

# MHT-CET Mathematics Sample Paper- 5

Duration: 90 Minutes

Maximum Marks: 100

## Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

**Q1.** If  $f(x) = \lim_{n \rightarrow \infty} \frac{\log(2+x) - x^{2n} \sin x}{1+x^{2n}}$ , then the value of  $f(1)$  so that  $f(x)$  is continuous at  $x = 1$  is:

- (A)  $\log 3$   
(B)  $\frac{1}{2}(\log 3 - \sin 1)$   
(C)  $\frac{1}{2}(\sin 1 - \log 3)$   
(D) 0

**Q2.** The value of  $\lim_{x \rightarrow 0} \left( \frac{1+5x^2}{1+3x^2} \right)^{1/x^2}$  is:

- (A)  $e^2$   
(B)  $e^{1/2}$   
(C)  $e^{-2}$   
(D)  $e$

**Q3.** If  $f(x) = [x^2] \sin(\pi x)$  (where  $[\cdot]$  denotes greatest integer function), then  $f(x)$  is discontinuous at:

- (A) All integers  
(B) All  $x = \sqrt{n}$  where  $n \in \mathbb{N}$   
(C)  $x = \sqrt{n}$  except when  $n$  is a perfect square



(D) No point in  $\mathbb{R}$

**Q4.** If  $x = \sqrt{a^{\sin^{-1} t}}$  and  $y = \sqrt{a^{\cos^{-1} t}}$ , then  $\frac{dy}{dx}$  is equal to:

(A)  $\frac{y}{x}$

(B)  $-\frac{y}{x}$

(C)  $\frac{x}{y}$

(D)  $-\frac{x}{y}$

**Q5.** If  $y = \tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$ , then  $y'(1)$  is:

(A)  $\frac{1}{4}$

(B)  $\frac{1}{2}$

(C) 1

(D) 0

**Q6.** Let  $f(x) = e^x g(x)$ ,  $g(0) = 2$  and  $g'(0) = 1$ , then  $f'(0)$  is:

(A) 1

(B) 3

(C) 2

(D) 0

**Q7.** The derivative of  $(\ln x)^x + x^{\ln x}$  at  $x = e$  is:

(A) 1

(B) 2

(C)  $\frac{2}{e}$

(D)  $e + 2$

**Q8.** If  $y = \sin(m \sin^{-1} x)$ , then  $(1 - x^2)y'' - xy'$  equals:

(A)  $m^2 y$

(B)  $-m^2 y$



(C)  $my$

(D) 0

**Q9.** Evaluate:  $\int \frac{dx}{x^2(x^4+1)^{3/4}}$

(A)  $-\frac{(x^4+1)^{1/4}}{x} + C$

(B)  $\frac{(x^4+1)^{1/4}}{x} + C$

(C)  $-\frac{(x^4+1)^{1/4}}{x^2} + C$

(D)  $\frac{(x^4+1)^{1/4}}{x^4} + C$

**Q10.** Evaluate:  $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$

(A)  $\frac{\pi}{2}$

(B)  $\frac{\pi}{4}$

(C)  $\pi$

(D) 0

**Q11.** Evaluate:  $\int \frac{dx}{\cos^6 x + \sin^6 x}$

(A)  $\tan^{-1}(\tan x - \cot x) + C$

(B)  $\tan^{-1}(\tan x + \cot x) + C$

(C)  $\sin^{-1}(\sin 2x) + C$

(D) None

**Q12.** Evaluate:  $\int_{-1}^1 \frac{dx}{1+e^{\sin x}}$

(A) 0

(B) 1

(C) 2

(D)  $\frac{1}{2}$

**Q13.** If  $I_n = \int_0^{\pi/4} \tan^n x dx$ , then  $I_n + I_{n-2}$  equals:

(A)  $\frac{1}{n}$



- (B)  $\frac{1}{n-1}$
- (C)  $\frac{1}{n+1}$
- (D)  $\frac{2}{n}$

**Q14.** Number of four-digit numbers greater than 4321 formed using digits {0, 1, 2, 3, 4, 5} (repetition allowed) is:

- (A) 360
- (B) 310
- (C) 311
- (D) 309

**Q15.** Three persons independently choose one of three houses. Probability all choose the same house is:

- (A)  $\frac{2}{9}$
- (B)  $\frac{1}{9}$
- (C)  $\frac{1}{3}$
- (D)  $\frac{7}{9}$

**Q16.** A bag has 4 red and 6 black balls. One ball is drawn and replaced with 2 more of same color. Then a ball is drawn. Probability of red is:

- (A)  $\frac{2}{5}$
- (B)  $\frac{3}{5}$
- (C)  $\frac{4}{7}$
- (D)  $\frac{1}{2}$

**Q17.** Number of ways to seat 5 boys and 5 girls so that no two girls sit together is:

- (A)  $5! \times 6!$
- (B)  $5! \times 5!$
- (C)  $2 \times 5!$



(D) 6!

**Q18.** Eccentricity of a hyperbola with given conditions is:

(A)  $\frac{4}{3}$

(B)  $\frac{4}{\sqrt{3}}$

(C)  $\frac{2}{\sqrt{3}}$

(D)  $\sqrt{3}$

**Q19.** Tangent to parabola  $y^2 = 8x$  perpendicular to  $x - y + 5 = 0$  is:

(A)  $x + y + 2 = 0$

(B)  $x + y - 2 = 0$

(C)  $x - y + 2 = 0$

(D)  $x - y - 2 = 0$

**Q20.** Product of perpendiculars from foci of ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  on any tangent is:

(A)  $a^2$

(B)  $b^2$

(C)  $ab$

(D)  $a^2 + b^2$

**Q21.** If  $y = mx + c$  is tangent to hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , then  $c^2$  equals:

(A)  $a^2m^2 + b^2$

(B)  $a^2m^2 - b^2$

(C)  $b^2m^2 - a^2$

(D)  $a^2 + b^2m^2$

**Q22.** The area of the region bounded by the curves  $y = |x - 1|$  and  $y = 3 - |x|$  is:

(A) 2 sq. units

(B) 3 sq. units



- (C) 4 sq. units
- (D) 6 sq. units

**Q23.** The area bounded by the parabola  $y^2 = 4x$  and its latus rectum is:

- (A)  $2/3$
- (B)  $4/3$
- (C)  $8/3$
- (D)  $16/3$

**Q24.** The solution of the differential equation  $\frac{dy}{dx} + \frac{y}{x} = x^2$  under the condition  $y(1) = 1$  is:

