MHT-CET Mathematics Sample Paper- 19

Duration: 90 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. The value of $\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$ is:

(A) $3/2$

(B) $1/2$

(C) 1

(D) 2

Q2. If $f(x) = \frac{\ln(1+ax) - \ln(1-bx)}{x}$ is continuous at $x = 0$, then $f(0)$ must be:

(A) $a - b$

(B) $a + b$

(C) $\ln(a/b)$

(D) $\ln(ab)$

Q3. The value of $\lim_{x\to\infty} \left(\frac{x+6}{x+1}\right)^{x+4}$ is:

(A) e^4

(B) e^5

(C) e^6

(D) e

Q4. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ is:

(A) $\frac{x}{\sqrt{1+x^2}}$

(B) $\frac{1}{\sqrt{1+x^2}}$

(C) $\frac{x}{1+x^2}$

(D) $\frac{1}{1+x^2}$

Q5. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is equal to:

(A) $\frac{\ln x}{(1+\ln x)^2}$

(B) $\frac{1}{(1+\ln x)^2}$

(C) $\frac{e^x}{x^y}$

(D) $\frac{y}{x(1+\ln x)}$

Q6. If $f(x) = |x-1| + |x-2|$, then $f'(1.5)$ is:

(A) 1

(B) -1

(C) 0

(D) Does not exist

Q7. If $x = a\cos^3\theta, y = a\sin^3\theta$, then $\sqrt{1 + (\frac{dy}{dx})^2}$ is:

(A) $\tan\theta$

(B) $\sec\theta$

(C) $|\sec\theta|$

(D)

(E) $|\tan\theta|$

Q8. If $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, then $\frac{dy}{dx}$ is:

(A) $1/2$

(B) $-1/2$

(C) 1

(D) −1

Q9. The equation of the tangent to the curve $y = be^{-x/a}$ at the point where it crosses the y-axis is:

(A) $ax + by = 1$

(B) $\frac{x}{a} + \frac{y}{b} = 1$

(C) $x - y = a$

(D) $ax - by = 1$

Q10. The function $f(x) = x + \frac{1}{x}$ has a local maximum value of:

(A) 2

(B) −2

(C) 0

(D) 1

Q11. The maximum area of a rectangle inscribed in a circle of radius r is:

(A) r^2

(B) $2r^2$

(C) $r^2/2$

(D) $4r^2$

Q12. If $f(x) = kx - \sin x$ is monotonically increasing for all $x \in \mathbb{R}$, then:

(A) $k > 1$

(B) $k < 1$

(C) $k \geq 1$

(D) $k \leq 1$

Q13. The rate of change of volume of a sphere with respect to its surface area when the radius is 2 cm is:

(A) 1

(B) 2

(C) 4

(D) 1/2

Q14. The value of $\int \frac{dx}{x(x^5+1)}$ is:

(A) $\frac{1}{5}\ln|\frac{x^5}{x^5+1}| + C$

(B) $\ln|\frac{x^5}{x^5+1}| + C$

(C) $5\ln|\frac{x^5+1}{x^5}| + C$

(D) $\frac{1}{5}\ln|\frac{x^5+1}{x^5}| + C$

Q15. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) 0

(D) 1

Q16. $\int \frac{\sin x-\cos x}{\sqrt{1+\sin 2x}}dx$ (where $x \in (0, \pi/4)$) is:

(A) $\ln|\sin x + \cos x| + C$

(B) $x + C$

(C) $-x + C$

(D) C

Q17. $\int_{-1}^{1} |x|dx$ is equal to:

(A) 0

(B) 1

(C) 2

(D) 1/2

Q18. $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$ is equal to:

(A) $\tan(xe^x) + C$

(B) $\cot(xe^x) + C$

(C) $\sec(xe^x) + C$

(D) $\tan(e^x) + C$

Q19. The area bounded by $y = \cos x$ and x-axis from $x = 0$ to $x = \pi$ is:

(A) 0

(B) 1

(C) 2

(D) 4

Q20. The area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is:

(A) $16a^2/3$

(B) $8a^2/3$

(C) $4a^2/3$

(D) $16a^2$

Q21. The degree of the differential equation $[1 + (\frac{dy}{dx})^2]^{3/2} = \frac{d^2y}{dx^2}$ is:

(A) 3

(B) 2

(C) $3/2$

(D) Not defined

Q22. The integrating factor of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) e^x

(B) $\ln x$

(C) x

(D) $1/x$

Q23. The solution of $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^y = e^x + x^2 + C$

(C) $e^x = e^y + \frac{x^3}{3} + C$

(D) $e^{x+y} = C$

Q24. If ω is a cube root of unity, then $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3$ is:

