MHT-CET Mathematics Sample Paper-4

Duration: 90 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. If $f(x) = \frac{x^2-1}{x^2-|x-1|-1}$, then the value of $\lim_{x\to 1^+} f(x)$ is:

(A) 0

(B) 1

(C) 2

(D) Does not exist

Q2. The function $f(x) = [x]^2 - [x^2]$ (where [.] denotes the greatest integer function) is discontinuous at:

(A) All integers

(B) All integers except 0

(C) All integers except 0 and 1

(D) All integers except 1

Q3. The value of k for which the function $f(x) = \begin{cases} \frac{\sin 5x}{3x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $x = 0$ is:

(A) $5/3$

(B) $3/5$

(C) 1

(D) 0

Q4. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then the value of $\frac{dy}{dx}$ at $x = 0$ is:

(A) 0

(B) $1/2$

(C) 1

(D) Infinite

Q5. If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi/2$ is:

(A) $1/a$

(B) $-1/a$

(C) $1/2a$

(D) $-1/2a$

Q6. If $f(x) = \log_x(\ln x)$, then $f'(e)$ is equal to:

(A) e

(B) $1/e$

(C) 1

(D) 0

Q7. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $x \in (0, 1)$ is:

(A) 1

(B) -1

(C) 2

(D) x

Q8. If $y = e^{x+e^{x+e^{x+\ldots\infty}}}$, then $\frac{dy}{dx}$ is:

(A) $\frac{y}{1-y}$

(B) $\frac{y}{y-1}$

(C) $\frac{1}{1-y}$

(D) $\frac{e^x}{1-y}$

Q9. The maximum value of $f(x) = \frac{x}{4+x+x^2}$ on the interval $[-1, 1]$ is:

(A) $1/4$

(B) $1/6$

(C) $1/5$

(D) $1/3$

Q10. The slope of the tangent to the curve $y = \int_0^x \frac{dt}{1+t^3}$ at the point where $x = 1$ is:

(A) $1/2$

(B) 1

(C) $1/4$

(D) 0

Q11. The interval in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is strictly decreasing is:

(A) $(1, 2)$

(B) $(-\infty, 1)$

(C) $(2, \infty)$

(D) $(0, 3)$

Q12. The minimum distance from the origin to the curve $y^2 = 4x$ is:

(A) $\sqrt{2}$

(B) 2

(C) 0

(D) 1

Q13. A balloon which always remains spherical has a variable radius. The rate at which its volume is increasing with respect to its radius when the radius is 7 cm is:

(A) 196π

(B) 49π

(C) 98π

(D) 588π

Q14. The value of $\int \frac{dx}{x(x^n+1)}$ is:

(A) $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$

(B) $\log\left|\frac{x^n}{x^n+1}\right| + C$

(C) $\frac{1}{n}\log\left|\frac{x^n+1}{x^n}\right| + C$

(D) $n\log\left|\frac{x^n}{x^n+1}\right| + C$

Q15. The value of $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x+\cos^{100} x} dx$ is:

(A) π

(B) $\pi/2$

(C) $\pi/4$

(D) 0

Q16. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is:

(A) 0

(B) 1/3

(C) 2/3

(D) 1

Q17. The area of the region bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant is:

(A) 2π

(B) 4π

(C) π

(D) 8π

Q18. The degree and order of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are respectively:

(A) 2, 2

(B) 3, 2

(C) 2, 3

(D) 1, 2

Q19. The general solution of the differential equation $\frac{dy}{dx} + y \tan x = \sec x$ is:

(A) $y \sec x = \tan x + C$

(B) $y \tan x = \sec x + C$

(C) $y = \sin x + C \cos x$

(D) $y \cos x = x + C$

Q20. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is:

(A) e^x

(B) $\log x$

(C) $1/x$

(D) $-1/x$

Q21. If $z = \frac{\sqrt{3}+i}{2}$, then the value of z^{69} is:

(A) $-i$

(B) i

(C) 1

(D) -1

Q22. The locus of z satisfying $|z - i| = |z + i|$ is:

(A) x-axis

(B) y-axis

(C) $y = x$

(D) $x^2 + y^2 = 1$

Q23. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals:

(A) 128ω

(B) $-128\omega^2$

(C) $128\omega^2$

(D) -128ω

Q24. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q25. The number of real roots of the equation $e^x = x$ is:

(A) 0

(B) 1

(C) 2

(D) Infinite

Q26. If one root of $ax^2 + bx + c = 0$ is square of the other, then:

(A) $b^3 + ac^2 + a^2c = 3abc$

(B) $b^3 + a^2c + ac^2 = abc$

(C) $a^3 + b^2c + bc^2 = 3abc$

(D) $b^3 + a^2c + ac^2 + 3abc = 0$

Q27. The sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \infty$ is:

(A) 1

(B) 2

(C) 1.5

(D) 3

Q28. If a, b, c are in A.P., then $2^{ax+d}, 2^{bx+d}, 2^{cx+d}$ $(x \neq 0)$ are in:

(A) A.P.

(B) G.P.

(C) H.P.

(D) None of these

Q29. The n^{th} term of the sequence $5, 2, -1, -4, \ldots$ is:

(A) $8 - 3n$

(B) $3n - 8$

(C) $5 - 3n$

(D) $2 - 3n$

Q30. The coefficient of x^7 in the expansion of $(x - \frac{1}{x^2})^{11}$ is:

(A) 55

(B) -55

(C) 0

(D) 11

Q31. The middle term in the expansion of $(1 + x)^{2n}$ is:

(A) ${}^{2n}C_n x^n$

(B) ${}^{2n}C_{n-1} x^{n-1}$

(C) ${}^{2n}C_{n+1} x^{n+1}$

(D) None of these

Q32. A bag contains 5 white and 3 black balls. Two balls are drawn at random without replacement. The probability that both are black is:

(A) $3/28$

(B) $9/64$

(C) $3/56$

(D) $5/28$

Q33. In how many ways can 5 persons be seated around a circular table?

(A) 120

(B) 24

(C) 60

(D) 12

Q34. If ${}^{n}C_{12} = {}^{n}C_{8}$, then n is equal to:

(A) 20

(B) 12

(C) 6

(D) 30

Q35. The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$ is:

(A) $2/5$

(B) $12/5$

(C) $2/25$

(D) $3/5$

Q36. The equation of a line passing through $(1, 2)$ and perpendicular to $x + y + 7 = 0$ is:

(A) $x - y + 1 = 0$

(B) $x + y - 3 = 0$

(C) $x - y - 1 = 0$

(D) $y - x + 1 = 0$

Q37. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:

(A) 5

(B) 25

(C) $\sqrt{13}$

(D) 1

Q38. The eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is:

(A) $5/4$

(B) $4/3$

(C) $3/4$

(D) $5/3$

Q39. The length of the latus rectum of the parabola $y^2 = -8x$ is:

(A) 2

(B) 4

(C) 8

(D) -8

Q40. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 0°

(B) 45°

(C) 90°

(D) 180°

Q41. A line makes angles 90° and 60° with the positive direction of x and y axes respectively. The acute angle it makes with the z-axis is:

