MHT-CET Mathematics Sample Paper-7

Duration: 90 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **50** Multiple Choice Questions.
- Each correct answer carries **+2 marks**.
- No negative marking for incorrect questions.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.
- No marks will be deducted for questions that are left unattempted.

Q1. If $f(x) = \frac{\cos x - \sin x}{\cos 2x}$ is continuous at $x = \frac{\pi}{4}$, then the value of $f\left(\frac{\pi}{4}\right)$ is:

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) $\frac{1}{2}$

(D) 2

Q2. The value of $\lim_{x \to 0} \dfrac{1 - \cos(1 - \cos x)}{x^4}$ is equal to:

(A) $\frac{1}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{16}$

Q3. If $f(x) = \log_x(\ln x)$, then $f'(e)$ is:

(A) 1

(B) e

(C) $1/e$

(D) 0

Q4. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, then $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$ is:

(A) $\frac{8\sqrt{2}}{a\pi}$

(B) $\frac{4\sqrt{2}}{a\pi}$

(C) $\frac{2}{a}$

(D) $\frac{a\pi}{4}$

Q5. The function $f(x) = x^x$ has a stationary point at x equal to:

(A) e

(B) $1/e$

(C) 1

(D) $\sqrt{e}$

Q6. The equation of the tangent to the curve $y = be^{-x/a}$ at the point where it crosses the y-axis is:

(A) $\frac{x}{a} + \frac{y}{b} = 1$

(B) $ax + by = 1$

(C) $ax - by = 1$

(D) $\frac{x}{a} - \frac{y}{b} = 1$

Q7. The value of $\int \frac{1}{\cos^6 x + \sin^6 x} dx$ is:

(A) $\tan^{-1}(\tan x - \cot x) + C$

(B) $\sin^{-1}(\sin x + \cos x) + C$

(C) $\tan^{-1}(\tan x + \cot x) + C$

(D) $\cos^{-1}(\sin 2x) + C$

Q8. The area bounded by the curve $y = \log_e x$, x-axis and the ordinate $x = e$ is:

(A) e sq. units

(B) 1 sq. unit

(C) $e - 1$ sq. units

(D) $1 - 1/e$ sq. units

Q9. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are respectively:

(A) $2, 2$

(B) $2, 3$

(C) $1, 2$

(D) $2, 1$

Q10. The value of $(1 + i)^{10} + (1 - i)^{10}$ is:

(A) $64i$

(B) 0

(C) 128

(D) -128

Q11. If α, β are the roots of $x^2 - p(x + 1) - c = 0$, then $(\alpha + 1)(\beta + 1)$ is equal to:

(A) $1 - c$

(B) $1 + c$

(C) c

(D) $p - c$

Q12. In an A.P., if m^{th} term is $1/n$ and n^{th} term is $1/m$, then the sum of mn terms is:

(A) $\frac{1}{2}(mn - 1)$

(B) $\frac{1}{2}(mn + 1)$

(C) $mn + 1$

(D) $mn - 1$

Q13. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is:

(A) ${}^nC_4$

(B) ${}^nC_4 +^n C_2$

(C) ${}^nC_4 +^n C_1 \cdot^n C_2 +^n C_2$

(D) ${}^nC_4 +^n C_3 +^n C_2$

Q14. The probability that a leap year selected at random contains 53 Sundays is:

(A) $1/7$

(B) $2/7$

(C) $53/366$

(D) $1/366$

Q15. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is:

(A) $3/10$

(B) $3/2$

(C) 6

(D) $1/2$

Q16. The number of real solutions of the equation $e^x + x - 1 = 0$ is:

(A) 0

(B) 1

(C) 2

(D) Infinitely many

Q17. If the area of the region bounded by $y^2 = 4x$ and $x^2 = 4y$ is k, then the value of $3k$ is:

(A) 16

(B) 8

(C) 32

(D) 4

Q18. The general solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^x = e^y + \frac{x^3}{3} + C$

(C) $e^y = e^x + x^3 + C$

(D) $e^x + e^y = \frac{x^3}{3} + C$

Q19. The argument of the complex number $z = \frac{1+i}{1-i}$ is:

(A) 0

(B) π

(C) $\pi/2$

(D) $-\pi/2$

Q20. If the sum of the roots of $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then bc^2, ca^2, ab^2 are in:

(A) A.P.

(B) G.P.

(C) H.P.

(D) None of these

Q21. The n^{th} term of the series $3, 7, 13, 21, \ldots$ is:

(A) $n^2 + n + 1$

(B) $n^2 - n + 3$

(C) $n^2 + n + 3$

(D) $2n^2 + 1$

Q22. The number of terms in the expansion of $(1 + 2x + x^2)^{20}$ is:

(A) 21

(B) 41

(C) 40

(D) 20

Q23. A box contains 3 red and 7 white balls. One ball is drawn at random and its color is noted. It is not put back. Then another ball is drawn. The probability that both balls are red is:

(A) $1/15$

(B) $9/100$

(C) $1/10$

(D) $3/10$

Q24. The angle between the lines $x + y - 3 = 0$ and $x - y + 3 = 0$ is:

(A) $0°$

(B) $45°$

(C) $60°$

(D) $90°$

Q25. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:

(A) 12

(B) 5

(C) $\sqrt{13}$

(D) 25

Q26. The eccentricity of the hyperbola $x^2 - y^2 = 16$ is:

(A) $\sqrt{2}$

(B) 2

(C) $1/2$

(D) 4

Q27. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 0°

(B) 45°

(C) 90°

(D) 180°

Q28. The distance of the point $(1, 2, 3)$ from the origin is:

(A) $\sqrt{6}$

(B) $\sqrt{14}$

(C) 6

(D) 14

Q29. If $f(x) = |x-1| + |x-2|$, then $f'(1.5)$ is:

(A) 1

(B) -1

(C) 0

(D) 2

Q30. The value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ is:

(A) π

(B) $\pi/2$

(C) $\pi/4$

(D) 0

Q31. The length of the latus rectum of the parabola $y^2 = 12x$ is:

(A) 3

(B) 6

(C) 12

(D) 4

Q32. The probability of getting a sum of 9 from two throws of a dice is:

(A) $1/6$

(B) $1/8$

(C) $1/9$

(D) $1/12$

Q33. If $z = 1 + i\sqrt{3}$, then $|z|$ is:

(A) 1

(B) 2

(C) 4

(D) $\sqrt{2}$

Q34. The sum of the infinite G.P. $1, 1/3, 1/9, \ldots$ is:

(A) $3/2$

(B) $2/3$

(C) $1/2$

(D) 3

Q35. The value of ${}^{n}C_{r} +^{n} C_{r-1}$ is:

(A) ${}^{n+1}C_{r}$

(B) ${}^{n}C_{r+1}$

(C) ${}^{n+1}C_{r-1}$

(D) ${}^{n-1}C_{r}$

Q36. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

(A) 3

(B) $1/3$

(C) -3

(D) $-1/3$

Q37. The value of $\int e^x(\tan x + \sec^2 x)dx$ is:

(A) $e^x \sec x + C$

(B) $e^x \tan x + C$

(C) $e^x \sec^2 x + C$

(D) $e^x \cot x + C$

Q38. The integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = \sec x$ is:

