MHT-CET Mathematics Sample Paper-18

Duration: 90 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. The value of $\lim_{x\to 0} \dfrac{\tan 4x - \sin 4x}{x^3}$ is:

(A) $\dfrac{64}{3}$

(B) $\dfrac{128}{3}$

(C) $\dfrac{32}{3}$

(D) 64

Q2. If $y = x^{\sin x}$, then $\dfrac{dy}{dx}$ equals:

(A) $x^{\sin x} \cos x$

(B) $x^{\sin x} \left(\cos x \log x + \dfrac{\sin x}{x} \right)$

(C) $x^{\sin x} (\sin x + \cos x)$

(D) $\sin x \cdot x^{\sin x - 1}$

Q3. The value of $\int_0^1 \dfrac{x^2}{1 + x^3}\, dx$ is:

(A) $\dfrac{1}{3} \log 2$

(B) $\log 2$

(C) $\frac{1}{2}\log 2$

(D) $\frac{2}{3}\log 2$

Q4. The minimum value of $x + \frac{16}{x}$ for $x > 0$ is:

(A) 4

(B) 8

(C) 12

(D) 16

Q5. The order and degree of $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + y = 0$ are:

(A) 2, 3

(B) 3, 2

(C) 2, 1

(D) 3, 1

Q6. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then A^2 equals:

(A) $\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix}$

Q7. The value of $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is:

(A) 0

(B) 18

(C) -18

(D) 9

Q8. Two dice are thrown simultaneously. The probability of getting sum greater than 9 is:

(A) $\frac{1}{6}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{5}{18}$

Q9. The middle term in the expansion of $(x + \frac{1}{x})^8$ is:

(A) 70

(B) $70x$

(C) $\frac{70}{x}$

(D) 56

Q10. If $z = 3 + 4i$, then the value of $\frac{z - \bar{z}}{z + \bar{z}}$ is:

(A) $\frac{4i}{3}$

(B) $\frac{3i}{4}$

(C) $\frac{8i}{3}$

(D) $\frac{2i}{3}$

Q11. The distance of the point $(2, -1)$ from the line $3x - 4y + 5 = 0$ is:

(A) $\frac{3}{5}$

(B) $\frac{7}{5}$

(C) $\frac{15}{5}$

(D) $\frac{4}{5}$

Q12. The radius of the circle $x^2 + y^2 - 6x + 8y + 9 = 0$ is:

(A) 3

(B) 4

(C) 5

(D) 6

Q13. The length of latus rectum of the parabola $y^2 = 8x$ is:

(A) 2

(B) 4

(C) 8

(D) 16

Q14. The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is:

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{5}$

Q15. The equation of asymptotes of $\frac{x^2}{9} - \frac{y^2}{16} = 1$ are:

(A) $y = \pm\frac{4}{3}x$

(B) $y = \pm\frac{3}{4}x$

(C) $y = \pm\frac{9}{4}x$

(D) $y = \pm\frac{4}{9}x$

Q16. If $\vec{a} = (1, 2, -1)$ and $\vec{b} = (2, 1, 3)$, then $\vec{a} \cdot \vec{b}$ equals:

(A) 1

(B) 2

(C) 3

(D) 4

Q17. If $\vec{a} = (1, 0, 1)$ and $\vec{b} = (2, 1, -1)$, then $\vec{a} \times \vec{b}$ equals:

(A) $(-1, 3, 1)$

(B) $(1, 3, -1)$

(C) $(1, -3, 1)$

(D) $(3, 1, -1)$

Q18. The direction ratios of a line perpendicular to both $(1, 2, 3)$ and $(2, 1, 1)$ are:

(A) $(-1, 5, -3)$

(B) $(1, -5, 3)$

(C) $(5, -1, 3)$

(D) $(3, 5, -1)$

Q19. The mean of the numbers 3, 7, 11, 15, 19 is:

(A) 9

(B) 10

(C) 11

(D) 12

Q20. If X follows binomial distribution with $n = 5$ and $p = \frac{1}{2}$, then $P(X = 2)$ is:

(A) $\frac{5}{16}$

(B) $\frac{10}{32}$

(C) $\frac{15}{32}$

(D) $\frac{5}{32}$

Q21. The value of $\sin 75°$ is:

(A) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(C) $\frac{\sqrt{3}+1}{2}$

(D) $\frac{\sqrt{3}-1}{2}$

Q22. The value of $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$ is:

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{4}$

(D) $\frac{\pi}{4}$

Q23. If $\log(x-1) + \log(x+1) = 1$, then x equals:

(A) $\sqrt{11}$

(B) $\sqrt{10}$

(C) $\sqrt{9}$

(D) $\sqrt{12}$

Q24. The sum of first 20 terms of the AP 3, 7, 11, ... is:

(A) 760

(B) 800

(C) 820

(D) 860

Q25. The sum to infinity of the GP $3, \frac{3}{2}, \frac{3}{4}, \ldots$ is:

(A) 3

(B) 4

(C) 5

(D) 6

Q26. If $f(x) = \frac{x^2 - 1}{x - 1}$, then $f(1)$ after continuity correction is:

(A) 0

(B) 1

(C) 2

(D) does not exist

Q27. The value of $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ is:

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

Q28. The value of $\int_0^{\pi/2} \cos x \, dx$ is:

(A) 0

(B) 1

(C) 2

(D) $\frac{\pi}{2}$

Q29. The general solution of $\frac{dy}{dx} = 3x^2$ is:

(A) $y = x^3 + C$

(B) $y = 3x^3 + C$

(C) $y = x^2 + C$

(D) $y = 6x + C$

Q30. The equation of tangent to $y = x^2$ at $(1, 1)$ is:

(A) $y = 2x - 1$

(B) $y = x + 1$

(C) $y = 2x + 1$

(D) $y = x - 1$

Q31. The equation $x^2 - y^2 = 0$ represents:

(A) perpendicular lines

(B) parallel lines

(C) coincident lines

(D) circle

Q32. If $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then Δ equals:

(A) $ab - cd$

(B) $ad - bc$

(C) $ac - bd$

(D) $bc - ad$

Q33. The transpose of $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is:

(A) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 4 & 2 \\ 5 & 3 & 6 \end{bmatrix}$

Q34. If $i^{35} =$

(A) i

(B) $-i$

(C) 1

(D) -1

Q35. The number of ways of arranging 5 distinct books on a shelf is:

(A) 25

(B) 60

(C) 120

(D) 240

Q36. The value of ${}^8C_2$ is:

(A) 16

(B) 28

(C) 32

(D) 56

Q37. The slope of the line $2x + 3y - 6 = 0$ is:

(A) $\frac{2}{3}$

(B) $-\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $-\frac{3}{2}$

Q38. The centre of the circle $x^2 + y^2 - 4x + 6y + 3 = 0$ is:

(A) $(2, -3)$

(B) $(-2, 3)$

(C) $(4, -6)$

(D) $(-4, 6)$

Q39. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ with angle between them $60°$, then $\vec{a} \cdot \vec{b} =$

(A) 3

(B) 6

(C) 9

(D) 12

Q40. The distance between points $(1, 2, 3)$ and $(4, 6, 3)$ is:

(A) 3

(B) 4

(C) 5

(D) 6

Q41. The median of 2, 4, 6, 8, 10 is:

(A) 4

(B) 5

(C) 6

(D) 8

Q42. A card is drawn from a pack of 52 cards. The probability of getting a king is:

(A) $\frac{1}{13}$

(B) $\frac{1}{26}$

(C) $\frac{1}{52}$

(D) $\frac{4}{13}$

Q43. The value of $\cos 60° \cos 30° - \sin 60° \sin 30°$ is:

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{4}$

Q44. If $y = \log(\sin x)$, then $\frac{dy}{dx}$ equals:

(A) $\sin x$

(B) $\cos x$

(C) $\cot x$

(D) $\tan x$

Q45. The value of $\int e^x \, dx$ is:

(A) $e^x + C$

(B) $\log x + C$

(C) $xe^x + C$

(D) $\frac{1}{x} + C$

Q46. The domain of $f(x) = \sqrt{5-x}$ is:

(A) $x < 5$

(B) $x \leq 5$

(C) $x > 5$

(D) all real numbers

Q47. The value of $\lim_{x\to\infty} \frac{2x^2+1}{x^2-3}$ is:

(A) 0

(B) 1

(C) 2

(D) ∞

Q48. If α, β are roots of $x^2 - 7x + 10 = 0$, then $\alpha + \beta$ equals:

(A) 5

(B) 7

(C) 10

(D) 17

Q49. The differential equation of the family $y = mx + c$ is of order:

(A) 1

(B) 2

(C) 3

(D) 0

Q50. A coin is tossed 3 times. The probability of getting exactly two heads is:

(A) $\frac{1}{8}$

(B) $\frac{3}{8}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

Detailed Solutions

Q1.

Solution

Concept: For limits resulting in the indeterminate form $\frac{0}{0}$, L'Hopital's Rule or standard trigonometric limits combined with algebraic manipulation are effective. The standard limits $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1$ are often useful. Trigonometric identities like $\sec^2\theta - 1 = \tan^2\theta$ and $1 - \cos\theta = 2\sin^2(\theta/2)$ are also key.

