\hat{i} - 5\hat{j} + 2\hat{k})$
2. Calculate  $\vec{a}_2 - \vec{a}_1$ :

$$\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (1 - 1)\hat{j} + (-1 - 0)\hat{k} = \hat{i} - \hat{k}$$

3. Calculate the cross product  $\vec{b}_1 \times \vec{b}_2$  using a determinant:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3) = 3\hat{i} - \hat{j} - 7\hat{k}$$

4. Calculate magnitudes and dot product:  $|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)| = |(1)(3) + (0)(-1) + (-1)(-7)| = |3 + 7| = 10$   $|\vec{b}_1 \times \vec{b}_2| = \sqrt{3^2 + (-1)^2 + (-7)^2} = \sqrt{9 + 1 + 49} = \sqrt{59}$
5. Shortest distance  $d = 10/\sqrt{59}$ .

**Final Answer:**

$$\boxed{10/\sqrt{59}}$$

**Answer: (A)**[Go Back to Question 42](#)

Q43.

**Solution****Concept:**

When a line makes angles  $\alpha, \beta, \gamma$  with the  $x, y,$  and  $z$  axes respectively, these are known as direction angles. Their cosines,  $l = \cos \alpha, m = \cos \beta, n = \cos \gamma$ , are the direction cosines (D.C.s). A fundamental property of D.C.s is  $l^2 + m^2 + n^2 = 1$ , which translates to the identity  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ .

**Solution:**

1. Given the direction angles:  $\alpha = 90^\circ$  (with x-axis)  $\beta = 60^\circ$  (with y-axis)  $\gamma = 30^\circ$  (with z-axis)
2. Calculate the direction cosines:  $l = \cos 90^\circ = 0$   $m = \cos 60^\circ = 1/2$   $n = \cos 30^\circ = \sqrt{3}/2$
3. Verify the fundamental property:

$$l^2 + m^2 + n^2 = (0)^2 + (1/2)^2 + (\sqrt{3}/2)^2 = 0 + 1/4 + 3/4 = 1$$

4. Since the property holds, these are the valid direction cosines for the line.

**Final Answer:**

$$0, 1/2, \sqrt{3}/2$$

**Answer: (A)**[Go Back to Question 43](#)

Q44.

**Solution****Concept:**

The negation of a logical statement  $P$  is denoted as  $\sim P$ . For a conditional statement  $P \implies Q$  (If  $P$  then  $Q$ ), the negation is  $P \wedge \sim Q$  ( $P$  and not  $Q$ ). This is because the only case where a conditional is false is when the premise is true but the conclusion is false. Applying this to verbal statements requires identifying the component propositions.

**Solution:**

1. Identify the components of the statement "If it rains, then the ground is wet":  $P$ : It rains.  $Q$ : The ground is wet.
2. The original statement is in the form  $P \implies Q$ .
3. Recall the rule for the negation of a conditional:  $\sim (P \implies Q) \equiv P \wedge \sim Q$ .
4. Translate  $P \wedge \sim Q$  back into a verbal statement: "It rains and the ground is not wet."

**Final Answer:**

$$\text{It rains and the ground is not wet.}$$
**Answer: (B)**[Go Back to Question 44](#)

Q45.

**Solution****Concept:**

Integration by parts is used for the product of two functions, based on the formula  $\int u dv = uv - \int v du$ . To choose which function is  $u$ , we use the **LIATE** rule (Logarithmic, Inverse Trig, Algebraic, Trigonometric, Exponential). For  $\int x \sin x dx$ , the algebraic part  $x$  is chosen as  $u$  because it simplifies upon differentiation, while  $\sin x$  is easily integrable.

**Solution:**

1. Given integral:  $\int x \sin x dx$ . Using LIATE, let  $u = x$  and  $dv = \sin x dx$ .
2. Differentiate  $u$  and integrate  $dv$ :  $du = dx$   $v = \int \sin x dx = -\cos x$
3. Apply the integration by parts formula:

$$\int u dv = uv - \int v du$$

$$I = x(-\cos x) - \int (-\cos x) dx$$

4. Simplify and integrate the remaining term:

$$I = -x \cos x + \int \cos x dx$$

$$I = -x \cos x + \sin x + C$$

**Final Answer:**

$$\boxed{\sin x - x \cos x + C}$$

**Answer: (A)**[Go Back to Question 45](#)

Q46.

**Solution****Concept:**

A matrix  $A$  is said to be singular if its determinant is zero ( $|A| = 0$ ). For a  $2 \times 2$  matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , the determinant is calculated as  $ad - bc$ . Setting the determinant of a matrix containing an unknown variable to zero allows us to solve for that variable. This is a common method for testing whether a matrix is invertible, as only non-singular matrices have inverses.