(A) $\tan x$

(B) $\sec x$

(C) $\cos x$

(D) $\sin x$

Q39. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then $|\vec{a} - \vec{b}|$ is:

(A) $2\cos(\theta/2)$

(B) $2\sin(\theta/2)$

(C) $\sin(\theta/2)$

(D) $\cos(\theta/2)$

Q40. The distance of the point $(3, 4, 5)$ from the x-axis is:

(A) 3

(B) 5

(C) $\sqrt{41}$

(D) $\sqrt{34}$

Q41. The value of $\int_{-1}^{1} |x|dx$ is:

(A) 0

(B) 1

(C) 2

(D) 1/2

Q42. If α and β are the roots of $x^2 - 5x + 6 = 0$, the equation whose roots are $1/\alpha$ and $1/\beta$ is:

(A) $6x^2 - 5x + 1 = 0$

(B) $x^2 - 5x + 1 = 0$

(C) $6x^2 + 5x + 1 = 0$

(D) $x^2 + 5x + 6 = 0$

Q43. The projection of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $8/\sqrt{6}$

(B) $10/\sqrt{6}$

(C) $5/\sqrt{3}$

(D) $4/\sqrt{6}$

Q44. The distance between the planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is:

(A) $2/\sqrt{29}$

(B) $4/\sqrt{29}$

(C) $8/\sqrt{29}$

(D) $1/2$

Q45. If A and B are independent events such that $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cup B)$ is:

(A) 0.7

(B) 0.12

(C) 0.58

(D) 0.82

Q46. The area of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is:

(A) 6π

(B) 13π

(C) 36π

(D) 5π

Q47. The value of $\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$ is:

(A) 0

(B) 1

(C) 2

(D) e

Q48. If the sum of n terms of an A.P. is $3n^2 + n$, then its common difference is:

(A) 3

(B) 6

(C) 9

(D) 4

Q49. The length of the intercept made by the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ on the x-axis is:

(A) 8

(B) 10

(C) 4

(D) 6

Q50. The number of ways in which 5 people can be seated around a circular table is:

(A) 120

(B) 24

(C) 60

(D) 10

Detailed Solutions

Q1.

Solution

Concept:

A function $f(x)$ is continuous at $x = a$ if $\lim_{x\to a} f(x) = f(a)$. For the given trigonometric limit, we use the identity $\cos 2x = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)$ to remove the indeterminate form.

Solution:

Step 1: Write the limit expression:

$$\lim_{x\to\pi/4} \frac{\cos x - \sin x}{\cos 2x}$$

Step 2: Expand the denominator using the double angle formula:

$$\lim_{x\to\pi/4} \frac{\cos x - \sin x}{(\cos x - \sin x)(\cos x + \sin x)}$$

Step 3: Cancel the common factor $(\cos x - \sin x)$ from numerator and denominator:

$$\lim_{x\to\pi/4} \frac{1}{\cos x + \sin x}$$

Step 4: Substitute $x = \pi/4$:

$$= \frac{1}{\cos(\pi/4) + \sin(\pi/4)} = \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}$$

$$= \frac{1}{\frac{2}{\sqrt{2}}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Since the function is continuous, $f(\pi/4)$ must equal this limit value.

Final Answer: $\boxed{1/\sqrt{2}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept:

This problem involves nested trigonometric limits. We use the standard result $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$. By substituting $\theta = 1 - \cos x$, we can break down the expression into manageable parts.

Solution:

Step 1: Identify the outer limit structure. Let $\theta = 1 - \cos x$. As $x \to 0$, $\theta \to 0$. The expression becomes:

$$\frac{1-\cos\theta}{x^4} = \frac{1-\cos\theta}{\theta^2} \cdot \frac{\theta^2}{x^4}$$

Step 2: Apply the limit to the first part:

$$\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$$

Step 3: Analyze the second part $\frac{\theta^2}{x^4}$:

$$\frac{(1-\cos x)^2}{x^4} = \left(\frac{1-\cos x}{x^2}\right)^2$$

Step 4: Apply the limit to the inner part:

$$\lim_{x\to 0} \left(\frac{1-\cos x}{x^2}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Step 5: Multiply the results from Step 2 and Step 4:

$$\text{Total Limit} = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

Final Answer: $\boxed{1/8}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept:

To differentiate $f(x) = \log_x(\ln x)$, we first use the base change formula for logarithms: $\log_a b = \frac{\ln b}{\ln a}$. Then we apply the quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$.

Solution:

Step 1: Rewrite the function using natural logarithms:

$$f(x) = \frac{\ln(\ln x)}{\ln x}$$

Step 2: Differentiate using the quotient rule where $u = \ln(\ln x)$ and $v = \ln x$:

$$u' = \frac{1}{\ln x} \cdot \frac{1}{x}, \quad v' = \frac{1}{x}$$

$$f'(x) = \frac{(\ln x)\left(\frac{1}{x \ln x}\right) - \ln(\ln x)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

Step 3: Simplify the numerator:

$$f'(x) = \frac{\frac{1}{x} - \frac{\ln(\ln x)}{x}}{(\ln x)^2} = \frac{1 - \ln(\ln x)}{x(\ln x)^2}$$

Step 4: Evaluate at $x = e$:

$$f'(e) = \frac{1 - \ln(\ln e)}{e(\ln e)^2}$$

Since $\ln e = 1$ and $\ln 1 = 0$:

$$f'(e) = \frac{1 - 0}{e(1)^2} = \frac{1}{e}$$

Final Answer: $\boxed{1/e}$

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept:

For parametric equations $x(t)$ and $y(t)$, the first derivative is $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. The second derivative is $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$.

Solution:

Step 1: Find dx/dt and dy/dt:

$$\frac{dx}{dt} = a(-\sin t + \sin t + t\cos t) = at\cos t$$

$$\frac{dy}{dt} = a(\cos t - (\cos t - t\sin t)) = at\sin t$$

Step 2: Find the first derivative:

$$\frac{dy}{dx} = \frac{at\sin t}{at\cos t} = \tan t$$

Step 3: Find the second derivative:

$$\frac{d^2y}{dx^2} = \frac{d}{dt}(\tan t) \cdot \frac{1}{dx/dt} = \sec^2 t \cdot \frac{1}{at\cos t} = \frac{\sec^3 t}{at}$$

Step 4: Evaluate at $t = \pi/4$:

$$\sec(\pi/4) = \sqrt{2} \implies \sec^3(\pi/4) = (\sqrt{2})^3 = 2\sqrt{2}$$

$$\frac{d^2y}{dx^2} = \frac{2\sqrt{2}}{a(\pi/4)} = \frac{8\sqrt{2}}{a\pi}$$

Final Answer: $\boxed{\frac{8\sqrt{2}}{a\pi}}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept:
A stationary point occurs where the first derivative $f'(x) = 0$. For a function of the form $y = x^x$, we use logarithmic differentiation to find the derivative.

Solution:
Step 1: Let $y = x^x$. Take natural logs on both sides:

$$\ln y = x \ln x$$

Step 2: Differentiate both sides with respect to x:

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y(\ln x + 1) = x^x(1 + \ln x)$$

Step 3: Set the derivative to zero to find the stationary point:

$$x^x(1 + \ln x) = 0$$

Since x^x is never zero for $x > 0$, we solve:

$$1 + \ln x = 0$$

Step 4: Solve for x:

$$\ln x = -1 \implies x = e^{-1} = \frac{1}{e}$$

The function has a stationary point (specifically a local minimum) at $x = 1/e$.