Solution: We need to evaluate $\lim\limits_{x\to 0}\dfrac{\tan 4x - \sin 4x}{x^3}$.
Substituting $x = 0$ gives $\frac{0}{0}$, an indeterminate form.

Method 1: Using L'Hopital's Rule
Apply L'Hopital's Rule:
$\lim_{x\to 0}\frac{\tan 4x - \sin 4x}{x^3} = \lim_{x\to 0}\frac{4\sec^2 4x - 4\cos 4x}{3x^2}$
This is still $\frac{0}{0}$. We can rewrite the expression:
$= \frac{4}{3}\lim_{x\to 0}\frac{\sec^2 4x - 1 - (1-\cos 4x)}{x^2}$
$= \frac{4}{3}\left(\lim_{x\to 0}\frac{\sec^2 4x - 1}{x^2} - \lim_{x\to 0}\frac{1-\cos 4x}{x^2}\right)$
Using $\sec^2\theta - 1 = \tan^2\theta$ and $1 - \cos\theta = 2\sin^2(\theta/2)$:
$= \frac{4}{3}\left(\lim_{x\to 0}\frac{\tan^2 4x}{x^2} - \lim_{x\to 0}\frac{2\sin^2 2x}{x^2}\right)$
Using standard limits $\lim_{\theta\to 0}\frac{\tan\theta}{\theta} = 1$ and
$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$:
$= \frac{4}{3}\left(\left(\lim_{x\to 0}\frac{\tan 4x}{4x}\cdot 4\right)^2 - 2\left(\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot 2\right)^2\right)$
$= \frac{4}{3}\left((1\cdot 4)^2 - 2(1\cdot 2)^2\right)$
$= \frac{4}{3}(16 - 2\cdot 4) = \frac{4}{3}(16-8) = \frac{4}{3}(8) = \frac{32}{3}$.

Method 2: Using Taylor Series (Conceptual Outline)
The Taylor expansions of $\tan(4x)$ and $\sin(4x)$ around $x = 0$ start with $4x$. The next terms involve x^3. Expanding these and substituting into the expression leads to cancellation of lower-order terms, leaving terms proportional to x^3, which upon division by x^3 yield a constant.

Final Answer: $\boxed{\dfrac{32}{3}}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: When differentiating a function of the form $y = [f(x)]^{g(x)}$, logarithmic differentiation is the standard method. This involves taking the natural logarithm of both sides to bring down the exponent, then differentiating implicitly.

Solution: We are given the function $y = x^{\sin x}$.
To differentiate this, we use logarithmic differentiation.

Step 1: Take the natural logarithm of both sides.
$\log y = \log(x^{\sin x})$
Using the logarithm property $\log a^b = b \log a$:
$\log y = (\sin x)(\log x)$

Step 2: Differentiate both sides with respect to x.
We apply the chain rule to the left side ($\frac{d}{dx}(\log y) = \frac{1}{y}\frac{dy}{dx}$) and the product rule to the right side ($\frac{d}{dx}(uv) = u'v + uv'$).
Let $u = \sin x$ and $v = \log x$. Then $u' = \cos x$ and $v' = \frac{1}{x}$.
$\frac{1}{y}\frac{dy}{dx} = (\cos x)(\log x) + (\sin x)\left(\frac{1}{x}\right)$

Step 3: Solve for $\frac{dy}{dx}$.
Multiply both sides by y:
$\frac{dy}{dx} = y\left(\cos x \log x + \frac{\sin x}{x}\right)$

Step 4: Substitute back the original expression for y.
Since $y = x^{\sin x}$, we have:
$\frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right)$

This matches option B.

Final Answer: $\boxed{x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right)}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: This integral is solved using the substitution method. The key observation is that the derivative of the denominator, $1 + x^3$, is $3x^2$, which is a constant multiple of the numerator x^2. The integral of $\frac{1}{u}$ with respect to u is $\log |u|$.

Solution: We need to evaluate the definite integral $\int_0^1 \frac{x^2}{1 + x^3}\, dx$.

Step 1: Perform the substitution.
Let $u = 1 + x^3$.
Differentiating both sides with respect to x, we get $du = 3x^2\, dx$.
Rearranging this, we have $x^2\, dx = \frac{1}{3}\, du$.

Step 2: Change the limits of integration.
The original limits are for x. We need to find the corresponding limits for u.
When $x = 0$, $u = 1 + (0)^3 = 1$.
When $x = 1$, $u = 1 + (1)^3 = 1 + 1 = 2$.

Step 3: Substitute into the integral.
The integral transforms into:

$$\int_1^2 \frac{1}{u} \left(\frac{1}{3}\, du \right)$$

Step 4: Evaluate the integral.
$= \frac{1}{3} \int_1^2 \frac{1}{u}\, du$
The antiderivative of $\frac{1}{u}$ is $\log |u|$.
$= \frac{1}{3} [\log |u|]_1^2$

Step 5: Apply the limits of integration.
$= \frac{1}{3} (\log |2| - \log |1|)$
Since $\log 1 = 0$:
$= \frac{1}{3} (\log 2 - 0)$
$= \frac{1}{3} \log 2$.

This matches option A.

Final Answer: $\boxed{\frac{1}{3} \log 2}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: We can find the minimum value of a function using calculus or the Arithmetic Mean-Geometric Mean (AM-GM) inequality. For a function $f(x)$, the minimum occurs at critical points where $f'(x) = 0$ and $f''(x) > 0$. The AM-GM inequality states that for non-negative numbers, the arithmetic mean is greater than or equal to the geometric mean. For two positive numbers a and b, $\frac{a+b}{2} \geq \sqrt{ab}$, with equality if and only if $a = b$.

Solution (using AM-GM Inequality):
We want to find the minimum value of $f(x) = x + \frac{16}{x}$ for $x > 0$. Since $x > 0$, both x and $\frac{16}{x}$ are positive. We can apply the AM-GM inequality to these two terms. Let $a = x$ and $b = \frac{16}{x}$. $\frac{x + \frac{16}{x}}{2} \geq \sqrt{x \cdot \frac{16}{x}}$ $\frac{x + \frac{16}{x}}{2} \geq \sqrt{16}$ $\frac{x + \frac{16}{x}}{2} \geq 4$ Multiplying by 2, we get: $x + \frac{16}{x} \geq 8$. The minimum value is 8. This minimum is achieved when $x = \frac{16}{x}$, which implies $x^2 = 16$. Since $x > 0$, $x = 4$.

Solution (using Calculus):
To find the minimum value using calculus, we first find the derivative of $f(x) = x + 16x^{-1}$.
$f'(x) = \frac{d}{dx}(x + 16x^{-1}) = 1 - 16x^{-2} = 1 - \frac{16}{x^2}$.
Set the derivative to zero to find critical points:
$f'(x) = 0 \implies 1 - \frac{16}{x^2} = 0 \implies x^2 = 16$.
Since $x > 0$, the critical point is $x = 4$.
Now, we find the second derivative to determine if this is a minimum:
$f''(x) = \frac{d}{dx}(1 - 16x^{-2}) = 0 - 16(-2)x^{-3} = \frac{32}{x^3}$.
Evaluate $f''(x)$ at $x = 4$:
$f''(4) = \frac{32}{4^3} = \frac{32}{64} = \frac{1}{2}$.
Since $f''(4) > 0$, the function has a local minimum at $x = 4$.
The minimum value is $f(4) = 4 + \frac{16}{4} = 4 + 4 = 8$.

Both methods confirm that the minimum value is 8.

Final Answer: $\boxed{8}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The order of a differential equation is determined by the highest order derivative present in the equation. The degree of a differential equation is the highest power of the highest order derivative, provided the equation is expressed as a polynomial in the derivatives.

Solution: The given differential equation is:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + y = 0$$

1. Determining the Order:

Identify all the derivatives present in the equation:

- $\frac{dy}{dx}$ is the first derivative.
- $\frac{d^2y}{dx^2}$ is the second derivative.

The highest order derivative is $\frac{d^2y}{dx^2}$. Therefore, the order of the differential equation is 2.

2. Determining the Degree:

Identify the highest order derivative, which is $\frac{d^2y}{dx^2}$.

Look at the power to which this highest order derivative is raised in the equation. In this case, $\left(\frac{d^2y}{dx^2}\right)^3$, the power is 3.

The equation is already polynomial in terms of the derivatives (no radicals involving derivatives, and no fractional powers of derivatives).

Therefore, the degree of the differential equation is 3.

The order is 2 and the degree is 3.

Final Answer: $\boxed{2, 3}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: Matrix multiplication is a fundamental operation where the element in the i-th row and j-th column of the product matrix $C = AB$ is obtained by taking the dot product of the i-th row of matrix A and the j-th column of matrix B. Calculating A^2 means performing matrix multiplication of A with itself ($A \times A$).