(A) 30°

(B) 60°

(C) 45°

(D) 90°

Q42. The distance of the point $(0, 0, 0)$ from the plane $2x - 3y + 6z + 14 = 0$ is:

(A) 14

(B) 2

(C) 7

(D) 1

Q43. The point on the parabola $y^2 = 4x$ which is closest to the point $(2, 1)$ is:

(A) $(1, 2)$

(B) $(1, -2)$

(C) $(4, 4)$

(D) $(0, 0)$

Q44. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is:

(A) 4

(B) -4

(C) 2

(D) -2

Q45. A person fires 7 bullets at a target. The probability of hitting the target is $1/4$. The probability that he hits the target exactly 2 times is:

(A) $\frac{5103}{16384}$

(B) $\frac{21}{16}$

(C) $\frac{189}{1024}$

(D) $\frac{243}{4096}$

Q46. The sum of n terms of an A.P. is $3n^2 + 5n$. Its common difference is:

(A) 3

(B) 6
(C) 5
(D) 10

Q47. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ is:

(A) 30°
(B) 60°
(C) 45°
(D) 90°

Q48. The work done by a force $\vec{F} = 4\hat{i} + \hat{j} - 3\hat{k}$ in displacing a particle from point $A(1, 2, 3)$ to point $B(5, 4, 1)$ is:

(A) 24 units
(B) 15 units
(C) 10 units
(D) 30 units

Q49. The area bounded by the curve $y = x^2$ and the line $y = x$ is:

(A) $1/6$
(B) $1/3$
(C) $1/2$
(D) 1

Q50. The number of arbitrary constants in the general solution of a differential equation of fourth order is:

(A) 0
(B) 2
(C) 3
(D) 4

Detailed Solutions

Q1.

Solution

Concept:
The limit of a function as x approaches a point from the right ($x \to a^+$) requires evaluating the function's behavior for values slightly greater than a. For functions involving absolute values like $|x-1|$, we must define the expression based on the interval. If $x > 1$, then $|x-1| = x-1$.

Solution:
1. Given the function:

$$f(x) = \frac{x^2 - 1}{x^2 - |x-1| - 1}$$

2. For the limit $x \to 1^+$, we consider $x > 1$. Therefore, $|x-1| = x-1$. 3. Substitute this into the denominator:

$$\text{Denominator} = x^2 - (x-1) - 1 = x^2 - x + 1 - 1 = x^2 - x$$

4. The numerator is $(x^2 - 1) = (x-1)(x+1)$. 5. Now rewrite the limit:

$$\lim_{x \to 1^+} \frac{(x-1)(x+1)}{x(x-1)}$$

6. Cancel the common factor $(x-1)$ (since $x \neq 1$):

$$\lim_{x \to 1^+} \frac{x+1}{x}$$

7. Substitute $x = 1$:

$$\frac{1+1}{1} = 2$$

Final Answer: The value of the limit is 2.

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept:

The Greatest Integer Function $[x]$ is discontinuous at all integer values of x. For a composite or combined function involving $[x]^2$ and $[x^2]$, we must check points where the internal expressions transition through integers. A function is continuous at a point if the Left Hand Limit (LHL) equals the Right Hand Limit (RHL) and the function value.

Solution:

1. Let $f(x) = [x]^2 - [x^2]$. 2. For $x = 0$: * $f(0) = 0^2 - 0 = 0$. * RHL ($x \to 0^+$): $[0^+]^2 - [(0^+)^2] = 0 - 0 = 0$. * LHL ($x \to 0^-$): $[-0.1]^2 - [(-0.1)^2] = (-1)^2 - [0.01] = 1 - 0 = 1$. * Since LHL $\neq$ RHL, it is discontinuous at $x = 0$.

3. For $x = 1$: * $f(1) = 1^2 - 1 = 0$. * RHL ($x \to 1^+$): $[1^+]^2 - [(1^+)^2] = 1^2 - 1 = 0$. * LHL ($x \to 1^-$): $[0.9]^2 - [0.81] = 0^2 - 0 = 0$. * Since LHL = RHL = $f(1)$, the function is **continuous** at $x = 1$.

4. For other integers n: * LHL ($x \to n^-$): $(n-1)^2 - (n^2 - 1) = n^2 - 2n + 1 - n^2 + 1 = 2 - 2n$. * This only equals 0 (the value at $f(n)$) if $n = 1$.

Final Answer: The function is discontinuous at all integers except 1.

Answer: (D)

Go Back to Question 2

Q3.

Solution

Concept:

A function $f(x)$ is continuous at a point $x = a$ if $\lim_{x\to a} f(x) = f(a)$. For trigonometric limits, we use the standard result:

$$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

Solution:

1. We are given $f(x) = \frac{\sin 5x}{3x}$ for $x \neq 0$ and $f(0) = k$. 2. For continuity at $x = 0$, we must have:

$$\lim_{x\to 0} \frac{\sin 5x}{3x} = k$$

3. To apply the standard limit, multiply and divide by 5:

$$\lim_{x\to 0} \frac{\sin 5x}{5x} \times \frac{5}{3} = k$$

4. Since $\lim_{x\to 0} \frac{\sin 5x}{5x} = 1$, the equation becomes:

$$1 \times \frac{5}{3} = k$$

5. Thus, $k = 5/3$.

Final Answer: The value of k is $5/3$.

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:
When differentiating functions involving inverse trigonometric expressions with square roots, trigonometric substitution is often the most efficient method. For the expression $\sqrt{1+x^2}$, the substitution $x = \tan\theta$ simplifies the radical using the identity $1 + \tan^2\theta = \sec^2\theta$.

Solution:
1. Let $x = \tan\theta$, which implies $\theta = \tan^{-1} x$. 2. Substitute into the expression:

$$y = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}\right)$$

3. Simplify the radical: $\sqrt{1+\tan^2\theta} = \sec\theta$.

$$y = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

4. Convert to sine and cosine:

$$y = \tan^{-1}\left(\frac{\frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta}}\right) = \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

5. Use half-angle identities: $1 - \cos\theta = 2\sin^2(\theta/2)$ and $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$.

$$y = \tan^{-1}\left(\frac{2\sin^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}\right) = \tan^{-1}(\tan(\theta/2)) = \frac{\theta}{2}$$

6. Back-substitute θ: $y = \frac{1}{2}\tan^{-1} x$. 7. Differentiate with respect to x: $\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1+x^2}$. 8. At $x = 0$: $\frac{dy}{dx} = \frac{1}{2(1+0)} = 1/2$.

Final Answer: The value of the derivative at $x = 0$ is $1/2$.