Final Answer: $\boxed{1/e}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept:

The tangent to a curve at a point (x_1, y_1) is given by $y - y_1 = m(x - x_1)$, where m is the derivative dy/dx at that point. A curve crosses the y-axis when $x = 0$.

Solution:

Step 1: Find the point of intersection with the y-axis. Set $x = 0$ in the equation $y = be^{-x/a}$:

$$y = be^0 = b$$

So, the point is $(0, b)$.

Step 2: Find the derivative dy/dx:

$$\frac{dy}{dx} = b \cdot e^{-x/a} \cdot \left(-\frac{1}{a}\right) = -\frac{b}{a}e^{-x/a}$$

Step 3: Evaluate the slope m at $(0, b)$:

$$m = \left.\frac{dy}{dx}\right|_{x=0} = -\frac{b}{a}e^0 = -\frac{b}{a}$$

Step 4: Form the equation of the tangent:

$$y - b = -\frac{b}{a}(x - 0)$$

$$y - b = -\frac{b}{a}x$$

Divide the entire equation by b:

$$\frac{y}{b} - 1 = -\frac{x}{a} \implies \frac{x}{a} + \frac{y}{b} = 1$$

Final Answer: $\boxed{\frac{x}{a} + \frac{y}{b} = 1}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:

To integrate expressions involving $\sin^6 x$ and $\cos^6 x$, we use the algebraic identity $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$. Here, let $a = \sin^2 x$ and $b = \cos^2 x$.

Solution:

Step 1: Simplify the denominator:

$$\sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)$$

Since $\sin^2 x + \cos^2 x = 1$:

$$= (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x = 1 - 3\sin^2 x \cos^2 x$$

Step 2: Rewrite the integral:

$$\int \frac{dx}{1 - 3\sin^2 x \cos^2 x} = \int \frac{\sec^4 x dx}{\sec^4 x - 3\tan^2 x}$$

Multiply numerator and denominator by $\sec^4 x$:

$$= \int \frac{(1+\tan^2 x)\sec^2 x dx}{(1+\tan^2 x)^2 - 3\tan^2 x} = \int \frac{(1+\tan^2 x)\sec^2 x dx}{\tan^4 x - \tan^2 x + 1}$$

Step 3: Substitute $t = \tan x$, then $dt = \sec^2 x dx$:

$$\int \frac{1+t^2}{t^4 - t^2 + 1} dt = \int \frac{1 + 1/t^2}{t^2 - 1 + 1/t^2} dt$$

Step 4: Let $u = t - 1/t$, then $du = (1 + 1/t^2)dt$:

$$\int \frac{du}{u^2+1} = \tan^{-1}(u) + C = \tan^{-1}(t - 1/t) + C$$

Substituting back $t = \tan x$:

$$= \tan^{-1}(\tan x - \cot x) + C$$

Final Answer: $\boxed{\tan^{-1}(\tan x - \cot x) + C}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept:

The area under a curve $y = f(x)$ from $x = a$ to $x = b$ is given by $\int_a^b y\,dx$. The curve $y = \ln x$ intersects the x-axis where $\ln x = 0$, which is $x = 1$.

Solution:

Step 1: Determine the limits of integration. The region is bounded by $x = 1$ (where it hits the axis) and $x = e$.

Step 2: Set up the definite integral:

$$\text{Area} = \int_1^e \ln x\,dx$$

Step 3: Use integration by parts $\int u\,dv = uv - \int v\,du$. Let $u = \ln x$ and $dv = dx$. Then $du = (1/x)dx$ and $v = x$.

$$\int \ln x\,dx = x\ln x - \int x \cdot \frac{1}{x}dx = x\ln x - x$$

Step 4: Evaluate from 1 to e:

$$[x\ln x - x]_1^e = (e\ln e - e) - (1\ln 1 - 1)$$

Since $\ln e = 1$ and $\ln 1 = 0$:

$$= (e - e) - (0 - 1) = 0 + 1 = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept:

The **order** of a differential equation is the order of the highest derivative present. The **degree** is the power of the highest order derivative, provided the equation is a polynomial in its derivatives (no fractional powers or radicals).

Solution:

Step 1: Identify the highest derivative. In the equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$, the highest derivative is $\frac{d^2y}{dx^2}$. Thus, **Order = 2**.

Step 2: Rationalize the equation to find the degree. Square both sides to remove the $3/2$ power:

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Step 3: Identify the power of the highest order derivative. The highest order derivative is $\frac{d^2y}{dx^2}$ and its power is 2. Thus, **Degree = 2**.

Final Answer: $\boxed{2, 2}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept:

To solve powers of complex numbers, we simplify the base first. $(1 \pm i)^2$ results in purely imaginary numbers, which makes high powers easier to calculate.

Solution:

Step 1: Calculate $(1+i)^2$ and $(1-i)^2$:

$$(1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$$

$$(1-i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$$

Step 2: Rewrite the expression $(1+i)^{10} + (1-i)^{10}$ using these squares:

$$((1+i)^2)^5 + ((1-i)^2)^5 = (2i)^5 + (-2i)^5$$

Step 3: Expand the powers:

$$(2i)^5 = 2^5 \cdot i^5 = 32 \cdot (i^4 \cdot i) = 32i$$

$$(-2i)^5 = (-2)^5 \cdot i^5 = -32 \cdot i = -32i$$

Step 4: Add the results:

$$32i + (-32i) = 0$$

Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept:
For a quadratic equation $ax^2 + bx + c = 0$, the sum of roots is $-b/a$ and the product of roots is c/a. We first rearrange the given equation into standard form to identify these coefficients.

Solution:
Step 1: Rewrite the equation $x^2 - p(x + 1) - c = 0$ in standard form:

$$x^2 - px - (p + c) = 0$$

Here, $a = 1$, $b = -p$, and constant term $= -(p + c)$.
Step 2: Find the sum and product of roots α and β:

$$\alpha + \beta = -(-p)/1 = p$$

$$\alpha\beta = -(p + c)/1 = -p - c$$

Step 3: Expand the required expression $(\alpha + 1)(\beta + 1)$:

$$(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$$

Step 4: Substitute the values from Step 2:

$$= (-p - c) + (p) + 1$$

$$= -p - c + p + 1 = 1 - c$$

Final Answer: $\boxed{1 - c}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept:

In an Arithmetic Progression (A.P.), the n^{th} term is $a_n = a + (n-1)d$. We use the two given conditions to solve for the first term a and the common difference d, then apply the sum formula $S_k = \frac{k}{2}[2a + (k-1)d]$.

Solution:

Step 1: Write the equations for the m^{th} and n^{th} terms:

$$a + (m-1)d = \frac{1}{n} \quad \text{— (i)}$$

$$a + (n-1)d = \frac{1}{m} \quad \text{— (ii)}$$

Step 2: Subtract (ii) from (i):

$$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

$$d = \frac{1}{mn}$$

Step 3: Substitute d into (i) to find a:

$$a + (m-1)\frac{1}{mn} = \frac{1}{n} \implies a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \implies a = \frac{1}{mn}$$

Step 4: Find the sum of mn terms (S_{mn}):

$$S_{mn} = \frac{mn}{2}\left[2\left(\frac{1}{mn}\right) + (mn-1)\frac{1}{mn}\right]$$

$$S_{mn} = \frac{mn}{2}\left[\frac{2 + mn - 1}{mn}\right] = \frac{mn}{2}\left[\frac{mn+1}{mn}\right] = \frac{mn+1}{2}$$

Final Answer: $\boxed{\frac{1}{2}(mn+1)}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept:
The expression $(1 + x + x^2 + x^3)^n$ can be simplified using the geometric series sum formula or by factoring. Since $1 + x + x^2 + x^3 = (1 + x)(1 + x^2)$, we can use the binomial theorem on each part.