- (A)  $4xy = x^4 + 3$
- (B)  $4xy = x^4 - 3$
- (C)  $xy = x^4 + 3$
- (D)  $y = x^3 + 3$

**Q25.** The integrating factor of the differential equation  $(1 - y^2)\frac{dx}{dy} + yx = ay$  is:

- (A)  $\frac{1}{y^2-1}$
- (B)  $\frac{1}{\sqrt{1-y^2}}$
- (C)  $\frac{1}{1-y^2}$
- (D)  $\sqrt{1 - y^2}$

**Q26.** The order and degree of the differential equation  $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$  are respectively:

- (A) 2, 2
- (B) 2, 3
- (C) 1, 2
- (D) 2, 1



- Q27.** If  $z$  is a complex number such that  $|z| \geq 2$ , then the minimum value of  $|z + \frac{1}{2}|$  is:
- (A) 1
  - (B)  $3/2$
  - (C)  $5/2$
  - (D) 0
- Q28.** If  $\omega$  is a complex cube root of unity, then the value of  $\sin [(\omega^{10} + \omega^{23})\pi - \frac{\pi}{4}]$  is:
- (A)  $1/\sqrt{2}$
  - (B)  $-1/\sqrt{2}$
  - (C) 1
  - (D)  $\sqrt{3}/2$
- Q29.** The locus of  $z$  satisfying  $\arg \left( \frac{z-1}{z+1} \right) = \frac{\pi}{4}$  is:
- (A) A circle
  - (B) A straight line
  - (C) A parabola
  - (D) An ellipse
- Q30.** If  $\alpha, \beta$  are the roots of  $x^2 - p(x + 1) - c = 0$ , then  $(\alpha + 1)(\beta + 1)$  is equal to:
- (A)  $c$
  - (B)  $1 - c$
  - (C)  $1 + c$
  - (D)  $p + c$
- Q31.** The values of  $a$  for which the quadratic equation  $3x^2 + 2(a^2 + 1)x + (a^2 - 3a + 2) = 0$  possesses roots of opposite signs lie in:
- (A)  $(-\infty, 1)$
  - (B)  $(2, \infty)$



(C) (1, 2)

(D) (0, 2)

**Q32.** If the roots of  $x^2 - bx + c = 0$  are two consecutive integers, then  $b^2 - 4c$  is:

(A) 0

(B) 1

(C) 2

(D) 4

**Q33.** If the sum of first  $n$  terms of an A.P. is  $3n^2 + 5n$  and its  $m^{\text{th}}$  term is 164, then  $m$  is:

(A) 26

(B) 27

(C) 28

(D) 25

**Q34.** The sum of the infinite geometric series  $1 + \frac{2}{3} + \frac{4}{9} + \dots$  is:

(A) 3

(B) 2

(C) 4

(D) 5

**Q35.** If  $a, b, c$  are in H.P., then  $\frac{1}{b-a} + \frac{1}{b-c}$  is equal to:

(A)  $\frac{1}{a} + \frac{1}{c}$

(B)  $\frac{1}{a} - \frac{1}{c}$

(C)  $\frac{1}{b}$

(D) None

**Q36.** The coefficient of  $x^7$  in the expansion of  $(1 + 3x - 2x^3)^{10}$  is:



- (A) 62640
- (B) 60240
- (C) 52640
- (D) 0

**Q37.** The term independent of  $x$  in the expansion of  $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$  is 405. The value of  $k$  is:

- (A)  $\pm 3$
- (B)  $\pm 2$
- (C)  $\pm 1$
- (D) 0

**Q38.** The distance between the parallel lines  $3x + 4y - 9 = 0$  and  $6x + 8y + 15 = 0$  is:

- (A) 3.3
- (B) 3.9
- (C) 2.7
- (D) 5

**Q39.** If  $x^y = e^{x-y}$ , then  $\frac{dy}{dx}$  at  $x = 1$  is equal to:

- (A) 0
- (B) 1
- (C)  $-1$
- (D)  $e$

**Q40.** The equation of the line passing through  $(1, 2)$  and perpendicular to  $x + y + 7 = 0$  is:

- (A)  $x - y + 1 = 0$
- (B)  $x - y - 1 = 0$
- (C)  $x + y - 3 = 0$



(D)  $x - y = 0$

**Q41.** The equation of the circle passing through the origin and cutting intercepts 10 and 24 from the positive  $x$  and  $y$  axes respectively is:

(A)  $x^2 + y^2 - 10x - 24y = 0$

(B)  $x^2 + y^2 + 10x + 24y = 0$

(C)  $x^2 + y^2 - 5x - 12y = 0$

(D)  $x^2 + y^2 - 24x - 10y = 0$

**Q42.** If the line  $3x - 4y = k$  touches the circle  $x^2 + y^2 - 4x - 8y - 5 = 0$ , then  $k$  can be:

(A) 15

(B) 10

(C) -5

(D) 25

**Q43.** The length of the tangent from the point  $(5, 1)$  to the circle  $x^2 + y^2 + 6x - 4y - 3 = 0$  is:

(A) 7

(B) 49

(C)  $\sqrt{52}$

(D)  $\sqrt{10}$

**Q44.** If  $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$ , then the angle between  $\vec{a}$  and  $\vec{b}$  is:

(A)  $\pi/2$

(B)  $\pi/4$

(C)  $\pi/3$

(D)  $\pi/6$



- Q45.** The volume of the parallelepiped whose coterminous edges are  $2i - 3j + 4k$ ,  $i + 2j - k$ , and  $3i - j + 2k$  is:
- (A) 7  
(B) 14  
(C) 2  
(D) 0
- Q46.** The angle between the lines  $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$  and  $\frac{x-1}{1} = \frac{y+3}{3} = \frac{z+5}{2}$  is:
- (A)  $\pi/2$   
(B)  $\pi/3$   
(C)  $\pi/4$   
(D) 0
- Q47.** The distance of the point  $(1, 2, 3)$  from the plane  $x + 2y - 2z + 5 = 0$  is:
- (A)  $4/3$   
(B) 2  
(C) 3  
(D)  $1/3$
- Q48.** The function  $f(x) = \frac{x}{1+x \tan x}$  is maximum when  $x$  is equal to:
- (A)  $\sin x$   
(B)  $\cos x$   
(C)  $\tan x$   
(D)  $\cot x$
- Q49.** A balloon, which always remains spherical, has a variable diameter  $\frac{3}{2}(2x + 3)$ . The rate of change of its volume with respect to  $x$  is:
- (A)  $27\pi(2x + 3)^2$   
(B)  $\frac{27}{8}\pi(2x + 3)^2$



(C)  $\frac{81}{8}\pi(2x + 3)^2$

(D)  $\frac{9}{4}\pi(2x + 3)^2$

**Q50.** The value of the integral  $\int \frac{\sin x}{\sin(x-\alpha)} dx$  is:

(A)  $x \cos \alpha + \sin \alpha \log |\sin(x - \alpha)| + C$

(B)  $x \sin \alpha + \cos \alpha \log |\sin(x - \alpha)| + C$

(C)  $x \cos \alpha - \sin \alpha \log |\sin(x - \alpha)| + C$

(D)  $\cos \alpha \log |\sin(x - \alpha)| + C$



## Detailed Solutions

Q1.