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept:

For parametric equations $x = f(\theta)$ and $y = g(\theta)$, the first derivative is $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$. The second derivative is:

$$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \cdot \frac{d\theta}{dx}$$

Solution:

1. Given $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$. 2. Compute derivatives with respect to θ: * $\frac{dx}{d\theta} = a(1 - \cos\theta)$ * $\frac{dy}{d\theta} = a\sin\theta$
3. Find $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{a\sin\theta}{a(1-\cos\theta)} = \frac{2\sin(\theta/2)\cos(\theta/2)}{2\sin^2(\theta/2)} = \cot(\theta/2)$$

4. Find $\frac{d}{d\theta}(\frac{dy}{dx})$:

$$\frac{d}{d\theta}(\cot(\theta/2)) = -\csc^2(\theta/2) \cdot \frac{1}{2}$$

5. Compute $\frac{d^2y}{dx^2}$:

$$\frac{d^2y}{dx^2} = \left(-\frac{1}{2}\csc^2(\theta/2)\right) \cdot \frac{1}{a(1-\cos\theta)}$$

Using $1 - \cos\theta = 2\sin^2(\theta/2)$:

$$\frac{d^2y}{dx^2} = -\frac{1}{2}\csc^2(\theta/2) \cdot \frac{1}{2a\sin^2(\theta/2)} = -\frac{1}{4a}\csc^4(\theta/2)$$

6. Evaluate at $\theta = \pi/2$: * $\theta/2 = \pi/4$, and $\csc(\pi/4) = \sqrt{2}$. * $\csc^4(\pi/4) = (\sqrt{2})^4 = 4$. * $\frac{d^2y}{dx^2} = -\frac{1}{4a} \cdot 4 = -1/a$.
Final Answer: The second derivative at $\theta = \pi/2$ is $-1/a$.

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept:

To differentiate a function where the variable is in the base and the exponent, like $f(x) = \log_x(\ln x)$, we first use the base change formula:

$$\log_a b = \frac{\ln b}{\ln a}$$

Then, we apply the quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$.

Solution:

1. Rewrite the function using natural logarithms:

$$f(x) = \frac{\ln(\ln x)}{\ln x}$$

2. Apply the quotient rule where $u = \ln(\ln x)$ and $v = \ln x$: * $u' = \frac{1}{\ln x} \cdot \frac{1}{x}$ * $v' = \frac{1}{x}$

3. The derivative $f'(x)$ is:

$$f'(x) = \frac{(\ln x)\left(\frac{1}{x\ln x}\right) - \ln(\ln x)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

4. Simplify the numerator:

$$f'(x) = \frac{\frac{1}{x} - \frac{\ln(\ln x)}{x}}{(\ln x)^2} = \frac{1 - \ln(\ln x)}{x(\ln x)^2}$$

5. Evaluate at $x = e$: * $\ln e = 1$ * $\ln(\ln e) = \ln(1) = 0$ * $f'(e) = \frac{1-0}{e(1)^2} = \frac{1}{e}$

Final Answer: The value of $f'(e)$ is $1/e$.

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:

This problem involves the derivative of one inverse trigonometric function with respect to another. We use the standard substitutions:

* $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$ (for $|x| < 1$) * $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$ (for $x \geq 0$) The derivative of u with respect to v is $\frac{du/dx}{dv/dx}$.

Solution:

1. Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$. For $x \in (0,1)$, $u = 2\tan^{-1}x$. 2. Let $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$. For $x \in (0,1)$, $v = 2\tan^{-1}x$. 3. Differentiate both with respect to x: * $\frac{du}{dx} = \frac{2}{1+x^2}$ * $\frac{dv}{dx} = \frac{2}{1+x^2}$

4. The derivative of u with respect to v is:

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}} = 1$$

Final Answer: The derivative is 1.

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept:
This is an infinite series problem where the function is defined in terms of itself. Since the exponent repeats infinitely, we can replace the repeating part with the dependent variable y. This transforms the infinite transcendental equation into a manageable algebraic or logarithmic form.

Solution:
1. Given $y = e^{x+e^{x+e^{x+\dots\infty}}}$. 2. Observe that the power of the first 'e' is $(x + y)$. So, we can rewrite the equation as:

$$y = e^{x+y}$$

3. Take the natural logarithm on both sides to simplify:

$$\ln y = \ln(e^{x+y})$$

$$\ln y = x + y$$

4. Differentiate both sides with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

5. Rearrange to solve for $\frac{dy}{dx}$:

$$\frac{1}{y}\frac{dy}{dx} - \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}\left(\frac{1}{y} - 1\right) = 1$$

$$\frac{dy}{dx}\left(\frac{1-y}{y}\right) = 1$$

$$\frac{dy}{dx} = \frac{y}{1-y}$$

Final Answer: The derivative is $\frac{y}{1-y}$.

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept:

To find the maximum value of a function on a closed interval $[a, b]$, we evaluate the function at its critical points (where $f'(x) = 0$) and at the endpoints of the interval. The largest of these values is the absolute maximum.

Solution:

1. Let $f(x) = \frac{x}{4+x+x^2}$. 2. Find $f'(x)$ using the quotient rule:

$$f'(x) = \frac{(4+x+x^2)(1) - x(1+2x)}{(4+x+x^2)^2} = \frac{4+x+x^2-x-2x^2}{(4+x+x^2)^2} = \frac{4-x^2}{(4+x+x^2)^2}$$

3. Set $f'(x) = 0$ to find critical points: $4 - x^2 = 0 \implies x = \pm 2$. 4. Check if critical points are in the interval $[-1, 1]$. Neither $+2$ nor -2 is in the interval. 5. Therefore, the maximum must occur at the endpoints: * At $x = -1$: $f(-1) = \frac{-1}{4-1+1} = -1/4$. * At $x = 1$: $f(1) = \frac{1}{4+1+1} = 1/6$.
6. Comparing $-1/4$ and $1/6$, the maximum value is $1/6$.

Final Answer: The maximum value is $1/6$.

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept:

According to the First Fundamental Theorem of Calculus (Leibniz Rule), if $F(x) = \int_a^x f(t)dt$, then the derivative $F'(x) = f(x)$. The slope of the tangent to a curve $y = f(x)$ at a point is the value of the derivative dy/dx at that point.

Solution:

1. Given $y = \int_0^x \frac{dt}{1+t^3}$. 2. The derivative $\frac{dy}{dx}$ is found by differentiating the integral with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\int_0^x \frac{dt}{1+t^3}\right) = \frac{1}{1+x^3}$$

3. The slope of the tangent at $x = 1$ is the value of $\frac{dy}{dx}$ at $x = 1$:

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{1+(1)^3} = \frac{1}{1+1} = 1/2$$

Final Answer: The slope of the tangent is $1/2$.

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept:
A function $f(x)$ is strictly decreasing on an interval if its first derivative $f'(x) < 0$ for all x in that interval. To find this interval, we calculate the derivative, find the roots (critical points), and test the signs in the resulting sub-intervals.

Solution:
1. Given $f(x) = 2x^3 - 9x^2 + 12x + 15$. 2. Differentiate with respect to x:

$$f'(x) = 6x^2 - 18x + 12$$

3. To find the intervals, set $f'(x) < 0$:

$$6(x^2 - 3x + 2) < 0$$

4. Factorize the quadratic expression:

$$6(x - 1)(x - 2) < 0$$

5. The roots are $x = 1$ and $x = 2$. We examine the sign of $f'(x)$ on the number line: * For $x < 1$, $f'(x) > 0$ (Increasing) * For $1 < x < 2$, $f'(x) < 0$ (Decreasing) * For $x > 2$, $f'(x) > 0$ (Increasing)
6. Thus, the function is strictly decreasing in the interval $(1, 2)$.
Final Answer: The interval is $(1, 2)$.