Solution:
Step 1: Factor the expression:

$$(1 + x + x^2 + x^3)^n = [(1 + x)(1 + x^2)]^n = (1 + x)^n(1 + x^2)^n$$

Step 2: Write the general expansion for both:

$$(1 + x)^n = \sum_{r=0}^{n} {}^nC_r x^r = 1 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + {}^nC_4x^4 + \dots$$

$$(1 + x^2)^n = \sum_{k=0}^{n} {}^nC_k(x^2)^k = 1 + {}^nC_1x^2 + {}^nC_2x^4 + \dots$$

Step 3: Identify combinations that result in x^4:
1. $({}^nC_4x^4)$ from first and (1) from second $\rightarrow {}^nC_4$ 2. $({}^nC_2x^2)$ from first and $({}^nC_1x^2)$ from second $\rightarrow {}^nC_2 \cdot {}^nC_1$ 3. (1) from first and $({}^nC_2x^4)$ from second $\rightarrow {}^nC_2$
Step 4: Sum the coefficients: Coefficient of $x^4 = {}^nC_4 + {}^nC_1 \cdot {}^nC_2 + {}^nC_2$
Final Answer: $\boxed{{}^nC_4 +^n C_1 \cdot^n C_2 +^n C_2}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept:
A leap year has 366 days. Since a week has 7 days, we divide 366 by 7 to find the number of complete weeks and the remaining extra days. The probability depends solely on these extra days.

Solution:
Step 1: Calculate the number of full weeks in 366 days:

$$366 \div 7 = 52 \text{ weeks and 2 days remainder}$$

Step 2: Analyze the 52 weeks. These 52 weeks guarantee 52 Sundays. To have 53 Sundays, one of the 2 extra days must be a Sunday.
Step 3: List the possible pairs for the 2 extra days:
1. (Monday, Tuesday) 2. (Tuesday, Wednesday) 3. (Wednesday, Thursday) 4. (Thursday, Friday) 5. (Friday, Saturday) 6. (Saturday, Sunday) 7. (Sunday, Monday)
Step 4: Identify the favorable outcomes. Out of the 7 possibilities, the pairs (Saturday, Sunday) and (Sunday, Monday) contain a Sunday. Number of favorable cases = 2. Total possible cases = 7.

$$\text{Probability} = \frac{2}{7}$$

Final Answer: $\boxed{2/7}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept:
The distance d between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by the formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$. We must first ensure the coefficients of x and y are identical for both lines.
Solution:
Step 1: Write the lines in a comparable form. Line 1: $3x + 4y - 9 = 0$ Line 2: $6x + 8y - 15 = 0$
Step 2: Divide Line 2 by 2 so that the x and y coefficients match Line 1:

$$\frac{6x}{2} + \frac{8y}{2} - \frac{15}{2} = 0 \implies 3x + 4y - 7.5 = 0$$

Step 3: Identify the parameters: $A = 3$, $B = 4$, $C_1 = -9$, $C_2 = -7.5$ (or $-15/2$).
Step 4: Apply the distance formula:

$$d = \frac{|-9 - (-7.5)|}{\sqrt{3^2 + 4^2}} = \frac{|-9 + 7.5|}{\sqrt{9 + 16}}$$

$$d = \frac{|-1.5|}{\sqrt{25}} = \frac{1.5}{5} = \frac{3/2}{5} = \frac{3}{10}$$

Final Answer: $\boxed{3/10}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept:
To find the number of real solutions for an equation like $f(x) = 0$, we analyze the behavior of the function $f(x)$ using its derivative to check for monotonicity (increasing or decreasing nature).
Solution:
Step 1: Define the function $f(x) = e^x + x - 1$. Step 2: Find the derivative of the function:

$$f'(x) = \frac{d}{dx}(e^x + x - 1) = e^x + 1$$

Step 3: Analyze the sign of $f'(x)$. Since e^x is always positive for all real x, $e^x + 1 > 0$ for all real x. This means $f(x)$ is a strictly increasing function throughout its domain. Step 4: Check the values at specific points or limits: As $x \to -\infty$, $f(x) \to -\infty$. As $x \to \infty$, $f(x) \to \infty$. Since the function is continuous and strictly increasing from $-\infty$ to ∞, it must cross the x-axis exactly once. Step 5: Identify the root by inspection: $f(0) = e^0 + 0 - 1 = 1 + 0 - 1 = 0$. So, $x = 0$ is the unique real solution.
Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept:

The area bounded between two parabolas $y^2 = 4ax$ and $x^2 = 4by$ is given by the standard formula Area $= \frac{16ab}{3}$. This is derived by finding the points of intersection and integrating the difference between the upper and lower curves.

Solution:

Step 1: Identify the parameters a and b. For $y^2 = 4x$, $4a = 4 \implies a = 1$. For $x^2 = 4y$, $4b = 4 \implies b = 1$.

Step 2: Use the standard area formula:

$$k = \frac{16(1)(1)}{3} = \frac{16}{3}$$

Step 3: Calculate the required value $3k$:

$$3k = 3 \times \frac{16}{3} = 16$$

Alternative Step: If the formula is not remembered, intersect the curves: $x^2 = 4(\sqrt{4x}/2)$ leads to $x = 0, 4$. Integrate $\int_0^4 (\sqrt{4x} - x^2/4)dx$ to get the same result.

Final Answer: $\boxed{16}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:
This is a first-order differential equation. We can use the method of separation of variables by factoring out terms that depend only on y and terms that depend only on x.

Solution:
Step 1: Rewrite the differential equation:

$$\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y}$$

Step 2: Factor out e^{-y} on the right-hand side:

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

Step 3: Separate the variables y and x:

$$\frac{dy}{e^{-y}} = (e^x + x^2)dx \implies e^y dy = (e^x + x^2)dx$$

Step 4: Integrate both sides:

$$\int e^y dy = \int (e^x + x^2)dx$$

$$e^y = e^x + \frac{x^3}{3} + C$$

This matches the structure in Option A.

Final Answer: $\boxed{e^y = e^x + \frac{x^3}{3} + C}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept:

The argument of a complex number $z = x + iy$ is $\theta = \tan^{-1}(y/x)$. For a quotient of complex numbers $z = z_1/z_2$, the argument is $\arg(z) = \arg(z_1) - \arg(z_2)$.

Solution:

Step 1: Simplify z by rationalizing the denominator:

$$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1^2 - i^2}$$

Step 2: Expand the numerator and denominator:

$$z = \frac{1+2i+i^2}{1-(-1)} = \frac{1+2i-1}{2} = \frac{2i}{2} = i$$

Step 3: Express i in the form $x+iy$: $z = 0+1i$. Here $x = 0$, $y = 1$. Step 4: Find the argument: Since the point lies on the positive y-axis, the argument is 90° or $\pi/2$. Alternatively, $\arg(1+i) = \pi/4$ and $\arg(1-i) = -\pi/4$. $\arg(z) = \pi/4 - (-\pi/4) = \pi/4 + \pi/4 = \pi/2$.