## Solution

**Concept:** The given function depends on the limit of  $x^{2n}$  as  $n \rightarrow \infty$ , which changes behavior based on  $|x| < 1$ ,  $|x| = 1$ , or  $|x| > 1$ .

**Solution:**

**Step 1: Define cases**

- If  $|x| < 1$ , then  $x^{2n} \rightarrow 0$
- If  $|x| > 1$ , then  $x^{2n} \rightarrow \infty$
- If  $x = 1$ , then  $x^{2n} = 1$

**Step 2: Function for different regions**

$$f(x) = \lim_{n \rightarrow \infty} \frac{\log(2+x) - x^{2n} \sin x}{1+x^{2n}}$$

- For  $x < 1$ :

$$f(x) = \log(2+x)$$

- For  $x > 1$ :

$$f(x) = -\sin x$$

**Step 3: Value at  $x = 1$**

$$f(1) = \frac{\log 3 - \sin 1}{1+1} = \frac{1}{2}(\log 3 - \sin 1)$$

**Step 4: Continuity condition** For continuity at  $x = 1$ , we define  $f(1)$  as above.

**Final Answer:**  $\frac{1}{2}(\log 3 - \sin 1)$

**Answer: (B)**

[Go Back to Question 1](#)



Q2.

**Solution****Concept:** Use logarithm and standard limit  $\lim_{x \rightarrow 0} (1 + ax)^{1/x} = e^a$ .**Solution:****Step 1: Take logarithm**

$$\ln L = \lim_{x \rightarrow 0} \frac{1}{x^2} [\ln(1 + 5x^2) - \ln(1 + 3x^2)]$$

**Step 2: Use expansion**

$$\ln(1 + ax^2) \approx ax^2$$

$$\ln L = \lim_{x \rightarrow 0} \frac{1}{x^2} (5x^2 - 3x^2) = 2$$

**Step 3: Exponentiate**

$$L = e^2$$

**Final Answer:**  $e^2$ **Answer:** (A)[Go Back to Question 2](#)

Q3.

**Solution****Concept:** Discontinuity occurs where either  $\lfloor x^2 \rfloor$  jumps or  $\sin(\pi x)$  is non-zero at jump points.**Solution:****Step 1: Floor function discontinuity**

$$\lfloor x^2 \rfloor \text{ is discontinuous at } x = \pm\sqrt{n}, n \in \mathbb{N}$$

**Step 2: Check multiplication with  $\sin(\pi x)$** 

$$\sin(\pi x) = 0 \text{ only when } x \in \mathbb{Z}$$

**Step 3: Cancellation condition** Discontinuity disappears only when  $\sqrt{n}$  is integer  $\Rightarrow n$  is perfect square.**Final Answer:**  $x = \sqrt{n}$ ,  $n$  not a perfect square**Answer:** (C)[Go Back to Question 3](#)

Q4.

**Solution****Concept:** Differentiate using logarithmic differentiation and inverse trigonometric relations.**Solution:****Step 1: Take logs**

$$\ln x = \frac{1}{2} \sin^{-1} t \ln a, \quad \ln y = \frac{1}{2} \cos^{-1} t \ln a$$

**Step 2: Differentiate**

$$\frac{1}{x} \frac{dx}{dt} = \frac{\ln a}{2\sqrt{1-t^2}}, \quad \frac{1}{y} \frac{dy}{dt} = -\frac{\ln a}{2\sqrt{1-t^2}}$$

**Step 3: Divide**

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-y}{x}$$

**Final Answer:**  $\boxed{-\frac{y}{x}}$ **Answer: (B)**[Go Back to Question 4](#)

Q5.

**Solution****Concept:** Use standard identity:

$$\tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right) = \frac{1}{2} \tan^{-1} x$$

**Solution:****Step 1: Differentiate**

$$y' = \frac{1}{2} \cdot \frac{1}{1+x^2}$$

**Step 2: Substitute  $x = 1$** 

$$y'(1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

**Final Answer:**  $\boxed{\frac{1}{4}}$ **Answer: (A)**[Go Back to Question 5](#)

Q6.

**Solution****Concept:** Use product rule for differentiation:

$$f(x) = e^x g(x)$$

**Solution:****Step 1: Differentiate**

$$f'(x) = e^x g(x) + e^x g'(x) = e^x [g(x) + g'(x)]$$

**Step 2: Substitute  $x = 0$** 

$$f'(0) = e^0 [g(0) + g'(0)] = 1(2 + 1) = 3$$

**Final Answer:** **Answer:** (B)[Go Back to Question 6](#)

Q7.

**Solution****Concept:** Differentiate both terms using logarithmic differentiation.**Solution:****Step 1: Differentiate  $(\ln x)^x$** 

$$y_1 = (\ln x)^x$$

$$\ln y_1 = x \ln(\ln x)$$

$$\frac{y_1'}{y_1} = \ln(\ln x) + \frac{1}{\ln x}$$

$$y_1' = (\ln x)^x \left[ \ln(\ln x) + \frac{1}{\ln x} \right]$$

At  $x = e$ :

$$(\ln e)^e = 1, \quad \ln(\ln e) = 0$$

$$y_1'(e) = 1 \cdot (0 + 1) = 1$$

**Step 2: Differentiate  $x^{\ln x}$** 

$$y_2 = x^{\ln x}$$

$$\ln y_2 = (\ln x)^2$$

$$\frac{y_2'}{y_2} = \frac{2 \ln x}{x}$$

$$y_2' = x^{\ln x} \cdot \frac{2 \ln x}{x}$$

At  $x = e$ :

$$y_2'(e) = 1 \cdot \frac{2}{e} = \frac{2}{e}$$

**Step 3: Add derivatives**

$$f'(e) = 1 + \frac{2}{e}$$

**Final Answer:**  $1 + \frac{2}{e}$ **Answer: (D)**[Go Back to Question 7](#)

Q8.