Solution: We are given the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.
We need to compute A^2, which is $A \times A$.

$$A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Let's compute each element of the resulting 2×2 matrix:

- The element in the first row, first column (Row 1 of A · Column 1 of A):
$(2 \times 2) + (1 \times 1) = 4 + 1 = 5$.

- The element in the first row, second column (Row 1 of A · Column 2 of A):
$(2 \times 1) + (1 \times 2) = 2 + 2 = 4$.

- The element in the second row, first column (Row 2 of A · Column 1 of A):
$(1 \times 2) + (2 \times 1) = 2 + 2 = 4$.

- The element in the second row, second column (Row 2 of A · Column 2 of A):
$(1 \times 1) + (2 \times 2) = 1 + 4 = 5$.

Combining these results, we get the matrix A^2:
$A^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$.

This matches option A.

Final Answer: $\boxed{\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: The determinant of a 3x3 matrix can be calculated using various methods, including cofactor expansion or row/column operations. Row operations are often used to simplify the matrix before calculating the determinant, as they can introduce zeros. The determinant calculation formula for a 3x3 matrix $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ is $a(ei - fh) - b(di - fg) + c(dh - eg)$.

Solution: We need to find the value of the determinant:

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

Method 1: Cofactor Expansion (along the first row)

$$D = 1 \cdot \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3 \cdot \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

$D = 1((3)(2) - (1)(1)) - 2((2)(2) - (1)(3)) + 3((2)(1) - (3)(3))$

$D = 1(6 - 1) - 2(4 - 3) + 3(2 - 9)$

$D = 1(5) - 2(1) + 3(-7)$

$D = 5 - 2 - 21$

$D = 3 - 21 = -18.$

Method 2: Using Row Operations

We can simplify the determinant by performing row operations to create zeros. Let's aim to get zeros in the first column below the first element.

$R_2 \to R_2 - 2R_1$ (Subtract 2 times the first row from the second row)

$R_3 \to R_3 - 3R_1$ (Subtract 3 times the first row from the third row)

The determinant becomes:

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 2-2(1) & 3-2(2) & 1-2(3) \\ 3-3(1) & 1-3(2) & 2-3(3) \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -5 & -7 \end{vmatrix}$$

Now, we can expand the determinant along the first column, as it has the most zeros:

$$D = 1 \cdot \begin{vmatrix} -1 & -5 \\ -5 & -7 \end{vmatrix} - 0 \cdot \begin{vmatrix} 2 & 3 \\ -5 & -7 \end{vmatrix} + 0 \cdot \begin{vmatrix} 2 & 3 \\ -1 & -5 \end{vmatrix}$$

$D = 1 \cdot ((-1)(-7) - (-5)(-5))$

$D = 1 \cdot (7 - 25)$

$D = 1 \cdot (-18) = -18.$

Both methods yield the same result.

Final Answer: $\boxed{-18}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: Probability is calculated as the ratio of the number of favorable outcomes to the total number of possible outcomes. When rolling two fair dice, the total number of possible outcomes is $6 \times 6 = 36$. Each outcome is an ordered pair (d_1, d_2), where d_1 and d_2 are the numbers shown on the first and second die, respectively.

Solution: We are interested in the event where the sum of the numbers on the two dice is greater than 9. This means the sum must be 10, 11, or 12.

Step 1: Identify the total number of possible outcomes.
When rolling two dice, there are $6 \times 6 = 36$ possible outcomes.

Step 2: Identify the favorable outcomes (sum > 9).
We need to list all pairs (d_1, d_2) such that $d_1 + d_2 > 9$.
- Sum = 10: The pairs are (4, 6), (5, 5), (6, 4). There are 3 outcomes.
- Sum = 11: The pairs are (5, 6), (6, 5). There are 2 outcomes.
- Sum = 12: The pair is (6, 6). There is 1 outcome.

Step 3: Count the total number of favorable outcomes.
The total number of outcomes where the sum is greater than 9 is $3 + 2 + 1 = 6$.

Step 4: Calculate the probability.
The probability is the number of favorable outcomes divided by the total number of possible outcomes.
$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{6}{36}$$

Step 5: Simplify the fraction.
$$\text{Probability} = \frac{1}{6}.$$

This matches option A.

Final Answer: $\boxed{\frac{1}{6}}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: The binomial theorem provides a formula for the expansion of $(a + b)^n$. The general term (or the $(k + 1)^{th}$ term) in the expansion is given by $T_{k+1} = \binom{n}{k}a^{n-k}b^k$. When n is an even integer, the expansion of $(a + b)^n$ has $n + 1$ terms (an odd number), and there is a single middle term. The position of this middle term is $\frac{n}{2} + 1$.

Solution: We are asked to find the middle term in the expansion of $(x + \frac{1}{x})^8$.
Here, $n = 8$. Since n is even, the number of terms in the expansion is $8 + 1 = 9$.
The middle term is the $(\frac{8}{2} + 1)^{th}$ term, which is the 5^{th} term.

To find the 5^{th} term, we use the formula for the general term T_{k+1}, where $k + 1 = 5$, so $k = 4$.
In this expansion, $a = x$ and $b = \frac{1}{x}$.
$T_{k+1} = \binom{n}{k}a^{n-k}b^k$
$T_5 = \binom{8}{4}x^{8-4}\left(\frac{1}{x}\right)^4$
$T_5 = \binom{8}{4}x^4\left(\frac{1}{x^4}\right)$
$T_5 = \binom{8}{4}x^4 \cdot \frac{1}{x^4}$
The x^4 terms cancel out:
$T_5 = \binom{8}{4}$

Now, we calculate the binomial coefficient $\binom{8}{4}$:
$\binom{8}{4} = \frac{8!}{4!(8-4)!} = \frac{8!}{4!4!}$
$\binom{8}{4} = \frac{8\times7\times6\times5\times4!}{4\times3\times2\times1\times4!}$
Cancel out the 4!:
$\binom{8}{4} = \frac{8\times7\times6\times5}{4\times3\times2\times1}$
$\binom{8}{4} = \frac{1680}{24}$
$\binom{8}{4} = 70.$

Thus, the middle term is 70.

Final Answer: $\boxed{70}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: For a complex number $z = a + bi$, its complex conjugate is $\bar{z} = a - bi$. The sum of a complex number and its conjugate is $z + \bar{z} = (a + bi) + (a - bi) = 2a$, which is twice its real part. The difference between a complex number and its conjugate is $z - \bar{z} = (a + bi) - (a - bi) = 2bi$, which is twice its imaginary part multiplied by i.

Solution: We are given the complex number $z = 3 + 4i$.
The complex conjugate of z is $\bar{z} = 3 - 4i$.

We need to evaluate the expression $\frac{z - \bar{z}}{z + \bar{z}}$.

Step 1: Calculate the numerator, $z - \bar{z}$.
$z - \bar{z} = (3 + 4i) - (3 - 4i)$
$z - \bar{z} = 3 + 4i - 3 + 4i$
Combine the real parts and the imaginary parts:
$z - \bar{z} = (3 - 3) + (4i + 4i)$
$z - \bar{z} = 0 + 8i = 8i$.

Step 2: Calculate the denominator, $z + \bar{z}$.
$z + \bar{z} = (3 + 4i) + (3 - 4i)$
Combine the real parts and the imaginary parts:
$z + \bar{z} = (3 + 3) + (4i - 4i)$
$z + \bar{z} = 6 + 0i = 6$.

Step 3: Substitute the results back into the expression.
$$\frac{z - \bar{z}}{z + \bar{z}} = \frac{8i}{6}$$

Step 4: Simplify the resulting fraction.
$$\frac{8i}{6} = \frac{4 \times 2i}{3 \times 2} = \frac{4i}{3}.$$

This matches option A.

Final Answer: $\boxed{\frac{4i}{3}}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: The distance of a point (x_0, y_0) from a line $Ax + By + C = 0$ is given by the formula:
$d = \dfrac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$.

Solution: We need to find the distance of the point $(2, -1)$ from the line $3x - 4y + 5 = 0$.
Here, $(x_0, y_0) = (2, -1)$ and the line is $Ax + By + C = 0$, with $A = 3$, $B = -4$, and $C = 5$.

Step 1: Substitute the coordinates of the point and the coefficients of the line into the distance formula.
$d = \dfrac{|(3)(2) + (-4)(-1) + 5|}{\sqrt{(3)^2 + (-4)^2}}$

Step 2: Calculate the numerator.
$|3 \times 2 - 4 \times (-1) + 5| = |6 + 4 + 5| = |15| = 15.$

Step 3: Calculate the denominator.
$\sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$

Step 4: Compute the distance.
$d = \dfrac{15}{5} = 3.$

Wait, let me recheck the options. Option C is $\dfrac{15}{5}$.
My calculation resulted in 3, which is indeed $\dfrac{15}{5}$.

Final Answer: $\boxed{\dfrac{15}{5}}$

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: The standard equation of a circle with center (h, k) and radius r is $(x-h)^2+(y-k)^2 = r^2$. The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. The center is $(-g, -f)$ and the radius is $r = \sqrt{g^2 + f^2 - c}$.

Solution: We are given the equation of the circle: $x^2 + y^2 - 6x + 8y + 9 = 0$.
To find the radius, we can compare this to the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$.