Final Answer: $\boxed{\pi/2}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept:

For roots α, β, we use $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$. The condition given is $(\alpha + \beta) = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

Solution:

Step 1: Write the given condition:

$$\alpha + \beta = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

Step 2: Substitute the sum and product formulas:

$$-\frac{b}{a} = \frac{(-b/a)^2 - 2(c/a)}{(c/a)^2} = \frac{(b^2/a^2) - (2c/a)}{c^2/a^2}$$

Step 3: Simplify the right side:

$$-\frac{b}{a} = \frac{b^2 - 2ac}{c^2} \implies -bc^2 = ab^2 - 2a^2c$$

Step 4: Rearrange the terms:

$$2a^2c = ab^2 + bc^2$$

Divide by 2: $a^2c = \frac{ab^2+bc^2}{2}$. This is the condition for bc^2, a^2c, ab^2 to be in Arithmetic Progression (A.P.).

Final Answer: $\boxed{A.P.}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:

To find the n^{th} term of a series where the differences between consecutive terms form an Arithmetic Progression (A.P.), we use the method of differences or assume a general quadratic form $T_n = an^2 + bn + c$.

Solution:

Step 1: Observe the differences between consecutive terms: $7 - 3 = 4$ $13 - 7 = 6$ $21 - 13 = 8$ The differences are $4, 6, 8, \ldots$, which are in A.P. This implies the n^{th} term is a quadratic in n.

Step 2: Let $T_n = an^2 + bn + c$. Substitute $n = 1, 2, 3$:

1. $a(1)^2 + b(1) + c = 3 \implies a + b + c = 3$ 2. $a(2)^2 + b(2) + c = 7 \implies 4a + 2b + c = 7$ 3. $a(3)^2 + b(3) + c = 13 \implies 9a + 3b + c = 13$

Step 3: Solve the system of equations: Subtract (1) from (2): $3a + b = 4$ Subtract (2) from (3): $5a + b = 6$ Subtract these results: $2a = 2 \implies a = 1$. Substitute $a = 1$ into $3a + b = 4 \implies 3(1) + b = 4 \implies b = 1$. Substitute $a = 1, b = 1$ into $a + b + c = 3 \implies 1 + 1 + c = 3 \implies c = 1$.

Step 4: Form the n^{th} term: $T_n = 1n^2 + 1n + 1 = n^2 + n + 1$.

Final Answer: $\boxed{n^2 + n + 1}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept:

The number of terms in the expansion of $(x + y)^n$ is $n + 1$. However, if the base expression can be simplified into a single binomial, we must simplify it first to avoid overcounting terms.

Solution:

Step 1: Analyze the base expression $1 + 2x + x^2$. Recognize that this is a perfect square:

$$1 + 2x + x^2 = (1 + x)^2$$

Step 2: Substitute this back into the original expression:

$$(1 + 2x + x^2)^{20} = ((1 + x)^2)^{20}$$

Step 3: Apply the power of a power rule $(a^m)^n = a^{mn}$:

$$= (1 + x)^{40}$$

Step 4: Determine the number of terms. The expansion of a binomial $(a + b)^N$ has $N + 1$ terms. Here $N = 40$. Number of terms $= 40 + 1 = 41$.

Final Answer: $\boxed{41}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept:
This is a probability problem involving dependent events (sampling without replacement). The probability of two events A and B occurring is $P(A \cap B) = P(A) \cdot P(B|A)$.

Solution:
Step 1: Define the total number of balls. Total balls = 3 (Red) + 7 (White) = 10 balls.
Step 2: Find the probability that the first ball is red ($P(R_1)$):

$$P(R_1) = \frac{\text{Number of Red balls}}{\text{Total balls}} = \frac{3}{10}$$

Step 3: Find the probability that the second ball is red given the first was red ($P(R_2|R_1)$). Since the ball was not put back, there are now 9 balls left in the box, and only 2 of them are red.

$$P(R_2|R_1) = \frac{2}{9}$$

Step 4: Calculate the joint probability:

$$P(R_1 \cap R_2) = \frac{3}{10} \times \frac{2}{9}$$

$$= \frac{6}{90} = \frac{1}{15}$$

Final Answer: $\boxed{1/15}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept:

The angle θ between two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$. If $m_1m_2 = -1$, the lines are perpendicular (90°).

Solution:

Step 1: Find the slope of the first line $x + y - 3 = 0$:

$$y = -x + 3 \implies m_1 = -1$$

Step 2: Find the slope of the second line $x - y + 3 = 0$:

$$y = x + 3 \implies m_2 = 1$$

Step 3: Check the product of the slopes:

$$m_1 \cdot m_2 = (-1) \cdot (1) = -1$$

Step 4: Determine the angle. Since the product of the slopes is -1, the two lines are perpendicular to each other. Therefore, the angle $\theta = 90°$.

Final Answer: $\boxed{90°}$

Answer: (D)

Go Back to Question 24

Q25.

Solution

Concept:

The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r is calculated using the formula $r = \sqrt{g^2 + f^2 - c}$.

Solution:

Step 1: Identify the coefficients from the given equation $x^2 + y^2 - 4x + 6y - 12 = 0$: $2g = -4 \implies g = -2$ $2f = 6 \implies f = 3$ $c = -12$

Step 2: Plug the values into the radius formula:

$$r = \sqrt{(-2)^2 + (3)^2 - (-12)}$$

Step 3: Simplify the expression:

$$r = \sqrt{4 + 9 + 12}$$

$$r = \sqrt{25}$$

Step 4: Calculate the final value:

$$r = 5$$

Final Answer: $\boxed{5}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept:
For a rectangular hyperbola of the form $x^2 - y^2 = a^2$, the lengths of the transverse and conjugate axes are equal ($a = b$). The eccentricity e of any hyperbola is given by $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Solution:
Step 1: Rewrite the given equation $x^2 - y^2 = 16$ in standard form:

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

Here, $a^2 = 16$ and $b^2 = 16$.
Step 2: Identify the type of hyperbola. Since $a = b$, this is a rectangular hyperbola.
Step 3: Use the eccentricity formula:

$$e = \sqrt{1 + \frac{16}{16}} = \sqrt{1 + 1}$$

$$e = \sqrt{2}$$

Step 4: Verify the property. It is a known geometric fact that the eccentricity of any rectangular hyperbola is always $\sqrt{2}$.

Final Answer: $\boxed{\sqrt{2}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept:

We use the properties of the dot product for vectors. The magnitude squared is given by $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. The relation $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ implies that the diagonals of a parallelogram formed by these vectors are equal.

Solution:

Step 1: Square both sides of the given equation:

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

Step 2: Expand using the dot product property:

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

Step 3: Cancel $|\vec{a}|^2$ and $|\vec{b}|^2$ from both sides:

$$2\vec{a} \cdot \vec{b} = -2\vec{a} \cdot \vec{b}$$

$$4\vec{a} \cdot \vec{b} = 0 \implies \vec{a} \cdot \vec{b} = 0$$

Step 4: Determine the angle θ. Since the dot product is zero:

$$|\vec{a}||\vec{b}| \cos\theta = 0 \implies \cos\theta = 0$$

$$\theta = 90^\circ$$

Final Answer: $\boxed{90^\circ}$

Answer: (C)

Go Back to Question 27

Q28.