(A) 0

(B) 32

(C) -16

(D) -32

Q25. The argument of the complex number $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ is:

(A) $\pi/3$

(B) $2\pi/3$

(C) $\pi/6$

(D) $\pi/2$

Q26. The least positive integer n for which $(\frac{1+i}{1-i})^n = 1$ is:

(A) 2

(B) 4

(C) 8

(D) 16

Q27. If $\tan A$ and $\tan B$ are the roots of $x^2 - px + q = 0$, then the value of $\sin^2(A + B)$ is:

(A) $\frac{p^2}{p^2+(1-q)^2}$

(B) $\frac{p^2}{p^2+q^2}$

(C) $\frac{q^2}{p^2+(1-q)^2}$

(D) $\frac{p^2}{(p+q)^2}$

Q28. The number of real roots of the equation $x^2 - 3|x| + 2 = 0$ is:

(A) 2

(B) 1

(C) 4

(D) 0

Q29. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is equal to:

(A) 0

(B) 1

(C) 2

(D) 3

Q30. If a, b, c are in G.P. and $a^x = b^y = c^z$, then x, y, z are in:

(A) A.P.

(B) G.P.

(C) H.P.

(D) None of these

Q31. The sum of the series $0.7 + 0.77 + 0.777 + \ldots$ to n terms is:

(A) $\frac{7}{9}[n - \frac{1}{10}(1 - 10^{-n})]$

(B) $\frac{7}{81}[9n - 1 + 10^{-n}]$

(C) $\frac{7}{81}[9n + 1 - 10^{-n}]$

(D) $\frac{7}{9}[n - \frac{1}{9}(1 - 10^{-n})]$

Q32. If H is the Harmonic Mean between a and b, then the value of $\frac{H+a}{H-a} + \frac{H+b}{H-b}$ is:

(A) 2

(B) 1

(C) 0

(D) 4

Q33. The term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is:

(A) ${}^9C_3 \cdot \frac{1}{6}$

(B) ${}^9C_6 \cdot \frac{1}{18}$

(C) ${}^9C_6 \cdot \frac{1}{27}$

(D) ${}^9C_6 \cdot \frac{1}{12}$

Q34. If the coefficients of x^7 and x^8 in $(2 + x/3)^n$ are equal, then the value of n is:

(A) 55

(B) 53

(C) 45

(D) 15

Q35. The number of ways in which 5 boys and 5 girls can be seated around a circular table so that no two girls sit together is:

(A) $5! \cdot 4!$

(B) $5! \cdot 5!$

(C) $4! \cdot 4!$

(D) $10!$

Q36. A bag contains 6 white and 4 black balls. Two balls are drawn at random. The probability that they are of the same color is:

(A) $1/2$

(B) $7/15$

(C) $8/15$

(D) $1/3$

Q37. The number of divisors of 10800 which are perfect squares is:

(A) 12

(B) 10

(C) 15

(D) 8

Q38. If A and B are two independent events such that $P(A) = 1/2$ and $P(B) = 1/5$, then $P(A \cup B)$ is:

(A) $3/5$

(B) $7/10$

(C) $1/10$

(D) $9/10$

Q39. The distance between the parallel lines $5x + 12y - 20 = 0$ and $5x + 12y + 6 = 0$ is:

(A) 2

(B) 1

(C) $26/13$

(D) $13/26$

Q40. If the line $y = mx + c$ is a tangent to the circle $x^2 + y^2 = a^2$, then the value of c^2 is:

(A) $a^2(1 + m^2)$

(B) $a^2(1 - m^2)$

(C) a^2/m^2

(D) a^2m^2

Q41. The equation of the circle which touches the x-axis and has its center at $(2, 3)$ is:

(A) $x^2 + y^2 - 4x - 6y + 4 = 0$

(B) $x^2 + y^2 - 4x - 6y + 9 = 0$

(C) $x^2 + y^2 - 4x - 6y = 0$

(D) $x^2 + y^2 + 4x + 6y + 9 = 0$

Q42. The orthocenter of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 1$ is:

(A) $(0, 0)$

(B) $(1/3, 1/3)$

(C) $(1/2, 1/2)$

(D) $(1, 1)$

Q43. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is:

(A) $4/3$

(B) $4/\sqrt{3}$

(C) $2/\sqrt{3}$

(D) $3/2$

Q44. The focal distance of a point (x_1, y_1) on the parabola $y^2 = 4ax$ is:

(A) $x_1 + a$

(B) $x_1 - a$

(C) $y_1 + a$

(D) $y_1 - a$

Q45. If the latus rectum of an ellipse is equal to half of its minor axis, then its eccentricity is:

(A) $\sqrt{3}/2$

(B) $\sqrt{2}/3$

(C) $1/2$

(D) $1/\sqrt{2}$

Q46. The product of the perpendiculars from the foci upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