**Solution****Concept:** Use known identity:

$$y = \sin(m \sin^{-1} x)$$

This satisfies a standard differential equation.

**Solution:****Step 1: Standard result** For  $y = \sin(m \sin^{-1} x)$ :

$$(1 - x^2)y'' - xy' + m^2y = 0$$

**Step 2: Rearranging**

$$(1 - x^2)y'' - xy' = -m^2y$$

**Final Answer:**  $-m^2y$ **Answer: (B)**[Go Back to Question 8](#)

Q9.

**Solution****Concept:** Use substitution:

$$u = \frac{(x^4 + 1)^{1/4}}{x}$$

**Solution:****Step 1: Differentiate substitution** After simplification,

$$du = -\frac{dx}{x^2(x^4 + 1)^{3/4}}$$

**Step 2: Rewrite integral**

$$\int \frac{dx}{x^2(x^4 + 1)^{3/4}} = -\int du$$

**Step 3: Integrate**

$$= -u + C$$

**Step 4: Substitute back**

$$= -\frac{(x^4 + 1)^{1/4}}{x} + C$$

**Final Answer:**  $-\frac{(x^4 + 1)^{1/4}}{x} + C$ **Answer: (A)**[Go Back to Question 9](#)

Q10.

**Solution****Concept:** Use symmetry of the integrand.

$$I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$$

**Solution:****Step 1: Substitute**  $x \rightarrow \frac{\pi}{2} - x$ 

$$I = \int_0^{\pi/2} \frac{\cos^{100} x}{\sin^{100} x + \cos^{100} x} dx$$

**Step 2: Add both forms**

$$2I = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$$

**Step 3: Solve**

$$I = \frac{\pi}{4}$$

**Final Answer:** **Answer: (B)**[Go Back to Question 10](#)

Q11.

**Solution****Concept:** Use identity:

$$\cos^6 x + \sin^6 x = (\cos^2 x + \sin^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x)$$

**Solution:****Step 1: Simplify**

$$\begin{aligned} &= 1 - 3 \sin^2 x \cos^2 x \\ &= 1 - \frac{3}{4} \sin^2 2x \end{aligned}$$

**Step 2: Integral becomes**

$$\int \frac{dx}{1 - \frac{3}{4} \sin^2 2x}$$

**Step 3: Standard form leads to non-matching options**

No given option matches the correct simplified antiderivative.

**Final Answer:** **Answer: (D)**[Go Back to Question 11](#)

Q12.

**Solution****Concept:** Use substitution symmetry  $x \rightarrow -x$ .**Solution:****Step 1: Define integral**

$$I = \int_{-1}^1 \frac{dx}{1 + e^{\sin x}}$$

**Step 2: Substitute**  $x \rightarrow -x$ 

$$I = \int_{-1}^1 \frac{dx}{1 + e^{-\sin x}}$$

**Step 3: Add both forms**

$$2I = \int_{-1}^1 \left( \frac{1}{1 + e^{\sin x}} + \frac{1}{1 + e^{-\sin x}} \right) dx$$

**Step 4: Simplify**

$$\frac{1}{1 + e^a} + \frac{1}{1 + e^{-a}} = 1$$

$$2I = \int_{-1}^1 1 dx = 2$$

**Step 5: Solve**

$$I = 1$$

**Final Answer:** **Answer:** (B)[Go Back to Question 12](#)

Q13.

**Solution****Concept:** Use standard reduction formula.

$$I_n = \int_0^{\pi/4} \tan^n x dx$$

**Solution:****Step 1: Known identity**

$$I_n = \frac{1}{n-1} - I_{n-2}$$

**Step 2: Rearranging**

$$I_n + I_{n-2} = \frac{1}{n-1}$$

**Final Answer:** **Answer:** (B)[Go Back to Question 13](#)

Q14.

**Solution**

**Concept:** Count 4-digit numbers using digits  $\{0, 1, 2, 3, 4, 5\}$ , repetition allowed.

**Solution:**

Total digits available:  $\{0, 1, 2, 3, 4, 5\}$  (Total 6 digits).

**Case 1: Numbers starting with 5**

The first digit is 5. The remaining three positions can be filled by any of the 6 digits.

Number of ways =  $1 \times 6 \times 6 \times 6 = 216$ .

**Case 2: Numbers starting with 44 or 45**

The first digit is 4. The second digit can be 4 or 5 (2 ways). The remaining two positions can be filled by any of the 6 digits.

Number of ways =  $1 \times 2 \times 6 \times 6 = 72$ .

**Case 3: Numbers starting with 43 and the third digit is 3, 4, or 5**

The first two digits are fixed as 4 and 3. The third digit can be  $\{3, 4, 5\}$  (3 ways). The last position can be any of the 6 digits.

Number of ways =  $1 \times 1 \times 3 \times 6 = 18$ .

**Case 4: Numbers starting with 432 and the last digit is  $> 1$**

The first three digits are fixed as 4, 3, 2. The last digit can be  $\{2, 3, 4, 5\}$  (4 ways).

Number of ways =  $1 \times 1 \times 1 \times 4 = 4$ .

**Total Count:**

Sum =  $216 + 72 + 18 + 4 = 310$ .

**Final Answer:**

**Answer:** (C)

[Go Back to Question 14](#)



Q15.

**Solution****Concept:** Total outcomes for independent choices.**Solution:****Step 1: Total outcomes** Each of 3 persons has 3 choices:

$$3^3 = 27$$

**Step 2: Favorable outcomes** All choose same house:

$$(1, 1, 1), (2, 2, 2), (3, 3, 3) \Rightarrow 3 \text{ ways}$$

**Step 3: Probability**

$$P = \frac{3}{27} = \frac{1}{9}$$

**Final Answer:**  $\frac{1}{9}$ **Answer: (B)**[Go Back to Question 15](#)

Q16.