Solution

Concept:

In 3D geometry, the distance d of a point $P(x, y, z)$ from the origin $O(0, 0, 0)$ is calculated using the 3D version of the Pythagorean theorem: $d = \sqrt{x^2 + y^2 + z^2}$.

Solution:

Step 1: Identify the coordinates of the point P: $x = 1, y = 2, z = 3$.

Step 2: Apply the distance formula:

$$d = \sqrt{(1)^2 + (2)^2 + (3)^2}$$

Step 3: Calculate the squares of the coordinates:

$$d = \sqrt{1 + 4 + 9}$$

Step 4: Sum the values to find the final distance:

$$d = \sqrt{14}$$

Final Answer: $\boxed{\sqrt{14}}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept:
To find the derivative of a function involving absolute values, we must first redefine the function without absolute value bars in the neighborhood of the point of interest (here, $x = 1.5$).

Solution:
Step 1: Analyze the behavior of each modulus term near $x = 1.5$. For $|x - 1|$: Since $1.5 > 1$, $(x - 1)$ is positive. Thus, $|x - 1| = x - 1$. For $|x - 2|$: Since $1.5 < 2$, $(x - 2)$ is negative. Thus, $|x - 2| = -(x - 2) = 2 - x$.
Step 2: Rewrite $f(x)$ for the interval $1 < x < 2$:

$$f(x) = (x - 1) + (2 - x)$$

Step 3: Simplify the expression:

$$f(x) = x - 1 + 2 - x = 1$$

Step 4: Differentiate the simplified function: Since $f(x)$ is a constant in this interval, its derivative is:

$$f'(x) = \frac{d}{dx}(1) = 0$$

Therefore, $f'(1.5) = 0$.
Final Answer: $\boxed{0}$

Answer: (C)

Go Back to Question 29

Q30.

Solution

Concept:

We use the definite integral property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. This is particularly useful for integrals involving $\sin x$ and $\cos x$ with limits 0 to $\pi/2$, as $\sin(\pi/2 - x) = \cos x$.

Solution:

Step 1: Let the integral be I:

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad \text{— (i)}$$

Step 2: Apply the property $x \to \pi/2 - x$:

$$I = \int_0^{\pi/2} \frac{\sin(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)} dx$$

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \quad \text{— (ii)}$$

Step 3: Add equations (i) and (ii):

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx = \int_0^{\pi/2} 1 dx$$

Step 4: Solve the simple integral:

$$2I = [x]_0^{\pi/2} = \pi/2$$

$$I = \pi/4$$

Final Answer: $\boxed{\pi/4}$

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept:

For a standard parabola of the form $y^2 = 4ax$ opening towards the right, the length of the latus rectum is defined as the chord passing through the focus perpendicular to the axis of symmetry. The length is always equal to the coefficient of x, which is $4a$.

Solution:

Step 1: Write the given equation of the parabola:

$$y^2 = 12x$$

Step 2: Compare this with the standard form $y^2 = 4ax$:

$$4a = 12$$

Step 3: Solve for the parameter a:

$$a = \frac{12}{4} = 3$$

Step 4: Identify the length of the latus rectum. By definition, for the equation $y^2 = 4ax$, the length of the latus rectum is $4a$. From Step 2, we already have $4a = 12$.

Therefore, the length is 12 units.

Final Answer: $\boxed{12}$

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept:
When two dice are thrown, the total number of possible outcomes (sample space) is $6 \times 6 = 36$. To find the probability, we must identify the specific pairs (d_1, d_2) whose sum equals the desired value.

Solution:
Step 1: Determine the total number of outcomes:

$$n(S) = 6^2 = 36$$

Step 2: List the favorable outcomes where the sum of the two faces is 9:
1. $(3, 6)$ 2. $(4, 5)$ 3. $(5, 4)$ 4. $(6, 3)$
Step 3: Count the number of favorable outcomes:

$$n(E) = 4$$

Step 4: Calculate the probability using the formula $P(E) = \frac{n(E)}{n(S)}$:

$$P(E) = \frac{4}{36}$$

Step 5: Simplify the fraction:

$$P(E) = \frac{1}{9}$$

Final Answer: $\boxed{1/9}$

Answer: (C)

Go Back to Question 32

Q33.

Solution

Concept:
The modulus (or absolute value) of a complex number $z = x + iy$ represents its distance from the origin in the complex plane. It is calculated using the formula $|z| = \sqrt{x^2 + y^2}$.
Solution:
Step 1: Identify the real and imaginary parts of $z = 1 + i\sqrt{3}$: Real part $(x) = 1$ Imaginary part $(y) = \sqrt{3}$
Step 2: Apply the modulus formula:

$$|z| = \sqrt{(1)^2 + (\sqrt{3})^2}$$

Step 3: Square the individual terms:

$$|z| = \sqrt{1 + 3}$$

Step 4: Sum the terms and find the square root:

$$|z| = \sqrt{4} = 2$$

The modulus of the given complex number is 2.
Final Answer: $\boxed{2}$

Answer: (B)
Go Back to Question 33

Q34.

Solution

Concept:
An infinite Geometric Progression (G.P.) has a finite sum if and only if the absolute value of the common ratio r is less than 1 ($|r| < 1$). The sum is given by the formula $S_\infty = \frac{a}{1-r}$, where a is the first term.

Solution:
Step 1: Identify the first term (a) and common ratio (r) of the series $1, 1/3, 1/9, \ldots$: First term $a = 1$ Common ratio $r = \frac{1/3}{1} = \frac{1}{3}$
Step 2: Check the condition for convergence: Since $|r| = 1/3 < 1$, the infinite sum exists.
Step 3: Substitute the values into the sum formula:

$$S_\infty = \frac{1}{1 - 1/3}$$

Step 4: Simplify the denominator:

$$S_\infty = \frac{1}{2/3}$$

Step 5: Calculate the final result:

$$S_\infty = \frac{3}{2} = 1.5$$

Final Answer: $\boxed{3/2}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept:
This is a fundamental property of binomial coefficients known as Pascal's Rule (or Pascal's Identity). It states that the sum of two adjacent entries in a row of Pascal's triangle results in the entry directly below them in the next row.

Solution:
Step 1: Write the expression:

$$^nC_r +^n C_{r-1}$$

Step 2: Expand using the factorial formula $^nC_k = \frac{n!}{k!(n-k)!}$:

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

Step 3: Take common terms out:

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

Step 4: Simplify the expression inside the brackets:

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right] = \frac{n!}{(r-1)!(n-r)!} \cdot \frac{n+1}{r(n-r+1)}$$

Step 5: Combine the terms:

$$= \frac{(n+1)!}{r!(n+1-r)!} =^{n+1} C_r$$

Final Answer: $\boxed{^{n+1}C_r}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept:

The slope of the tangent to a curve $y = f(x)$ at a point is $m_t = f'(x)$. The slope of the normal, m_n, is the negative reciprocal of the tangent's slope, provided $m_t \neq 0$, following the relation $m_t \cdot m_n = -1$.