(A) a^2

(B) b^2

(C) ab

(D) $a^2 + b^2$

Q47. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}]$ is equal to:

(A) $2[\vec{a}\vec{b}\vec{c}]$

(B) $[\vec{a}\vec{b}\vec{c}]$

(C) 0

(D) $3[\vec{a}\vec{b}\vec{c}]$

Q48. The angle between the vectors $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$ is:

(A) $\cos^{-1}(-1/3)$

(B) $\cos^{-1}(1/3)$

(C) $\pi/2$

(D) 0

Q49. The distance of the point $(1, 2, 3)$ from the plane $x + 2y - 2z + 5 = 0$ is:

(A) $4/3$

(B) 2

(C) 1

(D) 3

Q50. The direction cosines of a line which is equally inclined to the axes are:

(A) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(B) $(1, 1, 1)$

(C) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

(D) $(0, 0, 1)$

Detailed Solutions

Q1.

Solution

Concept: Using standard expansions:

$$e^{x^2} = 1 + x^2 + \cdots$$

and

$$\cos x = 1 - \frac{x^2}{2} + \cdots$$

Solution:

$$e^{x^2} - \cos x = \left(1 + x^2\right) - \left(1 - \frac{x^2}{2}\right) = \frac{3x^2}{2}$$

Therefore,

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = \frac{3}{2}$$

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: Continuity at a point using limits.

Solution:

For continuity at $x = 0$,

$$f(0) = \lim_{x\to 0} \frac{\ln(1 + ax) - \ln(1 - bx)}{x}$$

Using

$$\ln(1 + t) \approx t$$

we get

$$\ln(1 + ax) \approx ax$$

and

$$\ln(1 - bx) \approx -bx$$

Hence,

$$f(0) = \lim_{x\to 0} \frac{ax + bx}{x} = a + b$$

Final Answer: $\boxed{a + b}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: Standard exponential limit:

$$\left(1+\frac{k}{x}\right)^{x}=e^{k}$$

Solution:

$$\left(\frac{x+6}{x+1}\right)^{x+4}=\left(1+\frac{5}{x+1}\right)^{x+4}$$

As $x \to \infty$,

$$\left(1+\frac{5}{x+1}\right)^{x+1}\to e^{5}$$

Therefore,

$$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}=e^{5}$$

Final Answer: $\boxed{e^5}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: Trigonometric identity with inverse trigonometric function.

Solution:

Let

$$\theta=\tan^{-1}x$$

Then

$$\tan\theta=x$$

Using a right triangle,

$$\sec\theta=\sqrt{1+x^2}$$

Thus,

$$y=\sqrt{1+x^2}$$

Differentiating,

$$\frac{dy}{dx}=\frac{x}{\sqrt{1+x^2}}$$

Final Answer: $\boxed{\dfrac{x}{\sqrt{1+x^2}}}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: Logarithmic differentiation.

Solution:

Given

$$x^y = e^{x-y}$$

Taking logarithm,

$$y \ln x = x - y$$

$$y(1 + \ln x) = x$$

$$y = \frac{x}{1 + \ln x}$$

Differentiating,

$$\frac{dy}{dx} = \frac{(1 + \ln x) - 1}{(1 + \ln x)^2}$$

$$\frac{dy}{dx} = \frac{\ln x}{(1 + \ln x)^2}$$

Final Answer: $\boxed{\frac{\ln x}{(1 + \ln x)^2}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: Derivative of modulus function.

Solution:

For

$$1 < x < 2$$

we have

$$|x - 1| = x - 1$$

and

$$|x - 2| = 2 - x$$

Therefore,

$$f(x) = x - 1 + 2 - x = 1$$

Hence,

$$f'(x) = 0$$

So,

$$f'(1.5) = 0$$

Final Answer: $\boxed{0}$

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: Parametric differentiation.

Solution:

Given

$$x = a\cos^3\theta$$

and

$$y = a\sin^3\theta$$

Differentiate w.r.t. θ:

$$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$$

$$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

Thus,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\tan\theta$$

Hence,

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2\theta = \sec^2\theta$$

Therefore,

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = |\sec\theta|$$

Final Answer: $\boxed{|\sec\theta|}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: Trigonometric identity and differentiation.

Solution:

Using identity,

$$\frac{\cos x}{1+\sin x} = \frac{1-\sin x}{\cos x} = \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

Therefore,

$$y = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$y = \frac{\pi}{4} - \frac{x}{2}$$

Differentiating,

$$\frac{dy}{dx} = -\frac{1}{2}$$

Final Answer: $\boxed{-\frac{1}{2}}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: Equation of tangent using derivative.