Step 1: Identify the coefficients g, f, and c.
Comparing the terms:
$2g = -6 \implies g = -3$
$2f = 8 \implies f = 4$
$c = 9$

Step 2: Use the formula for the radius $r = \sqrt{g^2 + f^2 - c}$.
$r = \sqrt{(-3)^2 + (4)^2 - 9}$
$r = \sqrt{9 + 16 - 9}$
$r = \sqrt{16}$
$r = 4$.

Alternatively, we can complete the square to convert the general equation to the standard form.
$x^2 - 6x + y^2 + 8y = -9$
$(x^2 - 6x + 9) + (y^2 + 8y + 16) = -9 + 9 + 16$
$(x - 3)^2 + (y + 4)^2 = 16$
Comparing this to the standard form $(x - h)^2 + (y - k)^2 = r^2$, we have $r^2 = 16$.
Therefore, the radius $r = \sqrt{16} = 4$.

Final Answer: $\boxed{4}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: The standard equation of a parabola with its vertex at the origin and opening towards the positive x-axis is given by $y^2 = 4ax$. In this form, 'a' represents the distance from the vertex to the focus and from the vertex to the directrix. The latus rectum is the chord of the parabola that passes through the focus, is perpendicular to the axis of symmetry (the x-axis in this case), and has endpoints on the parabola. The length of the latus rectum is always $4a$.

Solution: The equation of the parabola given is $y^2 = 8x$.
This equation is in the standard form $y^2 = 4ax$, which represents a parabola opening to the right with its vertex at the origin $(0, 0)$.

Step 1: Identify the value of $4a$ by comparing the given equation with the standard form.
Comparing $y^2 = 8x$ with $y^2 = 4ax$, we can equate the coefficients of x:
$4a = 8$.

Step 2: Solve for the parameter 'a'.
Divide both sides by 4:
$a = \frac{8}{4}$
$a = 2$.
This value 'a' signifies that the focus of the parabola is located at $(a, 0)$, which is $(2, 0)$, and the directrix is the line $x = -a$, which is $x = -2$.

Step 3: Determine the length of the latus rectum.
The length of the latus rectum for a parabola in the form $y^2 = 4ax$ is defined as $4a$.
Since we found $a = 2$, the length of the latus rectum is:
Length = $4a = 4 \times 2 = 8$.

The length of the latus rectum is 8.

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: The standard equation of an ellipse centered at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If $a^2 > b^2$, the major axis lies along the x-axis, and $a > b$. The eccentricity e is a measure of how elongated the ellipse is and is calculated using the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution: The equation of the ellipse is given as $\frac{x^2}{25} + \frac{y^2}{9} = 1$.
This equation is in the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Step 1: Identify the values of a^2 and b^2.
By comparing the given equation with the standard form, we identify:
$a^2 = 25$
$b^2 = 9$.

Step 2: Determine the values of a and b.
Since $a^2 = 25$, $a = \sqrt{25} = 5$.
Since $b^2 = 9$, $b = \sqrt{9} = 3$.
Note that $a > b$ $(5 > 3)$, which confirms that the major axis is horizontal (along the x-axis).

Step 3: Calculate the eccentricity using the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.
Substitute the values of a^2 and b^2 into the formula:
$$e = \sqrt{1 - \frac{9}{25}}$$
To perform the subtraction, find a common denominator:
$$e = \sqrt{\frac{25}{25} - \frac{9}{25}}$$
$$e = \sqrt{\frac{25 - 9}{25}}$$
$$e = \sqrt{\frac{16}{25}}$$
Take the square root of the numerator and the denominator:
$$e = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}.$$

The eccentricity of the ellipse is $\frac{4}{5}$.

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: The standard equation of a hyperbola centered at the origin with a horizontal transverse axis is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The asymptotes are straight lines that the hyperbola approaches as it tends towards infinity. For a hyperbola in this standard form, the equations of the asymptotes are given by $y = \pm\frac{b}{a}x$.

Solution: The equation of the hyperbola is given as $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

This equation matches the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Step 1: Identify the values of a^2 and b^2.
Comparing the given equation with the standard form, we have:
$a^2 = 9$
$b^2 = 16$.

Step 2: Determine the values of a and b.
From $a^2 = 9$, we get $a = \sqrt{9} = 3$.
From $b^2 = 16$, we get $b = \sqrt{16} = 4$.

Step 3: Use the formula for the equations of the asymptotes, $y = \pm\frac{b}{a}x$.
Substitute the values of a and b:
$y = \pm\frac{4}{3}x$.

Thus, the equations of the asymptotes are $y = \frac{4}{3}x$ and $y = -\frac{4}{3}x$.

Final Answer: $\boxed{y = \pm\frac{4}{3}x}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: The dot product (or scalar product) of two vectors provides a scalar quantity. For two vectors $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ in component form, their dot product is calculated by summing the products of their corresponding components: $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$.

Solution: We are given two vectors in component form:
$\vec{a} = (1, 2, -1)$
$\vec{b} = (2, 1, 3)$

To find the dot product $\vec{a} \cdot \vec{b}$, we multiply the corresponding components and sum the results:
$\vec{a} \cdot \vec{b} = (1 \times 2) + (2 \times 1) + (-1 \times 3)$

Calculate each product:
$1 \times 2 = 2$
$2 \times 1 = 2$
$-1 \times 3 = -3$

Sum the results:
$\vec{a} \cdot \vec{b} = 2 + 2 + (-3)$
$\vec{a} \cdot \vec{b} = 4 - 3$
$\vec{a} \cdot \vec{b} = 1.$

The dot product of the two vectors is 1.

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: The cross product (or vector product) of two vectors $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$ results in a vector that is perpendicular to both $\vec{a}$ and $\vec{b}$. It can be calculated using the determinant of a specific matrix involving the standard basis vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ and the components of the vectors:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

Solution: We are given the vectors $\vec{a} = (1, 0, 1)$ and $\vec{b} = (2, 1, -1)$.
We need to compute their cross product $\vec{a} \times \vec{b}$.

Using the determinant formula:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

Expand the determinant along the first row:

$$= \mathbf{i} \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$$

Calculate the 2x2 determinants:
For the $\mathbf{i}$ component: $(0)(-1) - (1)(1) = 0 - 1 = -1$.
For the $\mathbf{j}$ component: $(1)(-1) - (1)(2) = -1 - 2 = -3$.
For the $\mathbf{k}$ component: $(1)(1) - (0)(2) = 1 - 0 = 1$.

Substitute these values back:
$\vec{a} \times \vec{b} = \mathbf{i}(-1) - \mathbf{j}(-3) + \mathbf{k}(1)$
$\vec{a} \times \vec{b} = -1\mathbf{i} + 3\mathbf{j} + 1\mathbf{k}$

The resulting vector is $(-1, 3, 1)$.

Final Answer: $\boxed{(-1, 3, 1)}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: A vector that is perpendicular to two given vectors lies in the direction of their cross product. The direction ratios of a line are the components of any vector parallel to that line. Therefore, the direction ratios of a line perpendicular to two vectors are the components of the cross product of those vectors.

Solution: We are given two vectors: $\vec{u} = (1, 2, 3)$ and $\vec{v} = (2, 1, 1)$.
We need to find the direction ratios of a line perpendicular to both these vectors. This direction is given by the cross product $\vec{u} \times \vec{v}$.

Using the formula for the cross product:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix}$$

Expand the determinant:

$$= \mathbf{i} \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

Calculate the 2x2 determinants:
For the $\mathbf{i}$ component: $(2)(1) - (3)(1) = 2 - 3 = -1$.
For the $\mathbf{j}$ component: $(1)(1) - (3)(2) = 1 - 6 = -5$.
For the $\mathbf{k}$ component: $(1)(1) - (2)(2) = 1 - 4 = -3$.

Substitute these values back:
$\vec{u} \times \vec{v} = \mathbf{i}(-1) - \mathbf{j}(-5) + \mathbf{k}(-3)$
$\vec{u} \times \vec{v} = -1\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$

The direction ratios of the line perpendicular to both vectors are the components of this cross product vector, which are $(-1, 5, -3)$.

Final Answer: $\boxed{(-1, 5, -3)}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: The mean (average) of a set of numbers is calculated by summing all the numbers and dividing by the count of numbers in the set. Mean = $\frac{\sum x_i}{n}$. For an arithmetic progression, if the number of terms is odd, the mean is equal to the middle term.

Solution: We are given the numbers: 3, 7, 11, 15, 19.

Method 1: Using the definition of mean

Step 1: Sum the numbers.

Sum = 3 + 7 + 11 + 15 + 19 = 55.

Step 2: Count the number of observations.

There are 5 numbers.

Step 3: Calculate the mean.

$$\text{Mean} = \frac{\text{Sum}}{\text{Number of observations}} = \frac{55}{5} = 11.$$

Method 2: Using properties of Arithmetic Progression

The sequence 3, 7, 11, 15, 19 is an arithmetic progression with a first term $a = 3$ and a common difference $d = 4$.