Solution:

Step 1: Differentiate the function $y = 2x^2 + 3\sin x$ with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2) + \frac{d}{dx}(3\sin x) = 4x + 3\cos x$$

Step 2: Find the slope of the tangent (m_t) at $x = 0$:

$$m_t = 4(0) + 3\cos(0) = 0 + 3(1) = 3$$

Step 3: Determine the slope of the normal (m_n): Using the relation $m_n = -1/m_t$:

$$m_n = -\frac{1}{3}$$

Step 4: Conclusion: The tangent at the origin has a positive slope of 3, meaning the line perpendicular to it (the normal) must have a slope of $-1/3$.

Final Answer: $\boxed{-1/3}$

Answer: (D)

Go Back to Question 36

Q37.

Solution

Concept:

This integral follows the special form $\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$. This property is derived using integration by parts on the first term and observing that it cancels the second term.

Solution:

Step 1: Identify $f(x)$ and $f'(x)$ in the expression $\int e^x(\tan x + \sec^2 x)dx$. Let $f(x) = \tan x$.

Step 2: Differentiate $f(x)$ to verify:

$$f'(x) = \frac{d}{dx}(\tan x) = \sec^2 x$$

Step 3: Match the expression with the standard form: The given integral is exactly in the form $\int e^x[f(x) + f'(x)]dx$.

Step 4: Apply the result:

$$\int e^x(\tan x + \sec^2 x)dx = e^x \tan x + C$$

Step 5: Reasoning: If we were to integrate $\int e^x \tan x dx$ by parts, we would get $e^x \tan x - \int e^x \sec^2 x dx$. Adding the second term of the original integral leads to the final result.

Final Answer: $\boxed{e^x \tan x + C}$

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept:

A linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ is solved using an Integrating Factor (I.F.). The formula for the Integrating Factor is $I.F. = e^{\int P(x)dx}$.

Solution:

Step 1: Identify $P(x)$ from the given equation $\frac{dy}{dx} + y\tan x = \sec x$:

$$P(x) = \tan x$$

Step 2: Set up the integral for the exponent:

$$\int P(x)dx = \int \tan x dx$$

Step 3: Solve the integral:

$$\int \tan x dx = \ln|\sec x|$$

Step 4: Calculate the Integrating Factor:

$$I.F. = e^{\ln|\sec x|}$$

Step 5: Simplify using the property $e^{\ln a} = a$:

$$I.F. = \sec x$$

The purpose of this factor is to make the left-hand side of the differential equation a perfect derivative of $(y \cdot I.F.)$.

Final Answer: $\boxed{\sec x}$

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept:

For unit vectors $\vec{a}$ and $\vec{b}$, we have $|\vec{a}| = |\vec{b}| = 1$. The magnitude of their difference is found using the dot product identity $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$ and trigonometric half-angle identities.

Solution:

Step 1: Write the expression for the square of the magnitude:

$$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$$

$$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

Step 2: Substitute the magnitudes and the dot product formula $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$:

$$= 1^2 + 1^2 - 2(1)(1)\cos\theta$$

$$= 2 - 2\cos\theta$$

Step 3: Factor out the 2:

$$= 2(1 - \cos\theta)$$

Step 4: Apply the trigonometric identity $1 - \cos\theta = 2\sin^2(\theta/2)$:

$$= 2[2\sin^2(\theta/2)] = 4\sin^2(\theta/2)$$

Step 5: Take the square root of both sides:

$$|\vec{a} - \vec{b}| = \sqrt{4\sin^2(\theta/2)} = 2\sin(\theta/2)$$

Final Answer: $\boxed{2\sin(\theta/2)}$

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept:

The distance of a point $P(x, y, z)$ from the coordinate axes is derived from the projection of the point on the axis. For the x-axis, the projection is $(x, 0, 0)$. The distance is the square root of the sum of the squares of the other two coordinates.

Solution:

Step 1: Identify the coordinates of the point P: $x = 3, y = 4, z = 5$.

Step 2: Identify the coordinates of the projection of P on the x-axis. The projection of $(3, 4, 5)$ on the x-axis is $P'(3, 0, 0)$.

Step 3: Use the distance formula between $P(3, 4, 5)$ and $P'(3, 0, 0)$:

$$D = \sqrt{(3-3)^2 + (4-0)^2 + (5-0)^2}$$

Step 4: Simplify the expression:

$$D = \sqrt{0^2 + 4^2 + 5^2}$$

$$D = \sqrt{16 + 25}$$

Step 5: Final Calculation:

$$D = \sqrt{41}$$

Final Answer: $\boxed{\sqrt{41}}$

Answer: (C)

Go Back to Question 40

Q41.

Solution

Concept:

The absolute value function $|x|$ is defined as x for $x \geq 0$ and $-x$ for $x < 0$. When integrating over an interval that includes zero, we must split the integral into two parts to remove the modulus.

Solution:

Step 1: Identify the point where the function changes its definition. Since the interval is $[-1, 1]$, zero is the point of interest.

Step 2: Split the integral at $x = 0$:

$$\int_{-1}^{1} |x| dx = \int_{-1}^{0} (-x) dx + \int_{0}^{1} x dx$$

Step 3: Integrate each term separately:

$$\left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

Step 4: Substitute the limits:

$$\left(0 - \left(-\frac{(-1)^2}{2}\right)\right) + \left(\frac{1^2}{2} - 0\right)$$

$$= \left(0 + \frac{1}{2}\right) + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

Geometrically, this represents the sum of the areas of two identical triangles, each with base 1 and height 1, which equals $2 \times (1/2 \times 1 \times 1) = 1$.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 41

Q42.

Solution

Concept:
To find an equation whose roots are reciprocals of the roots of a given equation $f(x) = 0$, we substitute x with $1/x$. This transforms the original equation into the new desired form.
Solution:
Step 1: Start with the given equation $x^2 - 5x + 6 = 0$. Step 2: Replace x with $1/x$ to find the equation with roots $1/\alpha$ and $1/\beta$:

$$\left(\frac{1}{x}\right)^2 - 5\left(\frac{1}{x}\right) + 6 = 0$$

Step 3: Simplify the expression:

$$\frac{1}{x^2} - \frac{5}{x} + 6 = 0$$

Step 4: Multiply the entire equation by x^2 to clear the denominators:

$$1 - 5x + 6x^2 = 0$$

Step 5: Rearrange into the standard form $ax^2 + bx + c = 0$:

$$6x^2 - 5x + 1 = 0$$

Alternatively, since sum of roots $\alpha + \beta = 5$ and product $\alpha\beta = 6$, the new sum is $\frac{\alpha+\beta}{\alpha\beta} = 5/6$ and new product is $1/6$. The equation is $x^2 - (5/6)x + 1/6 = 0$, which simplifies to the same result.
Final Answer: $\boxed{6x^2 - 5x + 1 = 0}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept:

The scalar projection of a vector $\vec{a}$ on vector $\vec{b}$ is defined as the component of $\vec{a}$ in the direction of $\vec{b}$. The formula is $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Solution:

Step 1: Calculate the dot product $\vec{a}\cdot\vec{b}$:

$$\vec{a}\cdot\vec{b} = (2\hat{i} + 3\hat{j} + 2\hat{k})\cdot(\hat{i} + 2\hat{j} + \hat{k})$$

$$= (2\times 1) + (3\times 2) + (2\times 1) = 2 + 6 + 2 = 10$$

Step 2: Calculate the magnitude of vector $\vec{b}$ ($|\vec{b}|$):

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$$

Step 3: Apply the projection formula:

$$\text{Projection} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{10}{\sqrt{6}}$$

Step 4: Rationalize or simplify if necessary. Since $10/\sqrt{6}$ is in the options, we stop here.