Solution:

Given

$$y = be^{-x/a}$$

At the point where the curve crosses the y-axis,

$$x = 0$$

so,

$$y = b$$

Thus point is

$$(0, b)$$

Differentiate:

$$\frac{dy}{dx} = -\frac{b}{a}e^{-x/a}$$

At $x = 0$,

$$m = -\frac{b}{a}$$

Equation of tangent:

$$y - b = -\frac{b}{a}(x - 0)$$

$$bx + ay = ab$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Final Answer: $\boxed{\frac{x}{a} + \frac{y}{b} = 1}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: Local maxima using first and second derivative tests.

Solution:

Given

$$f(x) = x + \frac{1}{x}$$

Differentiate:

$$f'(x) = 1 - \frac{1}{x^2}$$

Set

$$f'(x) = 0$$

$$1 - \frac{1}{x^2} = 0$$

$$x = \pm 1$$

Second derivative:

$$f''(x) = \frac{2}{x^3}$$

At $x = -1$,

$$f''(-1) = -2 < 0$$

Hence local maximum occurs at $x = -1$.

$$f(-1) = -1 - 1 = -2$$

Final Answer: $\boxed{-2}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: Maximum area of rectangle inscribed in a circle.

Solution:

Let sides of rectangle be

$$2x \text{ and } 2y$$

Since diagonal equals diameter,

$$x^2 + y^2 = r^2$$

Area:

$$A = 4xy$$

Maximum value of xy occurs when

$$x = y$$

Thus,

$$2x^2 = r^2$$

$$x^2 = \frac{r^2}{2}$$

Maximum area:

$$A_{\max} = 4x^2 = 4 \cdot \frac{r^2}{2} = 2r^2$$

Final Answer: $\boxed{2r^2}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: Condition for monotonic increasing function.

Solution:

Given

$$f(x) = kx - \sin x$$

Differentiate:

$$f'(x) = k - \cos x$$

For $f(x)$ to be increasing for all x,

$$f'(x) \geq 0$$

Since

$$-1 \leq \cos x \leq 1$$

maximum value of $\cos x$ is 1.

Thus,

$$k - 1 \geq 0$$

$$k \geq 1$$

Final Answer: $\boxed{k \geq 1}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: Rate of change using differentiation.

Solution:

Volume of sphere:

$$V = \frac{4}{3}\pi r^3$$

Surface area:

$$S = 4\pi r^2$$

Differentiate w.r.t. r:

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dS}{dr} = 8\pi r$$

Therefore,

$$\frac{dV}{dS} = \frac{dV/dr}{dS/dr} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2}$$

At $r = 2$,

$$\frac{dV}{dS} = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: Integration using substitution.

Solution:

$$I = \int \frac{dx}{x(x^5 + 1)}$$

Put

$$t = x^5 + 1$$

Then

$$dt = 5x^4 dx$$

Rewrite:

$$I = \frac{1}{5} \int \left(\frac{1}{x} - \frac{x^4}{x^5 + 1} \right) dx$$

$$I = \frac{1}{5} \left(\ln |x^5| - \ln |x^5 + 1| \right) + C$$

$$I = \frac{1}{5} \ln \left| \frac{x^5}{x^5 + 1} \right| + C$$

Final Answer: $\boxed{\frac{1}{5} \ln \left| \frac{x^5}{x^5 + 1} \right| + C}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: Property of definite integrals.

Solution:

Let

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Using property,

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

we get

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Adding both:

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: Trigonometric simplification in integration.

Solution:

$$1 + \sin 2x = 1 + 2\sin x\cos x = (\sin x + \cos x)^2$$

Hence,

$$\sqrt{1 + \sin 2x} = \sin x + \cos x$$

Therefore,

$$\int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x}}dx = \int \frac{\sin x - \cos x}{\sin x + \cos x}dx$$

Let

$$u = \sin x + \cos x$$

Then

$$du = (\cos x - \sin x)dx$$

So,

$$I = -\int \frac{du}{u}$$

$$I = -\ln|\sin x + \cos x| + C$$

Final Answer: $\boxed{-\ln|\sin x + \cos x| + C}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: Definite integral involving modulus function.

Solution:

$$\int_{-1}^{1} |x|dx = 2\int_{0}^{1} x\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1$$

$$= 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: Integration using substitution.

Solution:

Let

$$u = xe^x$$

Then

$$\frac{du}{dx} = e^x(1+x)$$

$$du = e^x(1+x)\,dx$$

Thus,

$$\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx = \int \sec^2 u\,du$$

$$= \tan u + C$$

$$= \tan(xe^x) + C$$

Final Answer: $\boxed{\tan(xe^x) + C}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: Area under curve using definite integration.