Since there are 5 terms (an odd number), the mean is equal to the middle term. The middle term is the $\frac{5+1}{2} = 3^{rd}$ term.

The 3rd term is 11.

Both methods yield the mean as 11.

Final Answer: $\boxed{11}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept: For a binomial distribution $B(n, p)$, the probability mass function (PMF) is given by $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$, where n is the number of trials, p is the probability of success in a single trial, and k is the number of successes.

Solution: We are given that X follows a binomial distribution with $n = 5$ and $p = \frac{1}{2}$.
We need to find the probability $P(X = 2)$.

Using the binomial probability formula $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$:
Here, $n = 5$, $k = 2$, $p = \frac{1}{2}$.
The probability of failure is $1 - p = 1 - \frac{1}{2} = \frac{1}{2}$.

$$P(X = 2) = \binom{5}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2}$$
$$P(X = 2) = \binom{5}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3$$

Step 1: Calculate the binomial coefficient $\binom{5}{2}$.
$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \times 4 \times 3!}{2 \times 1 \times 3!} = \frac{5 \times 4}{2} = \frac{20}{2} = 10.$$

Step 2: Calculate the powers of the probabilities.
$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$
$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Step 3: Substitute these values back into the formula.
$$P(X = 2) = 10 \times \frac{1}{4} \times \frac{1}{8}$$
$$P(X = 2) = 10 \times \frac{1}{32}$$
$$P(X = 2) = \frac{10}{32}.$$

This matches option B.

Final Answer: $\boxed{\frac{10}{32}}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: We can find the value of $\sin 75°$ using the angle addition formula for sine: $\sin(A + B) = \sin A \cos B + \cos A \sin B$. We can express $75°$ as a sum of two standard angles, such as $45° + 30°$.

Solution: We want to find the value of $\sin 75°$.
We can write $75°$ as the sum of two well-known angles: $75° = 45° + 30°$.

Using the sine addition formula $\sin(A + B) = \sin A \cos B + \cos A \sin B$:
Let $A = 45°$ and $B = 30°$.
$\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 45° \sin 30°$.

We know the values of the trigonometric functions for these angles:

$$\sin 45° = \frac{\sqrt{2}}{2}$$
$$\cos 30° = \frac{\sqrt{3}}{2}$$
$$\cos 45° = \frac{\sqrt{2}}{2}$$
$$\sin 30° = \frac{1}{2}$$

Substitute these values into the formula:

$$\sin 75° = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$
$$\sin 75° = \frac{\sqrt{2} \times \sqrt{3}}{4} + \frac{\sqrt{2} \times 1}{4}$$
$$\sin 75° = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$
$$\sin 75° = \frac{\sqrt{6} + \sqrt{2}}{4}.$$

This matches option A.

Final Answer: $\boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: The specific form depends on the product of the arguments:
If $xy < 1$, $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$. If $xy > 1$ and $x, y > 0$, $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$.
We also use the property $\tan^{-1}(-x) = -\tan^{-1} x$.

Solution: We need to calculate the value of $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$.

Let's first combine the terms $\tan^{-1} 2$ and $\tan^{-1} 3$.
Here, $x = 2$ and $y = 3$. The product $xy = 2 \times 3 = 6$.
Since $xy = 6 > 1$ and both x and y are positive, we use the formula:
$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$.

Applying this formula to $\tan^{-1} 2 + \tan^{-1} 3$:
$\tan^{-1} 2 + \tan^{-1} 3 = \pi + \tan^{-1}\left(\frac{2+3}{1-(2)(3)}\right)$
$= \pi + \tan^{-1}\left(\frac{5}{1-6}\right)$
$= \pi + \tan^{-1}\left(\frac{5}{-5}\right)$
$= \pi + \tan^{-1}(-1)$.

We know that $\tan^{-1}(-1)$ is the angle whose tangent is -1. In the principal value range $(-\frac{\pi}{2}, \frac{\pi}{2})$, this angle is $-\frac{\pi}{4}$.
So, $\tan^{-1}(-1) = -\frac{\pi}{4}$.

Substituting this back:
$\tan^{-1} 2 + \tan^{-1} 3 = \pi - \frac{\pi}{4} = \frac{4\pi - \pi}{4} = \frac{3\pi}{4}$.

Now, we need to add the first term, $\tan^{-1} 1$, to this result.
We know that $\tan^{-1} 1$ is the angle whose tangent is 1. This angle is $\frac{\pi}{4}$.

So, the total sum is:
$\tan^{-1} 1 + (\tan^{-1} 2 + \tan^{-1} 3) = \frac{\pi}{4} + \frac{3\pi}{4}$
$= \frac{\pi + 3\pi}{4} = \frac{4\pi}{4} = \pi$.

Final Answer: $\boxed{\pi}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: We use the logarithm property $\log A + \log B = \log(AB)$. Also, the definition of logarithm: if $\log_b x = y$, then $b^y = x$. Here, the base of the logarithm is assumed to be 10 since it's not specified. $\log 10 = 1$.

Solution: We are given the equation $\log(x - 1) + \log(x + 1) = 1$.

Step 1: Combine the logarithms on the left side using the property $\log A + \log B = \log(AB)$.
$\log((x - 1)(x + 1)) = 1$.

Step 2: Simplify the expression inside the logarithm.
Recall the difference of squares formula: $(a - b)(a + b) = a^2 - b^2$.
So, $(x - 1)(x + 1) = x^2 - 1^2 = x^2 - 1$.
The equation becomes $\log(x^2 - 1) = 1$.

Step 3: Convert the logarithmic equation to an exponential equation.
Assuming the base of the logarithm is 10 (common logarithm):
$10^1 = x^2 - 1$.

Step 4: Solve for x.
$10 = x^2 - 1$
$x^2 = 10 + 1$
$x^2 = 11$
$x = \pm\sqrt{11}$.

Step 5: Check for domain restrictions.
The original equation involves $\log(x - 1)$ and $\log(x + 1)$. For the logarithm to be defined, the arguments must be positive.
$x - 1 > 0 \implies x > 1$
$x + 1 > 0 \implies x > -1$
Both conditions must be met, so we require $x > 1$.
The solution $x = -\sqrt{11}$ is less than 1, so it is an extraneous solution.
The valid solution is $x = \sqrt{11}$.

Final Answer: $\boxed{\sqrt{11}}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: The sum of the first n terms of an arithmetic progression (AP) is given by the formula $S_n = \frac{n}{2}[2a + (n-1)d]$, where a is the first term and d is the common difference.

Solution: We are given the first 20 terms of the AP: 3, 7, 11, ...
We need to find the sum of these 20 terms (S_{20}).

Step 1: Identify the first term (a) and the common difference (d).
The first term is $a = 3$.
The common difference d is the difference between consecutive terms: $d = 7 - 3 = 4$. We can check: $11 - 7 = 4$.

Step 2: Identify the number of terms (n).
We need the sum of the first 20 terms, so $n = 20$.

Step 3: Use the formula for the sum of an AP.
$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{20} = \frac{20}{2}[2(3) + (20-1)(4)]$
$S_{20} = 10[6 + (19)(4)]$
$S_{20} = 10[6 + 76]$
$S_{20} = 10[82]$
$S_{20} = 820.$

Final Answer: $\boxed{820}$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept: The sum to infinity of a geometric progression (GP) exists if the absolute value of the common ratio r is less than 1 ($|r| < 1$). The formula for the sum to infinity is $S_\infty = \frac{a}{1-r}$, where a is the first term and r is the common ratio.

Solution: We are given the geometric progression: $3, \frac{3}{2}, \frac{3}{4}, \ldots$.
We need to find the sum to infinity.

Step 1: Identify the first term (a) and the common ratio (r).
The first term is $a = 3$.
The common ratio r is found by dividing any term by its preceding term:
$r = \frac{3/2}{3} = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$.
We can check: $\frac{3/4}{3/2} = \frac{3}{4} \times \frac{2}{3} = \frac{2}{4} = \frac{1}{2}$.

Step 2: Check if the sum to infinity exists.
The common ratio is $r = \frac{1}{2}$. Since $|r| = |\frac{1}{2}| < 1$, the sum to infinity exists.

Step 3: Use the formula for the sum to infinity $S_\infty = \frac{a}{1-r}$.
$S_\infty = \frac{3}{1 - \frac{1}{2}}$
$S_\infty = \frac{3}{\frac{1}{2}}$
$S_\infty = 3 \times 2$
$S_\infty = 6$.

Final Answer: $\boxed{6}$

Answer: (D)

Go Back to Question 25

Q26.

Solution

Concept: For a function $f(x)$ to be continuous at a point $x = c$, the following conditions must be met: 1. $f(c)$ must be defined.
2. $\lim_{x \to c} f(x)$ must exist.
3. $\lim_{x \to c} f(x) = f(c)$.
If a function is not defined at a point but the limit exists, we can define (or redefine) the function at that point to be equal to the limit, making it continuous. This is called continuity correction.

Solution: We are given the function $f(x) = \frac{x^2 - 1}{x - 1}$.
We need to find the value of $f(1)$ after continuity correction.