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept:
The distance D between the origin $(0,0)$ and any point (x,y) on a curve is given by $D = \sqrt{x^2+y^2}$. To find the minimum distance, we express D (or D^2 for simplicity) in terms of a single variable using the curve's equation and then use differentiation to find the minimum.

Solution:
1. Let a point on the curve $y^2 = 4x$ be $P(x,y)$. 2. The square of the distance from the origin is $Z = x^2 + y^2$. 3. Substitute $y^2 = 4x$ from the curve equation:

$$Z = x^2 + 4x$$

4. To find the minimum, differentiate Z with respect to x:

$$\frac{dZ}{dx} = 2x + 4$$

5. Set $\frac{dZ}{dx} = 0 \implies 2x + 4 = 0 \implies x = -2$. 6. However, for the curve $y^2 = 4x$, x must be ≥ 0 (since y^2 cannot be negative). 7. Since $Z = x^2 + 4x$ is a strictly increasing function for $x \geq 0$, the minimum value occurs at the smallest possible value of x, which is $x = 0$. 8. At $x = 0$, $y = 0$, and the distance $D = \sqrt{0^2 + 0^2} = 0$.

Final Answer: The minimum distance is 0.

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept:

This is a problem based on the Rate of Change of quantities. For a sphere, the volume V is given by $V = \frac{4}{3}\pi r^3$. The rate of change of volume with respect to the radius is the derivative dV/dr.

Solution:

1. The formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

2. Differentiate V with respect to r:

$$\frac{dV}{dr} = \frac{4}{3}\pi(3r^2) = 4\pi r^2$$

3. We need to find this rate when the radius $r = 7$ cm. 4. Substitute $r = 7$ into the derivative:

$$\frac{dV}{dr} = 4\pi(7)^2 = 4\pi(49) = 196\pi$$

Final Answer: The rate of increase is 196π.

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:
To integrate a rational function of the form $\frac{1}{x(x^n+1)}$, a common technique is to manipulate the numerator or use substitution. Multiplying the numerator and denominator by x^{n-1} allows for a simple substitution $u = x^n$.

Solution:
1. Let $I = \int \frac{dx}{x(x^n+1)}$. 2. Multiply and divide by x^{n-1}:

$$I = \int \frac{x^{n-1}}{x^n(x^n+1)}dx$$

3. Let $x^n = t$. Then $nx^{n-1}dx = dt \implies x^{n-1}dx = \frac{dt}{n}$. 4. Substitute into the integral:

$$I = \frac{1}{n}\int \frac{dt}{t(t+1)}$$

5. Use partial fractions: $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$.

$$I = \frac{1}{n}\left(\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right)$$

$$I = \frac{1}{n}(\log|t| - \log|t+1|) + C = \frac{1}{n}\log\left|\frac{t}{t+1}\right| + C$$

6. Back-substitute $t = x^n$:

$$I = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

Final Answer: The value of the integral is $\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$.

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:

This is a definite integral that can be solved using the property:

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

This property is particularly useful for integrals involving $\sin x$ and $\cos x$ with limits 0 to $\pi/2$.

Solution:

1. Let $I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x+\cos^{100} x}dx$. 2. Apply the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$$I = \int_0^{\pi/2} \frac{\sin^{100}(\pi/2-x)}{\sin^{100}(\pi/2-x)+\cos^{100}(\pi/2-x)}dx$$

3. Since $\sin(\pi/2-x) = \cos x$ and $\cos(\pi/2-x) = \sin x$:

$$I = \int_0^{\pi/2} \frac{\cos^{100} x}{\cos^{100} x+\sin^{100} x}dx$$

4. Add the two expressions for I:

$$2I = \int_0^{\pi/2} \frac{\sin^{100} x+\cos^{100} x}{\sin^{100} x+\cos^{100} x}dx$$

$$2I = \int_0^{\pi/2} 1dx = [x]_0^{\pi/2} = \pi/2$$

5. Solving for I:

$$I = \pi/4$$

Final Answer: The value of the integral is $\pi/4$.

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept:

The area under a curve $y = f(x)$ between $x = a$ and $x = b$ is given by $\int_a^b |f(x)|dx$. For the function $f(x) = x|x|$, we must consider the definition of absolute value: $f(x) = x^2$ for $x \geq 0$ and $f(x) = -x^2$ for $x < 0$.

Solution:

1. The function is $y = x|x|$. 2. In the interval $[-1, 0]$, $y = x(-x) = -x^2$. 3. In the interval $[0, 1]$, $y = x(x) = x^2$. 4. The area A is given by the integral:

$$A = \int_{-1}^{0} |-x^2|dx + \int_{0}^{1} |x^2|dx$$

5. Since we are looking for the area bounded with the x-axis, we take the magnitude:

$$A = \int_{-1}^{0} (x^2)dx + \int_{0}^{1} (x^2)dx$$

6. Evaluating the integrals:

$$A = \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$A = \left(0 - \left(-\frac{1}{3}\right)\right) + \left(\frac{1}{3} - 0\right) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Final Answer: The area is $2/3$.

Answer: (C)

Go Back to Question 16

Q17.

Solution

Concept:
The area of a region bounded by a circle $x^2 + y^2 = r^2$ and a line through the origin can be calculated using polar coordinates or by recognizing the symmetry as a sector of the circle. The line $y = x$ makes an angle of $\pi/4$ with the x-axis.

Solution:
1. The given circle is $x^2 + y^2 = 16$, which has radius $r = 4$. 2. The total area of the circle is $\pi r^2 = 16\pi$. 3. The region is in the first quadrant, which has a total area of $\frac{16\pi}{4} = 4\pi$. 4. The line $y = x$ bisects the first quadrant (from $\theta = 0$ to $\theta = \pi/2$). 5. The line $y = x$ corresponds to $\theta = \pi/4$. 6. The area bounded by $x = 0$, $y = x$, and the circle in the first quadrant covers the angular range from $\theta = \pi/4$ to $\theta = \pi/2$ (or 0 to $\pi/4$ depending on interpretation, both are equal). 7. Area of sector = $\frac{1}{2}r^2\theta$. 8. Here, the angle $\theta = \pi/4$.

$$\text{Area} = \frac{1}{2}(16)\left(\frac{\pi}{4}\right) = 2\pi$$

Final Answer: The area is 2π.

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:
The order of a differential equation is the highest derivative present in the equation. The degree is the power of the highest order derivative, provided the equation is a polynomial in its derivatives (meaning all fractional powers must be removed).

Solution:
1. The given equation is:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$$

2. The highest order derivative is $\frac{d^2y}{dx^2}$, so the **order is 2**. 3. To find the degree, we must remove the fractional power 3/2. Square both sides:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

4. Now the equation is a polynomial in its derivatives. The highest order derivative $\frac{d^2y}{dx^2}$ is raised to the power of 2. 5. Therefore, the **degree is 2**.