Final Answer: $\boxed{10/\sqrt{6}}$

Answer: (B)

Go Back to Question 43

Q44.

Solution

Concept:

The distance d between two parallel planes $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is calculated using $d = \frac{|D_1 - D_2|}{\sqrt{A^2+B^2+C^2}}$. Both equations must have identical A, B, C values.

Solution:

Step 1: Write the equations of the planes: Plane 1: $2x+3y+4z-4 = 0$ Plane 2: $4x+6y+8z-12 = 0$

Step 2: Make the coefficients of x, y, z in Plane 2 same as Plane 1 by dividing Plane 2 by 2:

$$\frac{4x}{2} + \frac{6y}{2} + \frac{8z}{2} - \frac{12}{2} = 0 \implies 2x + 3y + 4z - 6 = 0$$

Step 3: Identify the constants: $A = 2, B = 3, C = 4, D_1 = -4, D_2 = -6$.

Step 4: Apply the distance formula:

$$d = \frac{|-4-(-6)|}{\sqrt{2^2+3^2+4^2}} = \frac{|2|}{\sqrt{4+9+16}}$$

Step 5: Calculate the final result:

$$d = \frac{2}{\sqrt{29}}$$

Final Answer: $\boxed{2/\sqrt{29}}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept:

The Addition Theorem of probability states $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. For independent events, the probability of the intersection is the product of their individual probabilities: $P(A \cap B) = P(A) \cdot P(B)$.

Solution:

Step 1: Calculate $P(A \cap B)$ since the events are independent:

$$P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.4 = 0.12$$

Step 2: Substitute values into the addition formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.3 + 0.4 - 0.12$$

Step 3: Perform the addition and subtraction:

$$0.3 + 0.4 = 0.7$$

$$0.7 - 0.12 = 0.58$$

The probability that at least one of the events occurs is 0.58.

Final Answer: $\boxed{0.58}$

Answer: (C)

Go Back to Question 45

Q46.

Solution

Concept:

The area of a standard ellipse with the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is given by the formula Area $= \pi ab$. This is derived by integrating the function $y = b\sqrt{1 - (x/a)^2}$ from $-a$ to a and doubling the result (or using the area of a circle transformation).

Solution:

Step 1: Identify the values of a^2 and b^2 from the given equation:

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Here, $a^2 = 9 \implies a = 3$ and $b^2 = 4 \implies b = 2$.

Step 2: Apply the area formula for an ellipse:

$$\text{Area} = \pi \times a \times b$$

Step 3: Substitute the values:

$$\text{Area} = \pi \times 3 \times 2 = 6\pi$$

Step 4: Conclusion: The total area enclosed by the boundary of the ellipse is 6π square units.

Final Answer: $\boxed{6\pi}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept:

This limit can be solved using the standard exponential limit $\lim_{x\to 0}\frac{e^x-1}{x}=1$ or by applying L'Hôpital's Rule, since the expression takes the indeterminate form $0/0$ as $x\to 0$.

Solution:

Step 1: Check the indeterminate form: As $x\to 0$, $e^0-e^0=1-1=0$, and the denominator is 0. Form is $0/0$.

Step 2: Apply L'Hôpital's Rule (differentiate numerator and denominator):

$$\lim_{x\to 0}\frac{\frac{d}{dx}(e^x-e^{-x})}{\frac{d}{dx}(x)}$$

Step 3: Perform the differentiation:

$$\frac{d}{dx}(e^x)=e^x,\quad \frac{d}{dx}(-e^{-x})=e^{-x},\quad \frac{d}{dx}(x)=1$$

The expression becomes:

$$\lim_{x\to 0}\frac{e^x+e^{-x}}{1}$$

Step 4: Substitute $x=0$:

$$e^0+e^{-0}=1+1=2$$

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 47

Q48.

Solution

Concept:
The n^{th} term (a_n) of an A.P. is related to the sum of n terms (S_n) by the formula $a_n = S_n - S_{n-1}$. Once a_n is found, the common difference d is $a_n - a_{n-1}$. Alternatively, for $S_n = An^2 + Bn$, the common difference is always $2A$.

Solution:
Step 1: Calculate S_1 (which is the first term a_1):

$$S_1 = 3(1)^2 + 1 = 4 \implies a_1 = 4$$

Step 2: Calculate S_2:

$$S_2 = 3(2)^2 + 2 = 3(4) + 2 = 14$$

Step 3: Find the second term a_2:

$$a_2 = S_2 - S_1 = 14 - 4 = 10$$

Step 4: Find the common difference d:

$$d = a_2 - a_1 = 10 - 4 = 6$$

Shortcut: Given $S_n = 3n^2 + n$, comparing with $S_n = \frac{d}{2}n^2 + (a - d/2)n$, we see $d/2 = 3 \implies d = 6$.

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept:
The length of the intercept made by a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the x-axis is given by the formula $L = 2\sqrt{g^2 - c}$. If $g^2 < c$, the circle does not intersect the x-axis.

Solution:
Step 1: Identify the coefficients from the equation $x^2+y^2-4x-6y-12 = 0$: $2g = -4 \implies g = -2$
$c = -12$

Step 2: Apply the x-intercept formula:

$$L = 2\sqrt{g^2 - c}$$

Step 3: Substitute the values:

$$L = 2\sqrt{(-2)^2 - (-12)}$$

$$L = 2\sqrt{4 + 12}$$

Step 4: Simplify:

$$L = 2\sqrt{16} = 2 \times 4 = 8$$

The circle cuts the x-axis at two points, and the distance between them is 8 units.

Final Answer: $\boxed{8}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept:

Circular permutations differ from linear permutations because there is no fixed starting or ending point. To account for this symmetry, we fix one person's position and arrange the remaining $(n-1)$ people. The formula for n distinct objects is $(n-1)!$.

Solution:

Step 1: Identify the number of people to be seated:

$$n = 5$$

Step 2: Apply the circular permutation formula:

$$\text{Ways} = (n-1)!$$

Step 3: Substitute $n = 5$:

$$\text{Ways} = (5-1)! = 4!$$

Step 4: Calculate the factorial:

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Step 5: Reasoning: In a linear row, there would be $5! = 120$ ways. However, in a circle, rotating the table results in the same relative arrangement. Since there are 5 such rotations for every arrangement, we divide by 5: $120/5 = 24$.

Final Answer: $\boxed{24}$

Answer: (B)

Go Back to Question 50

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	C	4	A	5	B
6	A	7	A	8	B	9	A	10	B
11	A	12	B	13	C	14	B	15	A
16	B	17	A	18	A	19	C	20	A
21	A	22	B	23	A	24	D	25	B
26	A	27	C	28	B	29	C	30	C
31	C	32	C	33	B	34	A	35	A
36	D	37	B	38	B	39	B	40	C
41	B	42	A	43	B	44	A	45	C
46	A	47	C	48	B	49	A	50	B