Step 1: Check the definition of the function at $x = 1$.
If we substitute $x = 1$ directly into the function, we get $f(1) = \frac{1^2 - 1}{1 - 1} = \frac{0}{0}$, which is an indeterminate form. This means the function is not defined at $x = 1$.

Step 2: Simplify the function for $x \neq 1$.
The numerator $x^2 - 1$ can be factored as a difference of squares: $x^2 - 1 = (x - 1)(x + 1)$.
So, for $x \neq 1$, we can simplify $f(x)$:
$f(x) = \frac{(x - 1)(x + 1)}{x - 1} = x + 1$.

Step 3: Find the limit of the function as x approaches 1.
Now that we have the simplified form $f(x) = x + 1$ (for $x \neq 1$), we can find the limit as x approaches 1:
$\lim_{x \to 1} f(x) = \lim_{x \to 1} (x + 1)$.
Substituting $x = 1$ into the simplified expression:
$\lim_{x \to 1} (x + 1) = 1 + 1 = 2$.

Step 4: Apply continuity correction.
To make the function continuous at $x = 1$, we define $f(1)$ to be equal to the limit.
So, the corrected value $f(1) = 2$.

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 26

Q27.

Solution

Concept: This limit involves a trigonometric function and results in the indeterminate form $\frac{0}{0}$. We can evaluate it using standard trigonometric limits, Taylor series, or L'Hopital's Rule. The standard limit is $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$.

Solution: We need to evaluate $\lim_{x\to 0}\dfrac{1-\cos x}{x^2}$. Substituting $x = 0$ gives $\frac{1-\cos 0}{0^2} = \frac{1-1}{0} = \frac{0}{0}$, which is an indeterminate form.

Method 1: Using the Standard Limit

This is a well-known standard limit in calculus:
$\lim_{x\to 0}\dfrac{1-\cos x}{x^2} = \dfrac{1}{2}$.

Method 2: Using L'Hopital's Rule

Apply L'Hopital's Rule since the limit is of the form $\frac{0}{0}$:
$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0}\frac{-(-\sin x)}{2x} = \lim_{x\to 0}\frac{\sin x}{2x}$
Apply L'Hopital's Rule again (or use the standard limit $\lim_{x\to 0}\frac{\sin x}{x} = 1$):
$= \lim_{x\to 0}\frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(2x)} = \lim_{x\to 0}\frac{\cos x}{2}$
Substitute $x = 0$:
$= \frac{\cos 0}{2} = \frac{1}{2}$.

Method 3: Using Trigonometric Identities and Standard Limits

Multiply the numerator and denominator by $1 + \cos x$:
$\lim_{x\to 0}\frac{1-\cos x}{x^2} \times \frac{1+\cos x}{1+\cos x}$
$= \lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$
Using the identity $\sin^2 x = 1 - \cos^2 x$:
$= \lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}$
Rearrange the terms:
$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \cdot \lim_{x\to 0}\frac{1}{1+\cos x}$
Using the standard limit $\lim_{x\to 0}\frac{\sin x}{x} = 1$:
$= (1)^2 \cdot \frac{1}{1+\cos 0}$
$= 1 \cdot \frac{1}{1+1} = \frac{1}{2}$.

All methods yield the result $\dfrac{1}{2}$.

Final Answer: $\boxed{\dfrac{1}{2}}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: The definite integral of a function $f(x)$ from a to b is found by first finding the antiderivative $F(x)$ of $f(x)$, and then evaluating $F(b) - F(a)$. The antiderivative of $\cos x$ is $\sin x$.

Solution: We need to evaluate the definite integral $\int_0^{\pi/2} \cos x \, dx$.

Step 1: Find the antiderivative of $\cos x$.
The antiderivative of $\cos x$ is $\sin x$. Let $F(x) = \sin x$.

Step 2: Apply the fundamental theorem of calculus.
The value of the definite integral is $F(b) - F(a)$, where $a = 0$ and $b = \pi/2$.
$\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2}$

Step 3: Evaluate the antiderivative at the upper and lower limits and subtract.
$= \sin\left(\frac{\pi}{2}\right) - \sin(0)$

Step 4: Substitute the values of the sine function.
$\sin\left(\frac{\pi}{2}\right) = 1$
$\sin(0) = 0$

So, the value of the integral is $1 - 0 = 1$.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: To find the general solution of a simple differential equation like $\frac{dy}{dx} = f(x)$, we integrate $f(x)$ with respect to x. The general solution includes an arbitrary constant of integration, C.

Solution: We are given the differential equation $\frac{dy}{dx} = 3x^2$.
To find the general solution for y, we need to integrate both sides with respect to x.

Step 1: Integrate the right-hand side with respect to x.
$y = \int 3x^2\, dx$.

Step 2: Perform the integration.
Using the power rule for integration, $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.
$y = 3\int x^2\, dx$
$y = 3\left(\frac{x^{2+1}}{2+1}\right) + C$
$y = 3\left(\frac{x^3}{3}\right) + C$
$y = x^3 + C$.

The general solution is $y = x^3 + C$.

Final Answer: $\boxed{y = x^3 + C}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: The equation of the tangent line to a curve $y = f(x)$ at a point (x_1, y_1) is given by $y - y_1 = m(x - x_1)$, where m is the slope of the tangent line at that point. The slope m is equal to the derivative of the function evaluated at x_1, i.e., $m = f'(x_1)$.

Solution: We need to find the equation of the tangent line to the curve $y = x^2$ at the point $(1, 1)$.

Step 1: Find the derivative of the function $y = x^2$.
$\frac{dy}{dx} = \frac{d}{dx}(x^2) = 2x.$

Step 2: Find the slope of the tangent line at the given point $(1, 1)$.
The slope m is the value of the derivative at $x = 1$.
$m = \left.\frac{dy}{dx}\right|_{x=1} = 2(1) = 2.$

Step 3: Use the point-slope form of a line equation: $y - y_1 = m(x - x_1)$.
Here, $(x_1, y_1) = (1, 1)$ and $m = 2$.
$y - 1 = 2(x - 1)$

Step 4: Simplify the equation to the slope-intercept form ($y = mx + c$).
$y - 1 = 2x - 2$
$y = 2x - 2 + 1$
$y = 2x - 1.$

The equation of the tangent line is $y = 2x - 1$.

Final Answer: $\boxed{y = 2x - 1}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: The general equation of a pair of lines is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. The equation $x^2 - y^2 = 0$ is a special case where $g = f = c = 0$. This represents lines passing through the origin. We can factor the expression $x^2 - y^2$ as $(x - y)(x + y)$.

Solution: The given equation is $x^2 - y^2 = 0$.
We can factor this equation as a difference of squares:
$(x - y)(x + y) = 0$.

This equation is satisfied if either $(x - y) = 0$ or $(x + y) = 0$.
1. $x - y = 0 \implies y = x$. This is the equation of a straight line passing through the origin with a slope of 1.
2. $x + y = 0 \implies y = -x$. This is the equation of a straight line passing through the origin with a slope of -1.

These two lines, $y = x$ and $y = -x$, are distinct and intersect at the origin. They are not parallel, coincident, or part of a circle. The angle between the lines $y = m_1x$ and $y = m_2x$ is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$. Here, $m_1 = 1$ and $m_2 = -1$.
$\tan\theta = \left|\frac{1 - (-1)}{1 + (1)(-1)}\right| = \left|\frac{2}{1 - 1}\right| = \left|\frac{2}{0}\right|$, which indicates that the angle θ is $90°$ or $\frac{\pi}{2}$ radians.
Therefore, the lines are perpendicular.

Final Answer: $\boxed{perpendicular\ lines}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The determinant is a fundamental scalar value associated with a square matrix. For a 2x2 matrix, denoted as $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its determinant, often symbolized as $\det(A)$ or $|A|$, is computed using a specific rule: multiply the elements on the main diagonal (from top-left to bottom-right) and subtract the product of the elements on the anti-diagonal (from top-right to bottom-left).

Solution: We need to evaluate the determinant Δ of the 2x2 matrix:
$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

The elements are arranged as follows:
- The element in the first row, first column is a.
- The element in the first row, second column is b.
- The element in the second row, first column is c.
- The element in the second row, second column is d.

The main diagonal elements are a and d. Their product is $a \times d = ad$.
The anti-diagonal elements are b and c. Their product is $b \times c = bc$.

According to the rule for calculating the determinant of a 2x2 matrix, we subtract the product of the anti-diagonal elements from the product of the main diagonal elements:
$\Delta = (a \times d) - (b \times c)$
$\Delta = ad - bc$.

This result matches option B.

Final Answer: $\boxed{ad - bc}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: The transpose of a matrix is obtained by interchanging its rows and columns. If A is an $m \times n$ matrix, its transpose A^T is an $n \times m$ matrix where the element in the i-th row and j-th column of A^T is the element in the j-th row and i-th column of A.

Solution: We are given the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$.
This is a 2×3 matrix (2 rows, 3 columns).
Its transpose, denoted by A^T, will be a 3×2 matrix (3 rows, 2 columns).