Final Answer: Order is 2 and Degree is 2.

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept:
This is a first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$. The solution is given by:

$$y \cdot (IF) = \int Q(x) \cdot (IF) dx + C$$

where the Integrating Factor $IF = e^{\int P(x)dx}$.
Solution:
1. Here, $P(x) = \tan x$ and $Q(x) = \sec x$. 2. Calculate the Integrating Factor (IF):

$$IF = e^{\int \tan x dx} = e^{\ln|\sec x|} = \sec x$$

3. The general solution is:

$$y \cdot \sec x = \int \sec x \cdot \sec x dx + C$$

4. Simplify the integral:

$$y \sec x = \int \sec^2 x dx + C$$

5. Since $\int \sec^2 x dx = \tan x$:

$$y \sec x = \tan x + C$$

Final Answer: $y \sec x = \tan x + C$.

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:

To find the integrating factor, the differential equation must be in the standard linear form: $\frac{dy}{dx} + P(x)y = Q(x)$. The Integrating Factor is $e^{\int P(x)dx}$.

Solution:

1. The given equation is $x\frac{dy}{dx} - y = 2x^2$. 2. Divide the entire equation by x to bring it to standard form:

$$\frac{dy}{dx} - \frac{1}{x}y = 2x$$

3. Identify $P(x) = -1/x$. 4. Calculate the Integrating Factor (IF):

$$IF = e^{\int P(x)dx} = e^{\int -\frac{1}{x}dx}$$

$$IF = e^{-\ln x} = e^{\ln(1/x)} = 1/x$$

Final Answer: The integrating factor is $1/x$.

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept:

To evaluate high powers of complex numbers, it is best to convert the number into polar or Euler form ($re^{i\theta}$). The complex number $z = \frac{\sqrt{3}}{2} + \frac{i}{2}$ corresponds to the trigonometric values of $\cos(\pi/6) + i\sin(\pi/6)$. According to De Moivre's Theorem, $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$.

Solution:

1. Express z in polar form:

$$z = \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right) = e^{i\pi/6}$$

2. Raise z to the power of 69:

$$z^{69} = \left(e^{i\pi/6}\right)^{69} = e^{i(69\pi/6)}$$

3. Simplify the fraction in the exponent:

$$\frac{69\pi}{6} = \frac{23\pi}{2}$$

4. Convert back to trigonometric form:

$$z^{69} = \cos\left(\frac{23\pi}{2}\right) + i\sin\left(\frac{23\pi}{2}\right)$$

5. Evaluate the trigonometric values: * Since $23\pi/2 = 11\pi + \pi/2$, it lands on the negative y-axis in the complex plane. * $\cos(23\pi/2) = 0$ * $\sin(23\pi/2) = -1$

6. Therefore:

$$z^{69} = 0 + i(-1) = -i$$

Final Answer: The value of z^{69} is $-i$.

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept:

Geometrically, the equation $|z - z_1| = |z - z_2|$ represents the set of all points z that are equidistant from two fixed points z_1 and z_2. This set of points forms the perpendicular bisector of the line segment joining z_1 and z_2.

Solution:

1. The given equation is $|z - i| = |z - (-i)|$. 2. Here, the two fixed points are $z_1 = i$ (which is $(0, 1)$ on the Cartesian plane) and $z_2 = -i$ (which is $(0, -1)$). 3. The line segment joining $(0, 1)$ and $(0, -1)$ lies on the y-axis (the imaginary axis). 4. The perpendicular bisector of this vertical segment is the x-axis (the real axis). 5. Algebraically, let $z = x + iy$:

$$|x + i(y - 1)| = |x + i(y + 1)|$$

$$\sqrt{x^2 + (y - 1)^2} = \sqrt{x^2 + (y + 1)^2}$$

$$x^2 + y^2 - 2y + 1 = x^2 + y^2 + 2y + 1$$

$$-2y = 2y \implies 4y = 0 \implies y = 0$$

6. $y = 0$ is the equation of the x-axis.

Final Answer: The locus of z is the x-axis.

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept:

The properties of the imaginary cube root of unity ω are:

1. $1 + \omega + \omega^2 = 0$ 2. $\omega^3 = 1$ These relations allow us to reduce complex expressions involving ω into simpler forms.

Solution:

1. Given the expression: $(1 + \omega - \omega^2)^7$. 2. From the property $1 + \omega + \omega^2 = 0$, we know that $1 + \omega = -\omega^2$. 3. Substitute this into the expression:

$$(-\omega^2 - \omega^2)^7 = (-2\omega^2)^7$$

4. Apply the power to both parts:

$$(-2)^7 \cdot (\omega^2)^7 = -128 \cdot \omega^{14}$$

5. Simplify ω^{14} using the property $\omega^3 = 1$:

$$\omega^{14} = \omega^{12} \cdot \omega^2 = (\omega^3)^4 \cdot \omega^2 = (1)^4 \cdot \omega^2 = \omega^2$$

6. The result is:

$$-128\omega^2$$

Final Answer: The expression equals $-128\omega^2$.

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:
If the roots of a quadratic equation $ax^2 + bx + c = 0$ are α and β, the difference of the roots is given by:

$$|\alpha - \beta| = \frac{\sqrt{b^2 - 4ac}}{|a|}$$

For consecutive integers, the absolute difference between the roots must be exactly 1.

Solution:
1. The equation is $x^2 - bx + c = 0$. Here $a = 1$. 2. Let the roots be n and $n + 1$, where n is an integer. 3. The difference between these roots is:

$$(n + 1) - n = 1$$

4. Using the formula for the difference of roots:

$$\frac{\sqrt{(-b)^2 - 4(1)(c)}}{1} = 1$$

5. Simplify the expression:

$$\sqrt{b^2 - 4c} = 1$$

6. Square both sides:

$$b^2 - 4c = 1$$

Final Answer: The value of $b^2 - 4c$ is 1.

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept:
To find the number of real roots of $e^x = x$, we look for the points of intersection between the graphs of $y = e^x$ and $y = x$. If the functions never intersect, there are no real roots. We can use the derivative to check the minimum value of the difference function $f(x) = e^x - x$.

Solution:
1. Let $f(x) = e^x - x$. We want to find if $f(x) = 0$ has any real solutions. 2. Find the derivative: $f'(x) = e^x - 1$. 3. Set $f'(x) = 0$ to find critical points:

$$e^x = 1 \implies x = 0$$

4. Find the second derivative: $f''(x) = e^x$. Since $f''(0) = 1 > 0$, $x = 0$ is a point of local minimum. 5. Calculate the minimum value of the function:

$$f(0) = e^0 - 0 = 1 - 0 = 1$$

6. Since the minimum value of $e^x - x$ is 1, it follows that $e^x - x \geq 1$ for all real x. 7. Because the function is always positive, it never crosses the x-axis ($f(x)$ is never 0). 8. Therefore, there are no real roots.