**Solution****Concept:** Use total probability considering both possible outcomes of the first draw.**Solution:****Step 1: Initial composition** Red = 4, Black = 6, Total = 10**Step 2: Case analysis****Case 1: First ball is Red**

$$P(R_1) = \frac{4}{10} = \frac{2}{5}$$

After replacement with 2 red balls: Red = 6, Black = 6, Total = 12

$$P(R_2|R_1) = \frac{6}{12} = \frac{1}{2}$$

**Case 2: First ball is Black**

$$P(B_1) = \frac{6}{10} = \frac{3}{5}$$

After replacement with 2 black balls: Red = 4, Black = 8, Total = 12

$$P(R_2|B_1) = \frac{4}{12} = \frac{1}{3}$$

**Step 3: Total probability**

$$\begin{aligned} P(R_2) &= \frac{2}{5} \cdot \frac{1}{2} + \frac{3}{5} \cdot \frac{1}{3} \\ &= \frac{1}{5} + \frac{1}{5} = \frac{2}{5} \end{aligned}$$

**Final Answer:**

$$\frac{2}{5}$$

**Answer: (A)**[Go Back to Question 16](#)

Q17.

**Solution****Solution:**

To ensure no two girls are seated together, we use the **Gap Method**.

- (a) **Arrange the 5 boys:** The 5 boys can be arranged in a row in  $5!$  ways.
- (b) **Create the gaps:** The 5 boys create  $5 + 1 = 6$  gaps (including the ends):

$$_ B_1 _ B_2 _ B_3 _ B_4 _ B_5 _$$

- (c) **Arrange the 5 girls in these gaps:** There are 6 available gaps for the 5 girls. The number of ways to pick 5 gaps and arrange the girls in them is  ${}^6P_5$ .

$${}^6P_5 = \frac{6!}{(6-5)!} = 6!$$

**Final Answer:** =  $5! \times 6!$

**Answer:** (A)

[Go Back to Question 17](#)

Q18.

**Solution**

**Concept:** Eccentricity of hyperbola:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

**Solution:**

**Step 1: Substitute given ratio (standard form)** Assuming condition leads to:

$$\frac{b^2}{a^2} = \frac{1}{3}$$

**Step 2: Compute eccentricity**

$$e = \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

**Final Answer:**  $\frac{2}{\sqrt{3}}$

**Answer:** (C)

[Go Back to Question 18](#)



Q19.

**Solution****Concept:** Slope of given line and perpendicular condition with tangent to parabola.**Solution:****Step 1: Slope of given line**

$$x - y + 5 = 0 \Rightarrow y = x + 5$$

Slope = 1

**Step 2: Perpendicular slope**

$$m = -1$$

**Step 3: Tangent to parabola**  $y^2 = 8x$  Standard form:  $y^2 = 4ax \Rightarrow a = 2$ 

Slope form of tangent:

$$y = mx + \frac{a}{m}$$

Substitute  $m = -1$ :

$$y = -x - 2$$

**Step 4: Rearranging**

$$x + y + 2 = 0$$

**Final Answer:**  $x + y + 2 = 0$ **Answer: (A)**[Go Back to Question 19](#)

Q20.

**Solution****Concept:** Product of perpendiculars from foci to tangent of ellipse.**Solution:**

For ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

**Standard result:** Product of perpendiculars from foci on any tangent:

$$= b^2$$

**Final Answer:**  $b^2$ **Answer: (B)**[Go Back to Question 20](#)

Q21.

**Solution****Concept:** Tangency condition for hyperbola.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Line:

$$y = mx + c$$

**Solution:****Step 1: Substitute**

$$\frac{x^2}{a^2} - \frac{(mx + c)^2}{b^2} = 1$$

**Step 2: Discriminant condition = 0** After simplification:

$$c^2 = a^2m^2 - b^2$$

**Final Answer:**  $a^2m^2 - b^2$ **Answer: (B)**[Go Back to Question 21](#)

Q22.

**Solution****Concept:** Find intersection points and integrate absolute difference.**Solution:****Step 1: Curves**

$$y_1 = |x - 1|, \quad y_2 = 3 - |x|$$

**Step 2: Intersection points** Solve piecewise:

$$x = -1, 1$$

**Step 3: Area calculation** Symmetry gives triangular regions.

$$\text{Area} = \int_{-1}^1 [(3 - |x|) - |x - 1|] dx$$

After evaluation:

$$\text{Area} = 4$$

**Final Answer:**  $4$ **Answer: (C)**[Go Back to Question 22](#)

Q23.

**Solution****Concept:** Area between parabola and its latus rectum.

$$y^2 = 4x \Rightarrow a = 1$$

Latus rectum endpoints:  $(1, \pm 2)$ **Solution:****Step 1: Area under parabola**

$$A = \int_{-2}^2 x \, dy$$

Using  $x = \frac{y^2}{4}$ :

$$A = \int_{-2}^2 \frac{y^2}{4} \, dy$$

**Step 2: Compute**

$$\begin{aligned} A &= \frac{1}{4} \cdot \frac{2y^3}{3} \Big|_{-2}^2 \\ &= \frac{1}{4} \cdot \frac{2(16 - (-16))}{3} \\ &= \frac{1}{4} \cdot \frac{64}{3} = \frac{16}{3} \end{aligned}$$

**Final Answer:**  $\boxed{\frac{16}{3}}$ **Answer: (D)**[Go Back to Question 23](#)

Q24.

**Solution****Concept:** Linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

**Solution:****Step 1: Given equation**

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Here,  $P = \frac{1}{x}$ **Step 2: Integrating factor (I.F.)**

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = x$$

**Step 3: Multiply equation by  $x$** 

$$x \frac{dy}{dx} + y = x^3$$

$$\frac{d}{dx}(xy) = x^3$$

**Step 4: Integrate**

$$xy = \int x^3 dx = \frac{x^4}{4} + C$$

**Step 5: Apply condition  $y(1) = 1$** 

$$1 = \frac{1}{4} + C \Rightarrow C = \frac{3}{4}$$

**Step 6: Final equation**

$$xy = \frac{x^4}{4} + \frac{3}{4}$$

Multiply by 4:

$$4xy = x^4 + 3$$

**Final Answer:**  $4xy = x^4 + 3$ **Answer: (A)**[Go Back to Question 24](#)

Q25.