To find the transpose, we interchange the rows and columns:
- The first row of A, [1 2 3], becomes the first column of A^T.
- The second row of A, [4 5 6], becomes the second column of A^T.

So, the transpose is:
$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

This matches option A.

Final Answer: $\boxed{\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The powers of the imaginary unit i follow a cycle of 4:
$i^1 = i$
$i^2 = -1$
$i^3 = i^2 \cdot i = -i$
$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$
$i^5 = i^4 \cdot i = 1 \cdot i = i$, and so on.
To find i^n, we can find the remainder when n is divided by 4. Let $n = 4q + r$, where r is the remainder $(0 \le r < 4)$. Then $i^n = i^r$.

Solution: We need to find the value of i^{35}.
We find the remainder when 35 is divided by 4.
$35 \div 4$.
$4 \times 8 = 32$.
$35 - 32 = 3$.
So, $35 = 4 \times 8 + 3$. The remainder is $r = 3$.

Therefore, $i^{35} = i^3$.
We know that $i^3 = -i$.

Final Answer: $\boxed{-i}$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept: This problem falls under the domain of combinatorics, specifically permutations. A permutation deals with the arrangement of objects in a specific order. When we have n distinct objects, the number of ways to arrange all n of them in a sequence is given by the factorial function, $n!$. The factorial $n!$ is calculated as the product of all positive integers from 1 up to n. For example, $3! = 3 \times 2 \times 1 = 6$.

Solution: We are asked to determine the total number of different ways to arrange 5 distinct books on a shelf. The key information here is that the books are "distinct," meaning each book is unique (e.g., different titles, authors, etc.). The fact that they are being arranged "on a shelf" implies that the order in which they are placed matters. For instance, arranging Book A then Book B is different from arranging Book B then Book A. This scenario requires the use of permutations.

Let n represent the number of distinct objects to be arranged. In this problem, $n = 5$ (the 5 distinct books).

We need to find the number of possible arrangements (permutations) of these 5 books. The formula for the number of permutations of n distinct objects is $n!$.

Calculating 5!:
$5! = 5 \times (5 - 1) \times (5 - 2) \times (5 - 3) \times (5 - 4)$
$5! = 5 \times 4 \times 3 \times 2 \times 1$

Let's break down the calculation:
- Consider the first position on the shelf. We have 5 choices for which book to place there.
- Once the first book is placed, we have 4 remaining books. So, there are 4 choices for the second position.
- After placing the first two books, we have 3 remaining books, giving 3 choices for the third position.
- Then, there are 2 choices for the fourth position.
- Finally, there is only 1 book left, so there is 1 choice for the last position.

The total number of arrangements is the product of the number of choices for each position:
Total arrangements = $5 \times 4 \times 3 \times 2 \times 1 = 120$.

Therefore, there are 120 different ways to arrange the 5 distinct books on the shelf.

Final Answer: $\boxed{120}$

Answer: (C)

Go Back to Question 35

Q36.

Solution

Concept: The number of combinations of choosing k items from a set of n distinct items (where order does not matter) is denoted by ${}^nC_k$ or $\binom{n}{k}$, and is calculated using the formula: ${}^nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Solution: We need to find the value of ${}^8C_2$.
Here, $n = 8$ and $k = 2$.

Using the combination formula:
$${}^8C_2 = \frac{8!}{2!(8-2)!}$$
$${}^8C_2 = \frac{8!}{2!6!}$$

Expand the factorials:
$${}^8C_2 = \frac{8 \times 7 \times 6!}{(2 \times 1) \times 6!}$$

Cancel out the 6!:
$${}^8C_2 = \frac{8 \times 7}{2 \times 1}$$
$${}^8C_2 = \frac{56}{2}$$
$${}^8C_2 = 28.$$

Final Answer: $\boxed{28}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: The slope of a line given by the equation $Ax + By + C = 0$ can be found by rearranging the equation into the slope-intercept form $y = mx + c$, where m is the slope. Alternatively, the slope is given by $m = -\frac{A}{B}$.

Solution: We are given the equation of the line: $2x + 3y - 6 = 0$.

Method 1: Rearranging to slope-intercept form

Solve for y:

$3y = -2x + 6$

$y = \frac{-2x + 6}{3}$

$y = -\frac{2}{3}x + 2.$

Comparing this to $y = mx + c$, the slope m is $-\frac{2}{3}$.

Method 2: Using the formula $m = -\frac{A}{B}$

In the equation $2x + 3y - 6 = 0$, we have $A = 2$, $B = 3$, and $C = -6$.

The slope $m = -\frac{A}{B} = -\frac{2}{3}$.

Both methods give the same slope.

Final Answer: $\boxed{-\frac{2}{3}}$

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept: The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. The center of the circle is given by the coordinates $(-g, -f)$.

Solution: We are given the equation of the circle: $x^2 + y^2 - 4x + 6y + 3 = 0$.
Compare this equation with the general form $x^2 + y^2 + 2gx + 2fy + c = 0$.

Step 1: Identify the values of g and f.
$2g = -4 \implies g = -2.$
$2f = 6 \implies f = 3.$
The value of c is 3, but it is not needed to find the center.

Step 2: Determine the coordinates of the center.
The center is $(-g, -f)$.
Center $= (-(-2), -(3)) = (2, -3)$.

Final Answer: $\boxed{(2, -3)}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, where $|\vec{a}|$ and $|\vec{b}|$ are the magnitudes of the vectors and θ is the angle between them.

Solution: We are given:
Magnitude of vector $\vec{a}$, $|\vec{a}| = 2$.
Magnitude of vector $\vec{b}$, $|\vec{b}| = 3$.
The angle between them, $\theta = 60^\circ$.

We need to calculate the dot product $\vec{a} \cdot \vec{b}$.
Using the formula:
$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
$\vec{a} \cdot \vec{b} = (2)(3) \cos 60^\circ$

We know that $\cos 60^\circ = \frac{1}{2}$.
$\vec{a} \cdot \vec{b} = (2)(3) \left(\frac{1}{2}\right)$
$\vec{a} \cdot \vec{b} = 6 \times \frac{1}{2}$
$\vec{a} \cdot \vec{b} = 3$.

Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in 3D space is given by the distance formula:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Solution: We need to find the distance between the points $(1, 2, 3)$ and $(4, 6, 3)$.
Let $(x_1, y_1, z_1) = (1, 2, 3)$ and $(x_2, y_2, z_2) = (4, 6, 3)$.

Step 1: Substitute the coordinates into the distance formula.
$d = \sqrt{(4 - 1)^2 + (6 - 2)^2 + (3 - 3)^2}$

Step 2: Calculate the differences in each coordinate.
$x_2 - x_1 = 4 - 1 = 3$
$y_2 - y_1 = 6 - 2 = 4$
$z_2 - z_1 = 3 - 3 = 0$

Step 3: Square the differences.
$(3)^2 = 9$
$(4)^2 = 16$
$(0)^2 = 0$

Step 4: Sum the squares.
$9 + 16 + 0 = 25$.

Step 5: Take the square root of the sum.
$d = \sqrt{25} = 5$.

The distance between the two points is 5.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 40

Q41.

Solution

Concept: The median of a dataset is the middle value when the dataset is arranged in ascending or descending order. If the dataset has an odd number of observations, the median is the middle term. If it has an even number of observations, the median is the average of the two middle terms.

Solution: We are given the dataset: 2, 4, 6, 8, 10.
The dataset is already arranged in ascending order.

Step 1: Count the number of observations.
There are 5 observations in the dataset.

Step 2: Identify the middle term.
Since there is an odd number of observations (5), the median is the middle term. The position of the middle term is $\frac{n+1}{2}$, where n is the number of observations.
Position = $\frac{5+1}{2} = \frac{6}{2} = 3$.
The 3rd term in the ordered dataset is the median.

Step 3: Determine the median.
The 3rd term in the sequence 2, 4, 6, 8, 10 is 6.

Final Answer: $\boxed{6}$

Answer: (C)

Go Back to Question 41

Q42.

Solution

Concept: Probability is defined as the ratio of the number of favorable outcomes to the total number of possible outcomes. A standard deck of 52 cards has 4 suits (hearts, diamonds, clubs, spades), and each suit has 13 ranks (2, 3, ..., 10, J, Q, K, A). There are 4 kings in a standard deck (one for each suit).

Solution: We need to find the probability of drawing a king from a standard deck of 52 cards.

Step 1: Determine the total number of possible outcomes.
The total number of cards in the deck is 52. So, the total number of possible outcomes when drawing one card is 52.

Step 2: Determine the number of favorable outcomes.
The number of kings in a standard deck is 4 (King of Hearts, King of Diamonds, King of Clubs, King of Spades). These are the favorable outcomes.

Step 3: Calculate the probability.
$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$
$\text{Probability (getting a king)} = \frac{4}{52}.$

Step 4: Simplify the fraction.
Divide both the numerator and the denominator by their greatest common divisor, which is 4.
$\frac{4}{52} = \frac{4 \div 4}{52 \div 4} = \frac{1}{13}.$

Final Answer: $\boxed{\frac{1}{13}}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: We can use the cosine angle addition formula: $\cos(A + B) = \cos A \cos B - \sin A \sin B$. Alternatively, we can directly substitute the known values of cosine and sine for the given angles.