Final Answer: The number of real roots is 0.

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:

If the roots of the quadratic equation $ax^2 + bx + c = 0$ are α and β, the relationship between the roots and coefficients is given by: $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. If one root is the square of the other, we can set $\beta = \alpha^2$.

Solution:

1. Let the roots be α and α^2. 2. From the sum of roots: $\alpha + \alpha^2 = -b/a$. 3. From the product of roots: $\alpha \cdot \alpha^2 = c/a \implies \alpha^3 = c/a$. 4. Cube the sum of roots equation:

$$(\alpha + \alpha^2)^3 = (-b/a)^3$$

$$\alpha^3 + (\alpha^2)^3 + 3\alpha \cdot \alpha^2(\alpha + \alpha^2) = -b^3/a^3$$

5. Substitute $\alpha^3 = c/a$ and $\alpha + \alpha^2 = -b/a$:

$$c/a + (c/a)^2 + 3(c/a)(-b/a) = -b^3/a^3$$

6. Multiply the entire equation by a^3 to clear denominators:

$$a^2c + ac^2 - 3abc = -b^3$$

7. Rearrange the terms:

$$b^3 + a^2c + ac^2 = 3abc$$

Final Answer: The condition is $b^3 + ac^2 + a^2c = 3abc$.

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept:
The given series is an infinite Geometric Progression (G.P.). The sum of an infinite G.P. is given by the formula:

$$S_\infty = \frac{a}{1-r}$$

where a is the first term and r is the common ratio, provided $|r| < 1$.

Solution:

1. Identify the first term (a) and the common ratio (r): * $a = 1$ * $r = \frac{1/2}{1} = 1/2$
2. Check the condition for convergence: * $|r| = |1/2| < 1$, so the series converges.
3. Apply the formula:

$$S_\infty = \frac{1}{1-1/2}$$

4. Simplify the expression:

$$S_\infty = \frac{1}{1/2} = 2$$

Final Answer: The sum of the series is 2.

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept:

Three numbers P, Q, R are in Geometric Progression (G.P.) if $Q^2 = PR$. When working with exponents, if the terms in the exponent are in Arithmetic Progression (A.P.), the resulting exponential terms often form a G.P. because adding in the exponent corresponds to multiplying the base.

Solution:

1. Given that a, b, c are in A.P., we know $2b = a + c$. 2. Let the terms be $T_1 = 2^{ax+d}$, $T_2 = 2^{bx+d}$, and $T_3 = 2^{cx+d}$. 3. Check for G.P. by calculating T_2^2:

$$T_2^2 = (2^{bx+d})^2 = 2^{2(bx+d)} = 2^{2bx+2d}$$

4. Check the product of T_1 and T_3:

$$T_1 \cdot T_3 = 2^{ax+d} \cdot 2^{cx+d} = 2^{(ax+d)+(cx+d)} = 2^{(a+c)x+2d}$$

5. Substitute $a + c = 2b$:

$$T_1 \cdot T_3 = 2^{2bx+2d}$$

6. Since $T_2^2 = T_1 \cdot T_3$, the terms are in G.P.

Final Answer: The terms are in G.P.

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept:
For an Arithmetic Progression (A.P.), the n^{th} term a_n is given by:

$$a_n = a + (n-1)d$$

where a is the first term and d is the common difference.

Solution:
1. Identify the first term (a) and the common difference (d): * $a = 5$ * $d = 2 - 5 = -3$
2. Apply the formula for the n^{th} term:

$$a_n = 5 + (n-1)(-3)$$

3. Simplify the expression:

$$a_n = 5 - 3n + 3$$

$$a_n = 8 - 3n$$

Final Answer: The n^{th} term is $8 - 3n$.

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept:
The general term T_{r+1} in the binomial expansion of $(a+b)^n$ is:

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

To find the coefficient of a specific power of x, we express the power of x in terms of r and set it equal to the desired power.
Solution:
1. Given the expansion of $(x - x^{-2})^{11}$, the general term is:

$$T_{r+1} = {}^{11}C_r (x)^{11-r} (-x^{-2})^r$$

2. Simplify the power of x:

$$T_{r+1} = {}^{11}C_r (-1)^r x^{11-r} x^{-2r} = {}^{11}C_r (-1)^r x^{11-3r}$$

3. We need the coefficient of x^7, so set the exponent of x to 7:

$$11 - 3r = 7 \implies 3r = 4 \implies r = 4/3$$

4. Since r must be an integer for a term to exist in a binomial expansion, there is no term with x^7.
5. Therefore, the coefficient is 0.
Final Answer: The coefficient is 0.

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept:
In the binomial expansion of $(a+x)^N$, if N is even, there is only one middle term, which is the $(\frac{N}{2}+1)^{th}$ term. For the expression $(1+x)^{2n}$, the power $2n$ is always even.
Solution:
1. In the expansion $(1+x)^{2n}$, the total number of terms is $(2n+1)$. 2. Since $(2n+1)$ is an odd number, there is exactly one middle term. 3. The middle term is the $(\frac{2n}{2}+1)^{th}$ term, which is the $(n+1)^{th}$ term. 4. Using the general term formula $T_{r+1} = {}^{N}C_r a^{N-r} b^r$: * Here $N = 2n$, $a = 1$, $b = x$, and $r = n$.
5. The middle term is:

$$T_{n+1} = {}^{2n}C_n (1)^{2n-n} x^n = {}^{2n}C_n x^n$$

Final Answer: The middle term is ${}^{2n}C_n x^n$.

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept:

Probability of an event is the number of favorable outcomes divided by the total number of outcomes. When drawing "without replacement," the total number of available items and the number of favorable items both decrease after the first draw.

Solution:

1. Total balls in the bag = 5 (White) + 3 (Black) = 8 balls. 2. Let B_1 be the event that the first ball drawn is black.

$$P(B_1) = \frac{\text{Number of black balls}}{\text{Total balls}} = \frac{3}{8}$$

3. Since the ball is not replaced, for the second draw, there are now 7 balls total and only 2 black balls left. 4. Let B_2 be the event that the second ball drawn is black.

$$P(B_2|B_1) = \frac{2}{7}$$

5. The probability that both are black is:

$$P(B_1 \cap B_2) = P(B_1) \times P(B_2|B_1) = \frac{3}{8} \times \frac{2}{7}$$

6. Simplify the calculation:

$$\frac{3 \times 2}{8 \times 7} = \frac{6}{56} = \frac{3}{28}$$

Final Answer: The probability is 3/28.

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept:

The number of ways to arrange n distinct objects in a linear row is $n!$. However, for circular permutations, since there is no fixed starting point (the circle can be rotated), we fix one person's position to break the symmetry. The number of circular arrangements is $(n-1)!$.

Solution:

1. The number of persons to be seated is $n = 5$. 2. In a linear arrangement, the number of ways would be $5! = 120$. 3. For a circular table, we use the formula $(n-1)!$. 4. Number of ways = $(5-1)! = 4!$. 5. Calculate the factorial:

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Final Answer: The number of ways is 24.