Solution:

Required area:

$$\int_0^{\pi} |\cos x|\, dx$$

Since $\cos x$ is positive on $[0, \pi/2]$ and negative on $[\pi/2, \pi]$,

$$A = \int_0^{\pi/2} \cos x\, dx - \int_{\pi/2}^{\pi} \cos x\, dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi}$$

$$= (1 - 0) - (0 - 1)$$

$$= 2$$

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept: Area bounded between two curves.

Solution:

Given curves:

$$y^2 = 4ax$$

and

$$x^2 = 4ay$$

From

$$y^2 = 4ax$$

we get

$$x = \frac{y^2}{4a}$$

From

$$x^2 = 4ay$$

we get

$$x = 2\sqrt{ay}$$

Intersection points:

$$\frac{y^2}{4a} = 2\sqrt{ay}$$

Solving,

$$y = 0,\ 4a$$

Area:

$$A = \int_0^{4a} \left(2\sqrt{ay} - \frac{y^2}{4a}\right) dy$$

$$= 2\sqrt{a}\int_0^{4a} \sqrt{y}\, dy - \frac{1}{4a}\int_0^{4a} y^2\, dy$$

$$= 2\sqrt{a}\left[\frac{2y^{3/2}}{3}\right]_0^{4a} - \frac{1}{4a}\left[\frac{y^3}{3}\right]_0^{4a}$$

$$= \frac{32a^2}{3} - \frac{16a^2}{3}$$

$$= \frac{16a^2}{3}$$

Final Answer: $\boxed{\frac{16a^2}{3}}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: Degree of a differential equation.

Solution:

Given:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$$

To define degree, equation must be polynomial in derivatives.

Squaring both sides:

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3}=\left(\frac{d^2y}{dx^2}\right)^2$$

Highest order derivative is

$$\frac{d^2y}{dx^2}$$

Its power is 2.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: Integrating factor of linear differential equation.

Solution:

Given:

$$\frac{dy}{dx}+\frac{y}{x}=x^2$$

Comparing with

$$\frac{dy}{dx}+Py=Q$$

we get

$$P=\frac{1}{x}$$

Integrating factor:

$$IF=e^{\int \frac{1}{x}\,dx}$$

$$=e^{\ln x}$$

$$=x$$

Final Answer: $\boxed{x}$

Answer: (C)

Go Back to Question 22

Q23.

Solution

Concept: First-order differential equation using substitution.

Solution:

Given:

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

Multiply both sides by e^y:

$$e^y \frac{dy}{dx} = e^x + x^2$$

Since

$$\frac{d}{dx}(e^y) = e^y \frac{dy}{dx}$$

we get

$$\frac{d}{dx}(e^y) = e^x + x^2$$

Integrating,

$$e^y = \int (e^x + x^2)\, dx$$

$$e^y = e^x + \frac{x^3}{3} + C$$

Final Answer: $\boxed{e^y = e^x + \frac{x^3}{3} + C}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: Properties of cube roots of unity.

Solution:

Using

$$1 + \omega + \omega^2 = 0$$

we get

$$1 + \omega - \omega^2 = 1 + \omega + (1 + \omega) = 2(1 + \omega)$$

Also,

$$1 - \omega + \omega^2 = -(2\omega)$$

Now,

$$(1 + \omega)^3 = -1$$

Hence,

$$(1 + \omega - \omega^2)^3 = [2(1 + \omega)]^3 = 8(-1) = -8$$

and

$$(1 - \omega + \omega^2)^3 = (-2\omega)^3 = -8$$

Therefore,

$$(-8) - (-8) = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: Argument of a complex number.

Solution:

Given:

$$z = \frac{1 + i\sqrt{3}}{1 - i\sqrt{3}}$$

Argument of quotient:

$$\arg z = \arg(1 + i\sqrt{3}) - \arg(1 - i\sqrt{3})$$

Now,

$$\arg(1 + i\sqrt{3}) = \frac{\pi}{3}$$

and

$$\arg(1 - i\sqrt{3}) = -\frac{\pi}{3}$$

Therefore,

$$\arg z = \frac{\pi}{3} - \left(-\frac{\pi}{3}\right) = \frac{2\pi}{3}$$

Final Answer: $\boxed{\frac{2\pi}{3}}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: De Moivre's theorem and periodicity.

Solution:

$$\frac{1 + i}{1 - i} = i$$

Thus,

$$\left(\frac{1 + i}{1 - i}\right)^n = i^n$$

We need

$$i^n = 1$$

Powers of i:

$$i, \ -1, \ -i, \ 1$$

Smallest positive integer:

$$n = 4$$

Final Answer: $\boxed{4}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: Sum formula for tangent and trigonometric identities.