Solution: We need to evaluate the expression $\cos 60° \cos 30° - \sin 60° \sin 30°$.

We know the values of the trigonometric functions for 60° and 30°:

$\cos 60° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

Substitute these values into the expression:

$\text{Expression} = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$\text{Expression} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$

$\text{Expression} = 0.$

Alternatively, we can recognize that the expression is in the form of the cosine angle addition formula $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

Here, $A = 60°$ and $B = 30°$.

So, the expression is equal to $\cos(60° + 30°) = \cos(90°)$.

We know that $\cos 90° = 0$.

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept: To differentiate a composite function like $y = \log(f(x))$, we use the chain rule. The derivative of $\log u$ with respect to u is $\frac{1}{u}$, and the derivative of $f(x)$ with respect to x is $f'(x)$. So, $\frac{d}{dx}(\log(f(x))) = \frac{1}{f(x)} \cdot f'(x)$.

Solution: We are given the function $y = \log(\sin x)$.
We need to find the derivative $\frac{dy}{dx}$.

Step 1: Identify the outer and inner functions.
The outer function is the logarithm function (let's denote it as $\log u$).
The inner function is $f(x) = \sin x$.

Step 2: Find the derivative of the outer function with respect to its argument.
The derivative of $\log u$ with respect to u is $\frac{1}{u}$.

Step 3: Find the derivative of the inner function with respect to x.
The derivative of $\sin x$ with respect to x is $\cos x$.

Step 4: Apply the chain rule.
$\frac{dy}{dx}$ = (derivative of outer function) × (derivative of inner function)
$\frac{dy}{dx} = \frac{1}{\sin x} \times (\cos x)$
$\frac{dy}{dx} = \frac{\cos x}{\sin x}$.

Step 5: Simplify the result using trigonometric identities.
We know that $\cot x = \frac{\cos x}{\sin x}$.
So, $\frac{dy}{dx} = \cot x$.

Final Answer: $\boxed{\cot x}$

Answer: (C)

Go Back to Question 44

Q45.

Solution

Concept: The integral of the exponential function e^x with respect to x is e^x plus an arbitrary constant of integration C. That is, $\int e^x dx = e^x + C$.

Solution: We need to find the indefinite integral of e^x.

$$\int e^x \, dx.$$

The antiderivative of e^x is e^x. Since this is an indefinite integral, we add the constant of integration C.

$$\int e^x \, dx = e^x + C.$$

Final Answer: $\boxed{e^x + C}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: The domain of a function is the set of all possible input values (usually x) for which the function is defined. For a function involving a square root, like $f(x) = \sqrt{g(x)}$, the expression under the square root, $g(x)$, must be non-negative, i.e., $g(x) \geq 0$.

Solution: We are given the function $f(x) = \sqrt{5 - x}$.
For the function to be defined, the expression inside the square root must be greater than or equal to zero. $5 - x \geq 0$.

Step 1: Solve the inequality for x.
Add x to both sides:
$5 \geq x$.
This can also be written as $x \leq 5$.

This means that the function $f(x) = \sqrt{5 - x}$ is defined for all real numbers x that are less than or equal to 5.

The domain is $(-\infty, 5]$.

Final Answer: $\boxed{x \leq 5}$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept: To find the limit of a rational function as x approaches infinity, we can divide both the numerator and the denominator by the highest power of x present in the denominator. For a rational function where the degree of the numerator is equal to the degree of the denominator, the limit as $x \to \infty$ is the ratio of the leading coefficients.

Solution: We need to evaluate the limit $\lim_{x\to\infty} \frac{2x^2+1}{x^2-3}$.
The highest power of x in the denominator is x^2.

Step 1: Divide both the numerator and the denominator by x^2.
$\lim_{x\to\infty} \frac{\frac{2x^2}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{3}{x^2}}$
$= \lim_{x\to\infty} \frac{2+\frac{1}{x^2}}{1-\frac{3}{x^2}}$

Step 2: Evaluate the limit as $x \to \infty$.
As $x \to \infty$, terms like $\frac{1}{x^2}$ and $\frac{3}{x^2}$ approach 0.
$\lim_{x\to\infty} \frac{1}{x^2} = 0$
$\lim_{x\to\infty} \frac{3}{x^2} = 0$

Step 3: Substitute these values back into the expression.
$= \frac{2+0}{1-0} = \frac{2}{1} = 2.$

Alternatively, since the degree of the numerator (2) is equal to the degree of the denominator (2), the limit is the ratio of the leading coefficients: $\frac{2}{1} = 2$.

Final Answer: $\boxed{2}$

Answer: (C)

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Q48.

Solution

Concept: For a quadratic equation of the form $ax^2 + bx + c = 0$, if α and β are its roots, then the sum of the roots is $\alpha + \beta = -\frac{b}{a}$ and the product of the roots is $\alpha\beta = \frac{c}{a}$.

Solution: We are given the quadratic equation $x^2 - 7x + 10 = 0$.
We need to find the sum of its roots, $\alpha + \beta$.

Step 1: Identify the coefficients a, b, and c.
Comparing the equation with the standard form $ax^2 + bx + c = 0$:
$a = 1$
$b = -7$
$c = 10$

Step 2: Use the formula for the sum of the roots.
$\alpha + \beta = -\frac{b}{a}$
$\alpha + \beta = -\frac{-7}{1}$
$\alpha + \beta = \frac{7}{1}$
$\alpha + \beta = 7.$

Alternatively, we can find the roots by factoring the quadratic equation:
$x^2 - 7x + 10 = 0$
$(x - 2)(x - 5) = 0.$
The roots are $x = 2$ and $x = 5$.
So, $\alpha = 2$ and $\beta = 5$ (or vice versa).
The sum of the roots is $\alpha + \beta = 2 + 5 = 7$.

Final Answer: $\boxed{7}$

Answer: (B)

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Q49.

Solution

Concept: The order of a differential equation is the order of the highest derivative present. The general solution of a differential equation contains as many arbitrary constants as the order of the equation. To find the differential equation representing a family of curves, we differentiate the equation of the family with respect to the independent variable and eliminate the arbitrary constants.

Solution: The family of curves is given by the equation $y = mx + c$.
This equation has two arbitrary constants, m (slope) and c (y-intercept).

Step 1: Differentiate the equation with respect to x to eliminate one constant.
Differentiating $y = mx + c$ with respect to x:
$$\frac{dy}{dx} = m.$$

Step 2: Differentiate again to eliminate the second constant.
Differentiating $\frac{dy}{dx} = m$ with respect to x:
$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(m)$$
$$\frac{d^2y}{dx^2} = 0.$$

The resulting differential equation is $\frac{d^2y}{dx^2} = 0$. The highest derivative in this equation is the second derivative $(\frac{d^2y}{dx^2})$.
Therefore, the order of the differential equation representing the family $y = mx + c$ is 2.

Final Answer: $\boxed{2}$

Answer: (B)

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Q50.

Solution

Concept: This is a binomial probability problem. A coin toss is a Bernoulli trial (two outcomes: heads or tails, with constant probability). When a coin is tossed multiple times, the number of heads follows a binomial distribution.

Solution: A coin is tossed 3 times. We want to find the probability of getting exactly two heads.
Let X be the random variable representing the number of heads.
The number of trials is $n = 3$.
The probability of getting a head in a single toss (success) is $p = \frac{1}{2}$.
The probability of getting a tail (failure) is $1 - p = 1 - \frac{1}{2} = \frac{1}{2}$.
We want to find the probability of getting exactly two heads, so $k = 2$.

Using the binomial probability formula:
$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$
$P(X = 2) = \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{3-2}$
$P(X = 2) = \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1$

Step 1: Calculate the binomial coefficient $\binom{3}{2}$.
$\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3!}{2!1!} = \frac{3 \times 2!}{2! \times 1} = 3.$

Step 2: Calculate the powers of the probabilities.
$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$
$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$

Step 3: Substitute the values back into the formula.
$P(X = 2) = 3 \times \frac{1}{4} \times \frac{1}{2}$
$P(X = 2) = 3 \times \frac{1}{8}$
$P(X = 2) = \frac{3}{8}.$

Alternatively, we can list the sample space:
HHH, HHT, HTH, THH, HTT, THT, TTH, TTT (Total 8 outcomes)
The outcomes with exactly two heads are: HHT, HTH, THH. There are 3 such outcomes.
Probability $= \frac{3}{8}$.

Final Answer: $\boxed{\frac{3}{8}}$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	A	4	B	5	A
6	A	7	C	8	A	9	A	10	A
11	C	12	B	13	C	14	B	15	A
16	A	17	A	18	A	19	C	20	B
21	A	22	B	23	A	24	C	25	D
26	C	27	B	28	B	29	A	30	A
31	A	32	B	33	A	34	B	35	C
36	B	37	B	38	A	39	A	40	C
41	C	42	A	43	A	44	C	45	A
46	B	47	C	48	B	49	B	50	B