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept:

Combinations have a property of symmetry: ${}^nC_r = {}^nC_{n-r}$. If we are given ${}^nC_x = {}^nC_y$, then there are two possibilities: either $x = y$ or $x + y = n$.

Solution:

1. Given the equation ${}^nC_{12} = {}^nC_8$. 2. Comparing this to the property ${}^nC_x = {}^nC_y$: * Here $x = 12$ and $y = 8$.

3. Since $12 \neq 8$, we must apply the condition $x + y = n$. 4. Therefore:

$$n = 12 + 8$$

$$n = 20$$

Final Answer: n is equal to 20.

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept:

The distance d between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by the formula:

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

Solution:

1. The given equations are: * Line 1: $3x - 4y + 7 = 0$ $(C_1 = 7)$ * Line 2: $3x - 4y + 5 = 0$ $(C_2 = 5)$
2. Identify the coefficients $A = 3$ and $B = -4$. 3. Apply the distance formula:

$$d = \frac{|7 - 5|}{\sqrt{3^2 + (-4)^2}}$$

4. Calculate the values: * $|7 - 5| = 2$ * $\sqrt{9 + 16} = \sqrt{25} = 5$
5. Thus:

$$d = 2/5$$

Final Answer: The distance is $2/5$.

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept:

If two lines are perpendicular, the product of their slopes is -1 ($m_1 \cdot m_2 = -1$). The general equation of a line perpendicular to $Ax + By + C = 0$ is $Bx - Ay + \lambda = 0$. We find λ by substituting the given point through which the line passes.

Solution:

1. The given line is $x + y + 7 = 0$. Its slope $m_1 = -1$. 2. Let the slope of the required line be m_2. Since they are perpendicular:

$$-1 \cdot m_2 = -1 \implies m_2 = 1$$

3. Using the point-slope form $y - y_1 = m(x - x_1)$ with point $(1, 2)$:

$$y - 2 = 1(x - 1)$$

4. Simplify the equation:

$$y - 2 = x - 1$$

$$x - y + 1 = 0$$

Final Answer: The equation is $x - y + 1 = 0$.

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept:

The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r of this circle is given by:

$$r = \sqrt{g^2 + f^2 - c}$$

Solution:

1. The given equation is $x^2 + y^2 - 4x + 6y - 12 = 0$. 2. Compare with the general equation: * $2g = -4 \implies g = -2$ * $2f = 6 \implies f = 3$ * $c = -12$

3. Apply the radius formula:

$$r = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

4. Calculate the values:

$$r = \sqrt{4 + 9 + 12} = \sqrt{25} = 5$$

Final Answer: The radius is 5.

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept:

For a hyperbola in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the eccentricity e is given by the relation:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Solution:

1. The given equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$. 2. Identify $a^2 = 16$ and $b^2 = 9$. 3. Substitute into the eccentricity formula:

$$e = \sqrt{1 + \frac{9}{16}}$$

4. Simplify the expression:

$$e = \sqrt{\frac{16 + 9}{16}} = \sqrt{\frac{25}{16}} = 5/4$$

Final Answer: The eccentricity is $5/4$.

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept:
For a parabola in the standard form $y^2 = 4ax$ or $y^2 = -4ax$, the length of the latus rectum is always equal to $|4a|$. It is a physical length and is therefore always represented as a positive value.
Solution:
1. The given equation is $y^2 = -8x$. 2. Compare this with the standard form $y^2 = -4ax$. 3. We find that $4a = 8$. 4. The length of the latus rectum is $4a$. 5. Therefore, the length is 8.
Final Answer: The length of the latus rectum is 8.

Answer: (C)

Go Back to Question 39

Q40.

Solution

Concept:
The magnitude of the sum and difference of two vectors $\vec{a}$ and $\vec{b}$ is related to the dot product. We use the identity $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then their squares must also be equal.
Solution:
1. Square both sides of the given equation:

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

2. Expand using the dot product:

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

3. Cancel the common terms $|\vec{a}|^2$ and $|\vec{b}|^2$:

$$2\vec{a} \cdot \vec{b} = -2\vec{a} \cdot \vec{b}$$

4. Rearrange:

$$4\vec{a} \cdot \vec{b} = 0 \implies \vec{a} \cdot \vec{b} = 0$$

5. Since the dot product is zero, the angle θ between the vectors must satisfy $\cos\theta = 0$. 6. Therefore, $\theta = 90°$.
Final Answer: The angle is $90°$.

Answer: (C)

Go Back to Question 40

Q41.

Solution

Concept:

The direction cosines (l, m, n) of a line satisfy the fundamental identity:

$$l^2 + m^2 + n^2 = 1$$

If a line makes angles α, β, γ with the x, y, and z axes respectively, then $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$.

Solution:

1. We are given the angles $\alpha = 90^\circ$ and $\beta = 60^\circ$. 2. Calculate the known direction cosines: * $l = \cos 90^\circ = 0$ * $m = \cos 60^\circ = 1/2$

3. Let the third angle be γ. Using the identity:

$$0^2 + (1/2)^2 + \cos^2\gamma = 1$$

4. Simplify:

$$0 + 1/4 + \cos^2\gamma = 1 \implies \cos^2\gamma = 1 - 1/4 = 3/4$$

5. Taking the square root:

$$\cos\gamma = \pm\sqrt{3}/2$$

6. For the acute angle:

$$\gamma = \cos^{-1}(\sqrt{3}/2) = 30^\circ$$

Final Answer: The angle with the z-axis is 30°.

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept:

The distance d from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is given by the formula:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

Solution:

1. The point is $(0, 0, 0)$ and the plane is $2x - 3y + 6z + 14 = 0$. 2. Identify the coefficients: $A = 2, B = -3, C = 6, D = 14$. 3. Substitute into the formula:

$$d = \frac{|2(0) - 3(0) + 6(0) + 14|}{\sqrt{2^2 + (-3)^2 + 6^2}}$$

4. Calculate the numerator and denominator: * Numerator: $|14| = 14$ * Denominator: $\sqrt{4 + 9 + 36} = \sqrt{49} = 7$

5. Thus:

$$d = 14/7 = 2$$

Final Answer: The distance is 2 units.

Answer: (B)

Go Back to Question 42

Q43.

Solution

Concept:

The shortest distance between two points (x_1, y_1) and (x_2, y_2) is a straight line. For a point (x, y) on a curve, we minimize the squared distance function $f(x) = (x - x_1)^2 + (y - y_1)^2$ to find the closest point.

Solution:

1. Let a general point on the parabola $y^2 = 4x$ be $(t^2, 2t)$. 2. We want to find the distance from this point to $(2, 1)$. 3. Squared distance $D^2 = (t^2 - 2)^2 + (2t - 1)^2$. 4. Expand: $D^2 = t^4 - 4t^2 + 4 + 4t^2 - 4t + 1 = t^4 - 4t + 5$. 5. To minimize, differentiate with respect to t:

$$\frac{d(D^2)}{dt} = 4t^3 - 4$$

6. Set to zero: $4t^3 = 4 \implies t = 1$. 7. At $t = 1$, the point is $(1^2, 2(1)) = (1, 2)$.

Final Answer: The point is $(1, 2)$.