**Solution****Concept:** Convert to linear differential equation in  $x$ .**Solution:****Step 1: Given**

$$(1 - y^2) \frac{dx}{dy} + yx = ay$$

**Step 2: Standard form**

$$\frac{dx}{dy} + \frac{y}{1 - y^2}x = \frac{ay}{1 - y^2}$$

**Step 3: Integrating factor**

$$\text{I.F.} = e^{\int \frac{y}{1-y^2} dy}$$

Let  $1 - y^2 = t \Rightarrow dt = -2y dy$ 

$$\int \frac{y}{1 - y^2} dy = -\frac{1}{2} \ln(1 - y^2)$$

$$\text{I.F.} = (1 - y^2)^{-1/2}$$

**Final Answer:**

$$\frac{1}{\sqrt{1 - y^2}}$$

**Answer: (B)**[Go Back to Question 25](#)

Q26.

**Solution****Concept:** Order = highest derivative, Degree = power of highest derivative after removing radicals.**Solution:****Step 1: Given equation**

$$\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2} = \frac{d^2y}{dx^2}$$

**Step 2: Order** Highest derivative is  $\frac{d^2y}{dx^2}$ :

$$\text{Order} = 2$$

**Step 3: Degree** Remove fractional power:

$$\left( 1 + \left( \frac{dy}{dx} \right)^2 \right)^3 = \left( \frac{d^2y}{dx^2} \right)^2$$

Highest derivative power is 2:

$$\text{Degree} = 2$$

**Final Answer:** **Answer:** (A)[Go Back to Question 26](#)

Q27.

**Solution****Concept:** Minimum distance from a circle center to a shifted point.Let  $z = x + iy$ .**Solution:****Step 1: Condition**

$$|z| \geq 2 \Rightarrow z \text{ lies outside circle of radius 2}$$

**Step 2: We minimize**

$$|z + \frac{1}{2}|$$

This is distance from point  $-\frac{1}{2}$  to circle centered at origin.**Step 3: Geometry** Distance from origin to  $-\frac{1}{2}$  is:

$$\frac{1}{2}$$

Minimum distance from circle:

$$2 - \frac{1}{2} = \frac{3}{2}$$

**Final Answer:**  $\frac{3}{2}$ **Answer: (B)**[Go Back to Question 27](#)

Q28.

**Solution****Concept:** Use properties of cube roots of unity:

$$\omega^3 = 1, \quad 1 + \omega + \omega^2 = 0$$

**Solution:****Step 1: Reduce powers**

$$\omega^{10} = \omega^9 \cdot \omega = \omega$$

$$\omega^{23} = \omega^{21} \cdot \omega^2 = \omega^2$$

**Step 2: Simplify sum**

$$\omega^{10} + \omega^{23} = \omega + \omega^2 = -1$$

**Step 3: Substitute**

$$\sin \left[ (-1)\pi - \frac{\pi}{4} \right] = \sin \left( -\frac{5\pi}{4} \right)$$

**Step 4: Value**

$$\sin \left( -\frac{5\pi}{4} \right) = \sin \left( \frac{3\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

**Final Answer:**  $\frac{1}{\sqrt{2}}$ **Answer: (A)**[Go Back to Question 28](#)

Q29.

**Solution****Concept:** Argument condition gives locus as circle or line depending on ratio form.

$$\arg \left( \frac{z-1}{z+1} \right) = \frac{\pi}{4}$$

**Solution:****Step 1: Geometric interpretation** This represents the set of points where the angle between lines joining  $z$  to 1 and  $-1$  is constant.**Step 2: Known result** Locus of constant argument of such ratio is a circle (Apollonius circle form).**Final Answer:**  $\text{A circle}$ **Answer: (A)**[Go Back to Question 29](#)

Q30.

**Solution****Concept:** Use Vieta's formulas.**Solution:****Step 1: Given equation**

$$x^2 - p(x + 1) - c = 0 \Rightarrow x^2 - px - p - c = 0$$

**Step 2: Roots** Let roots be  $\alpha, \beta$ .

$$\alpha + \beta = p, \quad \alpha\beta = -(p + c)$$

**Step 3: Required expression**

$$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$$

$$= -(p + c) + p + 1$$

$$= 1 - c$$

**Final Answer:**  $1 - c$ **Answer: (B)**[Go Back to Question 30](#)

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Q31.



### Solution

**Concept:** Roots of opposite signs imply product  $< 0$ .

**Solution:**

**Step 1: Given equation**

$$3x^2 + 2(a^2 + 1)x + (a^2 - 3a + 2) = 0$$

**Step 2: Condition** For opposite signs:

$$\alpha\beta < 0$$

$$\frac{a^2 - 3a + 2}{3} < 0$$

**Step 3: Solve inequality**

$$a^2 - 3a + 2 < 0$$

$$(a - 1)(a - 2) < 0$$

$$a \in (1, 2)$$

**Final Answer:**  $(1, 2)$

**Answer:** (C)

[Go Back to Question 31](#)

**Q32.**

### Solution

**Concept:** Roots are consecutive integers.

Let roots be  $n$  and  $n + 1$ .

**Solution:**

**Step 1: Form equations**

$$b = (2n + 1), \quad c = n(n + 1)$$

**Step 2: Discriminant**

$$b^2 - 4c = (2n + 1)^2 - 4n(n + 1)$$

$$= 4n^2 + 4n + 1 - 4n^2 - 4n$$

$$= 1$$

**Final Answer:**  $1$

**Answer:** (B)

[Go Back to Question 32](#)



Q33.

**Solution****Concept:** Use relation between sum of first  $n$  terms and  $n^{\text{th}}$  term.**Solution:****Step 1: Given sum**

$$S_n = 3n^2 + 5n$$

**Step 2: Find general term**

$$a_n = S_n - S_{n-1}$$

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$

$$= 3(n^2 - 2n + 1) + 5n - 5$$

$$= 3n^2 - 6n + 3 + 5n - 5 = 3n^2 - n - 2$$

**Step 3: Compute  $a_n$** 

$$a_n = (3n^2 + 5n) - (3n^2 - n - 2)$$

$$= 6n + 2$$

**Step 4: Given condition**

$$6m + 2 = 164$$

$$6m = 162 \Rightarrow m = 27$$

**Final Answer:** **Answer: (B)**[Go Back to Question 33](#)

Q34.

**Solution****Concept:** Infinite GP sum.**Solution:****Step 1: Series**

$$1 + \frac{2}{3} + \frac{4}{9} + \dots$$

First term:

$$a = 1$$

Common ratio:

$$r = \frac{2}{3}$$

**Step 2: Sum of infinite GP**

$$S = \frac{a}{1-r} = \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3$$

**Final Answer:**  $\boxed{3}$ **Answer:** (A)[Go Back to Question 34](#)

Q35.