Solution:

Given roots are

$$\tan A, \tan B$$

From quadratic equation,

$$\tan A + \tan B = p$$

and

$$\tan A \tan B = q$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{p}{1-q}$$

Using identity,

$$\sin^2\theta = \frac{\tan^2\theta}{1+\tan^2\theta}$$

Hence,

$$\sin^2(A+B) = \frac{\left(\frac{p}{1-q}\right)^2}{1 + \left(\frac{p}{1-q}\right)^2}$$

$$= \frac{p^2}{p^2 + (1-q)^2}$$

Final Answer: $\boxed{\dfrac{p^2}{p^2 + (1-q)^2}}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: Solving equations involving modulus.

Solution:

Given:

$$x^2 - 3|x| + 2 = 0$$

Let

$$|x| = t$$

where $t \geq 0$.

Then,

$$t^2 - 3t + 2 = 0$$

$$(t-1)(t-2) = 0$$

Thus,

$$t = 1 \quad \text{or} \quad t = 2$$

Hence,

$$|x| = 1 \Rightarrow x = \pm 1$$

and

$$|x| = 2 \Rightarrow x = \pm 2$$

Total real roots:

$$4$$

Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept: Relation between roots and coefficients.

Solution:

Let roots be consecutive integers:

$$n, \ n+1$$

Then,

$$b = n + (n+1) = 2n + 1$$

and

$$c = n(n+1)$$

Now,

$$b^2 - 4c = (2n+1)^2 - 4n(n+1)$$

$$= 4n^2 + 4n + 1 - 4n^2 - 4n$$

$$= 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: Relation between G.P. and exponents.

Solution:

Given,

$$a^x = b^y = c^z = k$$

Taking logarithm,

$$x \ln a = y \ln b = z \ln c = \ln k$$

Thus,

$$x = \frac{\ln k}{\ln a}, \quad y = \frac{\ln k}{\ln b}, \quad z = \frac{\ln k}{\ln c}$$

Hence,

$$x : y : z = \frac{1}{\ln a} : \frac{1}{\ln b} : \frac{1}{\ln c}$$

Since a, b, c are in G.P.,

$$\ln a, \ln b, \ln c$$

are in A.P.

Reciprocals of numbers in A.P. are in H.P.

Therefore,

$$x, y, z$$

are in H.P.

Final Answer: $\boxed{\text{H.P.}}$

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept: Sum of a special decimal series.

Solution:

General term:

$$0.\underbrace{77\cdots7}_{r\text{ times}} = \frac{7}{9}\left(1 - \frac{1}{10^r}\right)$$

Therefore,

$$S_n = \frac{7}{9}\sum_{r=1}^{n}\left(1 - \frac{1}{10^r}\right)$$

$$= \frac{7}{9}\left[n - \sum_{r=1}^{n}\frac{1}{10^r}\right]$$

Now,

$$\sum_{r=1}^{n}\frac{1}{10^r} = \frac{1}{9}(1 - 10^{-n})$$

Hence,

$$S_n = \frac{7}{9}\left[n - \frac{1}{9}(1 - 10^{-n})\right]$$

Final Answer: $\boxed{\frac{7}{9}\left[n - \frac{1}{9}(1 - 10^{-n})\right]}$

Answer: (D)

Go Back to Question 31

Q32.

Solution

Concept: Harmonic mean formula.

Solution:

Harmonic mean:

$$H = \frac{2ab}{a+b}$$

Now,

$$\frac{H+a}{H-a} = \frac{\frac{2ab}{a+b}+a}{\frac{2ab}{a+b}-a}$$

Simplifying,

$$= \frac{a(a+3b)}{a(b-a)} = \frac{a+3b}{b-a}$$

Similarly,

$$\frac{H+b}{H-b} = \frac{3a+b}{a-b}$$

Adding,

$$\frac{a+3b}{b-a} + \frac{3a+b}{a-b}$$

$$= -\frac{a+3b}{a-b} + \frac{3a+b}{a-b}$$

$$= \frac{2a-2b}{a-b} = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: General term in binomial expansion.

Solution:

General term:

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$$

Power of x:

$$2(9-r) - r = 18 - 3r$$

For constant term,

$$18 - 3r = 0$$

$$r = 6$$

Constant term:

$$ {}^9C_6 \left(\frac{3}{2}\right)^3 \left(\frac{1}{3}\right)^6$$

$$= {}^9C_6 \cdot \frac{27}{8} \cdot \frac{1}{729}$$

$$= {}^9C_6 \cdot \frac{1}{216}$$

Final Answer: $\boxed{{}^9C_6 \cdot \frac{1}{216}}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: Equality of coefficients in binomial expansion.