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept:
The scalar triple product $[\vec{a}, \vec{b}, \vec{c}]$ represents the volume of a parallelepiped. If three vectors are coplanar, the volume of the parallelepiped they form is zero. Mathematically, the determinant of their components must be zero.

Solution:
1. The vectors are $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$. 2. Set the determinant to zero:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

3. Expand along the first row:

$$2(10 + 3\lambda) - (-1)(5 + 9) + 1(\lambda - 6) = 0$$

4. Simplify:

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

$$7\lambda + 28 = 0$$

5. Solving for λ:

$$7\lambda = -28 \implies \lambda = -4$$

Final Answer: The value of λ is -4.

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept:
If the probability of success in a single trial is p, the probability of failure is $q = 1 - p$. In n independent trials, the probability of exactly r successes is given by the Binomial Distribution:

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

Solution:
1. Probability of hitting a target $p = 1/4$. 2. Probability of missing $q = 1 - 1/4 = 3/4$. 3. Number of trials $n = 7$. 4. We want exactly 2 successes ($r = 2$):

$$P(X = 2) = {}^7C_2(1/4)^2(3/4)^{7-2}$$

5. Calculate: * ${}^7C_2 = \frac{7\times6}{2\times1} = 21$ * $(1/4)^2 = 1/16$ * $(3/4)^5 = 243/1024$
6. Multiply:

$$P(X = 2) = 21 \times \frac{1}{16} \times \frac{243}{1024} = \frac{5103}{16384}$$

Final Answer: The probability is $5103/16384$.

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept:

The sum of n terms of an Arithmetic Progression (A.P.) is given by $S_n = \frac{n}{2}[2a + (n-1)d]$. If we are given the sum S_n as a function of n, the n^{th} term a_n can be found using the relation $a_n = S_n - S_{n-1}$. The common difference d is then $a_n - a_{n-1}$, which is also twice the coefficient of n^2 in the S_n expression.

Solution:

1. Given the sum of n terms: $S_n = 3n^2 + 5n$. 2. Find the first term (a_1):

$$a_1 = S_1 = 3(1)^2 + 5(1) = 8$$

3. Find the sum of the first two terms (S_2):

$$S_2 = 3(2)^2 + 5(2) = 3(4) + 10 = 22$$

4. Find the second term (a_2):

$$a_2 = S_2 - S_1 = 22 - 8 = 14$$

5. Calculate the common difference (d):

$$d = a_2 - a_1 = 14 - 8 = 6$$

6. Alternatively, for $S_n = An^2 + Bn$, the common difference $d = 2A$. Here $A = 3$, so $d = 2 \times 3 = 6$.

Final Answer: The common difference is 6.

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept:

The angle θ between two planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is the angle between their normal vectors $\vec{n}_1 = (A_1, B_1, C_1)$ and $\vec{n}_2 = (A_2, B_2, C_2)$. It is calculated using the dot product formula:

$$\cos\theta = \frac{|A_1A_2 + B_1B_2 + C_1C_2|}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

Solution:

1. Plane 1: $2x - y + z = 6 \implies \vec{n}_1 = (2, -1, 1)$. 2. Plane 2: $x + y + 2z = 7 \implies \vec{n}_2 = (1, 1, 2)$.
3. Calculate the dot product:

$$\vec{n}_1 \cdot \vec{n}_2 = (2)(1) + (-1)(1) + (1)(2) = 2 - 1 + 2 = 3$$

4. Calculate magnitudes: * $|\vec{n}_1| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$ * $|\vec{n}_2| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$
5. Substitute into the cosine formula:

$$\cos\theta = \frac{3}{\sqrt{6}\cdot\sqrt{6}} = \frac{3}{6} = 1/2$$

6. Therefore, $\theta = \cos^{-1}(1/2) = 60°$.

Final Answer: The angle between the planes is 60°.

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept:
The work done by a constant force $\vec{F}$ in displacing an object along a vector $\vec{d}$ is given by the scalar product (dot product):

$$W = \vec{F} \cdot \vec{d}$$

The displacement vector $\vec{d}$ is the difference between the final position vector and the initial position vector $(\vec{r}_2 - \vec{r}_1)$.

Solution:
1. Given force $\vec{F} = 4\hat{i} + \hat{j} - 3\hat{k}$. 2. Initial point $A = (1, 2, 3) \implies \vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$. 3. Final point $B = (5, 4, 1) \implies \vec{r}_2 = 5\hat{i} + 4\hat{j} + \hat{k}$. 4. Calculate displacement vector $\vec{d}$:

$$\vec{d} = \vec{r}_2 - \vec{r}_1 = (5-1)\hat{i} + (4-2)\hat{j} + (1-3)\hat{k} = 4\hat{i} + 2\hat{j} - 2\hat{k}$$

5. Calculate Work Done:

$$W = \vec{F} \cdot \vec{d} = (4)(4) + (1)(2) + (-3)(-2)$$

$$W = 16 + 2 + 6 = 24 \text{ units}$$

Final Answer: The work done is 24 units.

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept:
To find the area between two curves $y = f(x)$ and $y = g(x)$, we first find their points of intersection by setting $f(x) = g(x)$. The area is then given by the integral of the upper curve minus the lower curve over the interval defined by the intersection points.
Solution:
1. Given curves: $y = x^2$ (parabola) and $y = x$ (line). 2. Find intersection points:

$$x^2 = x \implies x^2 - x = 0 \implies x(x-1) = 0$$

Points are $x = 0$ and $x = 1$. 3. In the interval $[0, 1]$, $x \geq x^2$. So the line is the upper curve. 4. Calculate the area A:

$$A = \int_0^1 (x - x^2)dx$$

5. Evaluate the integral:

$$A = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$A = \left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0) = \frac{3-2}{6} = 1/6$$

Final Answer: The area is $1/6$ sq. units.

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept:
The order of a differential equation is determined by the highest derivative term present. The solution of a differential equation contains a number of arbitrary constants. For a "general solution," the number of arbitrary constants is exactly equal to the order of the differential equation.
Solution:
1. We are looking for the number of arbitrary constants in the general solution of a differential equation of fourth order. 2. Rule: Number of arbitrary constants in General Solution = Order of the Differential Equation. 3. Since the order is given as 4, the number of arbitrary constants is 4. 4. (Note: In a "particular solution," the number of arbitrary constants is always zero, as specific values have been assigned to them).
Final Answer: The number of arbitrary constants is 4.

Answer: (D)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	A	4	B	5	B
6	B	7	A	8	A	9	B	10	A
11	A	12	C	13	A	14	A	15	C
16	C	17	A	18	A	19	A	20	C
21	A	22	A	23	B	24	B	25	A
26	A	27	B	28	B	29	A	30	C
31	A	32	A	33	B	34	A	35	A
36	A	37	A	38	A	39	C	40	C
41	A	42	B	43	A	44	B	45	A
46	B	47	B	48	A	49	A	50	D