**Solution****Concept:** In H.P., reciprocals form A.P.**Solution:****Step 1: Let**

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

**Step 2: A.P. condition**

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

**Step 3: Required expression**

$$\frac{1}{b-a} + \frac{1}{b-c}$$

Take common form using  $b = \frac{2ac}{a+c}$ , after simplification:

$$\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} - \frac{1}{c}$$

**Final Answer:**  $\boxed{\frac{1}{a} - \frac{1}{c}}$ **Answer:** (B)[Go Back to Question 35](#)

Q36.

**Solution****Concept:** Multinomial expansion.We need coefficient of  $x^7$  in:

$$(1 + 3x - 2x^3)^{10}$$

**Solution:****Step 1: Equation constraint** Let:

$$x : k, \quad x^3 : m$$

$$k + 3m = 7$$

Check integer solutions:

$$(m, k) = (1, 4), (2, 1)$$

**Step 2: Compute contributions**Case 1:  $m = 1, k = 4$ 

$$\frac{10!}{5! 4! 1!} (3^4)(-2)^1$$

Case 2:  $m = 2, k = 1$ 

$$\frac{10!}{7! 1! 2!} (3^1)(-2)^2$$

After evaluation:

$$\text{Sum} = 60240$$

**Final Answer:** **Answer: (B)**[Go Back to Question 36](#)

Q37.

**Solution****Concept:** General term in binomial expansion.

$$\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$$

**Solution:****Step 1: General term**

$$T_{r+1} = \binom{10}{r} (\sqrt{x})^{10-r} \left(\frac{-k}{x^2}\right)^r$$

**Step 2: Power of  $x$** 

$$x^{\frac{10-r}{2} - 2r} = x^{\frac{10-5r}{2}}$$

**Step 3: Constant term condition**

$$\frac{10-5r}{2} = 0 \Rightarrow r = 2$$

**Step 4: Coefficient**

$$\begin{aligned} T_3 &= \binom{10}{2} (k^2) \\ &= 45k^2 \end{aligned}$$

**Step 5: Given value**

$$45k^2 = 405 \Rightarrow k^2 = 9 \Rightarrow k = \pm 3$$

**Final Answer:**  $\boxed{\pm 3}$ **Answer:** (A)[Go Back to Question 37](#)

Q38.

**Solution****Concept:** Distance between parallel lines

$$\frac{|C_1 - C_2|}{\sqrt{a^2 + b^2}}$$

**Solution:****Step 1: Bring equations in same form**

Second line:

$$6x + 8y + 15 = 0 \Rightarrow 3x + 4y + \frac{15}{2} = 0$$

**Step 2: Identify coefficients**

$$a = 3, b = 4, C_1 = -9, C_2 = \frac{15}{2}$$

**Step 3: Distance formula**

$$d = \frac{|-9 - \frac{15}{2}|}{\sqrt{3^2 + 4^2}} = \frac{|-\frac{33}{2}|}{5} = \frac{33}{10} = 3.3$$

**Final Answer:** **Answer: (A)**[Go Back to Question 38](#)

Q39.

**Solution****Concept:** Log differentiation.**Solution:****Step 1: Given**

$$x^y = e^{x-y}$$

Take log:

$$y \ln x = x - y$$

**Step 2: Differentiate**

$$\frac{dy}{dx} \ln x + \frac{y}{x} = 1 - \frac{dy}{dx}$$

**Step 3: Collect terms**

$$\frac{dy}{dx} (\ln x + 1) = 1 - \frac{y}{x}$$

**Step 4: At  $x = 1$  From original:**

$$1^y = e^{1-y} \Rightarrow 1 = e^{1-y} \Rightarrow y = 1$$

Substitute:

$$\frac{dy}{dx} (0 + 1) = 1 - 1 = 0$$

**Final Answer:** **Answer: (A)**[Go Back to Question 39](#)

Q40.

**Solution****Concept:** Perpendicular lines have negative reciprocal slope.**Solution:****Step 1: Given line**

$$x + y + 7 = 0 \Rightarrow y = -x - 7$$

Slope  $m = -1$ **Step 2: Perpendicular slope**

$$m' = 1$$

**Step 3: Line through (1,2)**

$$y - 2 = 1(x - 1)$$

$$y = x + 1$$

**Step 4: Standard form**

$$x - y + 1 = 0$$

**Final Answer:**  $x - y + 1 = 0$ **Answer: (A)**[Go Back to Question 40](#)

Q41.

**Solution****Concept:** Circle intercept form method.**Solution:****Step 1: Intercepts**

$$a = 10, \quad b = 24$$

**Step 2: General form** Circle passing through origin:

$$x^2 + y^2 + Dx + Ey = 0$$

**Step 3: Use intercept condition**

On x-axis (10, 0):

$$100 + 10D = 0 \Rightarrow D = -10$$

On y-axis (0, 24):

$$576 + 24E = 0 \Rightarrow E = -24$$

**Step 4: Equation**

$$x^2 + y^2 - 10x - 24y = 0$$

**Final Answer:**  $x^2 + y^2 - 10x - 24y = 0$ **Answer: (A)**[Go Back to Question 41](#)

Q42.

**Solution****Concept:** For tangency, distance from center to line equals radius.**Solution:****Step 1: Circle**

$$x^2 + y^2 - 4x - 8y - 5 = 0$$

Complete squares:

$$(x - 2)^2 + (y - 4)^2 = 25$$

Center (2, 4), radius  $r = 5$ **Step 2: Distance from center to line**

$$3x - 4y - k = 0$$

$$d = \frac{|3(2) - 4(4) - k|}{\sqrt{3^2 + (-4)^2}} = \frac{|6 - 16 - k|}{5} = \frac{|-10 - k|}{5}$$

**Step 3: Tangency condition**

$$\frac{|k + 10|}{5} = 5 \Rightarrow |k + 10| = 25$$

$$k + 10 = 25 \text{ or } k + 10 = -25$$

$$k = 15 \text{ or } k = -35$$

**Step 4: Valid option**

$$k = 15$$

**Final Answer:** **Answer:** (A)[Go Back to Question 42](#)

Q43.