Solution:

Coefficient of x^7:

$$^nC_7 2^{n-7}\left(\frac{1}{3}\right)^7$$

Coefficient of x^8:

$$^nC_8 2^{n-8}\left(\frac{1}{3}\right)^8$$

Given equal,

$$^nC_7 2^{n-7}\frac{1}{3^7} = {}^nC_8 2^{n-8}\frac{1}{3^8}$$

$$2 \cdot 3 \cdot {}^nC_7 = {}^nC_8$$

$$6 \cdot \frac{n!}{7!(n-7)!} = \frac{n!}{8!(n-8)!}$$

$$6 \cdot 8(n-8+1) = n-7$$

Using

$$\frac{^nC_8}{^nC_7} = \frac{n-7}{8}$$

we get

$$6 = \frac{n-7}{8}$$

$$n - 7 = 48$$

$$n = 55$$

Final Answer: $\boxed{55}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: Circular permutation with restriction.

Solution:

Arrange 5 boys around a circular table:

$$(5-1)! = 4!$$

Now there are 5 gaps between boys.

To ensure no two girls sit together, place 5 girls in these 5 gaps.

Number of ways:

$$5!$$

Hence total arrangements:

$$4! \times 5!$$

Final Answer: $\boxed{4! \cdot 5!}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: Probability using combinations.

Solution:

Total balls:

$$6 + 4 = 10$$

Total ways to draw 2 balls:

$$^{10}C_2 = 45$$

Favorable cases:

Both white:

$$^{6}C_2 = 15$$

Both black:

$$^{4}C_2 = 6$$

Total favorable ways:

$$15 + 6 = 21$$

Therefore,

$$P = \frac{21}{45} = \frac{7}{15}$$

Final Answer: $\boxed{\frac{7}{15}}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: Number of perfect square divisors.

Solution:

Prime factorization:

$$10800 = 2^4 \cdot 3^3 \cdot 5^2$$

For a perfect square divisor, exponents must be even.

Possible exponents:

For 2^4:

$$0, 2, 4$$

(3 choices)

For 3^3:

$$0, 2$$

(2 choices)

For 5^2:

$$0, 2$$

(2 choices)

Total perfect square divisors:

$$3 \times 2 \times 2 = 12$$

Final Answer: $\boxed{12}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: Probability of union of independent events.

Solution:

Given:

$$P(A) = \frac{1}{2}, \qquad P(B) = \frac{1}{5}$$

Since events are independent,

$$P(A \cap B) = P(A)P(B) = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{5} - \frac{1}{10}$$

$$= \frac{5}{10} + \frac{2}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$$

Final Answer: $\boxed{\frac{3}{5}}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: Distance between parallel lines.

Solution:

Given lines:

$$5x + 12y - 20 = 0$$

and

$$5x + 12y + 6 = 0$$

Distance formula:

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Thus,

$$d = \frac{|(-20) - 6|}{\sqrt{5^2 + 12^2}}$$

$$= \frac{26}{13} = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: Condition of tangency of a line to a circle.

Solution:

Circle:

$$x^2 + y^2 = a^2$$

Center:

$$(0, 0)$$

Line:

$$y = mx + c$$

or

$$mx - y + c = 0$$

Distance from center to tangent equals radius:

$$\frac{|c|}{\sqrt{m^2 + 1}} = a$$

Squaring,

$$c^2 = a^2(1 + m^2)$$

Final Answer: $\boxed{a^2(1 + m^2)}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: Equation of a circle from center and radius.

Solution:

Center:

$$(2, 3)$$

Since circle touches the x-axis, radius is:

$$r = 3$$

Equation:

$$(x - 2)^2 + (y - 3)^2 = 3^2$$

Expanding,

$$x^2 - 4x + 4 + y^2 - 6y + 9 = 9$$

$$x^2 + y^2 - 4x - 6y + 4 = 0$$

Final Answer: $\boxed{x^2 + y^2 - 4x - 6y + 4 = 0}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: Orthocenter of a right triangle.

Solution:

Given lines:

$$x = 0, \qquad y = 0, \qquad x + y = 1$$

These form a right triangle with right angle at

$$(0, 0)$$

Orthocenter of a right triangle is the vertex containing the right angle.

Therefore,

$$\text{Orthocenter} = (0, 0)$$

Final Answer: $\boxed{(0, 0)}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: Relation between latus rectum, conjugate axis and eccentricity of hyperbola.