**Solution:**

1. Given the matrix:

$$A = \begin{bmatrix} 4 & x \\ 2 & 3 \end{bmatrix}$$

2. Since the matrix is singular, its determinant  $|A|$  must be zero:

$$|A| = (4 \times 3) - (x \times 2) = 0$$

3. Set up the linear equation:

$$12 - 2x = 0$$

4. Solve for  $x$ :

$$2x = 12$$

$$x = 6$$

**Final Answer:**

6
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<b>Answer: (B)</b>
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Q47.

**Solution****Concept:**

The sum to infinity of a Geometric Progression (G.P.) exists only if the common ratio  $r$  satisfies the condition  $|r| < 1$ . The formula for the sum to infinity is  $S_\infty = \frac{a}{1-r}$ , where  $a$  is the first term. If the common ratio is greater than or equal to 1, the series diverges and the sum to infinity is not defined.

**Solution:**

1. Given the series:  $1, 1/2, 1/4, 1/8, \dots$
2. Identify the first term  $a$ :  $a = 1$
3. Calculate the common ratio  $r$ :  $r = \frac{1/2}{1} = 1/2$
4. Check the condition for infinity: Since  $|1/2| < 1$ , the sum to infinity exists.
5. Apply the formula  $S_\infty = \frac{a}{1-r}$ :

$$S_\infty = \frac{1}{1 - 1/2} = \frac{1}{1/2} = 2$$

**Final Answer:**

2
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Answer: (A)
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Q48.

**Solution****Concept:**

The adjoint of a  $2 \times 2$  matrix  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  can be found using a very simple shortcut: swap the elements on the main diagonal ( $a$  and  $d$ ) and change the signs of the elements on the off-diagonal ( $b$  and  $c$ ). This results in  $\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ . This shortcut is essential for saving time during competitive exams like MHT-CET.

**Solution:**

1. Given the matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

2. Swap the elements of the principal diagonal: 1 and 4 become 4 and 1.
3. Change the signs of the off-diagonal elements: 2 and 3 become -2 and -3.
4. Write the resulting adjoint matrix:

$$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

**Final Answer:**

$$\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

**Answer: (A)**[Go Back to Question 48](#)

Q49.

**Solution****Concept:**

Trigonometric equations of the form  $\sin \theta = \sin \alpha$  have a general solution given by  $\theta = n\pi + (-1)^n \alpha$ , where  $n$  is any integer ( $\mathbb{Z}$ ). This formula accounts for the periodic nature of the sine function and its symmetry across the quadrants. For MHT-CET, always ensure  $\alpha$  is chosen within the principal value branch, which for sine is  $[-\pi/2, \pi/2]$ .

**Solution:**

1. Given the equation:

$$\sin \theta = 1/2$$

2. Find the principal value  $\alpha$ : We know  $\sin(\pi/6) = 1/2$ , so  $\alpha = \pi/6$ .

3. Apply the general solution formula for sine:

$$\theta = n\pi + (-1)^n \alpha$$

4. Substitute the value of  $\alpha$ :

$$\theta = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$$

**Final Answer:**

$$\boxed{n\pi + (-1)^n \pi/6}$$

**Answer: (A)**[Go Back to Question 49](#)

Q50.

**Solution****Concept:**

The projection of a vector  $\vec{a}$  on another vector  $\vec{b}$  is the scalar value that represents the "shadow" of  $\vec{a}$  along the direction of  $\vec{b}$ . It is calculated using the dot product formula:

$$\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Note that the denominator is always the magnitude of the vector \*onto which\* the projection is being made. This is a scalar projection; if a vector projection is required, you would multiply this result by the unit vector  $\hat{b}$ .

**Solution:**

1. Identify the vectors:  $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$   $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$
2. Calculate the dot product  $\vec{a} \cdot \vec{b}$ :

$$\vec{a} \cdot \vec{b} = (1 \times 2) + (3 \times -3) + (1 \times 6) = 2 - 9 + 6 = -1$$

3. Calculate the magnitude of vector  $\vec{b}$ :

$$|\vec{b}| = \sqrt{2^2 + (-3)^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

4. Apply the projection formula:

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{-1}{7}$$

5. Typically, for distance or length-based projection questions, we provide the absolute value, but based on vector rules, the value is  $-1/7$ .

**Final Answer:**

$$\boxed{-1/7}$$

**Answer: (B)**

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## Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	A	4	1	5	A
6	A	7	B	8	B	9	A	10	C
11	B	12	A	13	A	14	C	15	A
16	A	17	A	18	A	19	A	20	A
21	B	22	B	23	A	24	A	25	A
26	C	27	A	28	B	29	B	30	A
31	A	32	A	33	A	34	A	35	B
36	A	37	B	38	A	39	D	40	B
41	B	42	A	43	A	44	B	45	A
46	B	47	A	48	A	49	A	50	B