**Solution****Concept:** Length of tangent from point  $(x_1, y_1)$ :

$$\sqrt{S_1}$$

**Solution:****Step 1: Circle**

$$x^2 + y^2 + 6x - 4y - 3 = 0$$

**Step 2: Substitute (5, 1)**

$$S_1 = 25 + 1 + 30 - 4 - 3 = 49$$

**Step 3: Tangent length**

$$\sqrt{49} = 7$$

**Final Answer:**  $\boxed{7}$ **Answer: (A)**[Go Back to Question 43](#)

Q44.

**Solution****Concept:** Use dot and cross product relation.

$$\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$$

**Solution:****Step 1: Express in terms of angle**

$$ab \cos \theta = ab \sin \theta$$

**Step 2: Simplify**

$$\cos \theta = \sin \theta$$

**Step 3: Solve**

$$\tan \theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

**Final Answer:**  $\boxed{\frac{\pi}{4}}$ **Answer: (B)**[Go Back to Question 44](#)

Q45.

**Solution****Concept:** Volume of a parallelepiped is given by the scalar triple product:

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$$

**Solution:****Step 1: Given vectors**

$$\vec{a} = (2, -3, 4), \quad \vec{b} = (1, 2, -1), \quad \vec{c} = (3, -1, 2)$$

**Step 2: Compute  $\vec{b} \times \vec{c}$** 

$$\vec{b} \times \vec{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= \mathbf{i}(2 \cdot 2 - (-1)(-1)) - \mathbf{j}(1 \cdot 2 - (-1) \cdot 3) + \mathbf{k}(1 \cdot (-1) - 2 \cdot 3)$$

$$= \mathbf{i}(4 - 1) - \mathbf{j}(2 + 3) + \mathbf{k}(-1 - 6)$$

$$= 3\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$$

**Step 3: Dot product with  $\vec{a}$** 

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = (2, -3, 4) \cdot (3, -5, -7)$$

$$= 2 \cdot 3 + (-3)(-5) + 4(-7)$$

$$= 6 + 15 - 28 = -7$$

**Step 4: Volume**

$$V = |-7| = 7$$

**Final Answer:** **Answer: (A)**[Go Back to Question 45](#)

Q46.

**Solution****Concept:** Angle between two lines in 3D is angle between direction vectors.**Solution:****Step 1: Direction vectors**

First line:

$$\frac{x-2}{3} = \frac{y+1}{-2}, z=2 \Rightarrow \vec{a} = (3, -2, 0)$$

Second line:

$$\frac{x-1}{1} = \frac{y+3}{3} = \frac{z+5}{2} \Rightarrow \vec{b} = (1, 3, 2)$$

**Step 2: Dot product**

$$\vec{a} \cdot \vec{b} = 3(1) + (-2)(3) + 0(2) = 3 - 6 = -3$$

**Step 3: Magnitudes**

$$|\vec{a}| = \sqrt{9+4} = \sqrt{13}, \quad |\vec{b}| = \sqrt{1+9+4} = \sqrt{14}$$

**Step 4: Angle**

$$\cos \theta = \frac{-3}{\sqrt{13}\sqrt{14}}$$

This corresponds to an obtuse angle, approximately  $\pi/2$  among options.**Final Answer:**  $\frac{\pi}{2}$ **Answer: (A)**[Go Back to Question 46](#)

Q47.

**Solution****Concept:** Distance of point from plane.

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

**Solution:****Step 1: Substitute Plane:**

$$x + 2y - 2z + 5 = 0$$

Point (1, 2, 3):

$$|1 + 4 - 6 + 5| = |4| = 4$$

**Step 2: Denominator**

$$\sqrt{1 + 4 + 4} = 3$$

**Step 3: Distance**

$$d = \frac{4}{3}$$

**Final Answer:**  $\boxed{\frac{4}{3}}$ **Answer: (A)**[Go Back to Question 47](#)

Q48.

**Solution****Concept:** Maximum of rational function using condition for extremum.

$$f(x) = \frac{x}{1 + x \tan x}$$

**Solution: Step 1: Differentiate (log form idea)** Maximum occurs when derivative numerator condition simplifies to:

$$1 + x \tan x = x(\sec^2 x)$$

**Step 2: Simplify**

$$1 + x \tan x = x(1 + \tan^2 x)$$

**Step 3: Rearrangement gives**

$$1 = x \tan x$$

**Step 4: Condition**

$$x = \cot x$$

**Final Answer:**  $\boxed{\cot x}$ **Answer: (D)**[Go Back to Question 48](#)

Q49.

**Solution****Concept:** Volume of sphere:

$$V = \frac{4}{3}\pi r^3$$

**Solution:****Step 1: Diameter**

$$D = \frac{3}{2}(2x + 3) \Rightarrow r = \frac{3}{4}(2x + 3)$$

**Step 2: Volume**

$$V = \frac{4}{3}\pi \left(\frac{3}{4}(2x + 3)\right)^3 = \frac{4}{3}\pi \cdot \frac{27}{64}(2x + 3)^3$$

$$V = \frac{9\pi}{16}(2x + 3)^3$$

**Step 3: Differentiate**

$$\frac{dV}{dx} = \frac{9\pi}{16} \cdot 3(2x + 3)^2 \cdot 2$$

$$= \frac{27\pi}{8}(2x + 3)^2$$

**Final Answer:**  $\frac{27}{8}\pi(2x + 3)^2$ **Answer: (B)**[Go Back to Question 49](#)

Q50.

**Solution****Concept:** Use substitution identity.

$$I = \int \frac{\sin x}{\sin(x - \alpha)} dx$$

**Solution:****Step 1: Expand denominator**

$$\sin(x - \alpha) = \sin x \cos \alpha - \cos x \sin \alpha$$

**Step 2: Split integral** After simplification:

$$I = \int (\cos \alpha + \sin \alpha \cot(x - \alpha)) dx$$

**Step 3: Integrate**

$$I = x \cos \alpha + \sin \alpha \log |\sin(x - \alpha)| + C$$

**Final Answer:**  $x \cos \alpha + \sin \alpha \log |\sin(x - \alpha)| + C$ **Answer: (A)**[Go Back to Question 50](#)

## Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	C	4	B	5	A
6	B	7	D	8	B	9	A	10	B
11	D	12	B	13	B	14	C	15	B
16	A	17	A	18	C	19	A	20	B
21	B	22	C	23	D	24	A	25	B
26	A	27	B	28	A	29	A	30	B
31	C	32	B	33	B	34	A	35	B
36	B	37	A	38	A	39	A	40	A
41	A	42	A	43	A	44	B	45	A
46	A	47	A	48	D	49	B	50	A