Solution:

For hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Length of latus rectum:

$$\frac{2b^2}{a} = 8$$

$$b^2 = 4a$$

Conjugate axis:

$$2b$$

Distance between foci:

$$2ae$$

Given,

$$2b = \frac{1}{2}(2ae)$$

$$2b = ae$$

$$b = \frac{ae}{2}$$

Using

$$b^2 = a^2(e^2 - 1)$$

Substitute $b = \frac{ae}{2}$:

$$\frac{a^2e^2}{4} = a^2(e^2 - 1)$$

$$\frac{e^2}{4} = e^2 - 1$$

$$3e^2 = 4$$

$$e = \frac{2}{\sqrt{3}}$$

Final Answer: $\boxed{\frac{2}{\sqrt{3}}}$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept: Focal distance property of parabola.

Solution:

For parabola:

$$y^2 = 4ax$$

Focus is

$$(a, 0)$$

Focal distance of point (x_1, y_1):

$$\sqrt{(x_1 - a)^2 + y_1^2}$$

Using

$$y_1^2 = 4ax_1$$

$$= (x_1 - a)^2 + 4ax_1$$

$$= x_1^2 + 2ax_1 + a^2$$

$$= (x_1 + a)^2$$

Hence focal distance:

$$x_1 + a$$

Final Answer: $\boxed{x_1 + a}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: Relation between latus rectum and axes of ellipse.

Solution:

For ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Length of latus rectum:

$$\frac{2b^2}{a}$$

Minor axis:

$$2b$$

Given,

$$\frac{2b^2}{a} = \frac{1}{2}(2b)$$

$$\frac{2b^2}{a} = b$$

$$2b = a$$

$$b = \frac{a}{2}$$

Now,

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Final Answer: $\boxed{\frac{\sqrt{3}}{2}}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: Property of tangent to ellipse.

Solution:

Equation of tangent:

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

Distance from focus $(ae, 0)$ to tangent:

$$d_1 = \frac{|ax_1e - a^2|}{\sqrt{b^2x_1^2 + a^2y_1^2}}$$

Similarly distance from other focus:

$$d_2 = \frac{|ax_1e + a^2|}{\sqrt{b^2x_1^2 + a^2y_1^2}}$$

Their product simplifies to:

$$d_1d_2 = b^2$$

Hence the required product is:

$$b^2$$

Final Answer: $\boxed{b^2}$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept: Scalar triple product properties.

Solution:

Consider:

$$[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}]$$

Using linearity:

$$= [\vec{a},\vec{b},\vec{c}] + [\vec{b},\vec{c},\vec{a}]$$

All other terms contain repeated vectors, hence are zero.

Now,

$$[\vec{b},\vec{c},\vec{a}] = [\vec{a},\vec{b},\vec{c}]$$

Therefore,

$$= 2[\vec{a},\vec{b},\vec{c}]$$

Final Answer: $\boxed{2[\vec{a}\ \vec{b}\ \vec{c}]}$

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept: Angle between vectors using dot product.

Solution:

Let

$$\vec{a} = \hat{i} - \hat{j} + \hat{k}$$

and

$$\vec{b} = \hat{i} + \hat{j} - \hat{k}$$

Dot product:

$$\vec{a} \cdot \vec{b} = 1(1) + (-1)(1) + (1)(-1)$$

$$= 1 - 1 - 1 = -1$$

Magnitudes:

$$|\vec{a}| = \sqrt{1 + 1 + 1} = \sqrt{3}$$

$$|\vec{b}| = \sqrt{3}$$

Thus,

$$\cos\theta = \frac{-1}{3}$$

Therefore,

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

Final Answer: $\boxed{\cos^{-1}\left(-\frac{1}{3}\right)}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept: Distance of point from plane.

Solution:

Distance formula:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Given plane:

$$x + 2y - 2z + 5 = 0$$

and point:

$$(1, 2, 3)$$

Substitute:

$$d = \frac{|1 + 4 - 6 + 5|}{\sqrt{1^2 + 2^2 + (-2)^2}}$$

$$= \frac{4}{\sqrt{9}} = \frac{4}{3}$$

Final Answer: $\boxed{\frac{4}{3}}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: Direction cosines of a line.

Solution:

If a line is equally inclined to all three axes, then its direction cosines are equal.

Let

$$l = m = n = k$$

Using property:

$$l^2 + m^2 + n^2 = 1$$

$$3k^2 = 1$$

$$k = \frac{1}{\sqrt{3}}$$

Hence direction cosines are:

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Final Answer: $\boxed{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$

Answer: (A)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	B	4	A	5	A
6	C	7	C	8	B	9	B	10	B
11	B	12	C	13	A	14	A	15	B
16	A	17	B	18	A	19	C	20	A
21	B	22	C	23	A	24	A	25	B
26	B	27	A	28	C	29	B	30	C
31	D	32	A	33	B	34	A	35	A
36	B	37	A	38	A	39	A	40	A
41	A	42	A	43	C	44	A	45	A
46	B	47	A	48	A	49	A	50	A